

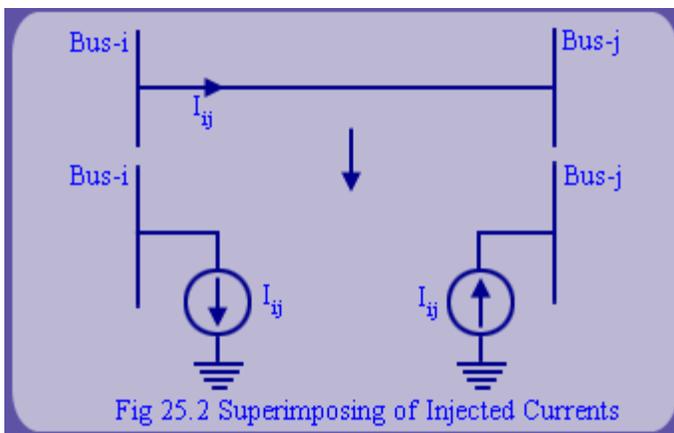
## Module 7 : Out of Step Protection

### Lecture 25 : Analysis of Power Swings in a Multi – Machine System

#### Objectives

In this lecture we will

- Analyse a given transmission network.
- Reduce it to a two source equivalent.
- Determine whether the power swing in R-X plane cuts through any transmission line.



Now let the current in the transmission line AB, when it is connected to the network be  $I_{ij}$  from the  $i^{\text{th}}$  end and  $I_{ji}$  at the  $j^{\text{th}}$  end respectively. Then, under the assumption of linearity, the effect of the transmission line connection can be evaluated by superimposing injected currents  $\Delta I_i = -I_{ij}$  and  $\Delta I_j = +I_{ij}$  on the network in which line  $i$ - $j$  was disconnected. Thus,  $I = [0, \dots, -I_{ij}, 0, \dots, +I_{ij}, 0, \dots, 0]^T$ . Since, we are only interested in characterizing behavior at busses  $i$  and  $j$ , we can ignore the remaining rows in (2).

$$\text{Thus, } \begin{bmatrix} \Delta V_i \\ \Delta V_j \end{bmatrix} = \begin{bmatrix} Z_{ii}^{\delta us} & Z_{ij}^{\delta us} \\ Z_{ji}^{\delta us} & Z_{jj}^{\delta us} \end{bmatrix} \begin{bmatrix} \Delta I_i \\ \Delta I_j \end{bmatrix}$$

(3)

$$\Delta V_i = -(Z_{ii}^{\delta us} - Z_{ij}^{\delta us}) I_{ij}$$

$$\Delta V_j = (Z_{jj}^{\delta us} - Z_{ji}^{\delta us}) I_{ij}$$

Now if we assure a simple series model of the line, then,

$$\Delta I_i = I_{ij} = Y_{ij} [V_i^{\text{new}} - V_j^{\text{new}}] = Y_{ij} [V_i^{\text{old}} - V_j^{\text{old}}] + [\Delta V_i - \Delta V_j] Y_{ij}$$

$$= Y_{ij} [V_i^{\text{old}} - V_j^{\text{old}}] - Y_{ij} [Z_{ii}^{\delta us} - Z_{ij}^{\delta us} + Z_{jj}^{\delta us} - Z_{ji}^{\delta us}] I_{ij}$$

$$I_{ij} = \frac{Y_{ij} [V_i^{\text{old}} - V_j^{\text{old}}]}{1 + Y_{ij} [Z_{ii}^{\delta us} + Z_{jj}^{\delta us} - Z_{ij}^{\delta us} - Z_{ji}^{\delta us}]}$$

where  $Y_{ij}$  is the primitive admittance of the transmission line

Equivalently, (3) can be expressed as follows.

$$\begin{bmatrix} \Delta I_i \\ \Delta I_j \end{bmatrix} = \begin{bmatrix} \hat{Y}_{ii} & \hat{Y}_{ij} \\ \hat{Y}_{ji} & \hat{Y}_{jj} \end{bmatrix} \begin{bmatrix} \Delta V_i \\ \Delta V_j \end{bmatrix}$$

(4)

where  $2 \times 2$   $\begin{bmatrix} \hat{Y} \end{bmatrix}$  matrix is the inverse of corresponding  $2 \times 2$  restricted  $Z^{\text{bus}}$  matrix. Under usual symmetry assumption,  $\hat{Y}_{ij} = \hat{Y}_{ji}$ .

