Module 9 : Numerical Relaying II : DSP Perspective

Lecture 37 : Estimation of System Frequency

Objectives

In this lecture,

- We will introduce the concept of DFT leakage, and use it to estimate magnitude and phase angle errors due to change

  in system frequency.

Error in Phasor Estimation due to Change in System Frequency

So far while discussing the phasor estimation problem, we have assumed that frequency of the power system remains fixed at it's nominal value ($f_0 = 50/60\ Hz$). Hence, we have also fixed the sampling frequency to $Nf_0 = f_s$, with N-point DFT being used for estimation.

However, during disturbance and even in steady state to a certain extent, the frequency varies. Thus, we expect the phasor estimation under constant frequency assumption to be erroneous. Under such situations, how good is our estimate of the phasor? We now plan to answer this question. As a by product of the analysis, we will also develop a frequency estimation technique which can be used in *under frequency* and *rate of change of frequency* relays. To simplify presentation, the analysis is developed gradually. First, we determine the DFT of complex exponential signal at frequency f_0 .

DFT of Complex Exponential

Let the signal be given by

$$x(t) = e^{j2\pi f_0 t} \qquad (1)$$

For this signal let N-sample be captured in P-cycles. P can be a positive integer or even a positive real number. Then, sampling interval (Δt) is given by P/Nf_0 , sampling speed by $Nf_0/P = f_s$. In the previous lectures, we had taken P = 1. The discrete sample at $t = n\Delta t$ is given by the following expression:

$$x_n = e^{j2\pi f_0 \frac{nP}{Nf_0}} = e^{\frac{j2\pi nP}{N}} \ (n \geq 0) \qquad (2)$$

Thus, m^{th} DFT component corresponding to frequency mf_0 , is given by

$$X(m) = \sum_{n=0}^{N-1} x_n e^{\frac{-j2\pi nm}{N}}$$

$$= \sum_{n=0}^{N-1} e^{\frac{j2\pi n}{N}(P-m)} \qquad (3)$$

It should now be fairly easy to compute the summation in (3)

Let $q = e^{j\frac{2\pi}{N}(P-m)}$, which is a constant once P and m are fixed. (4)

DFT of Complex Exponential (contd..)

Then $X(m) = \sum_{n=0}^{N-1} q^n = 1 + q + ------ + q^{N-1}$ (5)

$$= \frac{1-q^N}{1-q} = \frac{1-e^{j2\pi(P-m)}}{1-e^{j\frac{2\pi}{N}(P-m)}}$$

$$= \frac{e^{j\pi(P-m)}}{e^{j\frac{\pi}{N}(P-m)}} \times \frac{\left[e^{-j\pi(P-m)} - e^{j\pi(P-m)}\right]}{\left[e^{-j\frac{\pi}{N}(P-m)} - e^{j\frac{\pi}{N}(P-m)}\right]}$$

$$= e^{j\pi(P-m)(1-\frac{1}{N})} \times \frac{\left[e^{j\pi(P-m)} - e^{-j\pi(P-m)}\right] / 2j}{\left[e^{j\frac{\pi}{N}(P-m)} - e^{-j\frac{\pi}{N}(P-m)}\right] / 2j}$$

$$= e^{j\pi(P-m)(1-\frac{1}{N})} \times \frac{sin\pi(P-m)}{sin\frac{\pi}{N}(P-m)}$$

$$= e^{j\pi(P-m)(1-\frac{1}{N})} \frac{\sin c(\pi(P-m)) \times \pi(P-m)}{\sin c(\frac{\pi}{N}(P-m)) \times \frac{\pi}{N}(P-m)}$$

$$= N \times e^{j\pi(P-m)(1-\frac{1}{N})} \times \frac{sinc(\pi\overline{P-m})}{sinc(\frac{\pi}{N}\overline{P-m})} \quad (6)$$

Note that MATLAB defines $\sin c(x) = \sin(\pi x) / \pi x$. This convention differs from our convention viz. $\sin c(x) = \sin(x) / x$.

DFT of Complex Exponential (contd..)

