Topic

Convolution and Correlation

Convolution

- **convolution** is a mathematical operator which takes two functions *x* and *h* and produces a third function that represents the amount of overlap between *x* and a reversed and translated version of *h*.

- In signal processing, one of the functions (*h*) is taken to be a fixed filter *impulse response*, and is known as a *kernel*.

$$(h * x)(t) \equiv \int_a^b h(\tau)x(t-\tau)\,d\tau$$

Convolution operator

Discrete Convolution

- **convolution** is a mathematical operator which takes two functions *f* and *g* and produces a third function that represents the amount of overlap between *f* and a reversed and translated version of *g*.

- In signal processing, one of the functions is taken to be a fixed filter *impulse response*, and is known as a *kernel*.

$$(h * x)[m] \equiv \sum_{n} h[n]x[m-n]$$

Convolution In Matlab Code

```
function C= convolution(A,B)

lengthA= length(A);
lengthB= length(B);

C = zeros(1, lengthA + lengthB - 1);

for m = 1:lengthA
    for n = 1:lengthB
        C(m+n-1) = C(m+n-1) + A(m)*B(n);
    end
end
```

Cross-correlation

- **Cross-correlation** is a measure of similarity of two functions at time-lag Tau applied to one of them. It is a LOT like convolution…

Means "complex conjugate of f"

$$(h \text{ } \unicode{x2605} \text{ } x)(t) \equiv \int_{a}^{b} h^{*}(\tau)x(t+\tau)\,d\tau$$

Cross-correlation operator
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VERY Similar

- Convolution

$$(h * x)(t) \equiv \int_a^b h(\tau) x(t - \tau) d\tau$$

- Cross-correlation

$$(h \; \text{\Apple} \; x)(t) \equiv \int_a^b h^*(\tau) x(t + \tau) d\tau$$

Cross-correlation in Matlab Code

We can easily implement cross correlation with convolution as follows:

```
function x = crosscorrelation(A,B)
        return convolution(conj(A),B(end:-1:1))
```

Better yet, use the built in Matlab functions…

xcorr(A,B)   for cross correlation
conv(A,B)   for convolution

Auto-correlation

- **Auto-correlation** is a measure of similarity of a function to itself at time-lag Tau. It is just cross-correlation with yourself.

Means "complex conjugate of f"

$$(x \bullet x)(t) \equiv \int_a^b x^*(\tau)x(t+\tau)\,d\tau$$

Cross correlation
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Relating them all

Image from http://commons.wikimedia.org/wiki/File:Comparison_convolution_correlation.svg
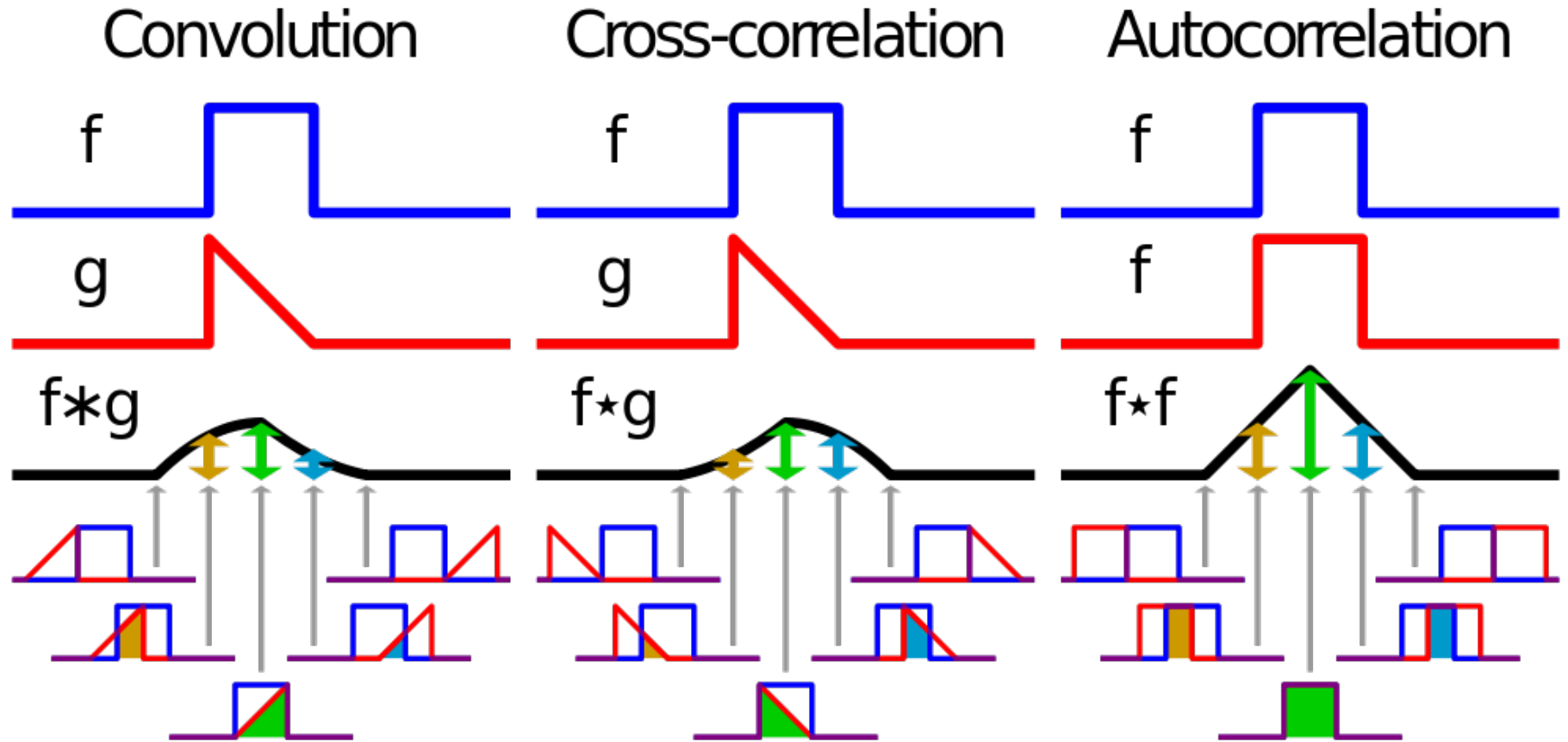
Convolution (Frequency Domain)

Convolution…
$$y(t) = h(t) * x(t)$$

is defined in the
time domain as…
$$y(t) = \int h(\tau)x(t-\tau)\,d\tau$$

and in frequency
domain as…..
$$Y(\omega) = H(\omega)X(\omega)$$