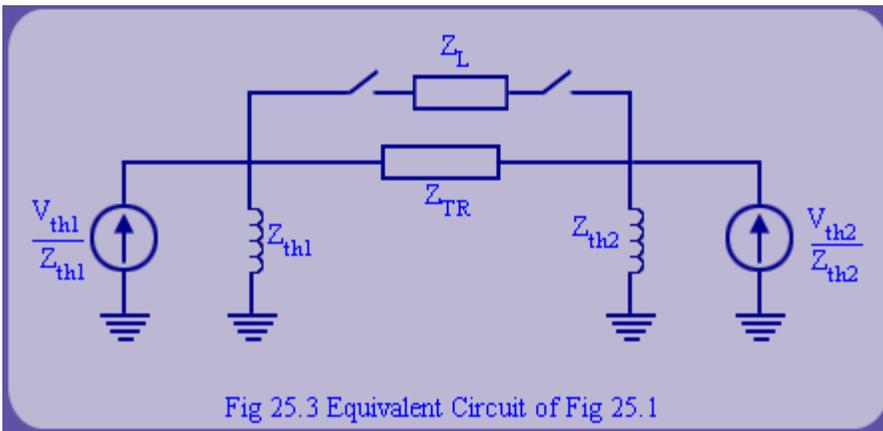
The equivalent circuit of fig 25.1 can be reduced as shown in fig 25.2. Now from the equivalent circuit of fig 25.2 we get that,

$$\begin{bmatrix} \frac{1}{Z_{th1}} + \frac{1}{Z_{TR}} & -\frac{1}{Z_{TR}} \\ -\frac{1}{Z_{TR}} & \frac{1}{Z_{th2}} + \frac{1}{Z_{TR}} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} = \begin{bmatrix} \frac{V_{th1}}{Z_{th1}} \\ \frac{V_{th2}}{Z_{th2}} \end{bmatrix}$$

Therefore, the incremental model is given by,

$$\begin{bmatrix} \Delta I_i \\ \Delta I_j \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_{th1}} + \frac{1}{Z_{TR}} & -\frac{1}{Z_{TR}} \\ -\frac{1}{Z_{TR}} & \frac{1}{Z_{th2}} + \frac{1}{Z_{TR}} \end{bmatrix} \begin{bmatrix} \Delta V_i \\ \Delta V_j \end{bmatrix}$$

(5)



Equating (5) to (4) we get,

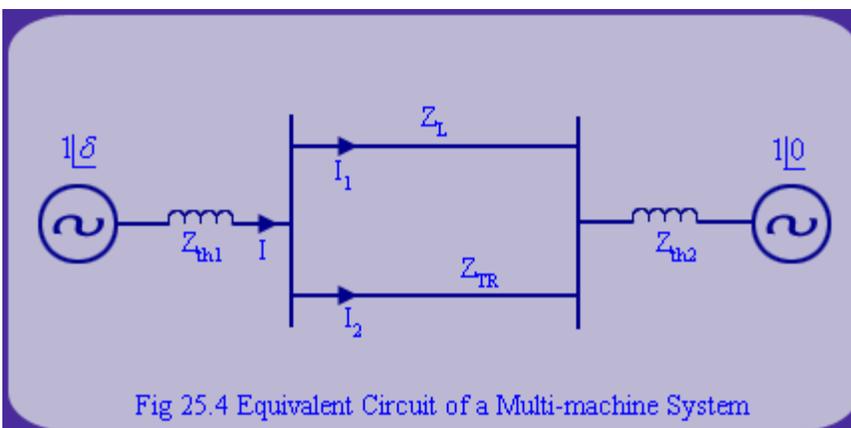
$$Z_{TR} = -\frac{1}{\hat{Y}_{ij}}$$

$$\frac{1}{Z_{th1}} = \hat{Y}_{ii} - \frac{1}{Z_{TR}} = \hat{Y}_{ii} + \hat{Y}_{ij}$$

$$\frac{1}{Z_{th2}} = \hat{Y}_{jj} - \frac{1}{Z_{TR}} = \hat{Y}_{jj} + \hat{Y}_{ij}$$

Thus the equivalent circuit is as shown in fig 25.3.

