EECE 360
Lecture 22

Frequency Response: Bode Diagrams

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Chapter 8.1-8.3, 8.5, 8.8

Outline

- Review
  - Frequency response
  - Bode diagrams for common elements
- Today
  - Sketching Bode diagrams
  - Performance requirements
  - Relationship to Root Locus
  - Gain and phase margin

Review: Bode Diagram

- Evaluate the gain and phase of a transfer function G(s) for s=jω
- General procedure:
  - Start a low frequencies
  - Identify break points
  - Approximate gain before and after break points
  - Approximate phase before and after break points
  - Effect is cumulative as frequency increases, for gain and phase

Review: Bode Diagram

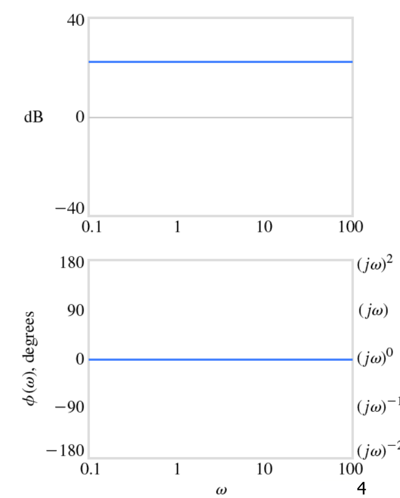
$G(s) = K$

- **1. Constant gain *K***
- Log gain

$$20\log|G(j\omega)| = 20\log K$$

- Phase

$$\angle G(j\omega) = 0^0$$



Review: Bode Diagram

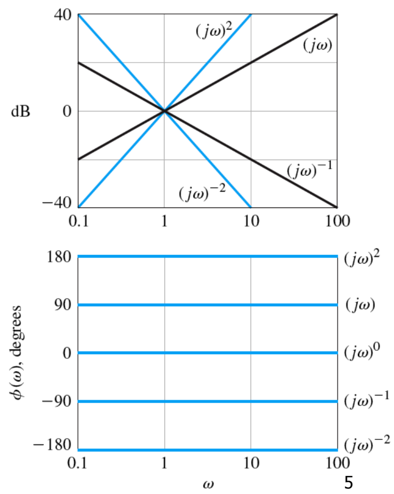
$$G(s) = \frac{1}{s^N}$$

- **2. Poles (or zeros) at the origin (jω)**
- Log gain

$$20\log|G(j\omega)| = -20N\log\omega$$

- Phase

$$\angle G(j\omega) = -90^0 N$$



Review: Bode Diagram

$$G(s) = \frac{1}{s\tau+1}$$

- **3. Poles (or zeros) on the real axis (jωτ+1)**
- Log gain

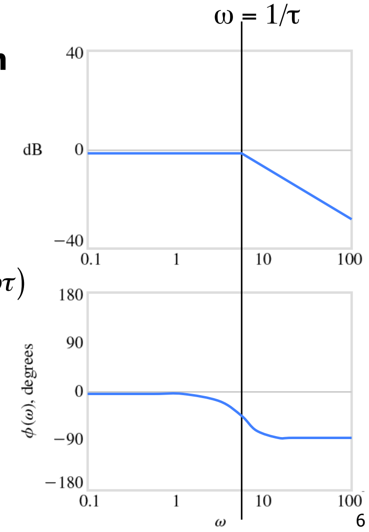
$$\omega\tau \ll 1,\ 20\log|G(j\omega)| \approx 0$$

$$\omega\tau \gg 1,\ 20\log|G(j\omega)| \approx 20\log(\omega\tau)$$

- Phase

$$\omega\tau \ll 1,\ \angle G(j\omega) \approx 0$$

$$\omega\tau \gg 1,\ \angle G(j\omega) \approx -90$$



Review: Bode Diagram

$$G(s) = \frac{1}{1 + 2\zeta/\omega_n\, s + (s/\omega_n)^2}$$

- **4. Complex conjugate poles (or zeros) $(1+(2\zeta/\omega_n)j\omega+(j\omega/\omega_n)^2)$**
- Log gain

