Sine Waves and Vectors

Math Session for Basic Electricity
A Fairfield University E-Course
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Module: Basic Electronics (AC Circuits and Impedance: two parts)

- Text: "Electricity One-Seven," Harry Mileaf, Prentice-Hall, 1996, ISBN 0-13-889585-6
  (Covers much more material than this section)
- References:
  - "Digital Mini Test: Principles of Electricity Lessons One and Two," SNET Home Study Coordinator, (203) 771-5400
  - Electronics Tutorial (Thanks to Alex Pounds)
  - Electronics Tutorial (Thanks to Mark Sokos)
  - Basic Math Tutorial (Thanks to George Mason University)
  - Vector Math Tutorial (Thanks to California Polytec at atom.physics.calpoly.edu)
- Alternating Current and Impedance
  - 5 on-line sessions plus one lab
- Resonance and Filters
  - 5 on-line sessions plus one lab

Section 3:
AC, Inductors and Capacitors

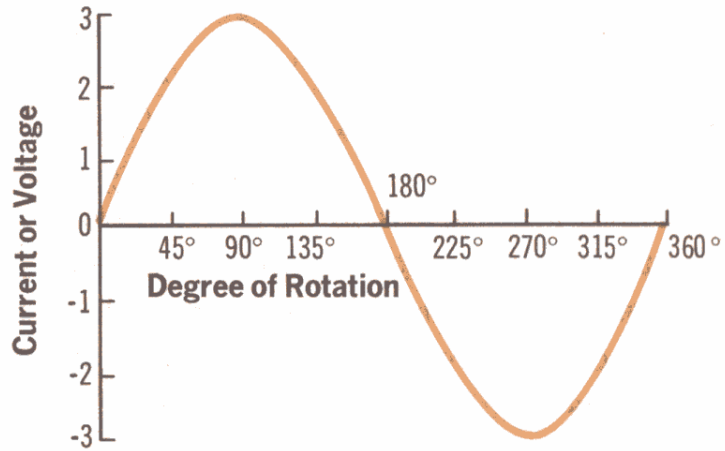
- 0BJECTIVES: This section introduces AC voltage / current and additional circuit components (inductors, transformers and capacitors).

Section 3 Schedule:

Mastery Test 1 – 05/03	Results and Discussion	
Mastery Test 1 – 05/06	Results and Discussion (cont.)	
Session 3a – 05/13	**Sine Waves, Magnitude, Phase and Vectors (again)**	**Text 4.1 – 4.24**
Session 3b – 05/15	R-L Circuits	Text 4.25 – 4.54
Session 3c – 05/17	R-C Circuits	Text 4.55 – 4.76
(lab - 05/18, Sat.)		
Session 3d – 05/20	Series LC Circuits	Text 4.77 – 4.88
Session 3e – 05/22	Parallel LC Circuits	Text 4.114 – 4.122
Quiz 3 (due 05/26)	(no class on 05/27)	
Session 3f – 05/29	Review (Discuss Quiz 3)	