### Determination of Power Swing Locus for Multi Machine System



$$Z_T = Z_{\#1} + Z_L \parallel Z_{TR} + Z_{\#2}$$

$$I = \frac{1|\underline{\delta} - 1|0}{Z_T}$$

$$I1 = \frac{Z_{TR}}{Z_L + Z_{TR}} - I$$

∴ Impedance seen by relay on line L,

$$\begin{aligned} Z_{seen(relay)} &= \frac{1|\underline{\delta} - Z_{\#1} I}{I_1} \\ &= \frac{1|\underline{\delta} - Z_{\#1} \left( \frac{Z_L + Z_{TR}}{Z_{TR}} \right)}{I_1} \\ &= \left[ -Z_{\#1} + \frac{1|\underline{\delta}}{I} \right] \frac{(Z_L + Z_{TR})}{Z_{TR}} \\ &= \left[ -Z_{\#1} + \frac{1|\underline{\delta}}{1|\underline{\delta} - 1|0} Z_T \right] \left( \frac{Z_L + Z_{TR}}{Z_{TR}} \right) \\ &= \left[ -Z_{\#1} + \frac{1}{1 - (\cos \delta - j \sin \delta)} Z_T \right] \left( \frac{Z_L + Z_{TR}}{Z_{TR}} \right) \end{aligned}$$

Determination of Power Swing Locus for Multi Machine System

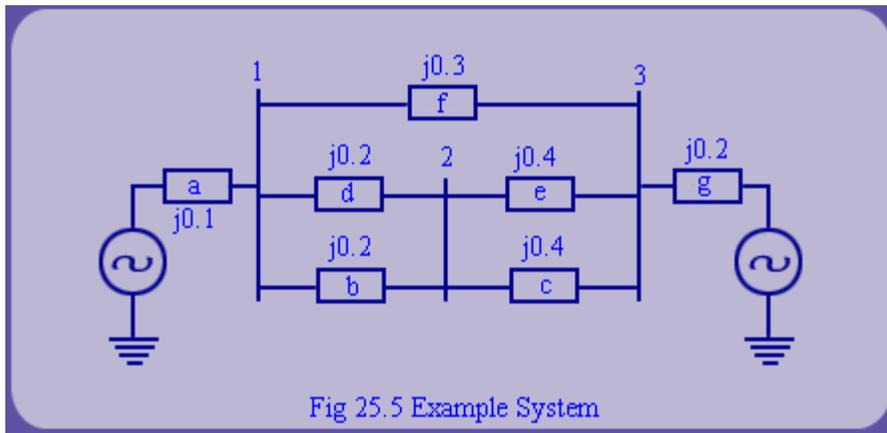
$$\begin{aligned} &= \left[ -Z_{\#1} + \frac{1}{2(\sin^2 \frac{\delta}{2} + j \sin \frac{\delta}{2} \cos \frac{\delta}{2})} Z_T \right] \left( \frac{Z_L + Z_{TR}}{Z_{TR}} \right) \\ &= \left[ -Z_{\#1} + \frac{Z_T}{2 \sin \frac{\delta}{2} \left( \sin \frac{\delta}{2} + j \cos \frac{\delta}{2} \right)} \right] \frac{Z_L + Z_{TR}}{Z_{TR}} \\ &= \frac{1}{k} \left[ -Z_{\#1} + \frac{Z_T}{2 \sin \frac{\delta}{2}} \left( \sin \frac{\delta}{2} - j \cos \frac{\delta}{2} \right) \right] \text{ where } k = \frac{Z_{TR}}{Z_L + Z_{TR}} \end{aligned}$$

Thus,

$$Z_{seen(relay)} = \frac{1}{k} \left[ -Z_{\#1} + \frac{Z_T}{2} - j \left( \cot \frac{\delta}{2} \right) \frac{Z_T}{2} \right] \quad (6)$$

So, it has to be checked whether  $\frac{1}{k} \left[ -Z_{\#1} + \frac{Z_T}{2} \right]$  is less than  $Z_L$  or not. If it is less than  $Z_L$  then electrical center is formed on line L. If it is greater than  $Z_L$ , then electrical center lies outside the transmission line L.

