EECE 360
Lecture 4

Op-Amps and Block Diagrams

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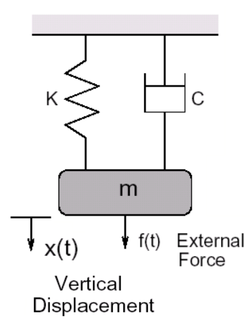
Chapter 2.5-2.6

Review

- Do you know:
  - How to obtain physical laws of process for a system?
  - What a transfer function represents?
- Do you know how to calculate:
  - the transfer function from system dynamics?
  - the steady-state gain of given transfer function?
  - the response (in the *s*-domain) of an LTI system to a given input?
  - the response (in the time-domain) of an LTI system to a given input?

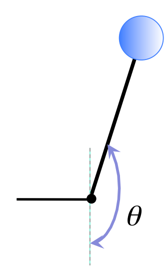
Example: SMD system

- Find the transfer function between the applied force (input) and the position of the mass (output)
- What is the steady-state gain of the system?
- Calculate and sketch the step response of the system (in the time domain).



Example: Inverted Pendulum

- Linearize the pendulum dynamics around the operating point that corresponds to the pendulum perfectly balanced in an inverted position.
- Find the transfer function between the applied torque (input) and the angular position of the mass (output)
- Calculate and sketch the impulse response of the system (in the time domain). Describe how the physical system will behave in this case.



Today

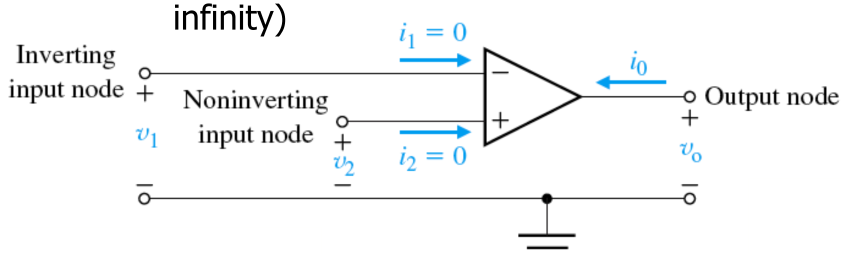
- Op-amps
- Block-diagram manipulation
- Example: DC Motor

Op-amps

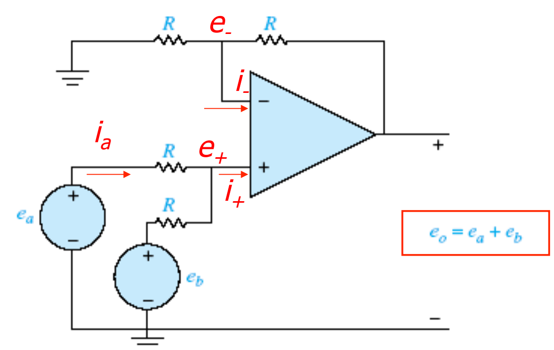
- Operational amplifiers, or op-amps, offer a convenient way to build, implement, or realize transfer functions.
- Building blocks:
  - High-order transfer functions can be implemented by connecting first-order op-amps
  - Many alternative op-amp configurations
- Analog controllers

Ideal Op-Amp

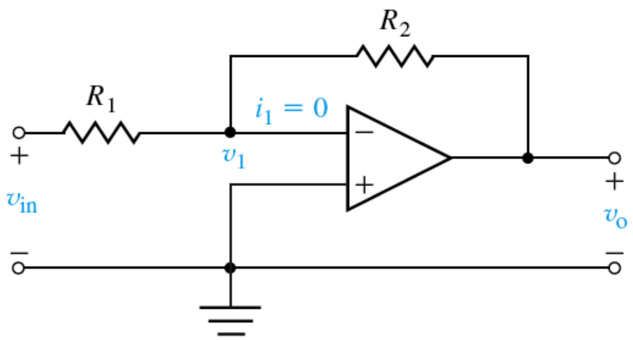
- Ideal operating conditions:
  - $i_1=0$ and $i_2=0$ (input impedance is infinite)
  - $v_1=v_2$
  - $v_0=K(v_2-v_1)$ (where gain K approaches infinity)