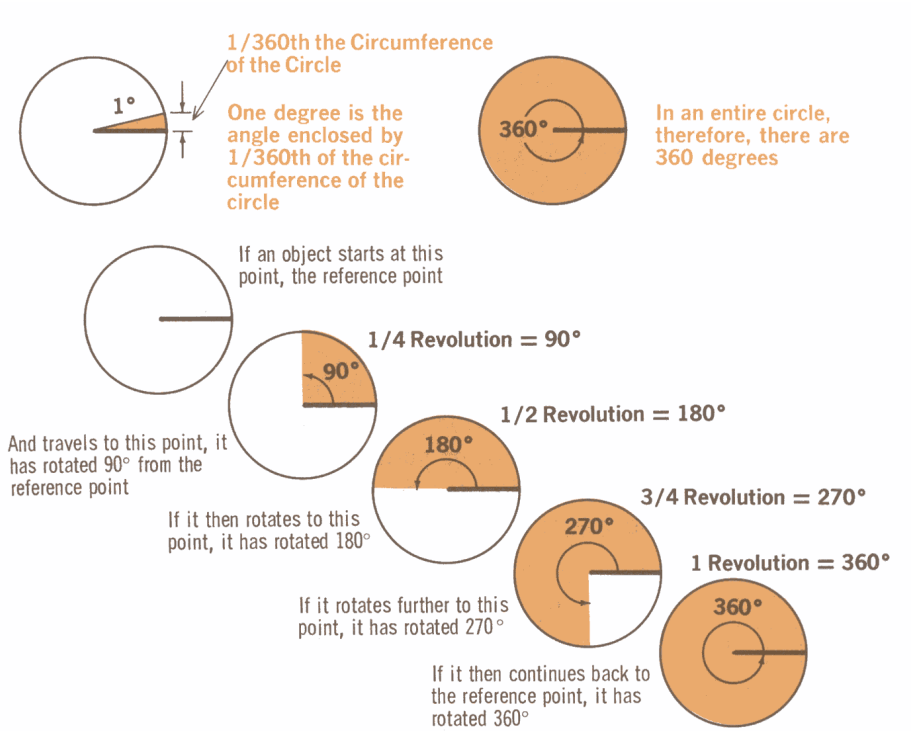
Session 2a Review

- AC vs. DC
- Transformers
- Ohm's Law
- AC Generators
- Sine Waves – $\sin(2\pi ft+\theta)$
- Frequency, Period, Wavelength and Magnitude
- Phase Angle
- Averages
  - Mean (DC)
  - RMS (Effective Value)



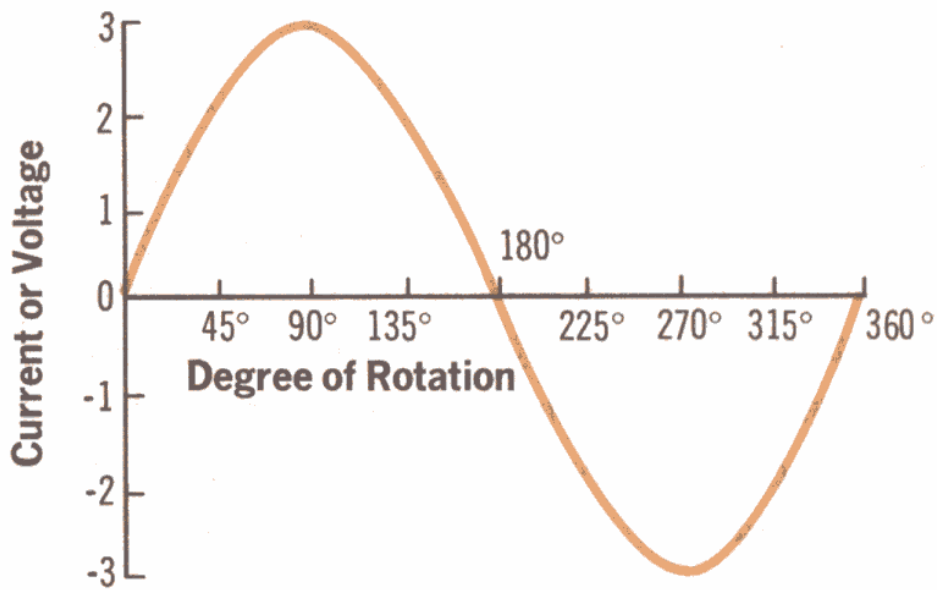
Angle: Degrees and Radians

- Degrees, minutes and seconds
  - 360° gets you around a circle
    - Invented by map makers in the middle ages
    - Reused for Time measurements
- Radians (in calculators)
  - 2 * π or 2 * 3.14159 gets you around a circle
    - The real angle measure
    - The distance traveled around the perimeter of a "unit" circle (r = 1)



Sine Waves and Angle

- V = 3*sine (angle)
  - Sine often shortened to sin [V = 3*sin(angle)]
  - 3 is the "Amplitude"
  - Starts at zero
  - Peak (3) at 90°(π/2)
  - Zero again at 180°(π)
  - Negative Peak (-3) at 270°(3*π/2)
  - Zero to Finish the "Cycle" at 360°(2*π)



