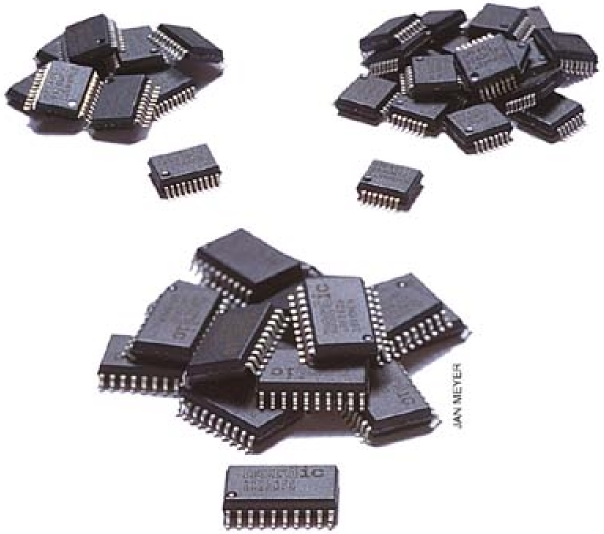
CIRCUITS 1

DEVELOP TOOLS FOR THE ANALYSIS AND DESIGN OF BASIC LINEAR ELECTRIC CIRCUITS



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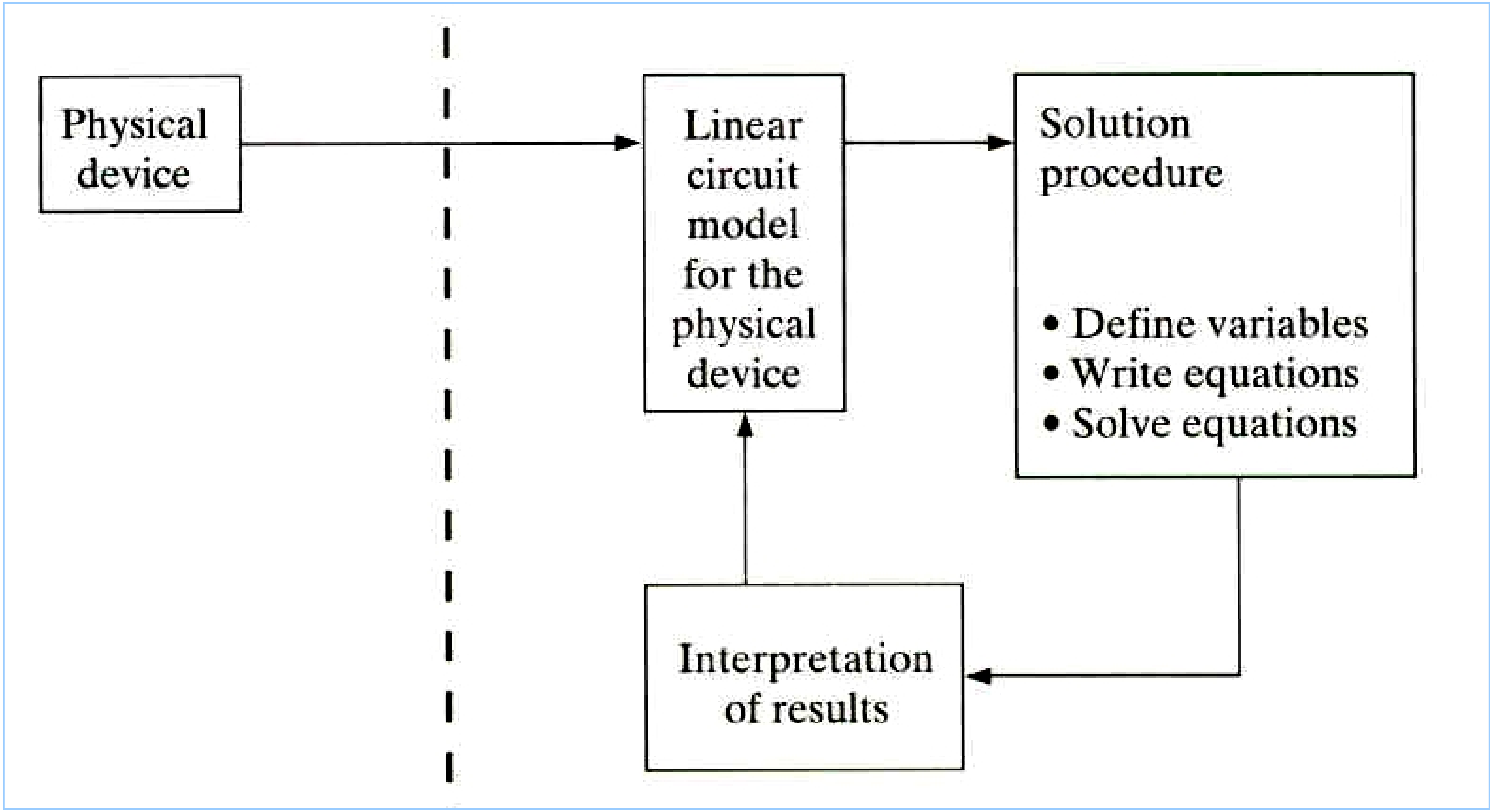
A FEW WORDS ABOUT ANALYSIS USING MATHEMATICAL MODELS

