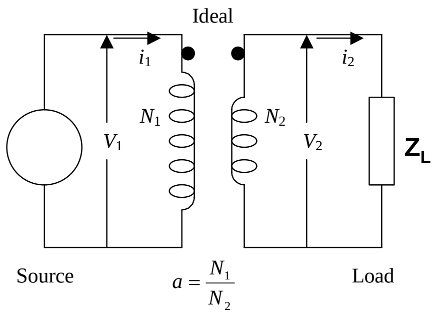
Ideal 1~ Transformer Model



Flux linkages (winding flux)

$$\lambda_1 = N_1\phi, \quad \lambda_2 = N_2\phi$$

$$v_1 = e_1, \quad v_2 = e_2$$

$$a = \frac{N_1}{N_2} = \frac{v_1}{v_2} = \frac{e_1}{e_2} = \frac{i_2}{i_1}$$

- **Dot convention**
  - Current into doted end produces positive MMF (N·I) or "ampere-turns"
  - Therefore, orientation of i_1 and i_2 must be as shown to cancel MMF which is necessary to maintain a finite flux in an "ideal (iron) core"

Loads on an Ideal Transformer

1. Solve for the secondary voltage

$$V_2 = \frac{|V_1|\angle 0°}{a} = |V_2|\angle 0°$$

2. Calculate the load current from the voltage and load impedance

$$I_2 = \frac{|V_2|\angle 0°}{|Z_L|\angle\theta} = |I_2|\angle -\theta$$

3. Solve for the primary current

$$I_1 = \frac{|I_2|\angle -\theta}{a} = |I_1|\angle -\theta$$

Equivalent modeling:

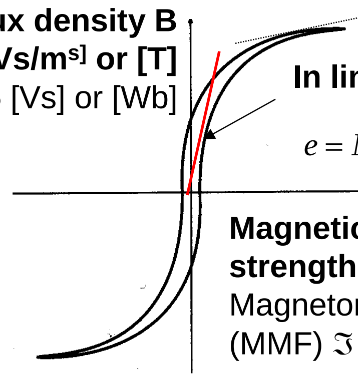
$$Z_{in} = \frac{V_1}{I_1} = \frac{aV_2}{I_2/a} = a^2\frac{V_2}{I_2} = a^2 Z_L$$

Example: Ideal 1~ Transformer

See Book, Example 3-1, p. 90

Real Transformer: Magnetizing Inductance

Flux density B [Vs/m^{s}] or [T]
Flux ϕ [Vs] or [Wb]



Saturation

In linear region $\phi = \Im/\Re = i_m \cdot N/\Re$

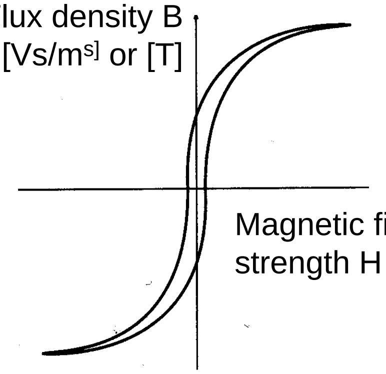
$$e = N\frac{d}{dt}\left(\frac{Ni_m}{\Re}\right) = \frac{N^2}{\Re}\frac{di_m}{dt} = L_m\frac{di_m}{dt}$$

Magnetic field strength H [A/m]
Magnetomotive force (MMF) $\Im$ [A-turns]

$$\Im = N_p i_p - N_S i_S$$

$\Re$… reluctance, “resistance” of magnetic circuit

Real Transformer: Core Losses



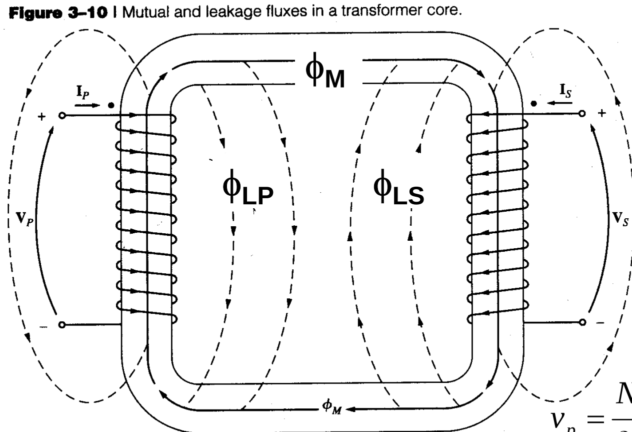
Energy term enclosed by hysteresis loop is lost each AC cycle