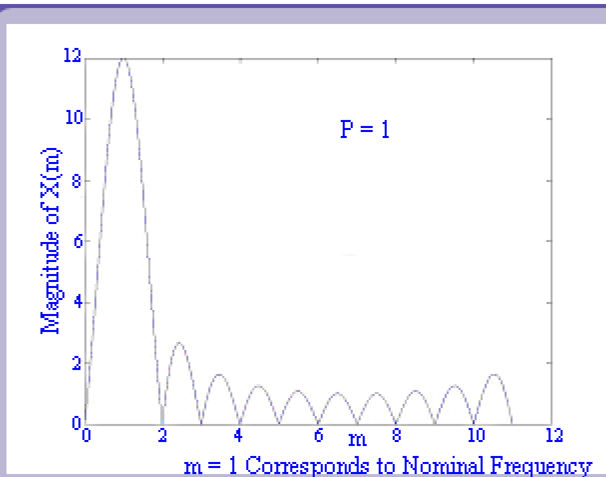

Fig 37.1 (a) DFT Gain with Frequency 50 Hz

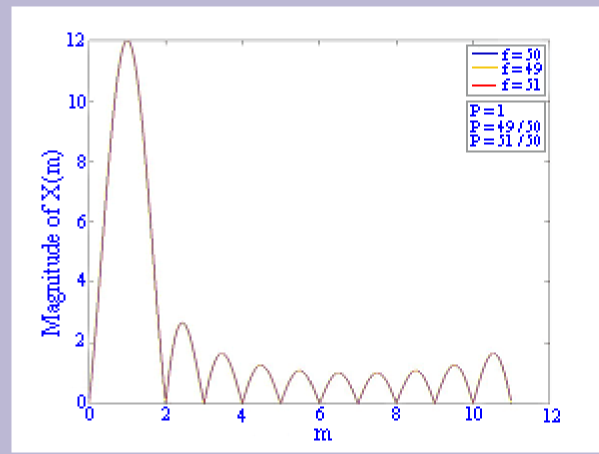

Fig 37.1 (b) DFT Gain at Frequencies (50 ± 1) Hz

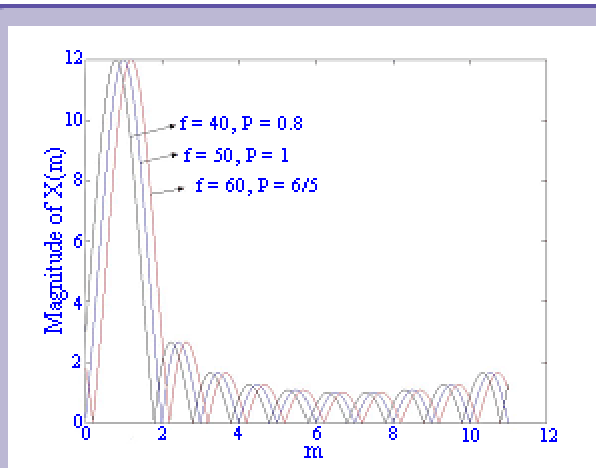

Fig 37.1 (c) DFT Gain at Frequencies (50 ± 10) Hz

DFT of Complex Exponential (contd..)

Fig 37.1(a) shows the envelope of response for $P=1$ as a function of m, where m is treated as a continuous variable. Since in DFT we have to restrict m to be a discrete number, this plot is sampled at discrete points ($m=0,1,2,.....,N-1$). The plot indicates that fundamental is extracted correctly as expected. At all harmonic frequencies, the DFT gain i.e. $|X(m)|$ is zero. Infact, fig 37.1(a) is identical to the frequency response of full-cycle Fourier algorithm. To be specific, we have chosen $f_0 = 50\,Hz$, N = 12.

Fig 37.1(b) shows the envelope of response for 3 different frequencies 49, 50, 51 Hz. The sampling rate is fixed at 12 samples per cycle at nominal frequency of 50 Hz. Thus, at 49 Hz, P = 49/50 and for 51Hz waveform P = 51/50. It is seen from the figure that magnitude response is more or less identical when frequency deviation is within ± 1 Hz.