BASIC STRATEGY USED IN ANALYSIS



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MATHEMATICAL ANALYSIS

DEVELOP A SET OF MATHEMATICAL EQUATIONS THAT REPRESENT THE CIRCUIT
- A MATEMATICAL MODEL -

LEARN HOW TO SOLVE THE MODEL TO DETERMINE HOW THE CIRCUIT WILL BEHAVE IN A GIVEN SITUATION

THIS COURSE TEACHES THE BASIC TECHNIQUES TO DEVELOP MATHEMATICAL MODELS FOR ELECTRIC CIRCUITS

THE MODELS THAT WILL BE DEVELOPED HAVE NICE MATHEMATICAL PROPERTIES.
IN PARTICULAR THEY WILL BE LINEAR WHICH MEANS THAT THEY SATISFY THE PRINCIPLE OF SUPERPOSITION

Model

$$y = Tu$$

Principle of Superposition

$$T(\alpha_1 u_1 + \alpha_2 u_2) = \alpha_1 T(u_1) + \alpha_2 T(u_2)$$

THE MATHEMATICS CLASSES - LINEAR ALGEBRA, DIFFERENTIAL EQUATIONS- PROVIDE THE TOOLS TO SOLVE THE MATHEMATICAL MODELS

FOR THE FIRST PART WE WILL BE EXPECTED TO SOLVE SYSTEMS OF ALGEBRAIC EQUATIONS

$$12V_1 - 9V_2 - 4V_3 = 8$$
$$-4V_1 + 16V_2 + V_3 = 0$$
$$-2V_1 - 4V_2 + 6V_3 = 20$$

LATER THE MODELS WILL BE DIFFERENTIAL EQUATIONS OF THE FORM

$$3\frac{dy}{dt} + y = f$$
$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 3\frac{df}{dt} + 4f$$

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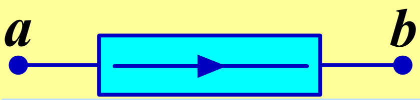
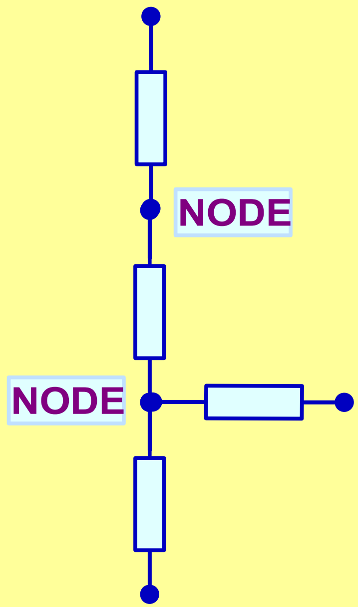
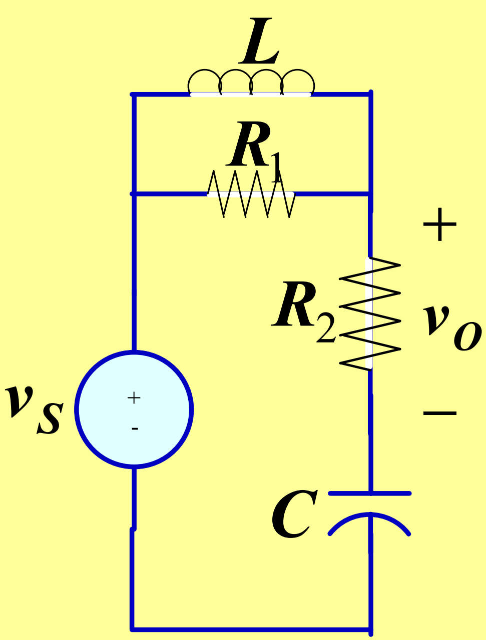
ELECTRIC CIRCUIT IS AN INTERCONNECTION OF ELECTRICAL COMPONENTS