Both are proportional B^2 and therefore to V^2 => modeled by resistor

dB/dt induces voltages in iron core which result in eddy currents producing losses

Real Transformer: Leakage Flux

Figure 3–10 | Mutual and leakage fluxes in a transformer core.



$$v_p = N_p \frac{d\phi_{LP}}{dt} + N_p \frac{d\phi_M}{dt}$$

$$v_S = N_S \frac{d\phi_{LS}}{dt} + N_S \frac{d\phi_M}{dt}$$

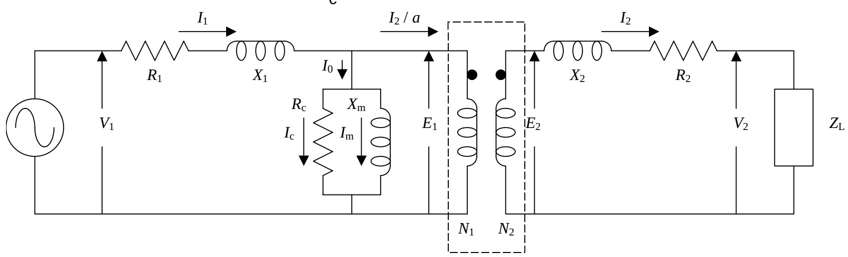
$$v_p = \frac{N_p^2}{\Re_1} \frac{di_P}{dt} + e_p = L_1 \frac{di_P}{dt} + e_p$$

$$v_S = \frac{N_S^2}{\Re_2} \frac{di_P}{dt} + e_S = L_2 \frac{di_P}{dt} + e_S$$

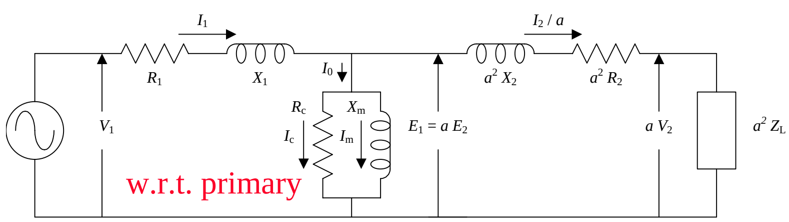
ϕ_{LP} is only associated with I_P, ϕ_{LS} only with I_S

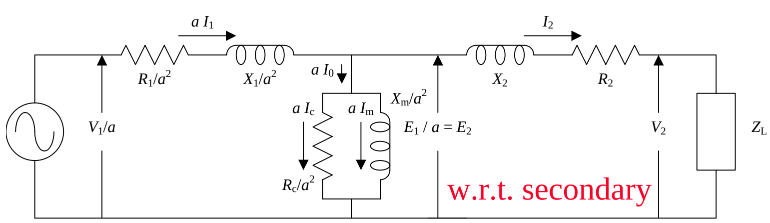
Real 1~ Transformer Model

- **Winding losses**
  - Winding resistances R_1, R_2
- **Non ideal coupling between windings**
  - Leakage reactances X_1, X_2
- **Magnetizing current**
  - Magnetizing reactance X_m
- **Core (hysteresis and eddy current) losses**
  - Core loss resistance R_c

