EECE 360

Lecture 26

Nyquist Stability Criterion (cont'd)

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Chapters 9.2-9.4

Today's lecture

- Review
  - Nyquist criterion
  - Examples with 0 RHP poles in the open-loop system
- Moving on…
  - Nyquist for open-loop unstable systems
  - Relative stability (gain margin, phase margin)
  - Examples

Review: The Nyquist Criterion**

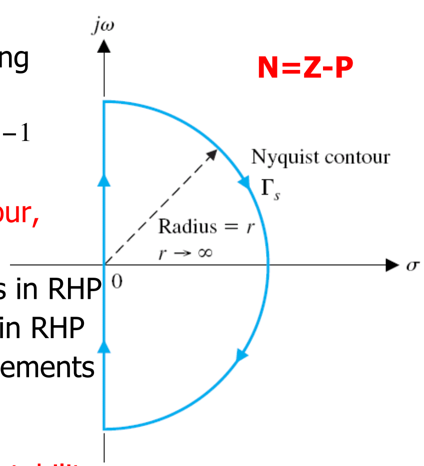
- **Equivalent** to evaluating

$$F'(s) = F(s) - 1$$
$$= 1 + KG_c(s)G(s) - 1$$
$$= KG_c(s)G(s)$$

along the Nyquist contour, with

- Z = # closed-loop poles in RHP
- P = # open-loop poles in RHP
- N = # clockwise encirclements of **-1**

N=Z-P



- **Z=0 for closed-loop stability

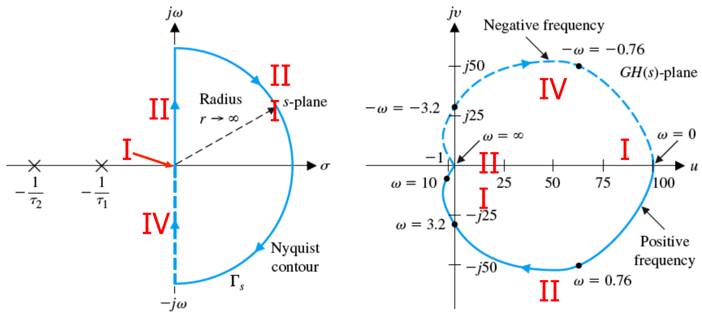
Review: The Nyquist Criterion

- The closed-loop system with is **stable** if and only if the number of counter-clockwise encirclements of -1 is equal to the number of open-loop poles in the right-half plane.
- The closed-loop system which is open-loop stable (no open-loop poles in RHP) is stable if and only if there are **no** encirclements of -1.

(Recall that Z = number of roots of characteristic equation of closed-loop system in the RHP, so for stability we want to have Z=0.)

Example 1: Two real poles

- Consider $G(s)H(s) = \dfrac{100}{(s+1)(s/10+1)}$



$P = 0$, hence for stability we require $Z = N = 0$,

i.e. the contour must not encircle the -1 point in the $GH(s)$-plane.

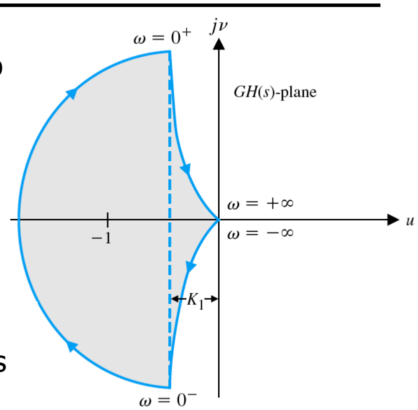
Procedure for Stability by Nyquist

- Map the Nyquist contour Γ_s to Γ_L using the loop gain $L(s)=G_c(s)G(s)$
- Count the net number of encirclements of the point (-1,0) by drawing a line from -1 to infinity in any direction and counting the left-to-right and right-to-left crossings. This is N.
- For a closed-loop system to be stable, N=-P, where P is the number of open-loop poles in the RHP.
- If N≠-P, the closed-loop system is not stable.