2 TERMINALS COMPONENT

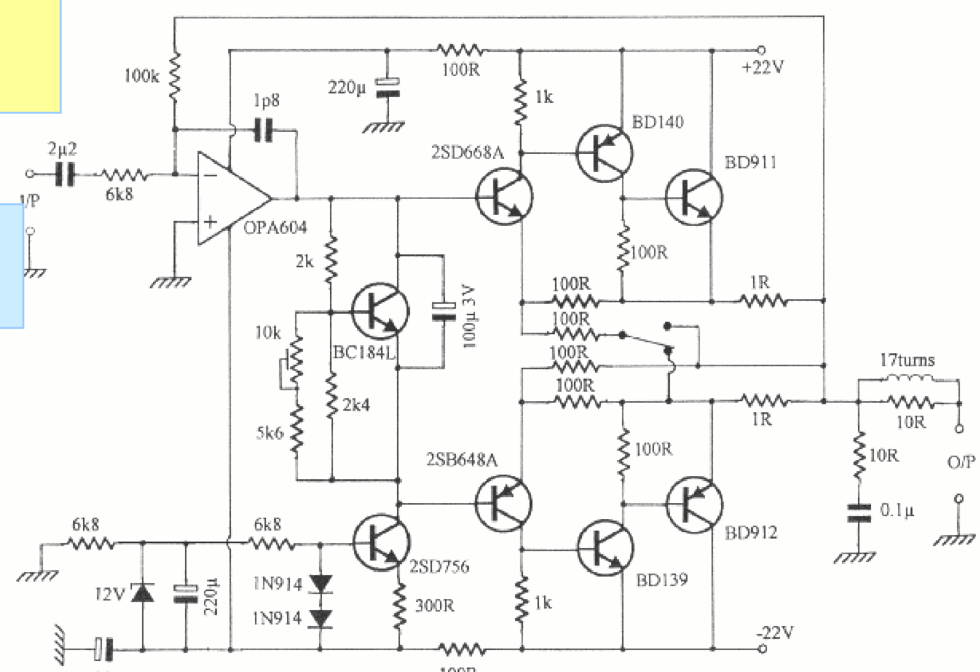


characterized by the current through it and the voltage difference betweeb terminals





TYPICAL LINEAR CIRCUIT



LOW DISTORTION POWER AMPLIFIER

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BASIC CONCEPTS

LEARNING GOALS

- System of Units: The SI standard system; prefixes
- Basic Quantities: Charge, current, voltage, power and energy
- Circuit Elements: Active and Passive

http://physics.nist.gov/cuu/index.html

The NIST Reference on Constants, Units, and Uncertainty

International System of Units (SI)

SI base units

The SI is founded on seven *SI base units* for seven *base quantities* assumed to be mutually independent, as given in Table 1.

Table 1. SI base units

	SI base unit	
Base quantity	**Name**	**Symbol**
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

For detailed information on the SI base units, see Definitions of the SI base units and their Historical contex

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Definitions of the SI base units

Unit of length	**meter**	The meter is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.
Unit of mass	**kilogram**	The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
Unit of time	**second**	The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.
Unit of electric current	**ampere**	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.
Unit of thermodynamic temperature	**kelvin**	The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
Unit of amount of substance	**mole**	1. The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is "mol."

The 20 SI prefixes used to form decimal multiples and submultiples of SI units are given in Table 5.