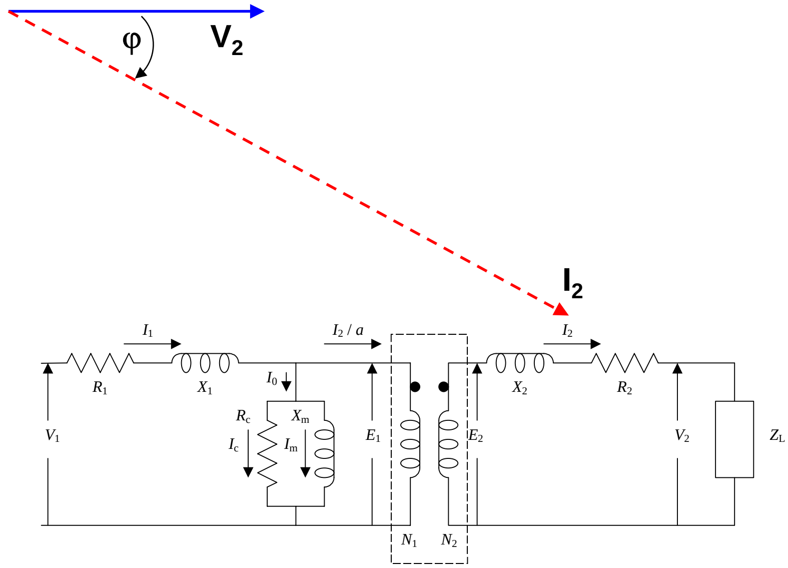


Other Equivalent Models

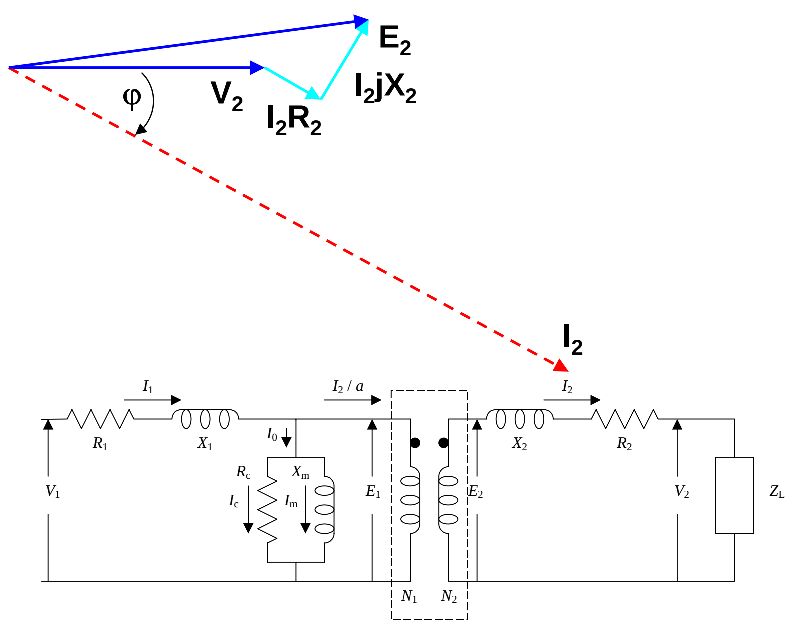




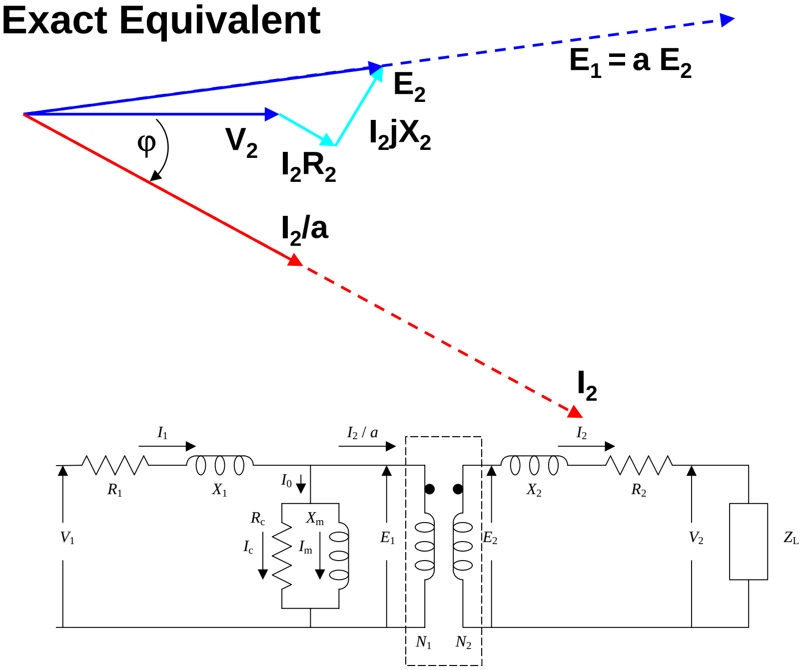
Phasor Diagram for Exact Equivalent



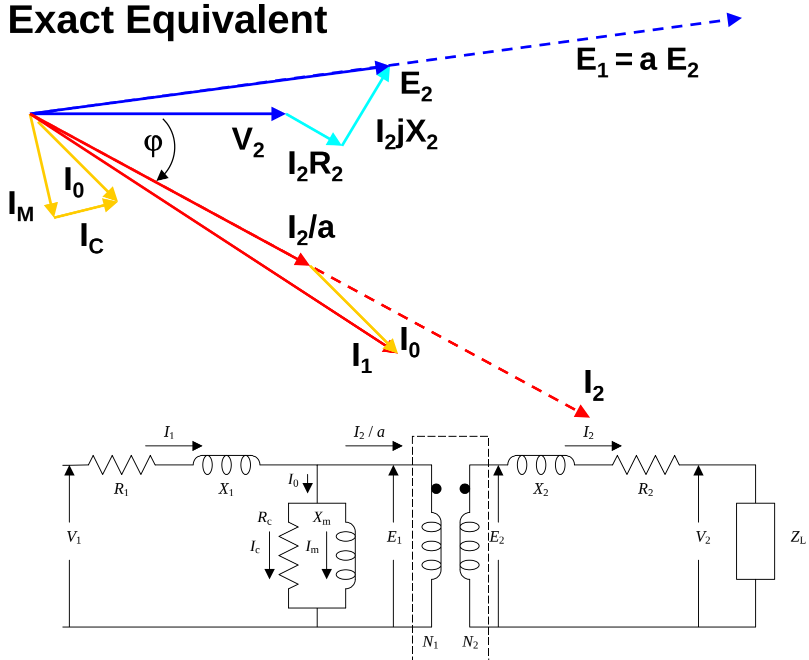
Phasor Diagram for Exact Equivalent



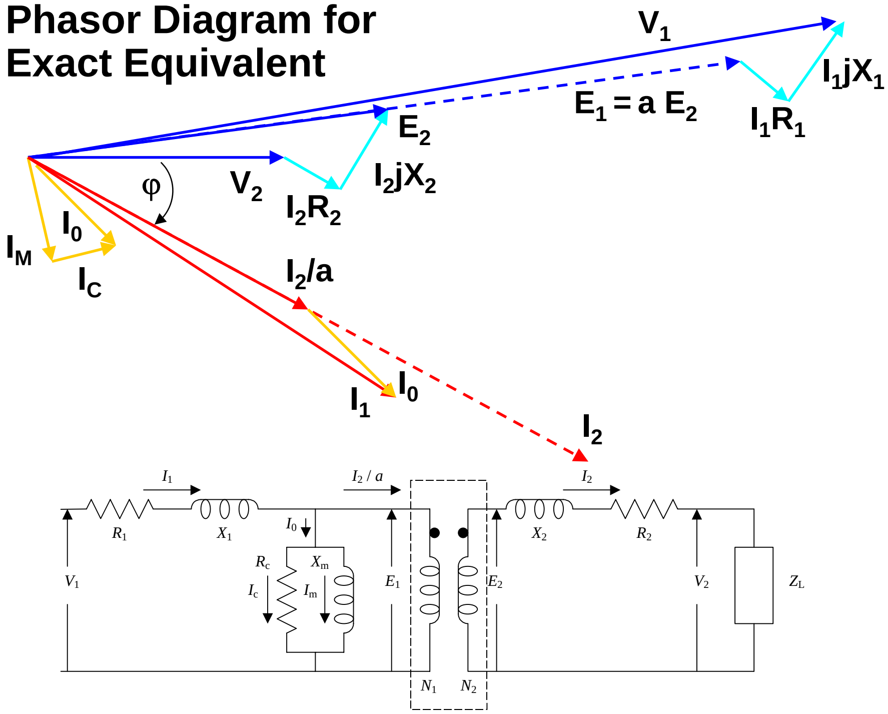
Phasor Diagram for Exact Equivalent



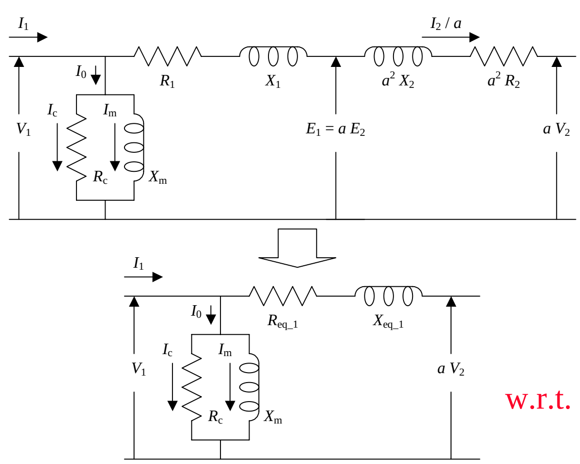
Phasor Diagram for Exact Equivalent



Phasor Diagram for Exact Equivalent

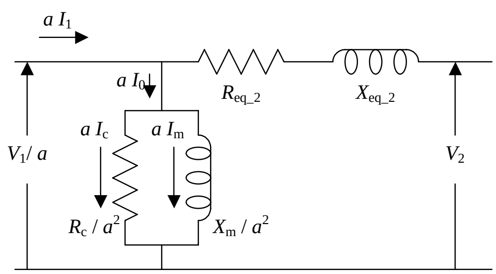


Approximate Circuit Model



w.r.t. primary

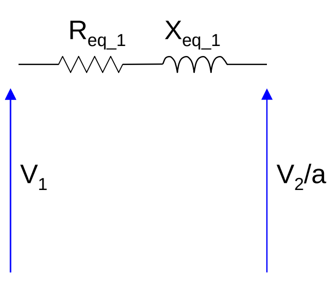
Approximate Circuit Model



