EECE 360
Lecture 2

Introduction to Modeling and the Control Design Process

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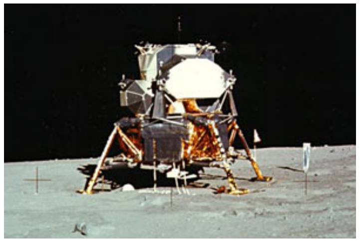
Chapters 1.1-1.10, 2.1-2.3

The Control Design Process

- Today:
  - Brief history
  - The principle of feedback *(review)*
  - Practical issues: sensors and actuators
  - Linear Systems and Linearization
  - Modeling physical systems with differential equations

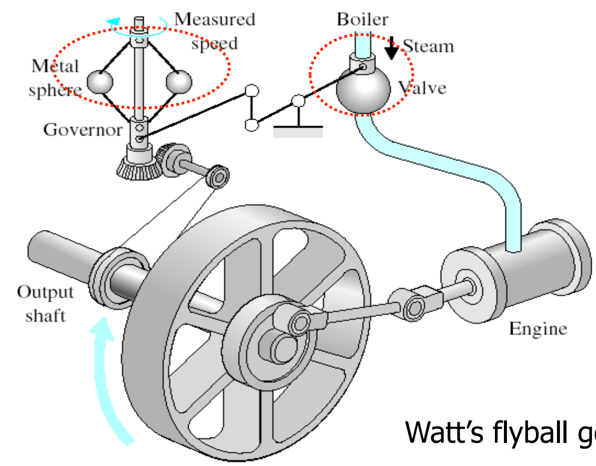
Brief History

- Ancient Egypt, Greece, and China, float regulator used in waterclocks
- Watts' flyball governor (1769); Maxwell's model (1868)
- "Classical" control (1940s)
- "Modern" control (1970s)
- Robust, Nonlinear, Optimal, H_∞, Discrete, Hybrid, Embedded…



Apollo 11 lunar module, *www.nasa.gov*

Brief History



Watt's flyball governor

Feedback

- Compare actual behavior with desired behavior
- Take corrective action based on the difference
- Deceivingly simple idea, but very powerful concept
- Feedback is a key idea in control

Open-Loop Control

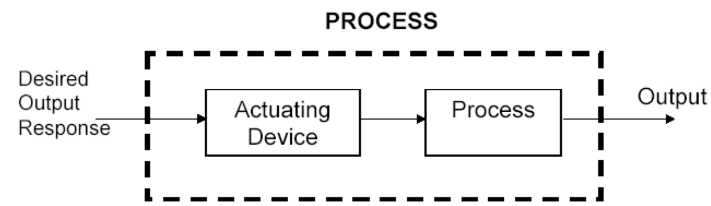


Figure 1.2 Open-loop Control System (without feedback)

An open-loop control system uses an actuating device to control the process directly.

Closed-Loop Control

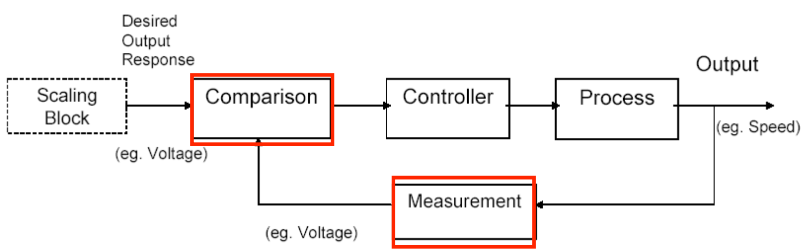
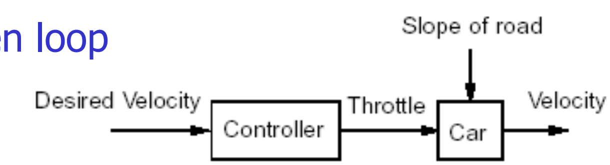


Figure 1.3 Closed-loop Control System (with feedback)

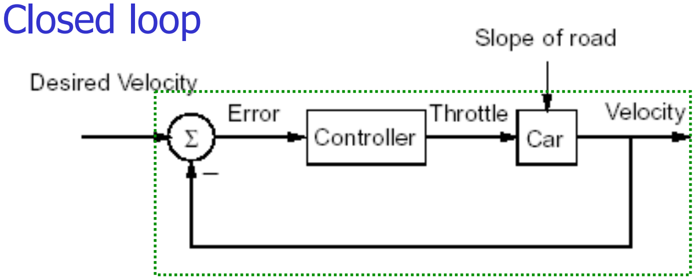
Through feedback, a closed-loop control system compares a **measurement** of the actual output with the **desired** output response.