Finally, fig 37.1(c) shows a set of response when frequency deviation from nominal is large enough i.e. when f = 40, 50 and 60 Hz. Now it can be seen easily that, if we sample the envelope in fig 37.1(c) (not the time domain signal) of 40 Hz signal at 50 Hz ($m=1$), then the P = 0.8, and gain at 50 Hz is a finite non-zero value different from unity. This effect is known as DFT leakage. It can be said that the energy in frequency bin $f=40\,Hz$ has leaked into frequency bin $f=50\,Hz$.

Remark 1: As f varies from f_0, with a constant sampling frequency f_s, we note that P varies proportionately. This is because we always obtain N-samples in T_0 seconds which is equivalent to f/f_0 cycles of Hz waveform.

f

Remark 2: From eqn. (6), it is clear that DFT gain is unity at $m = P$. Thus, the curve of fig 37.1(a), slides to the left when 'P' reduces below 1 and it slides to the right when $P > 1$. However, if we assume our waveform to be a 50 Hz signal, we always sample the envelope at $m = 1$.

In DSP literature, DFT leakage is considered to be undesirable. It means that we wrongly interpret a 40 or a 60Hz signal as a 50 Hz signal. However, by a little more analysis, we will show that deviation small enough from nominal frequency can be easily estimated from corresponding phase characteristic of DFT. This not only allows us to build underfrequency and rate of change of frequency relays but it simplifies hardware as sampling rate need not follow the system frequency. Consequently, no-zero crossing detectors are required. From the relaying perspective, it turns out to be a boon in disguise.

As seen in fig 37.1(b), the DFT magnitude leakage for ± 1 Hz frequency deviation around nominal frequency say 50Hz, is practically negligible. For f = 51 Hz, it equals $\left[\dfrac{\mathrm{sinc}(\pi(\frac{51}{50}-1))}{\mathrm{sinc}(\pi(\frac{50}{50}-1))} - 1 \right] pu$ = -0.0065. Thus, we conclude that effect of magnitude leakage on estimation of phasor magnitude can be neglected. However, the phase angle of DFT tells another story. Note that

For $P = \dfrac{51}{50}$ and $N = 12$ $\angle X(m) = \pi(P-m)(1-\dfrac{1}{N})$

Thus, $\angle X(1) = \dfrac{180^0}{50}(51-50)(1-\dfrac{1}{12}) = \dfrac{180}{50} \times \dfrac{11}{12} = 3.3^0$

Thus an error of is 3.3° introduced in phase computation due to 1 Hz frequency deviation.

DFT of real COSINE signal

So far, we have concentrated on DFT response of complex exponential. In practice, we are interested in DFT of real sine (or) cosine signal and not the complex exponential signal. However, deriving the response for real cosine (or) sine signal from response of complex exponential is not very difficult. From the fact that

$x(t) = \cos 2\pi f t = \dfrac{e^{j2\pi f t} + e^{-j2\pi f t}}{2}$ and by following similar steps as in the case of complex exponential, we get $x(n) = \dfrac{1}{2}\left[e^{j\frac{2\pi n P}{N}} + e^{-j\frac{2\pi n P}{N}} \right]$ where P is the number of cycles of the signal e.g. 1, 1.25, 2 etc, in which N-samples for DFT are obtained.

Thus,

$$X(m) = \frac{1}{2}\sum_{n=0}^{N-1}\left[e^{j\frac{2\pi n}{N}(P-m)} + e^{-j\frac{2\pi n}{N}(P+m)} \right] \qquad (7)$$

We have already deduced that

$$\sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(P-m)} = N e^{j\pi(P-m)(1-\frac{1}{N})} \frac{sinc\left(\pi(P-m)\right)}{sinc\left(\frac{\pi}{N}(P-m)\right)}$$

Similarly it can be shown that

$$\sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}(P+m)} = N e^{-j\left[\pi(P+m)-\pi(P+m)/N\right]} \frac{sinc\left(\pi(P+m)\right)}{sinc\left(\pi(P+m)/N\right)}$$

