Induced Voltage in Stator Coil – P-Poles

- **For a P-pole machine the flux per pole remains the same (integrating $d\phi = rlB_m \cos(P/2\alpha)d\alpha$ over one pole pitch $-\pi/P$ to $+\pi/P$)**

$$\phi = 2rlB_M$$

- **The electrical frequency becomes**

$$\omega_{el} = \frac{P}{2}\omega_m = \omega$$

- **Therefore, the induced voltage per pole-pair (one sub-winding) is**

$$e_{ind,pole} = N_C \phi \omega_m \cos(\frac{P}{2}\omega_m t)$$

- **The entire phase winding consists of P/2 sub-windings connected in series resulting in the <u>total induced phase voltage</u>**

$$e_{ind} = \frac{P}{2} N_C \phi \omega_m \cos(\frac{P}{2}\omega_m t) \qquad \boxed{e_{ind} = N_C \phi \omega \cos(\omega t)}$$

Example 4-2 (Book)

- **Additional question 2**
  - Assuming the same field from the rotor, a rated current of I=10 A (RMS) in the stator at PF=0.8, a stator magnetic field of 0.4 mT/A-turn, and a stator resistance of 0.3 Ω per phase, calculate the torque constant (k in Nm/T^2) in

$$\tau_{ind} = kB_R B_S \sin(\gamma)$$

    if the angle between the rotor field and the stator field space vectors is $\gamma = -100°$ (assume k >0).
  - What is the direction of rotation (CW, CCW) if this is a motor?

Example 4-2 (Book)

- **Additional question 2**

$$P_{el} = \sqrt{3}VI\cdot PF = \sqrt{3}\cdot 208\cdot 10\cdot 0.8 = 2882W$$

$$P_{Loss,R} = 3I^2R = 3\cdot 10^2 0.3 = 90W$$

$$\omega_m\tau_{ind} = P_{conv} \rightarrow \tau_{ind} = \frac{60\left(P_{el} - P_{Loss,Cu}\right)}{2\pi n} = \frac{60\cdot 2792}{2\pi 3600} = 7.41\,Nm$$

$B_R = 0.2T$

-100°

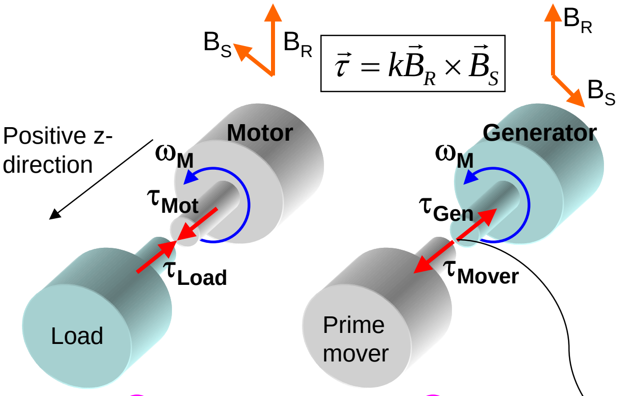
τ is CW (see x-product) and therefore negative with respect to the positive mathematical counting direction (I.e. CCW). The speed must also be CW since this is a motor.

$$B_S = 0.4\cdot 10^{-4}\cdot 15\sqrt{2}\cdot 10$$

$$= 0.085T$$

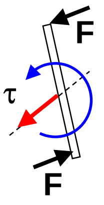
$$k = \frac{\tau_{ind}}{B_R B_S \sin(\gamma)} = \frac{-7.41}{0.2\cdot 0.085\sin(-100°)} = 442.6\frac{Nm}{T^2}$$

Comments on Power, Torque, and Speed on the Shaft of Rotating Machines



$$\vec{\tau} = k\vec{B}_R \times \vec{B}_S$$

Rated power
Mot: at shaft
Gen: at terminals



Attempts to accelerate CCW

$$P_{Shaft} = \textcircled{+}|\omega_M|\cdot|\tau_{Mot}|$$

> 0 > 0

Motor supplies power to load

$$P_{Shaft} = \textcircled{-}|\omega_M|\cdot|\tau_{Gen}|$$

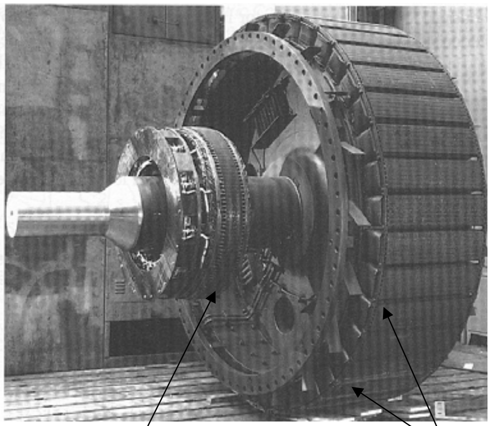
> 0 < 0

Generator absorbs power from prime mover

For ω_M=const ($d\omega_M/dt=0$) the sum of the torques at the shaft connection must be zero.