Open-loop unstable systems

- Consider the open-loop unstable system

$$G(s) = \frac{K_1}{s(s-1)}$$

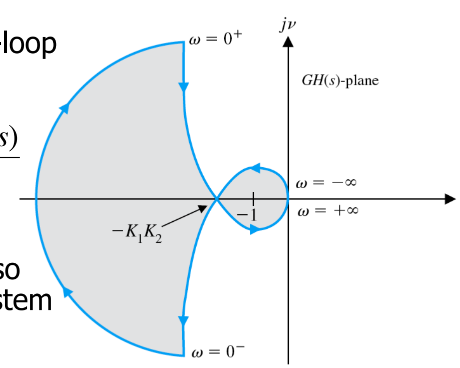


- The number of closed-loop roots in the RHP is Z=N+P=1+1=2, therefore the closed-loop system is unstable.

Open-loop unstable systems

- Consider the open-loop unstable system

$$G(s) = \frac{K_1(1+K_2 s)}{s(s-1)}$$

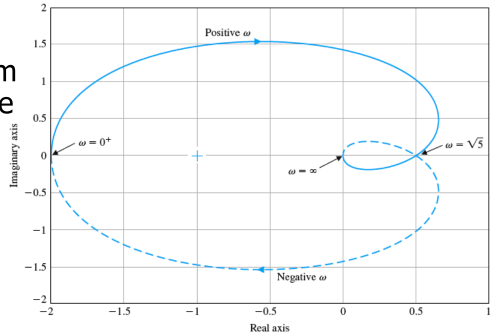


- For $-K_1K_2<-1$, Z=N+P=-1+1=0, so the closed-loop system is stable.
- For $-K_1K_2>-1$, Z=N+P=+1+1=2, so the closed-loop system is unstable.

Non-minimum phase system

- Consider the open-loop system with a pole in the RHP.

$$G(s)=\frac{K(s-2)}{(s+1)^2}$$

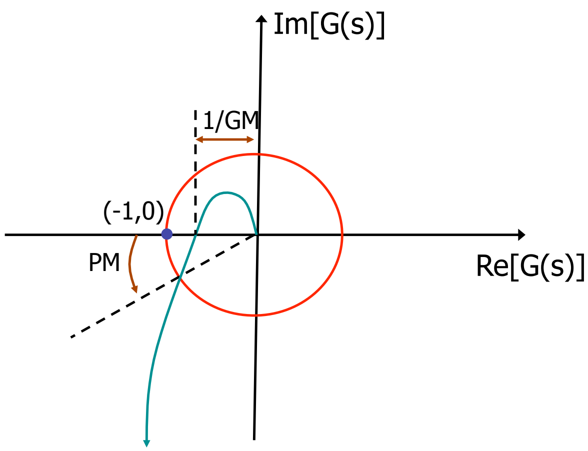


- For -2K<-1, Z=N+P=1+0=1, so the closed-loop system is unstable.
- For -2K ≥ -1, Z=N+P=0+0=0, so the closed-loop system is stable.

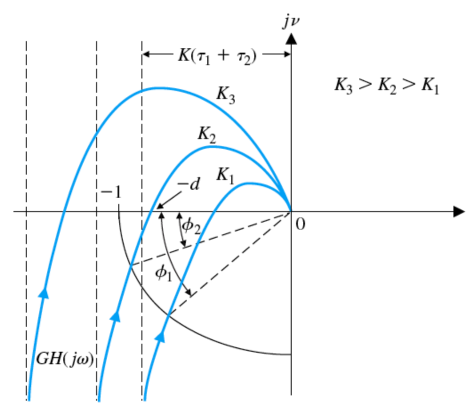
Relative Stability

- The distance between the (-1,0) point and the Nyquist diagram of the open-loop system is a measure of the relative stability of the closed-loop system
- **Gain and phase margin** can be measured on the Nyquist diagram from the (-1,0) point
- The (-1,0) point corresponds to the frequencies with 0dB gain and -180^0 phase.