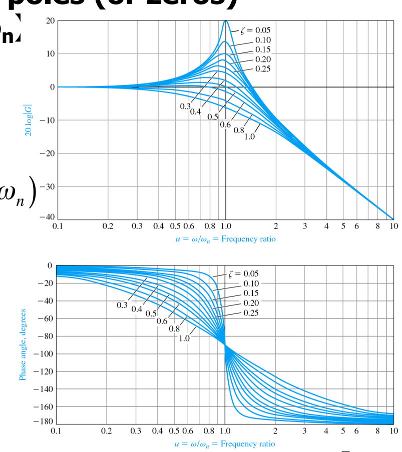
$$\omega/\omega_n \ll 1,\ 20\log|G(j\omega)| \approx 0$$

$$\omega/\omega_n \gg 1,\ 20\log|G(j\omega)| \approx -40\log(\omega/\omega_n)$$

- Phase

$$\omega/\omega_n \ll 1,\ \angle G(j\omega) \approx 0$$

$$\omega/\omega_n \gg 1,\ \angle G(j\omega) \approx -180$$



Review: Bode Diagram

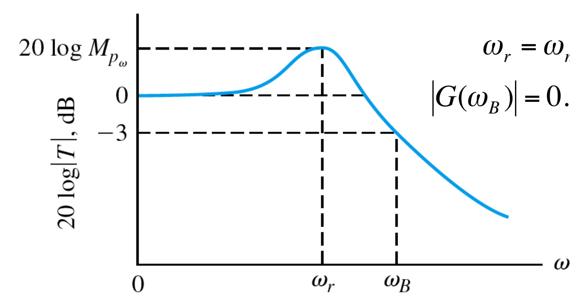
$$G(s) = \frac{1}{1 + 2\zeta/\omega_n\, s + (s/\omega_n)^2}$$

- **4. Complex conjugate poles (or zeros) $(1+(2\zeta/\omega_n)j\omega+(j\omega/\omega_n)^2)$**
- Important landmarks

$$M_{p\omega} = |G(\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

$$|G(\omega_B)| = 0.707$$



Sketching Bode Diagrams

- 1. Factor transfer function

$$G(s)=\frac{K_0\prod_i(j\omega/z_i+1)}{(j\omega)^n\prod_j(j\omega/p_j+1)\prod_k\left(1+2\zeta_k\omega/\omega_{n,k}+(\omega/\omega_{n,k})^2\right)}$$

- 2. Plot $K_0/(j\omega)^n$
  - Gain slope *n* through K_0 at $\omega=1$
  - Phase is -*n**90 degrees at low frequencies

Sketching Bode Diagrams

- 3. Plot remaining terms in ascending break point frequencies.
  - Extend $K_0/(j\omega)^n$ slope until first freq. break point.
  - Change gain slope by ±20dB/decade) for each zero/pole
  - Change phase by ±90 degrees for each zero/pole
- 4. Identify known points (gain at break points and resonant frequencies, phase at break points)
- 5. Smooth linear approximation in gain and phase.

- **See example in Lecture 21**

Performance specifications

- For 2nd-order systems, transient response characteristics can be estimated from Bode diagrams
- Maximum gain $M_{p\omega}=|G(\omega_r)|=\frac{1}{2\zeta\sqrt{1-\zeta^2}}$

  occurs at the resonant frequency

$$\omega_r=\omega_n\sqrt{1-2\zeta^2}$$

- Also note the gain at the natural frequency is

$$|G(j\omega_n)|=\frac{1}{2\zeta}$$

Performance specifications

- Bandwidth of 1st and 2nd order systems is a good measure of the speed of the transient response
- In first-order systems, the breakpoint frequency is the bandwidth

$$\omega_B=\frac{1}{\tau}$$

- In second-order systems,

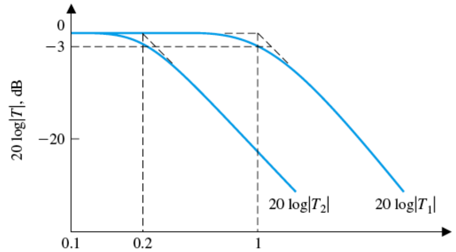
$$\frac{\omega_B}{\omega_n}=-1.19\zeta+1.85 \quad \text{for} \quad 0.3\le\zeta\le0.8$$