Automobile Cruise Control

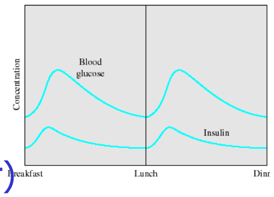
- Open loop



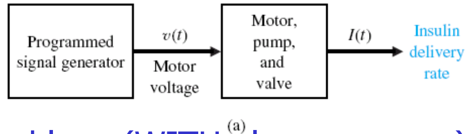
- Closed loop



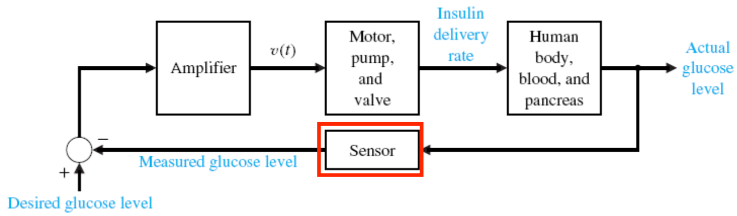
Blood sugar regulation



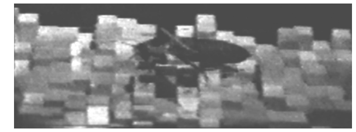
- Open loop (no glucose sensor)

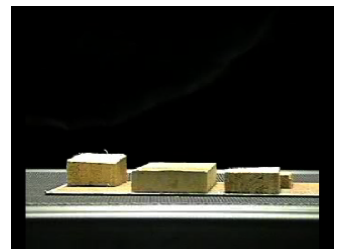


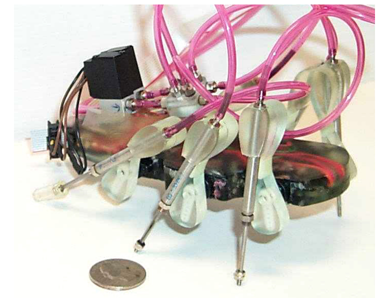
- Closed loop (WITH glucose sensor)



Robotic Hexapod



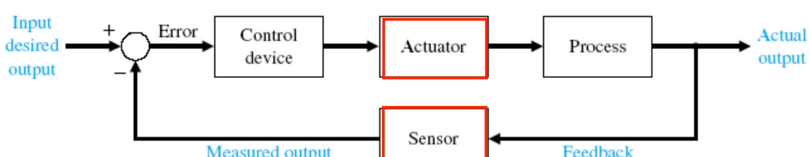




Images and video courtesy J. Cham

Feedback

- The sensor and the actuator are key components of a typical feedback loop



Feedback: Sensors

Sensor Attributes	Types of Sensors
▪ Reliability	▪ Encoder
▪ Accuracy	▪ Rate gyro
▪ Responsiveness	▪ Accelerometer
▪ Noise immunity	▪ Potentiometer
▪ Linearity	▪ Piezo-electrics
▪ Non-intrusiveness	▪ Chemical
	▪ Acoustic
	▪ more…

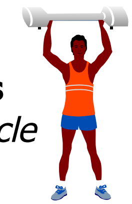
Feedback: Actuators

- Actuators provide the ability to affect or actuate the process in order to move from its current state to the desired state
- Actuators should be sized appropriately and be as linear as possible
  - Backlash
  - Stiction
  - Hysteresis

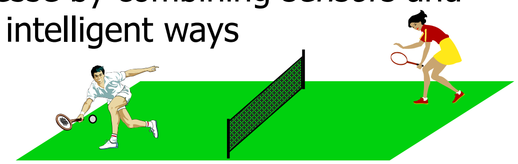
Feedback, Sensors, Actuators

- **Better Sensors**
  Provide better *vision*

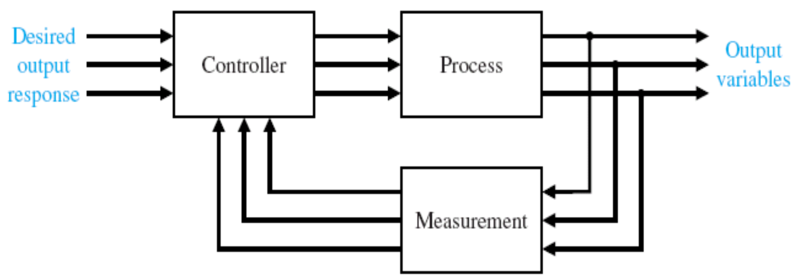
- **Better Actuators**
  Provide more *muscle*