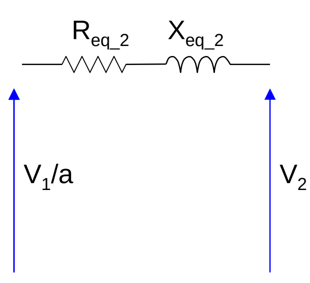
w.r.t. secondary

Approximate Circuit Model for Large Transformers

I_0 becomes relatively small for large, modern transformers and can be approximated by $I_0 = 0$.



w.r.t. primary

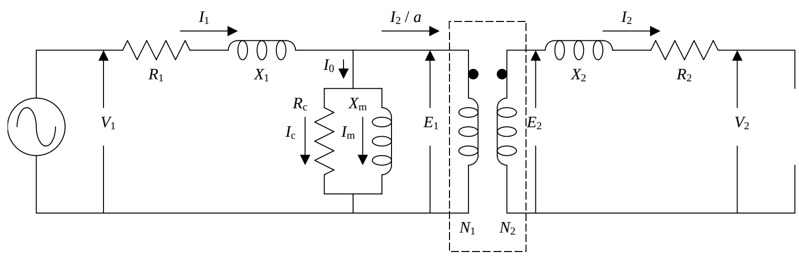


w.r.t. secondary

$R_{eq\_1} = a^2 R_{eq\_2}$
$X_{eq\_1} = a^2 X_{eq\_2}$

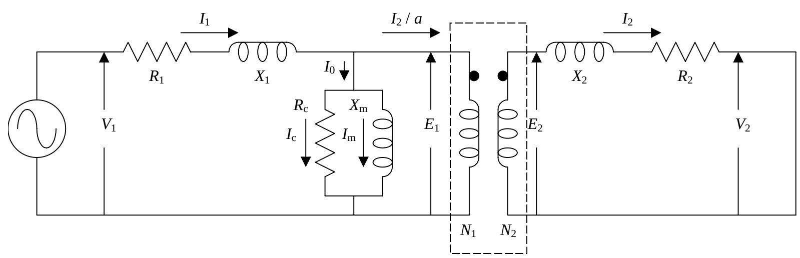
Determining Transformer Impedances

- **Open circuit test**
  - $V_1 = V_{1\text{-rated}}$, $I_2 = 0$
  - No-load current $I_1 = I_0 << I_{Rated}$
  - Turns ratio $a = N_1:N_2 = E_1:E_2 \cong V_1:V_2$
  - Core losses $P_{core} \cong V_1 I_1 \cos(\varphi)$, $I^2_0 \cdot R_1 \cong 0$