Thus,

$$X_c(m) = \frac{N}{2} e^{j\pi(P-m)(1-\frac{1}{N})} \frac{sinc[\pi(P-m)]}{sinc[\pi(P-m)/N]} + \frac{N}{2} e^{-j\pi(P+m)(1-\frac{1}{N})} \frac{sinc\left(\pi(P+m)\right)}{sinc\left(\pi(P+m)/N\right)} \tag{8}$$

With P $\approx$ 1, $(f/f_0 \approx 1)$, it can be seen that

$sinc(\pi(P+m)) \approx sinc\, 2\pi = 0$ while, $sinc(\pi(P-m)) \approx sinc(0) = 1$ and $sinc(\pi \overline{\frac{P-m}{N}}) = sinc(0) = 1$.

Hence, following approximation can be made to equation (8).

$$X_c(m) = \frac{N}{2} e^{j\pi(P-m)(1-\frac{1}{N})} \frac{sinc[\pi(P-m)]}{sinc[\pi(P-m)/N]} \tag{9}$$

We conclude that even with real sine or cosine signal, any reasonable deviation of the signal from the nominal value, leads to negligible magnitude leakage. However, phase angle error for 1Hz deviation in frequency is approximately 3.3° which is not negligible.

DFT of real COSINE signal

Fig 37.2 shows the envelope of magnitude response of $X_c(m)$ for different frequencies 40, 50, 60 Hz as per eqn. (9). For these plots, we have used N = 32 and f_s = 160 Hz.