Relative Stability **



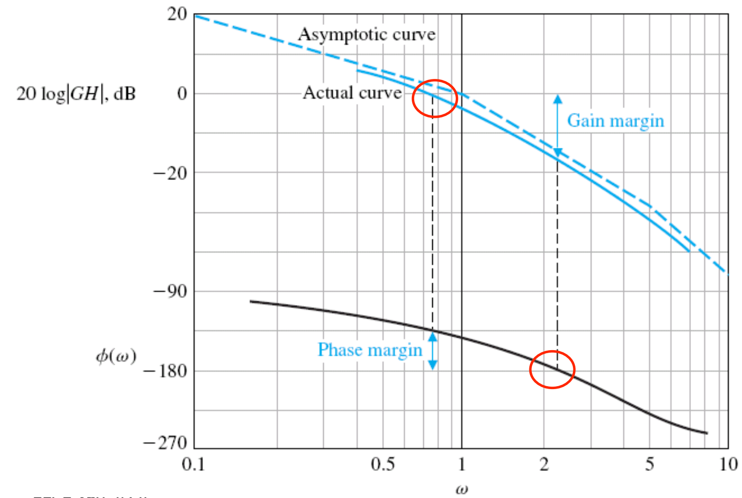
Relative Stability



Relative Stability

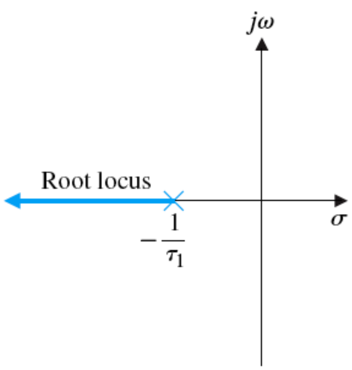
- The **gain margin** is the increase in the system gain when phase =-180º that will result in a marginally stable system with intersection of the -1+j0 on the Nyquist diagram
- The **phase margin** is the amount of phase shift of the GH Nyquist plot at unity magnitude that will result in a marginally stable system with intersection of the -1+j0 point on the Nyquist diagram