Table 5. SI prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	µ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

SI DERIVED BASIC ELECTRICAL UNITS

power, radiant flux	watt	W	J/s	$m^2 \cdot kg \cdot s^{-3}$
electric charge, quantity of electricity	coulomb	C	-	$s \cdot A$
electric potential difference, electromotive force	volt	V	W/A	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$
capacitance	farad	F	C/V	$m^{-2} \cdot kg^{-1} \cdot s^4 \cdot A^2$
electric resistance	ohm	Ω	V/A	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-2}$
electric conductance	siemens	S	A/V	$m^{-2} \cdot kg^{-1} \cdot s^3 \cdot A^2$
magnetic flux	weber	Wb	V·s	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-1}$
magnetic flux density	tesla	T	Wb/m^2	$kg \cdot s^{-2} \cdot A^{-1}$
inductance	henry	H	Wb/A	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-2}$

ONE AMPERE OF CURRENT CARRIES ONE COULOMB OF CHARGE EVERY SECOND.

$$A = C \times s$$

$$1 \ \text{COULOMB} = 6.28 \times 10^{18} \ (e)$$

(e) IS THE CHARGE OF ONE ELECTRON

VOLT IS A MEASURE OF ENERGY PER CHARGE.
TWO POINTS HAVE A VOLTAGE DIFFERENCE OF ONE VOLT IF ONE COULOMB OF CHARGE GAINS ONE JOULE OF CHARGE WHEN IT IS MOVED FROM ONE POINT TO THE OTHER.

$$V = \frac{J}{C}$$

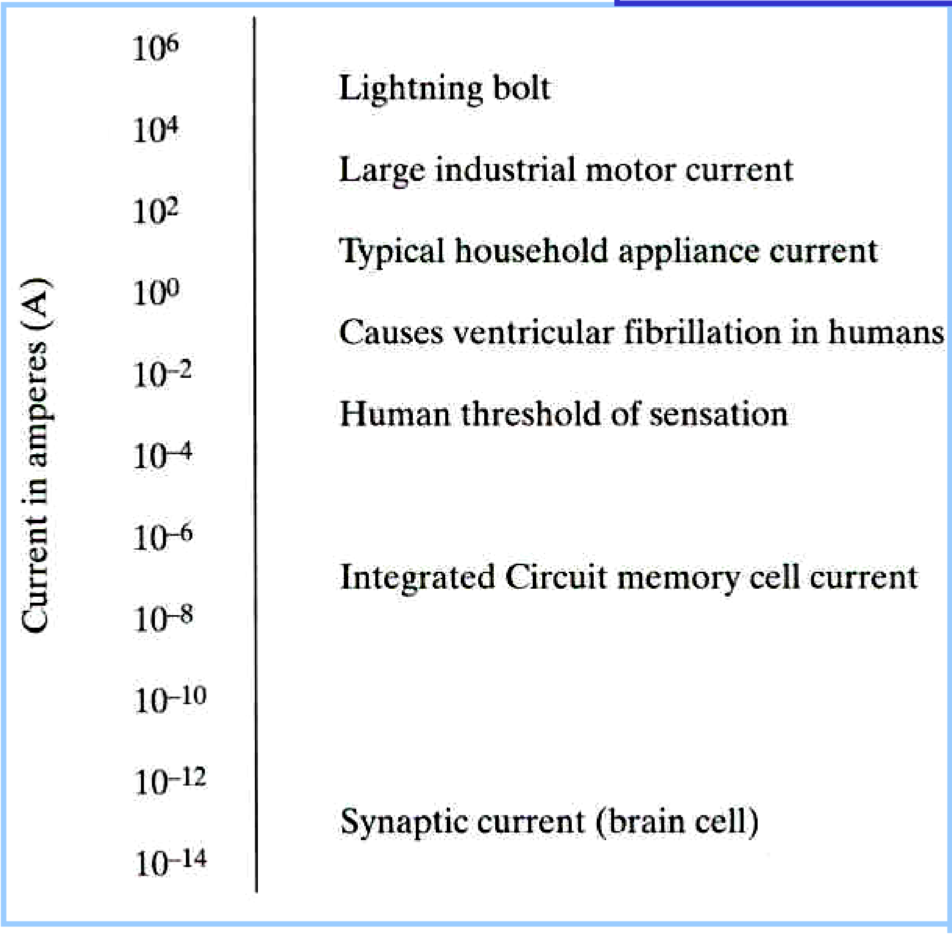
OHM IS A MEASURE OF THE RESISTANCE TO THE FLOW OF CHARGE.
THERE IS ONE OHM OF RESISTENCE IF IT IS REQUIRED ONE VOLT OF ELECTROMOTIVE FORCE TO DRIVE THROUGH ONE AMPERE OF CURRENT