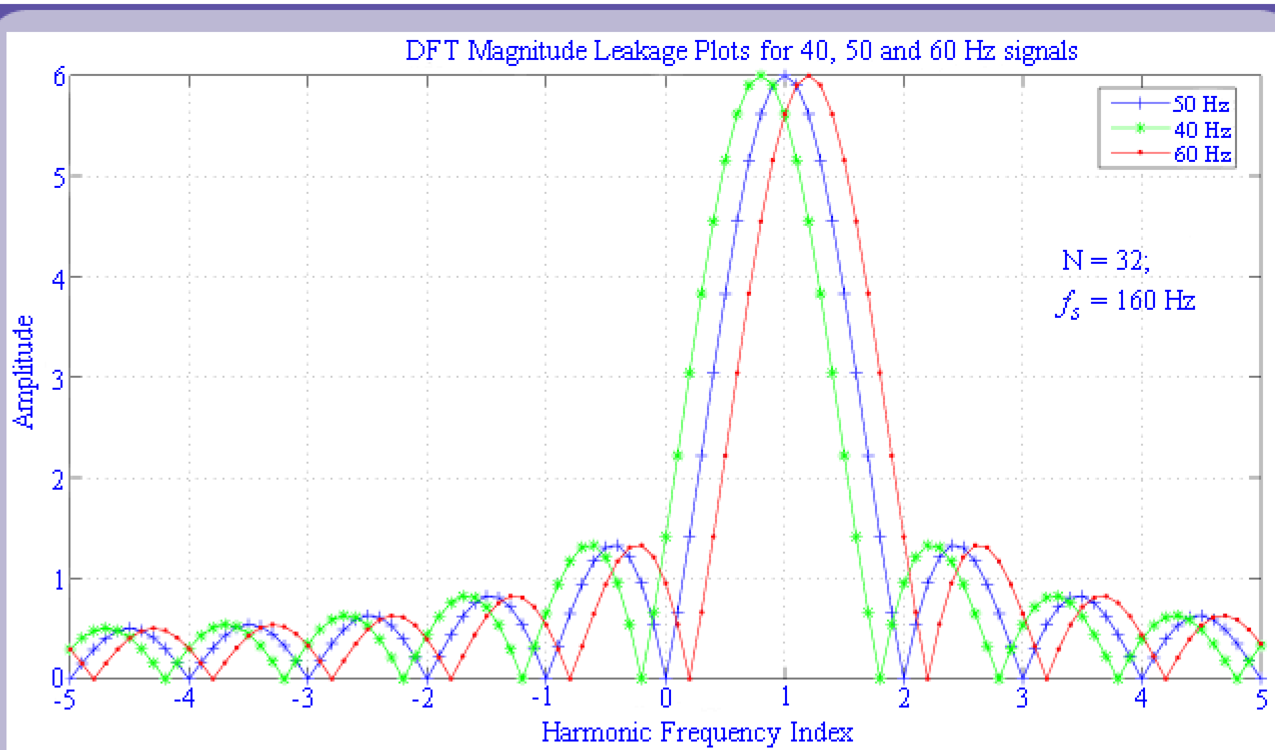

Fig 37.2 Amplitude Phase of a Cosine Signal for different frequency

Estimation of Frequency

Now our aim, is to develop a method of estimating power system frequency using the recursive DFT approach to phasor estimation. We plan to show that in the moving window approach the phase angle estimated by recursive DFT approach rotates at a speed proportional to the deviation from the nominal frequency. In turn, this deviation can be assessed by measuring the rate of change of phase angle.

In the previous section, we have derived the DFT leakage for complex exponential and real cosine signals. We will now generalize the DFT computation of complex exponential so that it can handle moving window concept.

Generalized DFT of Complex Exponential

Following the methodology used while generalizing DFT, we can write generalized DFT for w^{th} window (window number corresponds to the sample number of first sample in the window) as:

$$X^w(m) = \sum_{n=w}^{N+w-1} x_n e^{\frac{-j2\pi n}{N}m} \qquad (10)$$

where $x_n = e^{\frac{j2\pi nP}{N}}$

Substituting, x_n in (9), we get

$$X^w(m) = \sum_{n=w}^{N+w-1} e^{\frac{j2\pi n}{N}(P-m)}$$

$$= e^{\frac{j2\pi}{N}(P-m)w}[1+q+....+q^{N-1}]$$

where $q = e^{\frac{j2\pi}{N}(P-m)}$

Summation $\sum_{i=0}^{N-1} q^i$ is the DFT corresponds to window number 1. Substituting from eqn. (8),

we obtain $X^w(m) = e^{\frac{j2\pi n}{N}(P-m)w} X_c(m)$ (11)

If m = P, then it is clear that DFT $X^w(m)$ is stationary i.e. independent of window number. In relaying applications where we are interested in fundamental phasor estimation, we choose $f_s = Nf_0$. Hence, we can expect obtain stationary DFT in (10) when m = P = 1 i.e. when signal corresponds to fundamental frequency.

With the knowledge of the generalized DFT of the complex exponential, now we can derive the generalized DFT expression for real cosine signal.

Generalized DFT of Real Cosine Signal

For a real cosine signal, with moving window concept eqn (7) can be generalized as follows:

$$X^w(m) = \frac{1}{2}\sum_{n=w}^{N+w-1}\left[e^{\frac{j2\pi n}{N}(P-m)} + e^{\frac{-j2\pi n}{N}(P+m)}\right] \qquad (12)$$

$$= \frac{1}{2}e^{\frac{j2\pi w}{N}(P-m)}\sum_{n=0}^{N-1} e^{\frac{j2\pi n}{N}(P-m)} + \frac{1}{2}e^{\frac{-j2\pi w}{N}(P+m)}\sum_{n=0}^{N-1} e^{\frac{-j2\pi n}{N}(P+m)}$$

$$= \frac{N}{2}e^{\frac{j2\pi w}{N}(P-m)} e^{j\pi(P-m)\left(1-\frac{1}{N}\right)} \frac{\sin c\left(\pi\overline{P-m}\right)}{\sin c\left(\frac{\pi}{N}\overline{P-m}\right)} + \frac{N}{2}e^{\frac{-j2\pi w}{N}(P+m)} e^{-j\pi(P+m)\left(1-\frac{1}{N}\right)} \frac{\sin c\left(\pi\overline{P+m}\right)}{\sin c\left(\pi\frac{\overline{P+m}}{N}\right)} \qquad (13)$$

Following the similar reasoning as outlined in the previous section of the DFT of real cosine signal, we can neglect the second term in (13). This leads to following approximation.