## Example 1



For the system shown in fig 25.3, determine the two port equivalent and find out whether the power swing locus passes through

- transmission line 'c'
- transmission line 'b'

Solution:

- In order to analyze whether the power swing will pass through the transmission line 'c', we need to develop a two

machine equivalent across the line. For that, the transmission line 'c' is disconnected from the network and  $Z^{bus}$  matrix is formed.

Let us form the nodal admittance matrix or  $Y^{bus}$  for the given system with transmission line 'c' disconnected.

$$Y^{bus} = -j \begin{bmatrix} \left( \frac{1}{0.1} + \frac{1}{0.2} + \frac{1}{0.2} + \frac{1}{0.3} \right) & -\left( \frac{1}{0.2} + \frac{1}{0.2} \right) & -\left( \frac{1}{0.3} \right) \\ -\left( \frac{1}{0.2} + \frac{1}{0.2} \right) & \left( \frac{1}{0.2} + \frac{1}{0.2} + \frac{1}{0.4} \right) & -\left( \frac{1}{0.4} \right) \\ -\left( \frac{1}{0.3} \right) & -\left( \frac{1}{0.4} \right) & \left( \frac{1}{0.3} + \frac{1}{0.4} + \frac{1}{0.2} \right) \end{bmatrix}$$

$$= \begin{bmatrix} -j23.33 & j10 & j3.33 \\ j10 & -j12.5 & j2.5 \\ j3.33 & j2.5 & -j10.83 \end{bmatrix}$$

Now  $Z^{bus}$  is obtained by inverting  $Y^{bus}$ .

$$Z^{bus} = [Y^{bus}]^{-1} = \begin{bmatrix} j0.0795 & j0.0718 & j0.041 \\ j0.0718 & j0.1487 & j0.0564 \\ j0.041 & j0.0564 & j0.1179 \end{bmatrix}$$

## Example 1 (contd..)

Solution:

Since we are considering the transmission line 'c' which is connected between buses 2 and 3, we can ignore the first row and column of the  $Z^{bus}$  matrix and the reduced model will be,

$$\begin{bmatrix} \Delta V_2 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} Z_{22} & Z_{23} \\ Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} \Delta I_2 \\ \Delta I_3 \end{bmatrix}$$

$$= \begin{bmatrix} j0.1487 & j0.0564 \\ j0.0564 & j0.1179 \end{bmatrix} \begin{bmatrix} \Delta I_2 \\ \Delta I_3 \end{bmatrix}$$

Therefore,

$$\begin{bmatrix} \Delta I_2 \\ \Delta I_3 \end{bmatrix} = \begin{bmatrix} \hat{Y}_{22} & \hat{Y}_{23} \\ \hat{Y}_{32} & \hat{Y}_{33} \end{bmatrix} \begin{bmatrix} \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

$$= \begin{bmatrix} -j8.214 & j3.93 \\ j3.93 & -j10.36 \end{bmatrix} \begin{bmatrix} \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

$$\begin{bmatrix} \Delta I_2 \\ \Delta I_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_{th1}} + \frac{1}{Z_{TR}} & -\frac{1}{Z_{TR}} \\ -\frac{1}{Z_{TR}} & \frac{1}{Z_{th2}} + \frac{1}{Z_{TR}} \end{bmatrix} \begin{bmatrix} \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

$$\text{i.e.} \begin{bmatrix} \frac{1}{Z_{\#1}} + \frac{1}{Z_{TR}} & -\frac{1}{Z_{TR}} \\ -\frac{1}{Z_{TR}} & \frac{1}{Z_{\#2}} + \frac{1}{Z_{TR}} \end{bmatrix} = \begin{bmatrix} -j8.214 & j3.93 \\ j3.93 & -j10.36 \end{bmatrix}$$

$$Z_{TR} = -\frac{1}{j3.93} = j0.255$$

$$\frac{1}{Z_{th1}} = \hat{Y}_{22} - \frac{1}{Z_{TR}} = -j8.214 + j3.93 = -j4.284$$

$$Z_{th1} = j0.233$$

$$\frac{1}{Z_{th2}} = \hat{Y}_{33} - \frac{1}{Z_{TR}} = -j10.36 + j3.93 = -j6.43$$

$$Z_{th2} = j0.156$$

### Example 1 (contd..)

Solution:

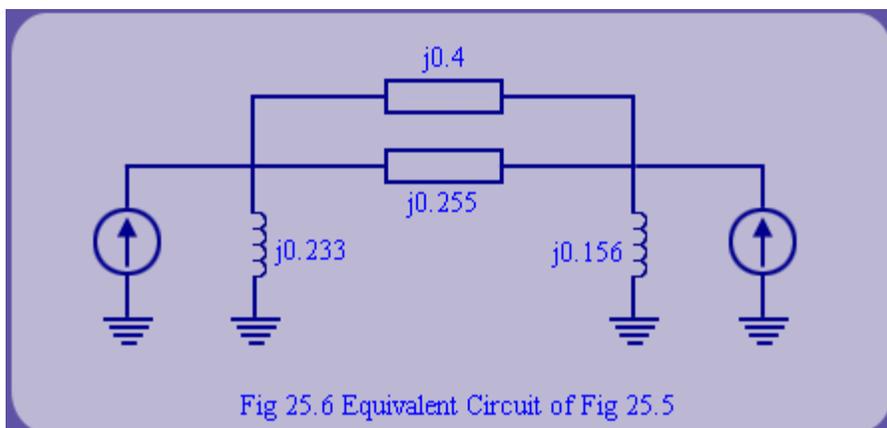
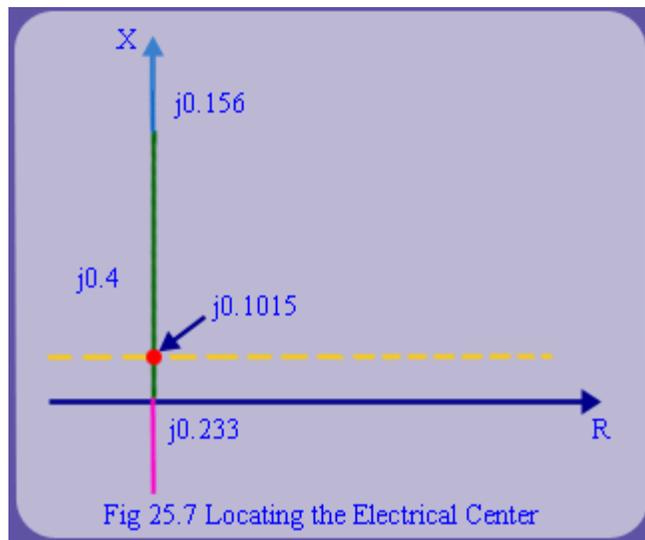


Fig 25.6 Equivalent Circuit of Fig 25.5

Hence the network will be as shown in fig 25.4, with transmission line 'c' connected across it. The total impedance between the two sources is given by,

$$Z_T = Z_{\#1} + \frac{Z_L \times Z_{TR}}{Z_L + Z_{TR}} + Z_{\#2} = j0.233 + \frac{j0.4 \times j0.255}{j0.4 + j0.255} + j0.156 = j0.545.$$

$$k = \frac{Z_{TR}}{Z_L + Z_{TR}} = \frac{j0.255}{j0.4 + j0.255} = 0.3893$$



The swing impedance trajectory intersects the impedance line at

$$\frac{1}{k} \left( -Z_{\#1} + \frac{Z_T}{2} \right) = \frac{1}{k} \left( -j0.233 + \frac{j0.545}{2} \right) = \frac{j0.0395}{0.3893} = j0.1015, \text{ which is the location of electrical center on R-X plane. From fig 25.5 it can be seen that electrical center of the swing lies on the transmission line 'c'.$$

### Example 1 (contd..)

Solution:

- (b) Now consider the transmission line 'b'. For forming the two machine equivalent, transmission line 'b' is disconnected

from the system and  $Z^{\text{bus}}$  is formed by inverting  $Y^{\text{bus}}$ .