Example: Additive Op-Amp



Example: Inverting Op-Amp



Inverting Op-Amps

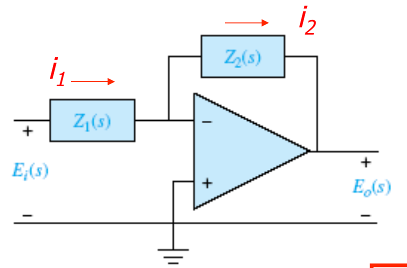


Figure E-3 Inverting op-amp configuration.

$$G(s) = \frac{E_o(s)}{E_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Inverting Op-Amps

TABLE E-1 Inverting Op-Amp Transfer Functions

	Input Element Z_1	Feedback Element Z_2	Transfer Function	Comments
(a)	R_1; $Z_1 = R_1$	R_2; $Z_2 = R_2$	$-\frac{R_2}{R_1}$	Inverting gain. e.g., if $R_1 = R_2$, $e_o = -e_1$.
(b)	R_1; $Z_1 = R_1$	C_2; $Y_2 = sC_2$	$\left(\frac{-1}{R_1C_2}\right)\frac{1}{s}$	Pole at the origin. i.e., an integrator.
(c)	C_1; $Y_1 = sC_1$	R_2; $Z_2 = R_2$	$(-R_2C_1)s$	Zero at the origin. i.e., a differentiator.
(d)	R_1; $Z_1 = R_1$	R_2, C_2; $Y_2 = \frac{1}{R_2} + sC_2$	$\frac{-\frac{1}{R_1C_2}}{s + \frac{1}{R_2C_2}}$	Pole at $\frac{-1}{R_2C_2}$ with a dc gain of $-R_2/R_1$.

Inverting Op-Amps

TABLE E-1 Inverting Op-Amp Transfer Functions

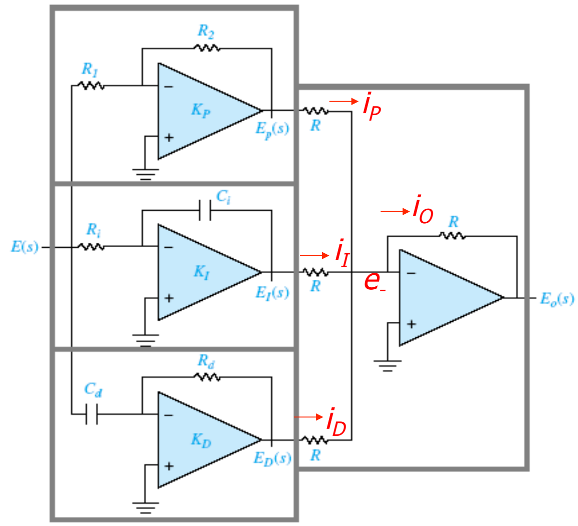
	Input Element	Feedback Element	Transfer Function	Comments
(e)	R_1; $Z_1 = R_1$	R_2, C_2; $Z_2 = R_2 + \frac{1}{sC_2}$	$\frac{-R_2}{R_1}\left(\frac{s + 1/R_2C_2}{s}\right)$	Pole at the origin and a zero at $-1/R_2C_2$, i.e., a PI Controller.
(f)	R_1, C_1; $Y_1 = \frac{1}{R_1} + sC_1$	R_2; $Z_2 = R_2$	$-R_2C_1\left(s + \frac{1}{R_1C_1}\right)$	Zero at $s = \frac{-1}{R_1C_1}$, i.e., a PD controller.
(g)	R_1, C_1; $Y_1 = \frac{1}{R_1} + sC_1$	R_2, C_2; $Y_2 = \frac{1}{R_2} + sC_2$	$\frac{\frac{-C_1}{C_2}\left(s + \frac{1}{R_1C_1}\right)}{s + \frac{1}{R_2C_2}}$	Pole at $s = \frac{-1}{R_2C_2}$ and a zero at $s = \frac{-1}{R_1C_1}$, i.e., a lead or lag controller.

PID

Proportional

Integral

Derivative



PID

Proportional: $\frac{E_P(s)}{E(s)} = -\frac{R_2}{R_1}$

Integral: $\frac{E_I(s)}{E(s)} = -\frac{1}{R_i C_i s}$

Derivative: $\frac{E_D(s)}{E(s)} = -R_d C_d s$

The output voltage is

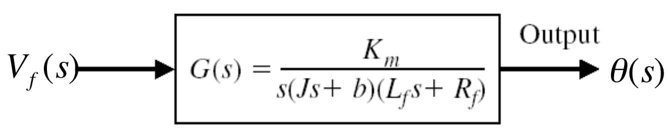
$$E_o(s) = -[E_P(s) + E_I(s) + E_D(s)]$$