- **Better Control**
  Provides more finesse by combining *sensors* and *actuators* in more intelligent ways



Multivariable Control (MIMO)



As systems become more complicated, the interrelationship of many controlled variables must be considered.

Common Control Objectives

- Regulation
  - Controlled output is held as close as possible to a **constant setpoint**, despite disturbances and errors
  - Examples: Control of basis weight and moisture on paper machine; Automobile cruise control
- Tracking
  - Controlled output follows as closely as possible a **time-varying command**, despite disturbances and errors
  - Examples: Aircraft autopilot, radar tracking device

Control Engineering: Linearity

- Feedback theory
- Linear system analysis
  - A function $f(x)$ is ***linear*** if it fulfills the following two properties:

$$
\begin{aligned}
&1.\ \text{Superposition:} && f(x+y) = f(x) + f(y) \\
&2.\ \text{Scaling:} && f(ax) = a\,f(x)
\end{aligned}
$$

Control Engineering: Linearity

- Feedback theory
- Linear system analysis
  - Linear systems are much easier to understand and the solutions are generally well-behaved.
  - We can add simple solutions to get more complex ones.
  - Linear systems can tell us about the stability of stationary states.

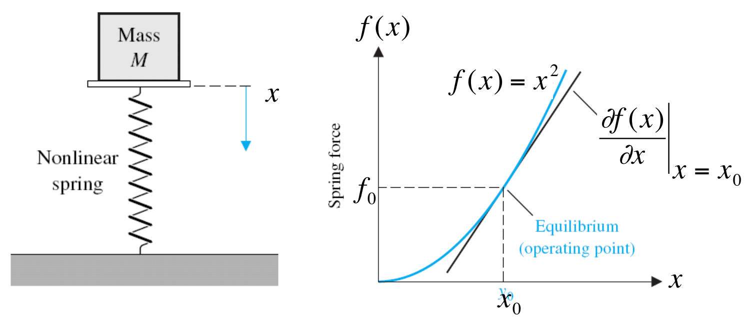
Linearization of Nonlinear Models

- Although many systems are nonlinear, they can be approximated by linear systems for small deviations around a chosen operating point.
- Note that the derived approximation will be valid only around this operating point.
- Control theory for Linear Time-Invariant (LTI) systems is more complete and much simpler than that for nonlinear systems.

How to Linearize

$$\Delta y = \left.\frac{\partial f(x)}{\partial x}\right|_{x = x_0} \Delta x$$

- Identify an operating point
- Perform Taylor series expansion and keep only constant and 1st derivative terms



How to Linearize

- Identify an operating point
- Perform Taylor series expansion and keep only constant and 1st derivative terms
- For a nonlinear function $y = f(x)$ linearized around x_0

$$y = \underbrace{f(x_0) + \frac{\partial f(x_0)}{\partial x}\bigg|_{x=x_0}(x - x_0)} + \underbrace{\frac{\partial^2 f(x_0)}{\partial x^2}\bigg|_{x=x_0}\frac{(x-x_0)^2}{2!} + \ldots}_{\text{H.O.T.}}$$

$$\tilde{y} = f(x)\big|_{x = x_0} + (x - x_0)\frac{\partial f(x)}{\partial x}\bigg|_{x = x_0}$$