Synchronous Machines (SM)

- **The majority of generators are of the synchronous machine type**
- **The rotor provides permanent magnetic excitation**
  - Permanent magnet
  - Electromagnet (DC)
- **The speed of the shaft is always in synchronism with the electrical frequency**

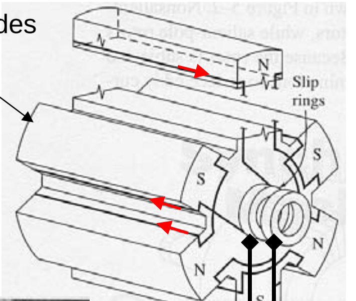


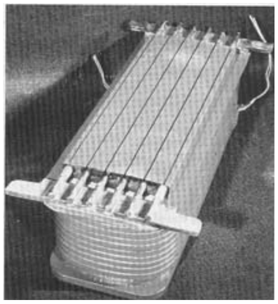
Excitation system for rotor field current

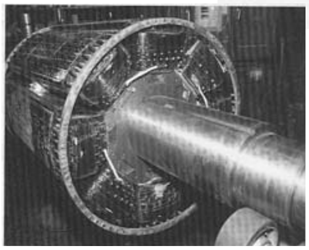
Salient poles

SM Rotor Construction

Pole shape provides sinusoidal B_R distribution



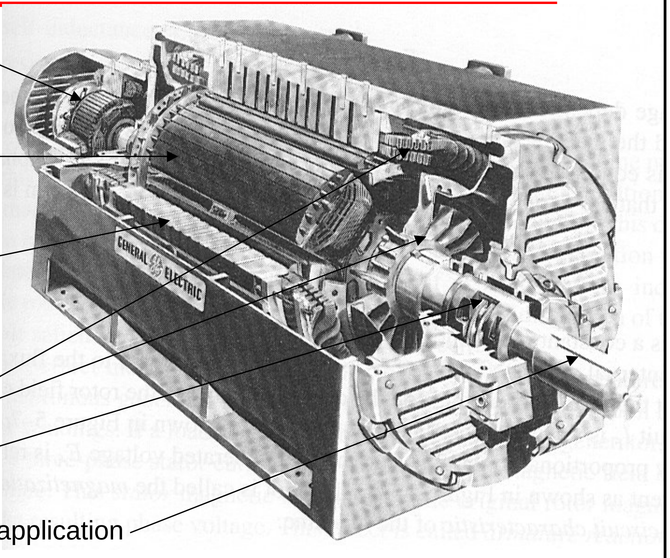




DC field current source I_F

Also possible: round rotor (no saliency)

SM Construction



DC excitation system

Rotor poles with DC filed winding

Stator iron to close magnetic path and slots for AC windings

AC stator windings

Cooling fan

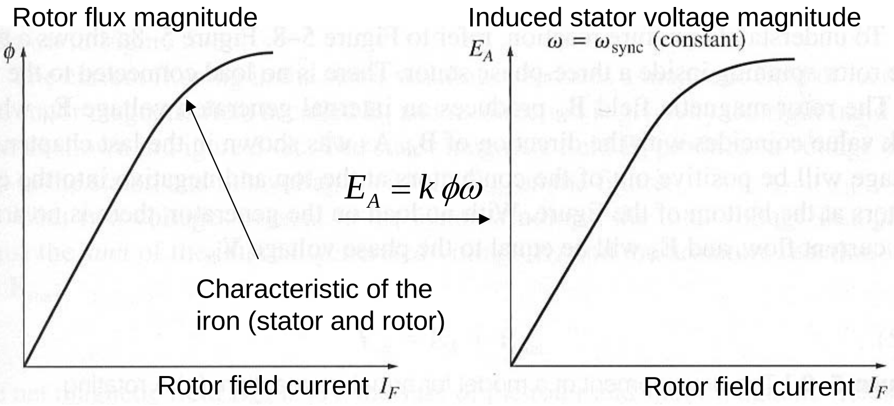
Bearings

Shaft stub for torque application

SM Modeling for Steady State Operation

- **Purpose: describing the behavior of the SM under different steady state load conditions**
- **The model must account for the**
  - Resistance of the stator (armature) winding
  - Self-inductance (reactance) of the armature winding
  - Effect of the armature B-field on the total B-field (armature reaction)
- **Assumptions, simplifications**
  - Balanced conditions
  - No spatial distortions of the B-field (infinite number of slots, perfect pole shapes), i.e. “perfect” sinusoidal B-field distribution in the air gap

SM No-Load Characteristic



Rotor flux magnitude

Induced stator voltage magnitude

$$E_A = k\,\phi\omega$$

Characteristic of the iron (stator and rotor)

Rotor field current I_F

Rotor field current I_F

Effect of saturation: for large field currents the no-load armature voltage magnitude scales sub-proportionally with I_F and the voltage waveform becomes non sinusoidal (distorted).