Thus, the transfer function of the PID op-amp circuit is

$$G(s) = \frac{E_o(s)}{E(s)} = \frac{R_2}{R_1} + \frac{1}{R_i C_i s} + R_d C_d s$$

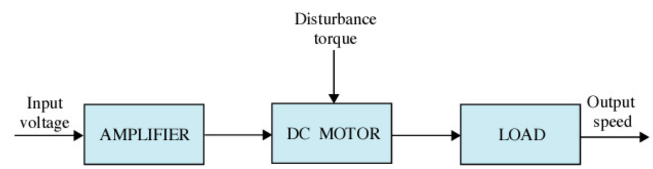
Block Diagrams

- Graphical representation of dynamic system relationships
- Consist of unidirectional, operational blocks representing transfer functions of components or subsystems

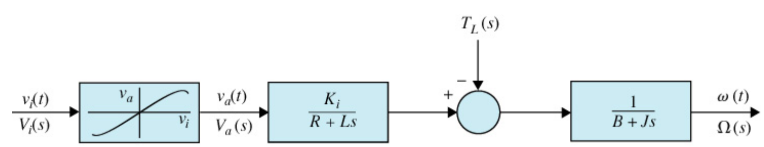


Block diagram of DC motor (field current controlled)

Block Diagrams

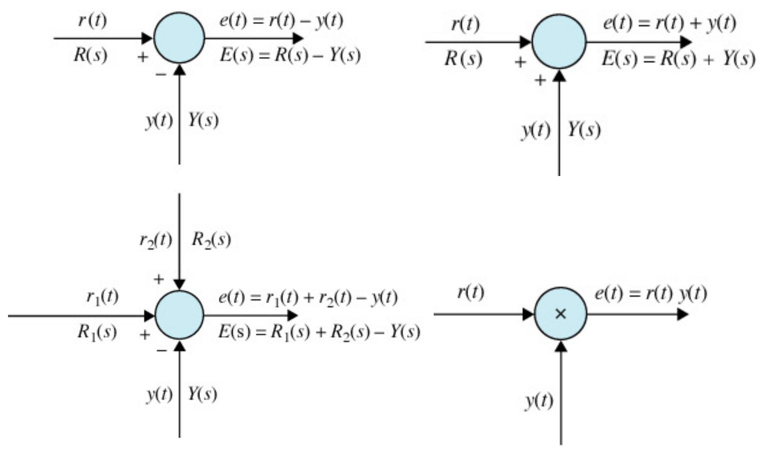


(a)



(b)

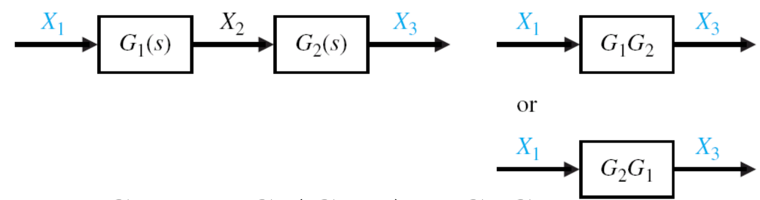
Summers and Multipliers



Linear?

Block Diagram Transformations

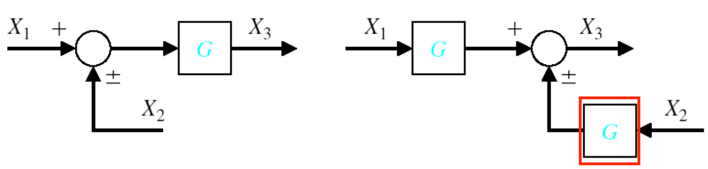
- Combining blocks in cascade: multiplication is commutative
- **When combining blocks, the input-output relationship (transfer function) should not change.**



