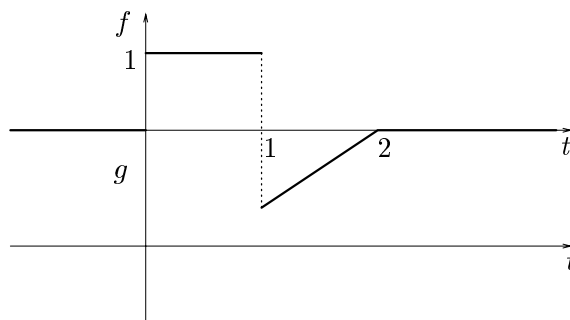


1. Review of integration.

(a) Evaluate the integrals $\int_0^\pi t \cos(t) dt$ and $\int_0^\pi t^2 \sin(t) dt$.(b) For a differentiable function f , derive the identity

$$\int_0^t f(t - \tau) d\tau = tf(t) - \int_0^t \tau f'(\tau) d\tau$$

(c) The figure below contains a picture of a function $f(t)$. Find the function $g(t) = \int_{-\infty}^t f(\tau) d\tau$ and sketch it under $f(t)$.

2. Review of complex numbers

(a) Find the following complex numbers (real and imaginary parts):

$$(1) e^{-\frac{27}{2}\pi i}, \quad (2) (i)^{i^6}$$

(b) Change these complex numbers into exponential form:

$$(1) \alpha = \sqrt{3} - i, \quad (2) \beta = -i.$$

(c) For the numbers in part (b), compute $\alpha^3 / \bar{\beta}$, where $\bar{\beta}$ is the complex conjugate of β .(d) Find the complex roots to the polynomial equation $z^6 - 27 = 0$.

3. Given the differential equation for $t \geq 0$

$$\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} - 2x(t)$$

• Let $x(0) = 0$ and $y(0) = 0$; solve for $y(t)$ in terms of $x(t)$.

4. For each of the following systems with input $x(t)$ and output $y(t)$, find out whether they are (i) linear, (ii) time invariant, (iii) causal. Justify your answer.

(a) $y(t) = x(t + 1) - 3$.

(b) $y(t) = e^t x(t)$.

(c) $y(t) = \int_t^\infty x(\tau) d\tau$.

(d) The system where $y(t)$ is equal to $x(t)$ when $x(t) > 0$, and zero otherwise.