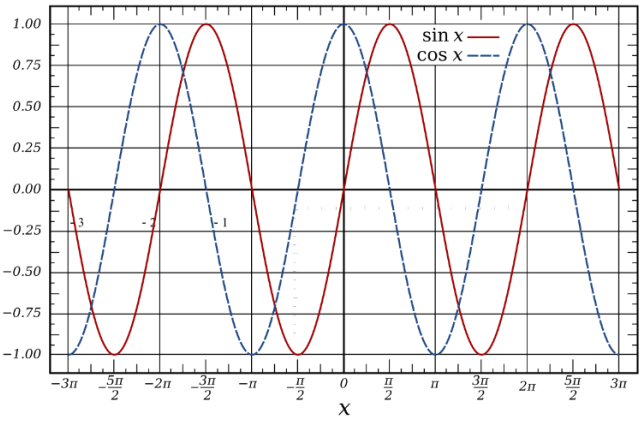
Sinusoids and RMS

Introduction

A sine wave is defined by three parameters:



A*cos(2πft + θ)

1. Amplitude (A) – the peak value of the sinusoid (happens when the total within the parenthesis is a multiple of 360° or 2π radians for cosine or that plus 90° or π/2 radians in the case of a sine wave)
2. Frequency (f in Hertz, cycles/second, or ω = 2πf in radians/second) – how rapidly the sinusoid goes through its cycles. An alternate parameter is the period (T = 1/f) which is how much time it takes for one full cycle of the sinusoid.
3. Phase (θ) – A shifting of the timing of the sinusoid. Notes:
    a. sin(x + 90°) = cos(x)
    b. You can measure the phase by comparing the sinusoid to a reference sinusoid and determine the offset of a zero crossing as a percent of a full period (T = 1 millisecond for f = 1000 Hertz). Then just apply that fraction of a cycle to either 360° or 2π radians to get the Phase shift between the two sinusoids.

Amplitude

There are three ways that the amplitude parameter of a sinusoid can be specified.

1. Peak amplitude, A, as described above.
2. Peak to peak which is 2*A and measured from the maximum negative value of the sinusoid to the maximum positive vale of the sinusoid.
3. RMS (Root-Mean-Square) or effective value of the sinusoid which is a measure that would cause the same heating of a resistor as a DC voltage equal to the RMS vaue.

Calculating the RMS

1. Square the sinusoid
   $A^2 * [cos(2\pi ft + \theta)]^2$
2. Average (take the mean value) the square over one full period of the original sinusoid
   $\frac{1}{T}\int_0^T A^2 * [cos(2\pi ft + \theta)]^2 dt = \frac{A^2}{T}\int_0^T [cos(2\pi ft + \theta)]^2 dt$

   but $\cos^2(x) = [1 + \cos(2x)]/2$

   $$\frac{A^2}{2T}\int_0^T [1 + cos(4\pi ft + 2\theta)]dt$$

   But that integral = the integral of 1 (T) and the area under two full cycles of the sinusoid which is zero so we have just $\frac{A^2}{2}$
3. Now take the square root and we have the RMS of the sinusoid is $A/\sqrt{2}$ or $A * (\sqrt{2}/2)$
   Notes:
   a. This is ONLY true for a sinusoid, the RMS of other waveforms will differ.
   b. the square root of two is 1.4142 and dividing by 2 yields 0.7071

Notes about adding voltage signals:

- Two voltage signals at the same frequency add point by point
  can be difficult with sinusoids with different amplitudes or phases – more later
- Two voltage signals at different frequencies add as powers
  (square each, add and then take the square root)