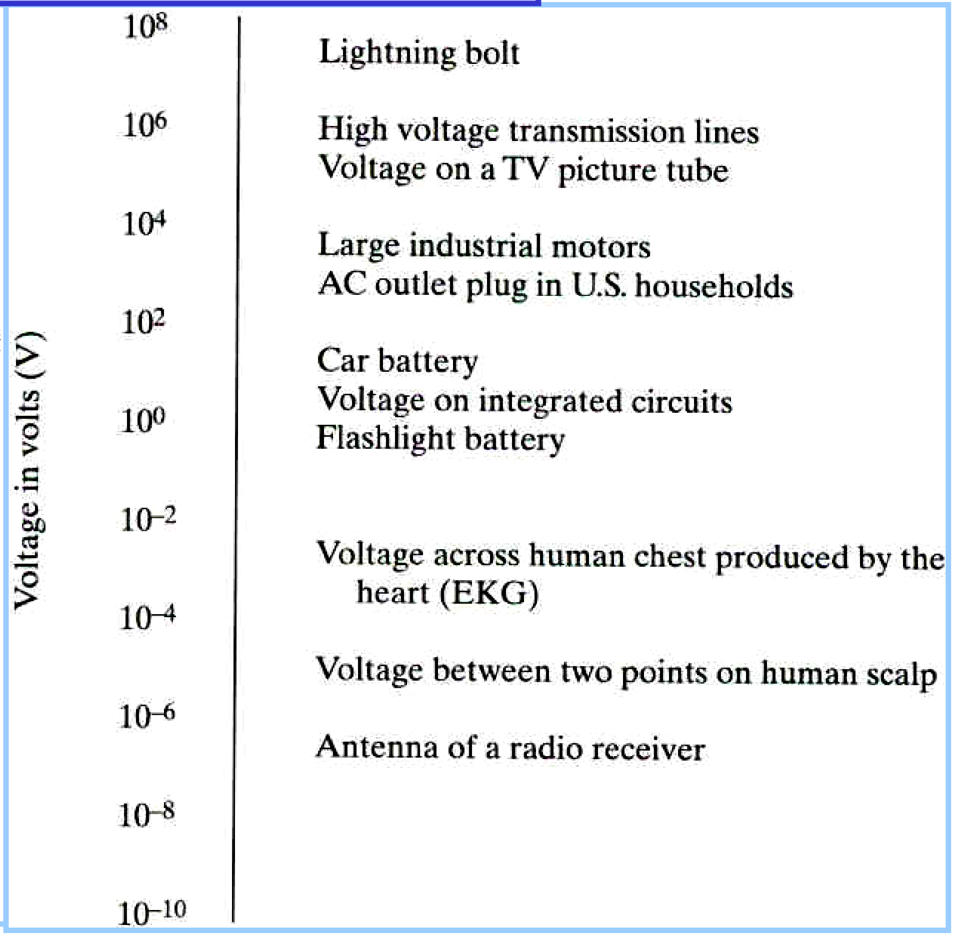
$$\Omega = \frac{V}{A}$$

IT IS REQUIRED ONE WATT OF POWER TO DRIVE ONE AMPER OF CURRENT AGAINST AN ELECTROMOTIVE DIFFERENCE OF ONE VOLTS

$$W = V \times A$$

CURRENT AND VOLTAGE RANGES



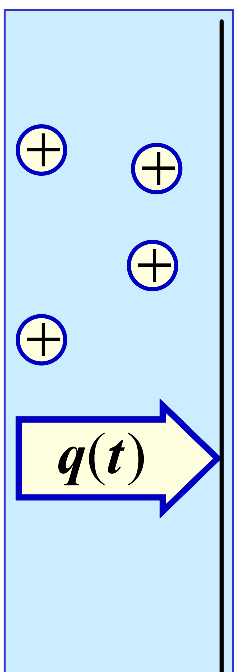


Strictly speaking current is a basic quantity and charge is derived. However, physically the electric current is created by a movement of charged particles.

An electric circuit is essentially a pipeline that facilitates the transfer of charge from one point to another. The time rate of change of charge constitutes an electric *current*. Mathematically, the relationship is expressed as

$$i(t) = \frac{dq(t)}{dt} \quad \text{or} \quad q(t) = \int_{-\infty}^{t} i(x)\,dx$$

Although we know that current flow in metallic conductors results from electron motion, the conventional current flow, which is universally adopted, represents the movement of positive charges.