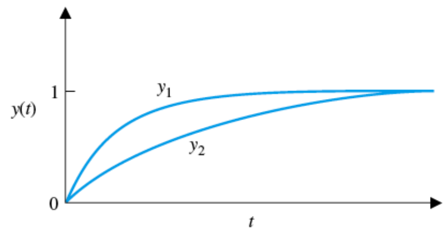
Performance specifications

- Higher bandwidth is indicative of a faster rise time

$$T_1 = \frac{1}{s+1}$$
$$\omega_B = 1, \quad \tau = 1$$

$$T_2 = \frac{1}{5s+1}$$
$$\omega_B = 0.2, \quad \tau = 5$$



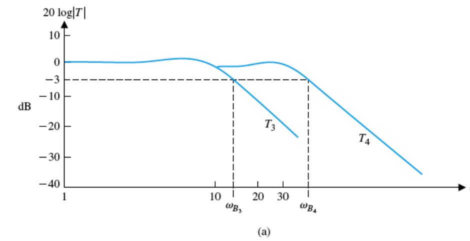


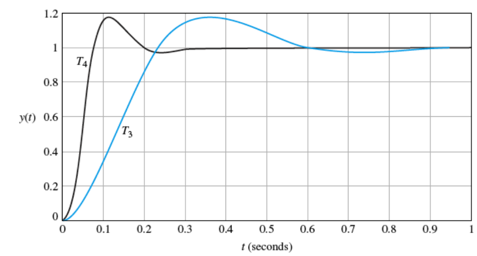
Performance specifications

- Higher bandwidth is indicative of a faster rise time

$$T_3 = \frac{100}{s^2 + 10s + 100}$$
$$\omega_B \approx 15, \quad \omega_n = 10$$

$$T_4 = \frac{900}{s^2 + 30s + 100}$$
$$\omega_B \approx 40, \quad \omega_n = 30$$





Performance specifications

- Steady-state error can be determined from a Bode diagram
- At low frequencies

$$G(j\omega) \approx \frac{K_0}{(j\omega)^n}$$

- Evaluate the gain K_0 at $\omega = 1$
  - For type 0 systems, $K_0 = K_p$
  - For type 1 systems, $K_0 = K_v$
  - For type 2 systems, $K_0 = K_a$

Relationship to Root Locus

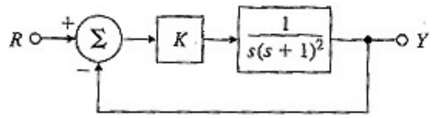
- In Bode plots, we are interested in s = $j\omega$
- In root locus plots, we are interested in s = $\sigma + j\omega$ such that 1+KG(s)=0 (assuming unity feedback).

- For systems whose root locus intersects the imaginary axis:
  - The crossover frequency can be identified on the Bode plots where the phase is -180 degrees.
  - Recall that *s* which satisfy the characteristic equation have a phase

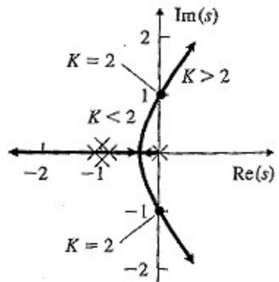
$$\angle G(s) = \frac{-1}{K} = 180 \pm 360n, \quad n \in \{0,1,2,\ldots\}$$