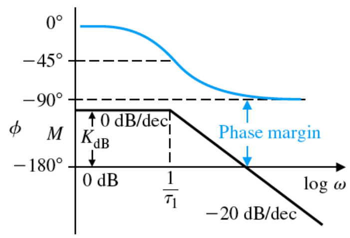
Relative Stability

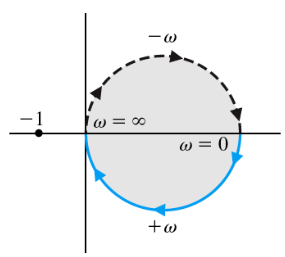


Common Transfer Functions

$$G(s) = \frac{K}{s\tau_1 + 1}$$

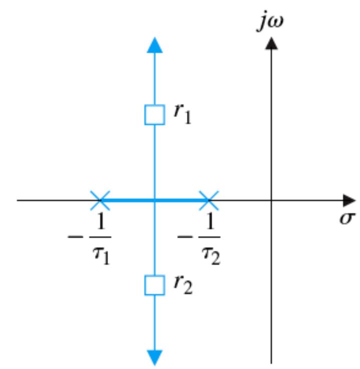


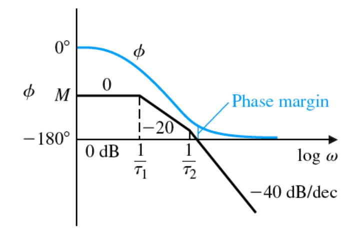


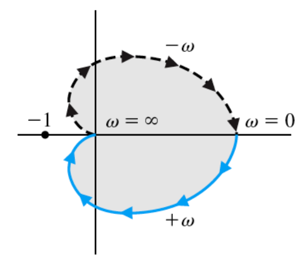


Common Transfer Functions

$$G(s) = \frac{K}{(s\tau_1 + 1)(s\tau_2 + 1)}$$

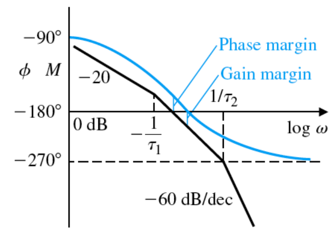


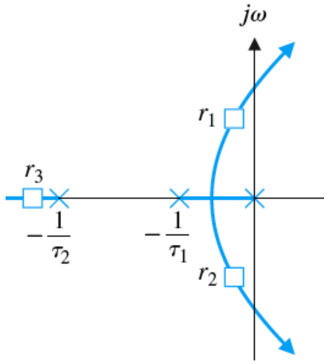


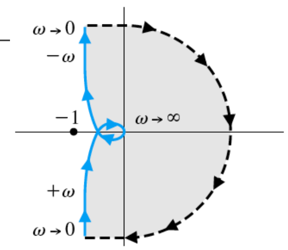


Common Transfer Functions

$$G(s) = \frac{K}{s(s\tau_1 + 1)(s\tau_2 + 1)}$$



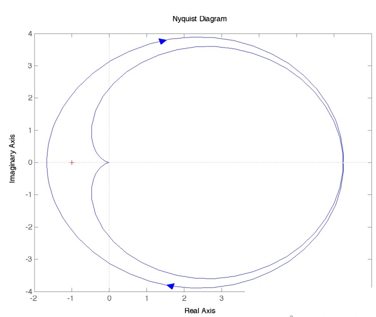


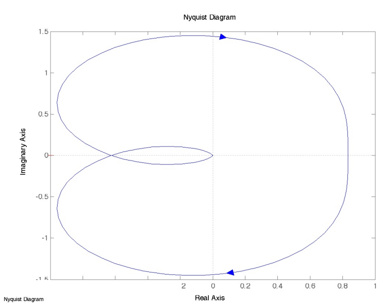


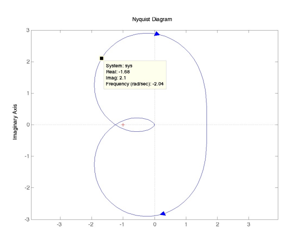
Common Transfer Functions

- For more functions, see *Dorf* Table 9.6.
- Matlab: 'nyquist'

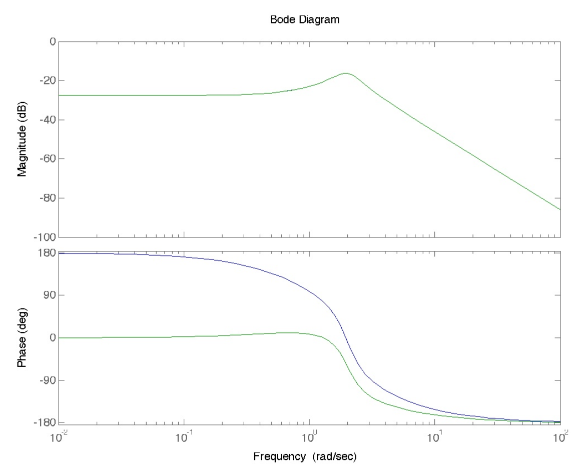
Examples



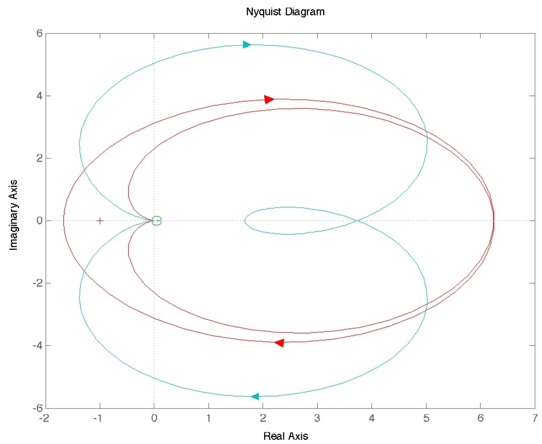




Examples



Examples



Nyquist Diagram
Imaginary Axis
Real Axis
6
4
2
0
-2
-4
-6
-2
-1
0
1
2
3
4
5
6
7