## Digital Design Lecture 7

Combinatorial Logic

## Combinatorial Logic Circuit

- General Logic Block Without Memory
- Outputs are only a function of the current inputs
- No storage elements to remember "state"
- There are timing delays from input to output due to gate delays


Fig. 4-1 Block Diagram of Combinational Circuit

## Analysis Example

$$
\begin{aligned}
& \mathrm{F}_{2}=\mathrm{AB}+\mathrm{AC}+\mathrm{BC} \\
& \mathrm{~T}_{1}=\mathrm{A}+\mathrm{B}+\mathrm{C} \\
& \mathrm{~T}_{2}=\mathrm{ABC} \\
& \mathrm{~T}_{3}=\mathrm{F}_{2}^{\prime} \mathrm{T}_{1} \\
& \mathrm{~F}_{1}=\mathrm{T}_{3}+\mathrm{T}_{2}
\end{aligned}
$$

Or


Fig. 4-2 Logic Diagram for Analysis Example
$\mathrm{F}_{1}=\mathrm{A}^{\prime} \mathrm{BC}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}+\mathrm{AB}^{\prime} \mathrm{C}^{\prime}+\mathrm{ABC}$

## Truth Table 4-1: Analysis Example

| A | B | C | $\mathrm{F}_{2}$ | $\mathrm{~F}_{2}{ }^{\prime}$ | $\mathrm{T}_{1}$ | $\mathrm{~T}_{2}$ | $\mathrm{~T}_{3}$ | $\mathrm{~F}_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |

## Truth Table 4-2: Code Conversion

Input BCD

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |

Output Excess-3 Code

| W | X | Y | Z |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |

## Code Conversion Karnaugh Maps




Fig. 4-3 Maps for BCD to Excess-3 Code Converter

## BCD to Excess-3 Code Converter

$$
\begin{aligned}
\mathrm{z} & =\mathrm{D}^{\prime} \\
\mathrm{y} & =\mathrm{CD}+\mathrm{C}^{\prime} \mathrm{D}^{\prime} \\
& =\mathrm{CD}+(\mathrm{C}+\mathrm{D})^{\prime} \\
\mathrm{x} & =\mathrm{B}^{\prime} \mathrm{C}+\mathrm{BD}^{\prime}+\mathrm{BC}^{\prime} \mathrm{D}^{\prime} \\
& =\mathrm{B}^{\prime}(\mathrm{C}+\mathrm{D})+\mathrm{B}(\mathrm{C}+\mathrm{D})^{\prime} \\
\mathrm{w} & =\mathrm{A}+\mathrm{BC}+\mathrm{BD} \\
& =\mathrm{A}+\mathrm{B}(\mathrm{C}+\mathrm{D})
\end{aligned}
$$



Fig. 4-4 Logic Diagram for BCD to Excess-3 Code Converter

## Half Adder

| x | y | C | S |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |


$S=x^{\prime} y^{\prime} z+x^{\prime} y z^{\prime}+x y^{\prime} z^{\prime}+x y z$


$$
\begin{aligned}
S & =x y+x z+y z \\
& =x y+x y^{\prime} z+x^{\prime} y z
\end{aligned}
$$

Fig. 4-6 Maps for Full Adder

## Half Adder (cont.)


(a) $\begin{aligned} S & =x y^{\prime}+x^{\prime} y \\ C & =x y\end{aligned}$
(b) $S=x \oplus y$
$C=x y$

Fig. 4-5 Implementation of Half-Adder

## Full Adder



## Full Adder: Sum of Products



Fig. 4-7 Implementation of Full Adder in Sum of Products

## Full Adder: Using Half Adders



Fig. 4-8 Implementation of Full Adder with Two Half Adders and an OR Gate

## 4-Bit Adder



Fig. 4-9 4-Bit Adder

## Carry Propagation

- Carry bits must "ripple" through each stage of a multi-bit adder before the output settles down to the correct result (two gate delays per bit)


Fig. 4-10 Full Adder with P and G Shown

## Carry Look Ahead



Fig. 4-11 Logic Diagram of Carry Lookahead Generator

## 4-Bit Adder with Carry Look Ahead



Fig. 4-12 4-Bit Adder with Carry Lookahead

## Binary Subtraction



Fig. 4-13 4-Bit Adder Subtractor

## BCD Adder

- Carry 0110 when first adder result overflows


Fig. 4-14 Block Diagram of a BCD Adder