Relationship to Root Locus

- Consider a simple example



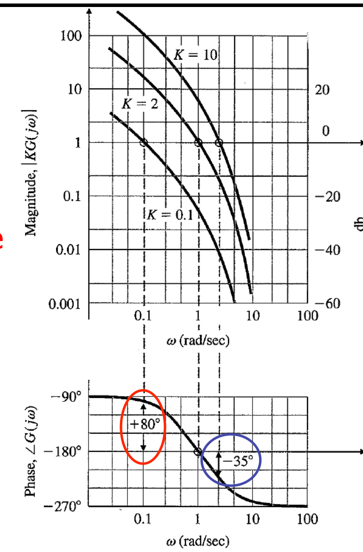
- with root locus plot



- The gain that results in **marginal stability** is *K*=2

Relationship to Root Locus

- Bode diagram for various gains K
- Notice that for K<2 (closed-loop system is stable), the phase of $G(j\omega_c)$ is greater than -180 degrees
- For K>2 (unstable) the phase is less than -180 degrees



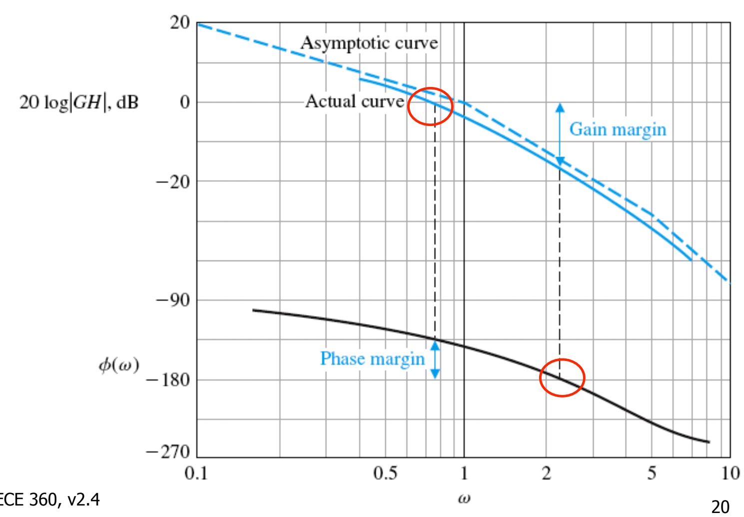
Relative Stability

Informal definitions:

- The **gain margin** is the factor by which the gain can be increased before instability results.
- The **phase margin** is the amount of phase by which G(jω) exceeds -180 degrees when |KG(jω)|=1
- These are easily measured on Bode diagrams.

Derivations and formal definitions will be provided when we investigate the Nyquist criterion (next week).

Gain and Phase Margins



Gain Margin

- Factor by which the gain can be increased in order to achieve marginal stability
- Measured where the phase is -180 degrees.

$$\text{Gain Margin} = \frac{1}{|KG(j\omega_{180})|}, \quad \omega_{180} = \arg\left(\angle G(j\omega) = -180\right)$$

$$M_G = -20\log G(j\omega_{180}) \text{ dB}$$

- Can be stated as absolute value or in dB
- **For stability, M_G > 0 dB**
- Reasonable values are often 2-5, or between 6dB-14dB on a Bode diagram

Phase Margin

- The phase margin is the difference between -180 degrees and the phase of the system at the crossover frequency

$$\text{Phase Margin} = M_\phi = 180^\circ + \arg\left(\angle G(j\omega_c)\right)$$

- For stability, $M_\phi > 0$
- Reasonable values are in the range 30^0-60^0
- **What about systems with multiple crossings?
- **Need to be very careful in analyzing stability through M_G and M_ϕ on Bode plots. (e.g., what about systems which require a minimum gain for stability?)