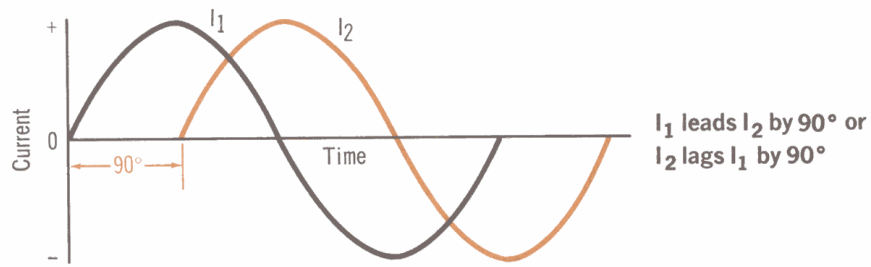
Phase Difference

- Waveforms can be “out of phase”
- Note:

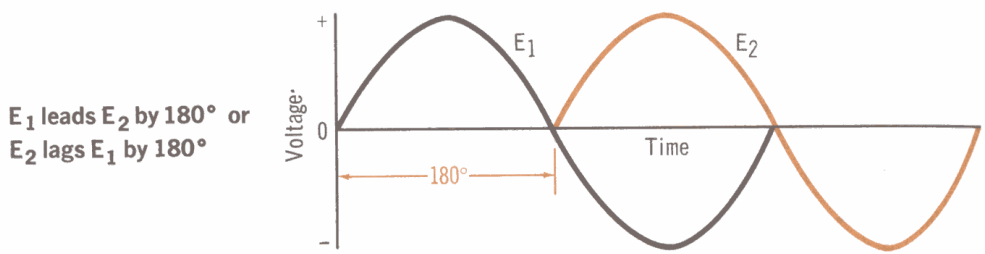
$\sin(2\pi ft - \pi/2) = \cos(2\pi ft)$

Cosine is the full name

  - Starts at 1 at t = 0
  - Looks just like sine but at a different phase



When maximum and minimum points of one voltage or current occur before the corresponding points of another voltage or current, the two are out of phase. When such a phase difference exists, one of the voltages or currents leads, and the other lags

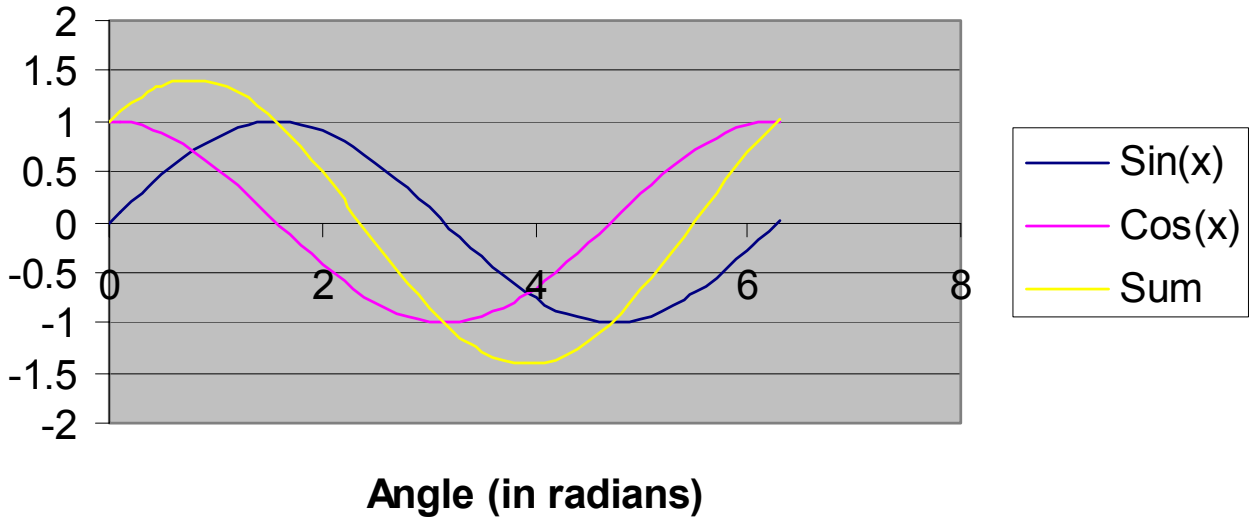


Adding Two Sine Waves

Adding two sine waves at the same frequency but different phases results in a sine wave with the same frequency, new amplitude, and new phase