$$Y^{\text{bus}} = -j \begin{bmatrix} \left( \frac{1}{0.1} + \frac{1}{0.2} + \frac{1}{0.3} \right) & -\left( \frac{1}{0.2} \right) & -\left( \frac{1}{0.3} \right) \\ -\left( \frac{1}{0.2} \right) & \left( \frac{1}{0.2} + \frac{1}{0.4} + \frac{1}{0.4} \right) & -\left( \frac{1}{0.4} + \frac{1}{0.4} \right) \\ -\left( \frac{1}{0.3} \right) & -\left( \frac{1}{0.4} + \frac{1}{0.4} \right) & \left( \frac{1}{0.3} + \frac{1}{0.4} + \frac{1}{0.4} + \frac{1}{0.2} \right) \end{bmatrix}$$

$$= \begin{bmatrix} -j18.33 & j5 & j3.33 \\ j5 & -j10 & j5 \\ j3.33 & j5 & -j13.33 \end{bmatrix}$$

$$Z^{\text{bus}} = \begin{bmatrix} j0.0788 & j0.0606 & j0.0424 \\ j0.0606 & j0.1697 & j0.0788 \\ j0.0424 & j0.0788 & j0.1152 \end{bmatrix}$$

Since we are considering only the buses 1 and 2 across which line 'b' is connected, we can ignore the third row and third column of the  $Z^{\text{bus}}$  matrix. Thus, the reduced system model is given by,

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \Delta I_1 \\ \Delta I_2 \end{bmatrix}$$

and

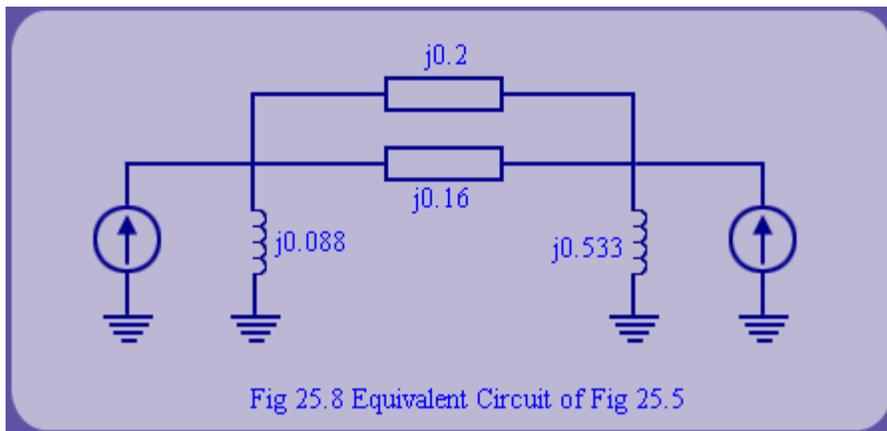
$$\begin{aligned} \begin{bmatrix} \Delta I_1 \\ \Delta I_2 \end{bmatrix} &= \begin{bmatrix} \hat{Y}_{11} & \hat{Y}_{12} \\ \hat{Y}_{21} & \hat{Y}_{22} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \end{bmatrix} \\ &= \begin{bmatrix} -j17.5 & j6.25 \\ j6.25 & -j8.125 \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \end{bmatrix} \end{aligned}$$

Since,

$$\begin{aligned} \begin{bmatrix} \Delta I_1 \\ \Delta I_2 \end{bmatrix} &= \begin{bmatrix} \frac{1}{Z_{th1}} + \frac{1}{Z_{TR}} & -\frac{1}{Z_{TR}} \\ -\frac{1}{Z_{TR}} & \frac{1}{Z_{th2}} + \frac{1}{Z_{TR}} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \end{bmatrix} \\ \begin{bmatrix} \frac{1}{Z_{th1}} + \frac{1}{Z_{TR}} & -\frac{1}{Z_{TR}} \\ -\frac{1}{Z_{TR}} & \frac{1}{Z_{th2}} + \frac{1}{Z_{TR}} \end{bmatrix} &= \begin{bmatrix} -j17.5 & j6.25 \\ j6.25 & -j8.125 \end{bmatrix} \end{aligned}$$