$$X^{w}(m) \approx \frac{N}{2} e^{\frac{j2\pi w}{N}(P-m)} e^{j\pi(P-m)\left(1-\frac{1}{N}\right)} \frac{\sin c\left(\pi \overline{P-m}\right)}{\sin c\left(\frac{\pi}{N}\overline{P-m}\right)} \quad (14)$$

Observe that P = m = 1, we obtain stationary DFT because $e^{\frac{j2\pi w}{N}(P-m)}$ = 1. However, when frequency deviates from nominal frequency i.e $P \neq 1$, then $X^{w}(1)$, the DFT estimate for nominal frequency, will start rotating along with the window. From, (14) we can derive that

$$\frac{X^{w+1}(1)}{X^{w}(1)} \approx e^{\frac{j2\pi}{N}(P-1)}$$

Since fundamental phasor estimated for window number ‘w' is given by

$\overline{X}^{w} = \frac{j2}{N\sqrt{2}} X^{w}(1)$, we deduce that $\frac{\overline{X}^{w+1}}{\overline{X}^{w}} = e^{\frac{j2\pi}{N}(P-1)} = e^{\frac{j2\pi}{N}\frac{f-f_0}{f_0}} = e^{\frac{j2\pi T_0}{N}\Delta f}$

$$\Rightarrow \frac{\overline{X}^{w+1}}{\overline{X}^{w}} = e^{j2\pi\Delta f\Delta t} \quad (15)$$

where Δt is the sampling time interval. Summarizing, if we set sampling frequency for a sinusoidal signal to provide N-samples per cycle at the nominal frequency f_0, (say 50 or 60 Hz), and keep the sampling frequency invariant of the actual frequency of sinusoid, then the phasor estimated by moving window approach rotates at a speed proportional to Δf . This rotation will be in anti-clockwise direction if $\Delta f > 0$ i.e. $f > f_0$. Conversely, it will be in the clockwise direction if $\Delta f < 0$ i.e. $f < f_0$.

Generalized DFT of Real Cosine Signal (contd..)

If we monitor, this phase rotation, then from the proportionality relationship of (15), we can estimate the frequency ‘f'. If ψ denotes phase-angle, then from (15), we can obtain the rate of change of ψ as follows:

$$\frac{d\psi}{dt} = \frac{\psi^{w+1} - \psi^{w}}{\Delta t} = 2\pi\Delta f = \Delta\omega \quad (16)$$

Discussion

There are many advantages associated with the above algorithm. The method is not based on zero crossing time and it is immune to noise and harmonics. Instead of using single phase quartiles, one can estimate the positive sequence component and derive frequency from it. Such an approach will use all the three phase voltages and hence will have better noise rejection properties. At harmonics of nominal frequency $X^{w}(m) = 0$ and hence, the above approach will reject frequencies m f_0 completely.

Measurement of Frequency

To measure f_0 accurately, we integrate eqn. (16) until $\Delta\psi$ equal to 0.5 radians. Let ‘T' be the time to accumulate this change:

$$\Delta f = \frac{0.5}{2\pi T}$$

If we assume that frequency computation will be further averaged over four measurements to smoothen out noise, then time to compute deviation Δf is given by

$$T = \frac{0.32}{\Delta f} \sec$$

Thus, a frequency deviation of 1 Hz is detected in 0.32sec while a frequency deviation of 0.1 Hz is detected in 3.2sec. This is an illustration of standard speed versus accuracy conflict in relaying. This technique can be recommended for development of under frequency and rate of change of frequency relays.

Recap

In this lecture we have learnt the following:

- We introduced DFT leakage. It was shown that when fixed sampling frequency is used, typically N-samples in 1 - cycle

at nominal frequency, then, DFT leakage is zero. This means that there are no magnitude and phase angle errors in estimation. However, when the system frequency deviates from the nominal (of the order $\pm 2\,Hz$ say), then errors introduced in estimating the amplitude of the signal is negligible. However, now phase angle errors are not negligible.

It was shown that if the frequency deviates from the nominal value, with constant sampling frequency, the phasor starts rotating at a speed proportional to it. This can be used for frequency estimation.