Determining Transformer Impedances

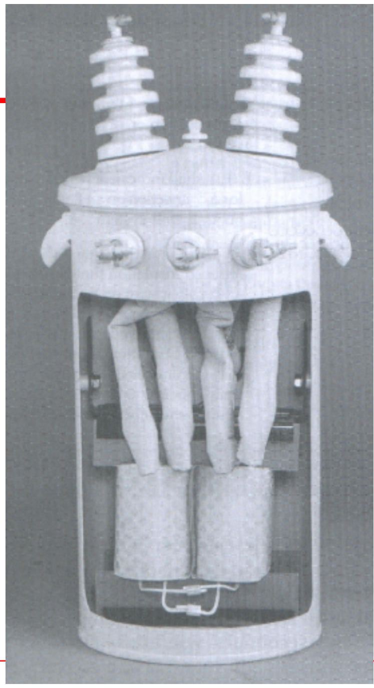
- **Short circuit test**
  - $V_2=0$, $I_2=I_{2\text{-rated}}$, $V_1 << V_{1\text{-rated}}$
  - $I_0 \cong 0$, $P_{core} \cong 0$
  - Copper losses $P_{Coper} \cong V_1 I_1 \cos(\varphi)$
  - typical: $R_1 \cong a^2 R_2$, $X_1 \cong a^2 X_2$ $R_{eq\_1} \cong 2 \cdot R_1 \cong P_{Coper}/I^2_1$
  - $V_1 = I_1 \cdot Z_{eq\_1} = I_1 (R_{eq\_1} + j X_{eq\_1}) => X_{eq\_1}$



Example: Transformer test

See Book, Example 3-2, p. 109

Pole top transformer



From: T. A. Short, "Electric Power Distribution Handbook", CRC Press, N. Y., 2004