Phase Margin

- Consider the open-loop system

$$G(s) = \frac{\omega_n^{\,2}}{s(s + 2\zeta\omega_n)}$$

- With unity feedback, this results in a standard 2nd order system

$$\frac{G(s)}{1 + G(s)} = \frac{1}{1 + 2\zeta/\omega_n\, s + (s/\omega_n)^2}$$

- with phase margin

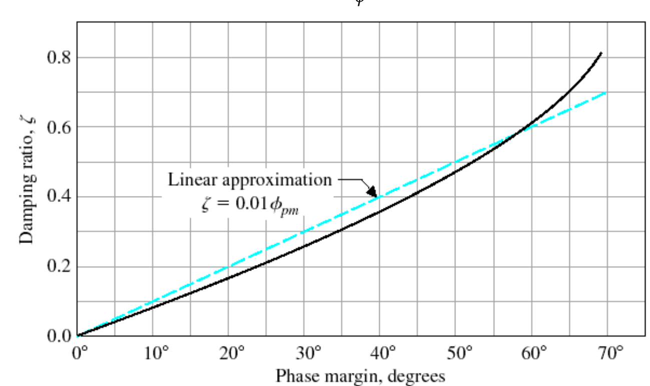
$$M_\phi = 180^\circ - 90^\circ - \tan^{-1}\left(\frac{\omega_c}{2\zeta\omega_n}\right)$$

$$= \tan^{-1}\left(2\zeta\left(\frac{1}{(4\zeta^4 + 1)^{1/2} - 2\zeta^2}\right)^{1/2}\right)$$

Phase Margin

- This can be linearly approximated by

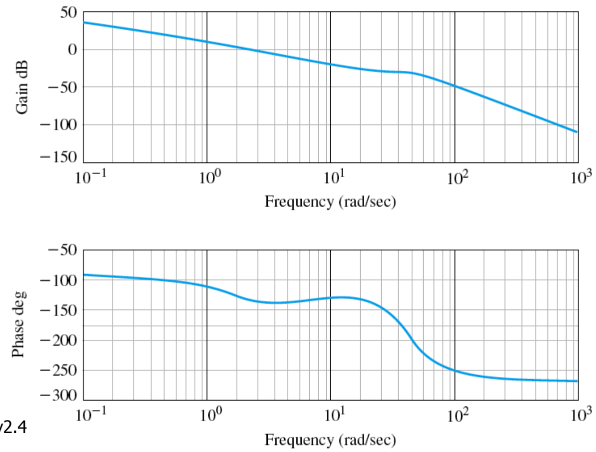
$$\zeta = 0.01M_\phi$$



Example

$$G(s) = \frac{5(1+s/10)}{s(1+s/2)\left(1+(0.6/50)s+(1/50^2)s^2\right)}$$

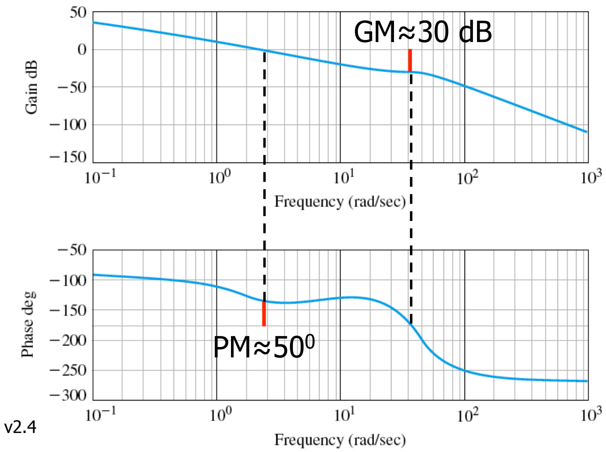
- Consider the transfer function with Bode plot



Example

$$G(s) = \frac{5(1+s/10)}{s(1+s/2)\left(1+(0.6/50)s+(1/50^2)s^2\right)}$$

- Phase margin and gain margin



Example

$$G(s) = \frac{5(1+s/10)}{s(1+s/2)\left(1+(0.6/50)s+(1/50^2)s^2\right)}$$

- Relevant Matlab functions:

```
% Bode plot script for Figure 8.22
%
num=5*[0.1 1];
f1=[1 0]; f2=[0.5 1]; f3=[1/2500 .6/50 1];
den=conv(f1,conv(f2,f3));
%
sys=tf(num,den);
bode(sys)
```

Compute $s(1+0.5s)\left(1+\frac{0.6}{50}s+\frac{1}{50^2}s^2\right)$

- Try 'margin' to find gain and phase margins
- Try 'logspace' to create a frequency vector with log-scale spacing

Summary

- Today
  - Sketching Bode diagrams
  - Performance requirements
  - Phase and gain margin
- Next time
  - Lead and lag control