### Example 1 (contd..)

Solution:



Solving the above, we get  $Z_{TR} = j0.16$ ,  $Z_{\#1} = j0.088$  and  $Z_{\#2} = j0.533$ . Now, connect the transmission line 'b' across the network as shown in fig 25.6 and then the total impedance across the sources will be,

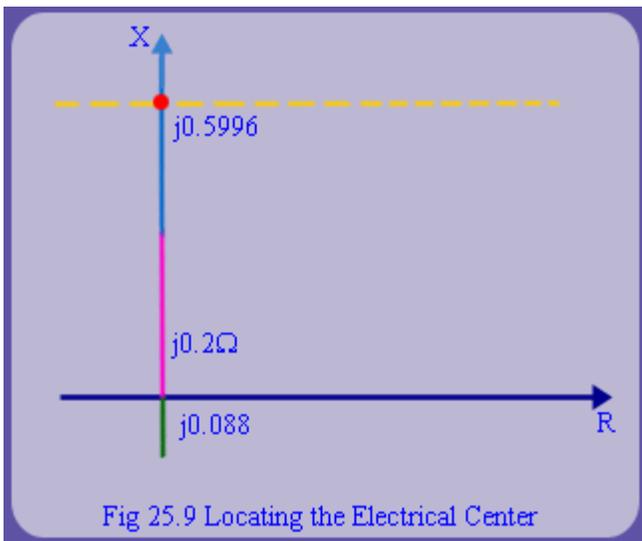
$$Z_T = Z_{\#1} + \frac{Z_L \times Z_{TR}}{Z_L + Z_{TR}} + Z_{\#2} = j0.088 + \frac{j0.2 \times j0.16}{j0.2 + j0.16} + j0.533 = j0.709,$$

$$k = \frac{Z_{TR}}{Z_L + Z_{TR}} = \frac{j0.16}{j0.2 + j0.16} = 0.4444$$

The swing impedance trajectory intersects the impedance line at

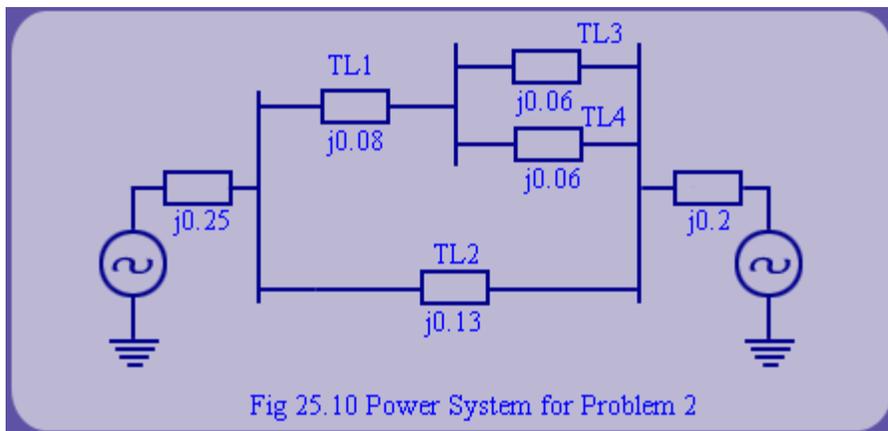
$$\frac{1}{k} \left( -Z_{th1} + \frac{Z_T}{2} \right) = \frac{1}{k} \left( -j0.088 + \frac{j0.709}{2} \right) = \frac{j0.2665}{0.4444}$$

$= j0.5996$ . Since the line impedance is  $j0.2$ , we can see from fig 25.7 we can see that the intersecting point of swing impedance trajectory with impedance line lies outside the transmission line 'b'.



### Review Questions

1. For the system shown in fig 25.3, determine whether swing locus passes through transmission line 'f'.
2. For the system shown in fig 25.8, find out whether power swing passes through any of the transmission lines?



### Recap

In this lecture we have learnt the following:

- Developed a two source equivalent of the power system.
- Analysed the system to determine whether swing locus passes through any transmission line.
- Found that swing locus passes through one of the transmission line.