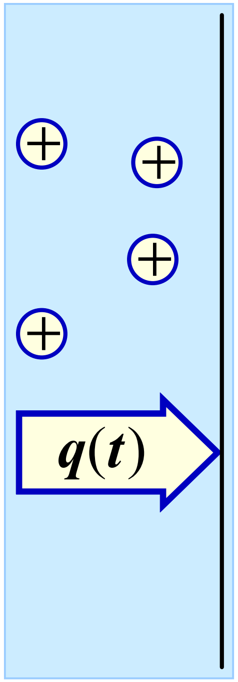


What is the meaning of a negative value for *q(t)?*

PROBLEM SOLVING TIP
IF THE CHARGE IS GIVEN DETERMINE THE CURRENT BY DIFFERENTIATION
IF THE CURRENT IS KNOWN DETERMINE THE CHARGE BY INTEGRATION

A PHYSICAL ANALOGY THAT HELPS VISUALIZE ELECTRIC CURRENTS IS THAT OF WATER FLOW.
CHARGES ARE VISUALIZED AS WATER PARTICLES

EXAMPLE



$$q(t) = 4\times 10^{-3}\sin(120\pi t)[C]$$

$$i(t) = 4\times 10^{-3}\times 120\pi\cos(120\pi t)\,[A]$$

$$i(t) = 0.480\pi\cos(120\pi t)[mA]$$

EXAMPLE

$$i(t) = \begin{cases} 0 & t < 0 \\ e^{-2t}mA & t \ge 0 \end{cases}$$

FIND THE CHARGE THAT PASSES DURING IN THE INTERVAL 0<t<1

$$q = \int_0^1 e^{-2x}dx = -\frac{1}{2}e^{-2x}\Big|_0^1 = -\frac{1}{2}e^{-2} - (-\frac{1}{2}e^{0})$$

$$q = \frac{1}{2}(1-e^{-2})$$

Units?

FIND THE CHARGE AS A FUNCTION OF TIME

$$q(t) = \int_{-\infty}^{t} i(x)dx = \int_{-\infty}^{t} e^{-2x}dx$$

$$t \le 0 \Rightarrow q(t) = 0$$

$$t > 0 \Rightarrow q(t) = \int_0^t e^{-2x}dx = \frac{1}{2}(1-e^{-2t})$$

And the units for the charge?...

