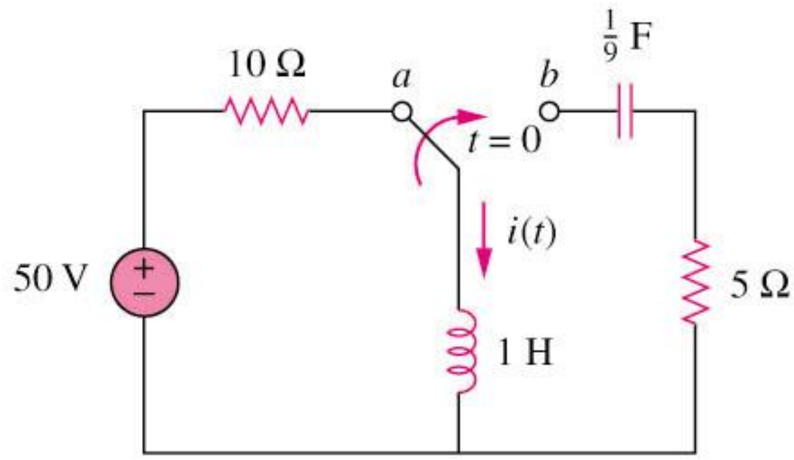
DoctorD's Solution to Practice problem 8.4 of the text

Note that the battery is 50v here, not 100v – this changes the coefficient of sine in the answer.

Example 2

The circuit shown below has reached steady state at t = 0-.

If the make-before-break switch moves to position b at t = 0, calculate i(t) for t > 0.



The Author is assuming that the voltage on the capacitor is 0 at t = 0-

Please refer to lecture or textbook for more detail elaboration.

Answer: $i(t) = e^{-2.5t}[5\cos 1.6583t - 7.538\sin 1.6583t]$ A

1. Find the initial current in the inductor

   i(0-) = 50/10 = 5 amps (down)

2. Determine the differential equation after the switch is thrown

Use a loop equation: (I assume the loop current flows counter clockwise)

$V_R + V_C + V_L = 0$

$$Ri + \frac{1}{C}\int i\,dt + L\frac{di}{dt} = 0$$

or, after taking the derivative of the equation,

$$R\frac{di}{dt} + \frac{1}{C}i + L\frac{d^2i}{dt^2} = 0$$

$$5\frac{di}{dt} + 9i + \frac{d^2i}{dt^2} = 0$$

3. Let $i = A\varepsilon^{st}$ and we get the characteristic equation $s^2 + 5s + 9 = 0$
4. Complete the square

$(s + 2.5)^2 + 2.75 = 0$

$(s + 2.5)^2 = -2.75$

$s + 2.5 = \pm 1.6583j$

$s = -2.5 \pm 1.6583j$ We have an underdamped system so the answer is:

$$i(t) = A_1\varepsilon^{-(2.5+1.6583j)t} + A_2\varepsilon^{-(2.5-1.6583j)t}$$

5. Find A_1 and A_2

$i(0) = 5 = A_1 + A_2$, or $A_2 = 5 – A_1$
but we need another equation so take the derivative of the solution

$$\frac{di}{dt} = -(2.5 + 1.6583j)A_1\varepsilon^{-(2.5+1.6583j)t} - (2.5 - 1.94j)A_2\varepsilon^{-(2.5-1.6583j)t}$$

but the initial voltage on the inductor is -25 volts (resistor current) and V_L=L*di/dt

$$25 = (2.5 + 1.6583j)A_1 + (2.5 - 1.96583j)A_2$$

or

$$5 = (0.5 + .3317j)A_1 + (0.5 - .3317j)A_2$$

Substituting

$$5 = (0.5 + .3317j)A_1 + (0.5 - .3317j) * (5 - A_1)$$
$$5 = (0.5 + .3317j)A_1 + (2.5 - 1.6583j) - (0.5 - .3317j) * A_1$$
$$2.5 + 1.6583j = [(0.5 + .3317j) - (0.5 - .3317j)] * A_1$$
$$2.5 + 1.6583j = [(.3317j) + (.3317j)] * A_1$$

$A_1 = \frac{2.5+1.6538j}{0.6633j} = 2.5 - 3.75j$ and $A_2 = 2.5 + 3.75j$

So the answer is (in exponential form) is

$$i(t) = (2.5 - 3.75j)\varepsilon^{-(2.5+1.6583j)t} + (2.5 + 3.75j)\varepsilon^{-(2.5-1.6583j)t}$$

6. To get the sine-cosine form we need to do some complex (this doesn't mean hard, but it can be tedious) algebra.

   Collecting terms:

$$i(t) = \varepsilon^{-2.5t}\left[(2.5 - 3.75j)\varepsilon^{-1.6583jt} + (2.5 + 3.75j)\varepsilon^{+1.6583jt}\right]$$

$$i(t) = \varepsilon^{-2.5t}\left[2.5\varepsilon^{-1.6583jt} + 2.5\varepsilon^{+1.6583jt} + (-3.75j)\varepsilon^{-1.6583jt} + (3.75j)\varepsilon^{+1.6583jt}\right]$$

   or

$$i(t) = \varepsilon^{-2.5t}\left[\frac{5\varepsilon^{1.6583jt} + 5\varepsilon^{-1.6583jt}}{2} - \frac{7.5\varepsilon^{1.6583jt} - 7.5\varepsilon^{-1.6583jt}}{2j}\right]$$

   or

   $i(t) = \varepsilon^{-2.5t}[5\cos(1.6583t) - 7.5\sin(1.6583t)]$ amps