Each point in the graph adds separately

Here the two sine waves are 90° apart. with equal amplitude

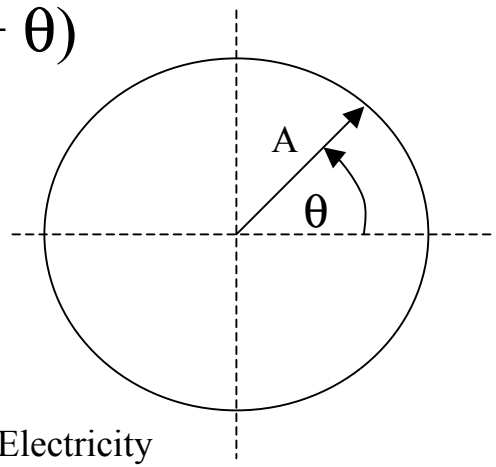


The result is a sine wave at 45° ($\pi/4$) with an amplitude of 1.414 (the square root of 2)

The Vector Analogy

- We can make the task of adding sine waves with the same frequency easier using vectors
- Treat a sine wave with Amplitude "A" and phase θ as a vector of length "A" at an angle of θ (the frequency is implicit)
  note: by convention cos(2πft) has a zero angle

$A*\cos(2\pi ft + \theta)$



Adding Sine and Cosine: Using Vectors

Angle = arctan(3/3) = π/4 (45°)

Length = sqrt(3^2 + 3^2) (Pythagorus)

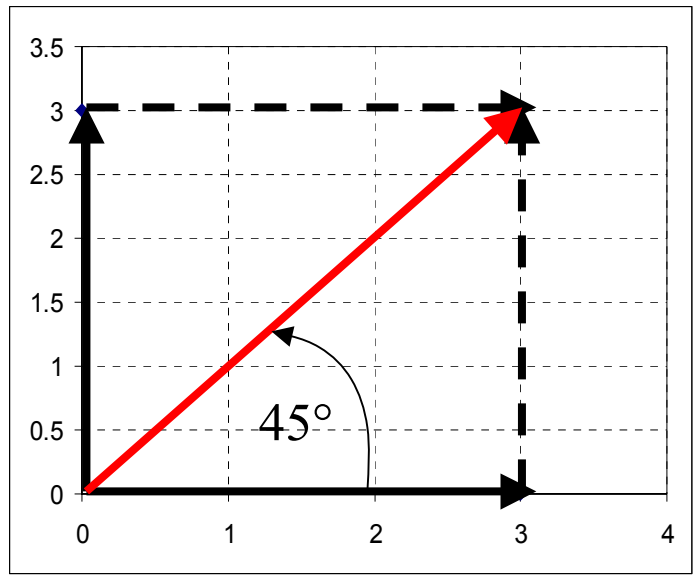
Length = sqrt(9 + 9)

Length = sqrt(9*2)

Length = sqrt(9)*sqrt(2)

Length = 3*sqrt(2)

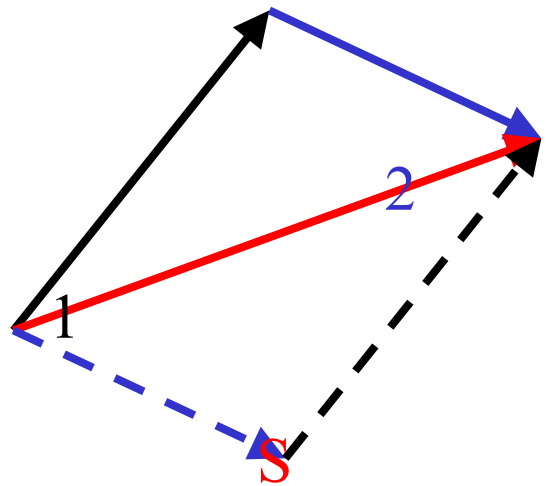
Length = 3*1.414 = 4.243

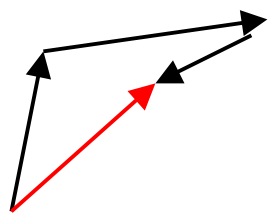


Sum = 4.243 * cos(x + 45°)