$$X_3 = G_2 X_2 = G_2(G_1 X_1) = G_2 G_1 X_1$$

Block Diagram Transformations

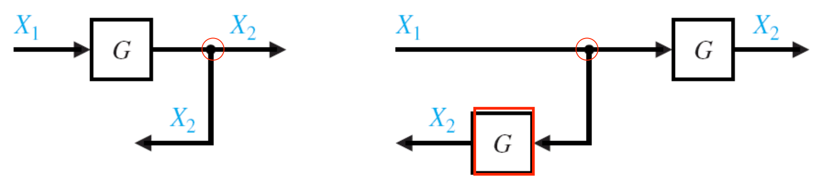
- Moving a summing point behind a block



$$X_3 = G(X_1 \pm X_2) = GX_1 \pm GX_2$$

Block Diagram Transformations

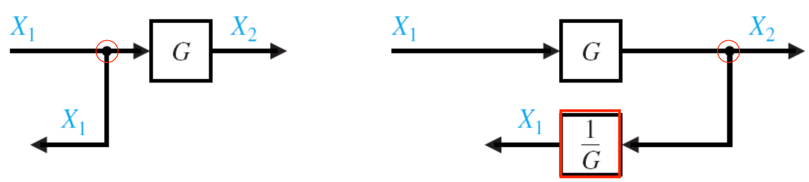
- Moving a pickoff point ahead of a block



$$X_2 = GX_1$$

Block Diagram Transformations

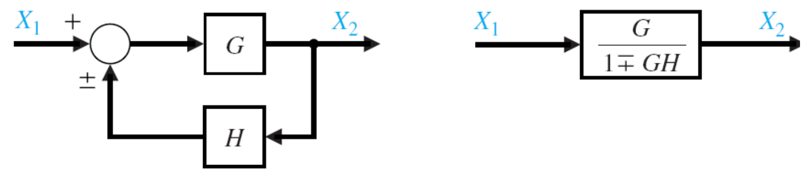
- Moving a pickoff point behind a block



- Try on your own: move a summing point ahead of a block

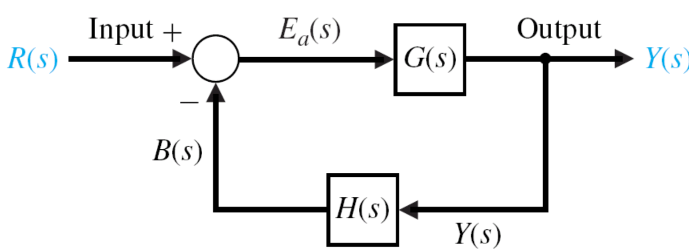
Block Diagram Transformations**

- Eliminating a feedback loop (positive or negative)



$$X_2 = G(X_1 \pm HX_2)$$
$$GX_1 = (1 \mp GH)X_2$$

Closed-Loop Transfer Function**

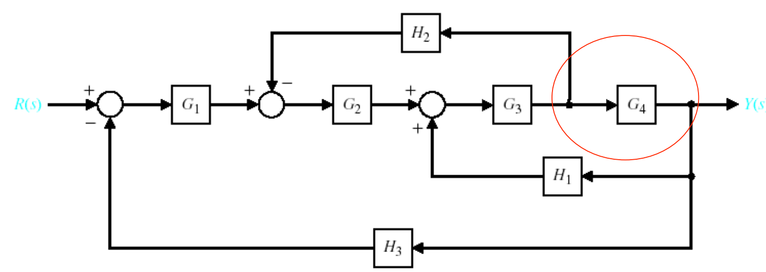


$$\frac{Y(s)}{R(S)} = \frac{G(s)}{1+G(s)H(s)}$$

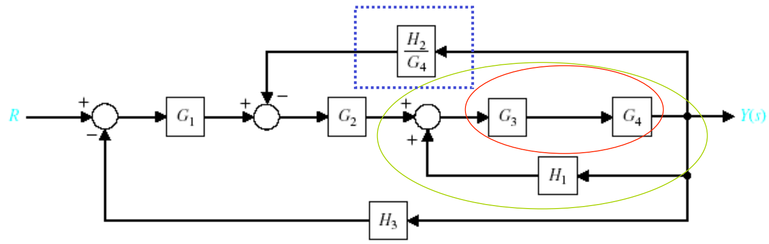
Extremely important!

Block Diagram Reduction

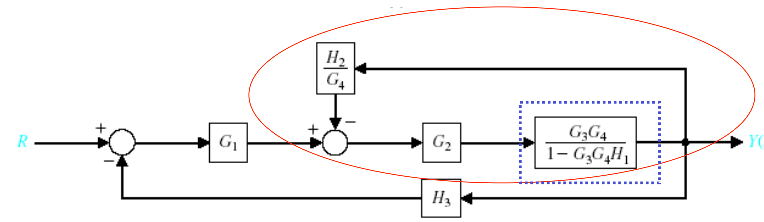
- Goal: Reduce to a block diagram with fewer blocks