$$\boxed{\Delta y = \frac{\partial f(x)}{\partial x}\bigg|_{x = x_0}\Delta x}$$

How to Linearize

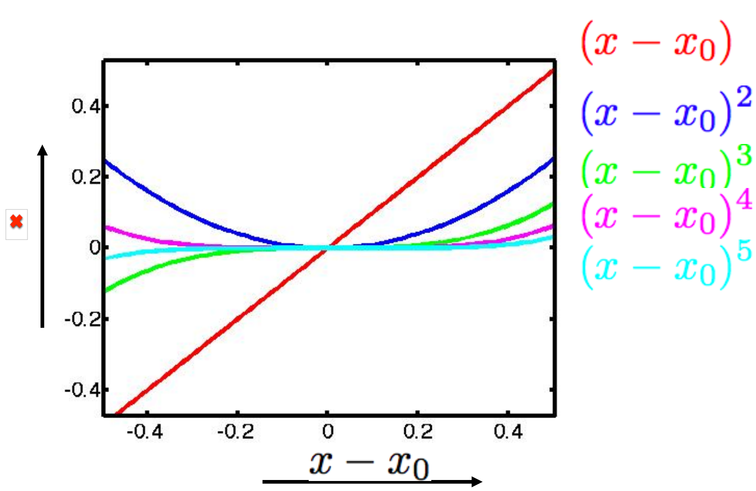
- For a nonlinear function $y = f(x)$ linearized around x_0

$$y = f(x)\big|_{x = x_0} + (x - x_0)\frac{\partial f(x)}{\partial x}\bigg|_{x = x_0} + \frac{(x-x_0)^2}{2!}\frac{\partial^2 f(x)}{\partial x^2}\bigg|_{x = x_0} + \frac{(x-x_0)^3}{3!}\frac{\partial^3 f(x)}{\partial x^3}\bigg|_{x = x_0} + \ldots$$

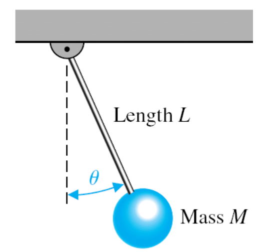
$$= a_0 + (x - x_0)a_1 + \underbrace{(x-x_0)^2}_{\text{very small}} a_2 + \underbrace{(x-x_0)^3}_{\text{very, very small}} a_3 + \underbrace{(x-x_0)^4}_{\text{even smaller yet}} a_4 + \ldots$$

(in comparison to the 0th and 1st order terms)

Why Linearization Works



Example of Linearization



- Pendulum oscillator model

  $T = MgL\sin\theta$

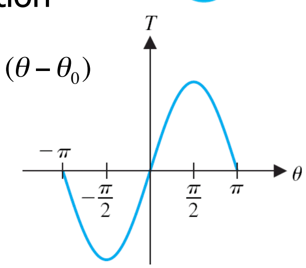
- Equilibrium point

  $\theta_0 = 0, \quad T(\theta_0) = 0$

- Taylor's series approximation

$$T \approx T(\theta_0) + MgL\frac{\partial \sin\theta}{\partial \theta}\bigg|_{\theta = \theta_0}(\theta - \theta_0)$$

$$\boxed{T \approx MgL\theta}$$



- (Accurate for small θ)

Control Design Process

1. Identify
   - what you want to control (process)
   - how you can control it (actuators)
   - how you can measure it (sensors)
2. Formulate a mathematical model
3. Linearize around desired operating point, if necessary
4. Design a controller for the linear system
5. Optimize its parameters to meet control objectives

Model Development

- High order models can only be justified where there is little uncertainty
- Control-relevant models are often quite simple compared to the true system and generally combine physical reasoning with experimental data
- Actuators should be included as they often are nonlinear and have their own dynamic behavior

Model Development

- Essential step of the "Design Procedure"
- Three different choices for modeling:
  - ***Analysis:*** Mathematical models based on first principles, including differential equations.
  - **Grey-Box:** Model is developed and then parameters are inferred from experiments.
  - **Black-Box:** Input-output data is used to infer a dynamic relationship.
- Nonlinear models can be linearized
- *Linear models* are most often used in analysis and design

Examples of Models

- Nonlinear time varying models
- Linear Time Invariant (LTI) models
- Continuous time models
- Discretized time models
- Transfer functions models
- State space models
- Analogies between mechanical, electrical, hydraulic and biological models

Modeling Procedure

- Given a physical system
  - Look up relevant physical laws
  - Define inputs and outputs
  - Formulate the mathematical model; list assumptions
  - Derive equations of motion
  - Establish initial conditions
  - Solve and verify existence of solutions
  - If necessary, re-analyze or re-design the system
- Works well for many mechanical and electrical problems

Important Items

- Role of actuators and sensors in effective feedback
- Definition of linearity
- Linearization through Taylor's Series approximation
- General process for modeling and control