Adding Vectors:
Head-to-Tail, Parallelogram

- Head-to-Tail Method
  - Redraw vectors so that one starts where the other ends
  - Draw the sum vector from the free tail to the free head.
  - Good for multiple vectors
- Parallelogram Method
  - Complete the parallelogram
  - The sum is the diagonal of the Parallelogram





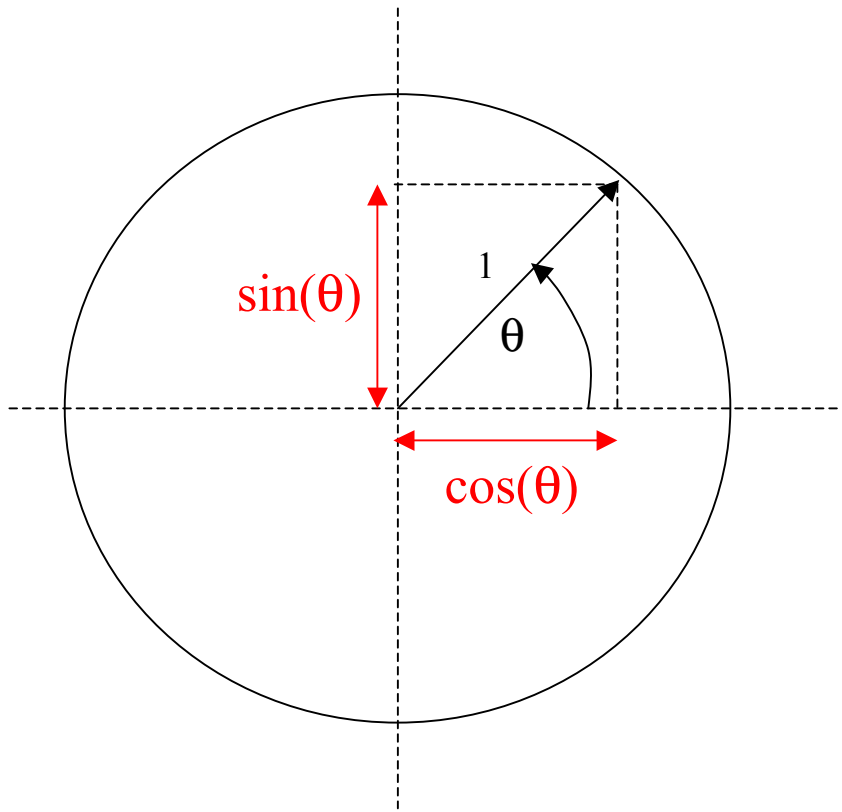
Vectors and Trig: The Unit Circle

- sin(θ) = opposite/hypotenuse
- cos(θ) = adjacent/hypotenuse
- tan(θ) = opposite/adjacent
- θ = arcsin(opposite/hypotenuse)
- θ = arccos(adjacent/hypotenuse)
- θ = arctan(opposite/adjacent)

Remember, if your calculator is in:

degree mode - θ is in degrees

radian mode - θ is in radians

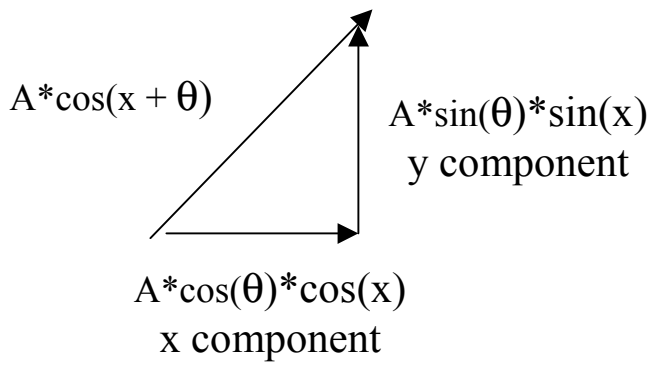


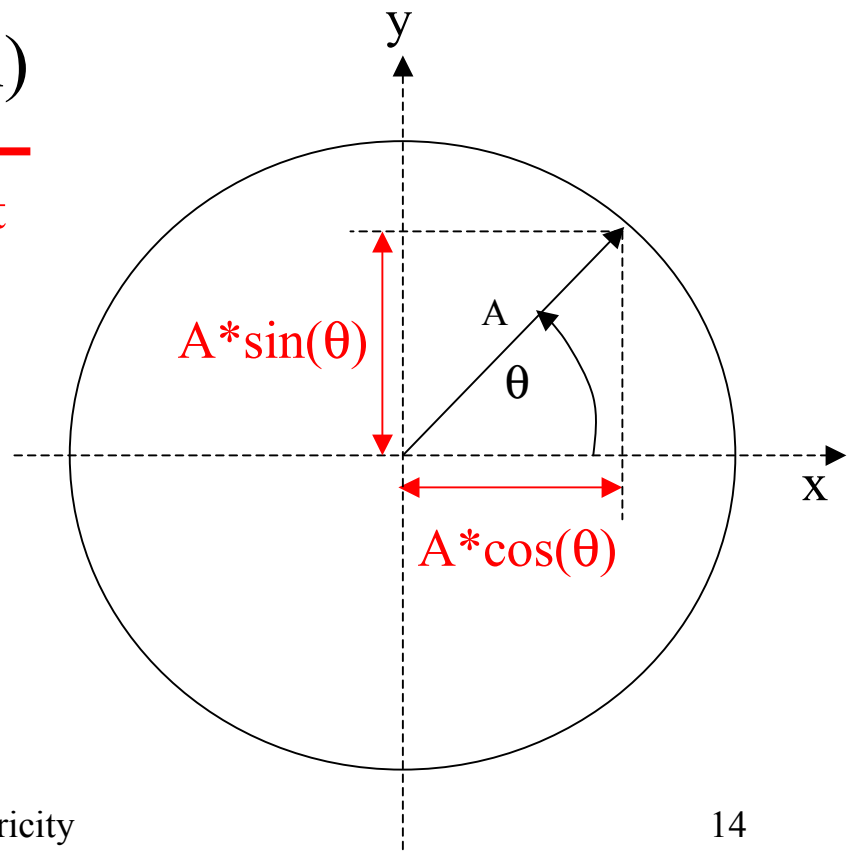
Vector Components
(Cartesian)

$A\cos(x + \theta) =$

$A\cos(\theta)\cos(x) + A\sin(\theta)\sin(x)$

x - component y - component





You can add vectors by adding their components ($x_1 + x_2$, $y_1 + y_2$)

Adding Vectors by Components
p. 4-16

- V1=4∠75°, V2=2∠45°, V3=3∠30°
- X components
  - V1x = 4*cos(75°) = 4*0.2588 = 1.035 **error in book**
  - V2x = 2*cos(45°) = 2*0.7071 = 1.414 sqrt(2)
  - V3x = 3*cos(30°) = 3*0.866 = 2.6
  - **Vtx** = 1.035 + 1.414 + 2.6 = **5.05**
- Y components
  - V1y = 4*sin(75°) = 4*0.966 = 3.86
  - V2y = 2*sin(45°) = 2*0.7071 = 1.414 **sqrt(2)**
  - V3y = 3*sin(30°) = 3*0.500 = 1.5
  - **Vty** = 3.86 + 1.414 + 1.5 = **6.77**
- **Vt** = 5.05**i** + 6.77**j**
  where **i** and **j** are the Cartesian unit vectors
  Vt = 8.5 ∠ 53°

Changing Component Form into Sign-Magnitude Form

Find the Magnitude

$A = \sqrt{(5.05)^2 + (6.77)^2}$

$A = \sqrt{72.3} = 8.5$

Find the Angle

$\theta = \arctan(6.77/5.05)$

$\theta = \arctan(1.34)$

$\theta = 0.93$ radians

$\theta = 0.93*180/\pi = 53.3°$

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(MT1 Review Labs 05/08 pm, 05/11 am)	and Vectors (again)	
Session 3b – 05/15	**R-L Circuits**	**Text 4.25 – 4.54**
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