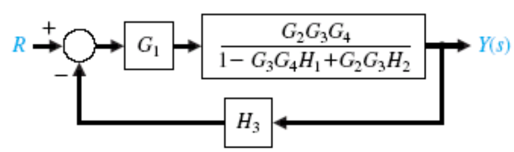
Block Diagram Reduction

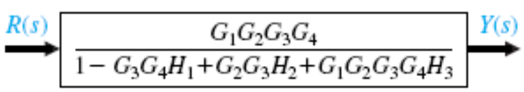


Block Diagram Reduction



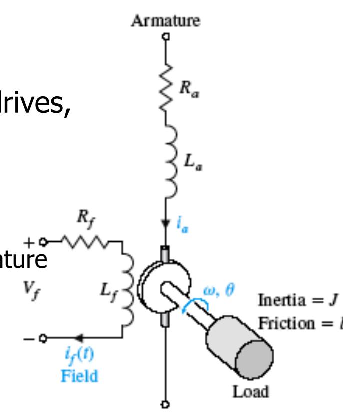
Block Diagram Reduction





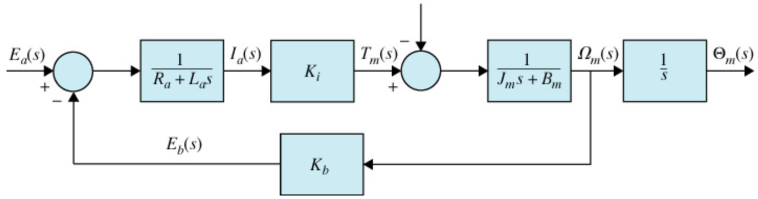
Example: DC Motor

- Power actuator
- Robotic manipulators, disk drives, machine tools, etc.
- Armature-controlled
  - Input: voltage applied to armature V_a $(=E_a)$
  - Output: angle of rotation θ
  - Control variable: current i_a



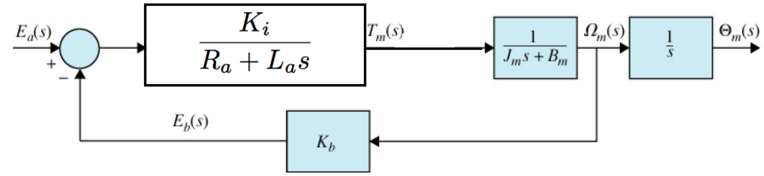
Example: DC Motor

- Block diagram for armature-controlled DC motor



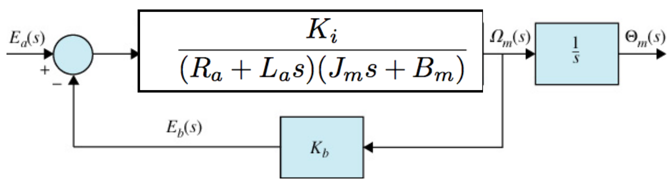
Example: DC Motor

- Consider the case when T_d=0



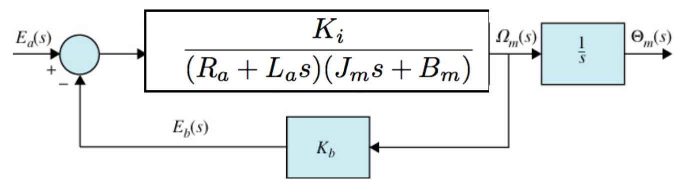
Example: DC Motor

- Combine blocks in cascade



Example: DC Motor

- Remove feedback loop: *G/(1-GK)*

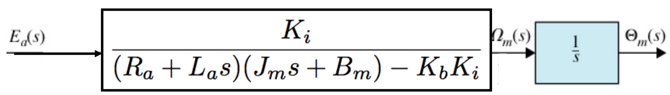


$$\Omega_m(s) = \frac{K_i}{(R_a + L_a s)(J_m s + B_m)} (E_a(s) - K_b \Omega_m(s))$$

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{K_i}{(R_a + L_a s)(J_m s + B_m) - K_b K_i}$$

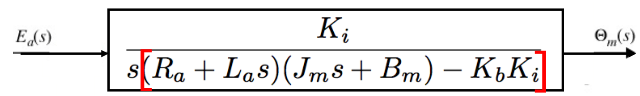
Example: DC Motor

- Combine cascade blocks



Example: DC Motor

- Fully reduced form



- How different would this be if $T_d \neq 0$?

Summary

- ***Today***
  - Operational amplifiers
  - Block diagram models
- ***Next***
  - State-space descriptions