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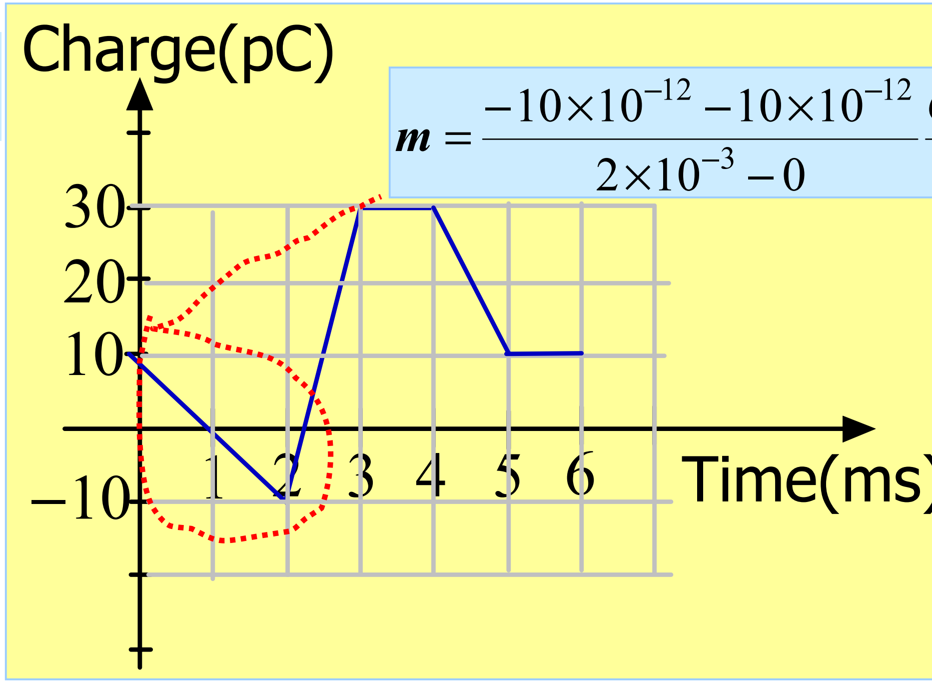
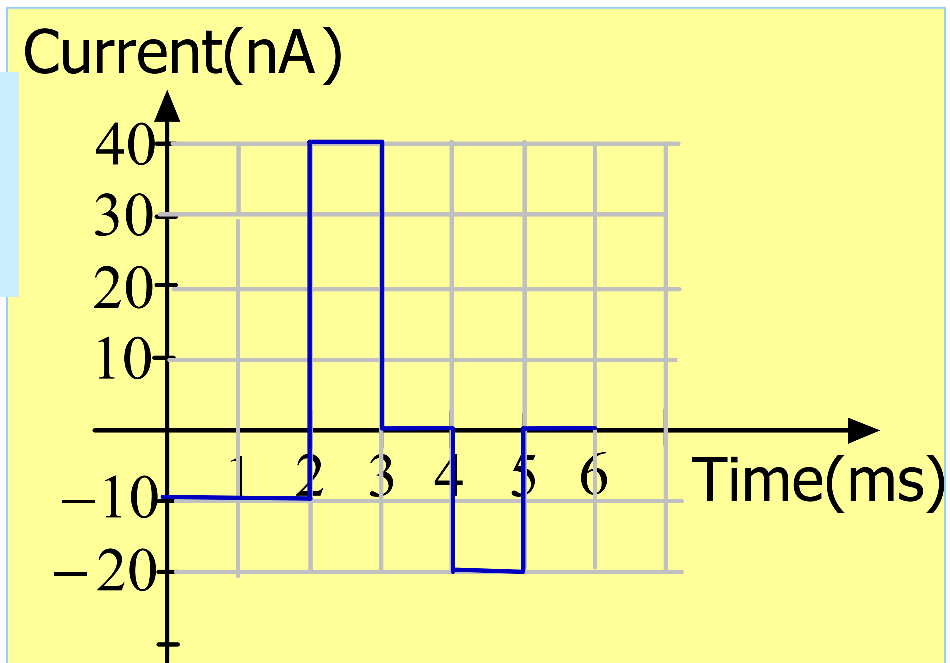
DETERMINE THE CURRENT

Here we are given the charge flow as function of time.



$$m = \frac{-10\times10^{-12} - 10\times10^{-12}}{2\times10^{-3} - 0}\,\frac{C}{s} = -10\times10^{-9}(C/s)$$

To determine current we must take derivatives.
<u>PAY ATTENTION TO UNITS</u>

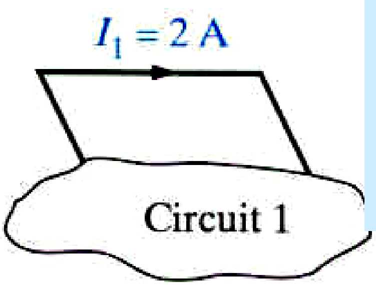


CONVENTION FOR CURRENTS

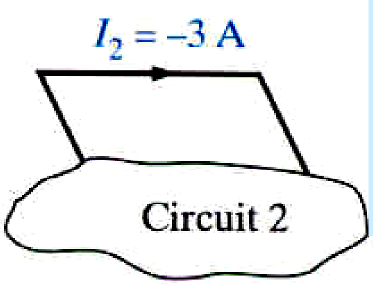
IT IS ABSOLUTELY NECESSARY TO INDICATE THE DIRECTION OF MOVEMENT OF CHARGED PARTICLES.

THE UNIVERSALLY ACCEPTED CONVENTION IN ELECTRICAL ENGINEERING IS THAT CURRENT IS FLOW OF POSITIVE CHARGES.
AND WE INDICATE THE DIRECTION OF FLOW FOR POSITIVE CHARGES

-THE REFERENCE DIRECTION-



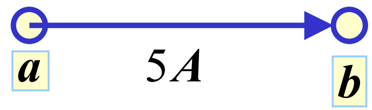
A POSITIVE VALUE FOR THE CURRENT INDICATES FLOW IN THE DIRECTION OF THE ARROW (THE REFERENCE DIRECTION)



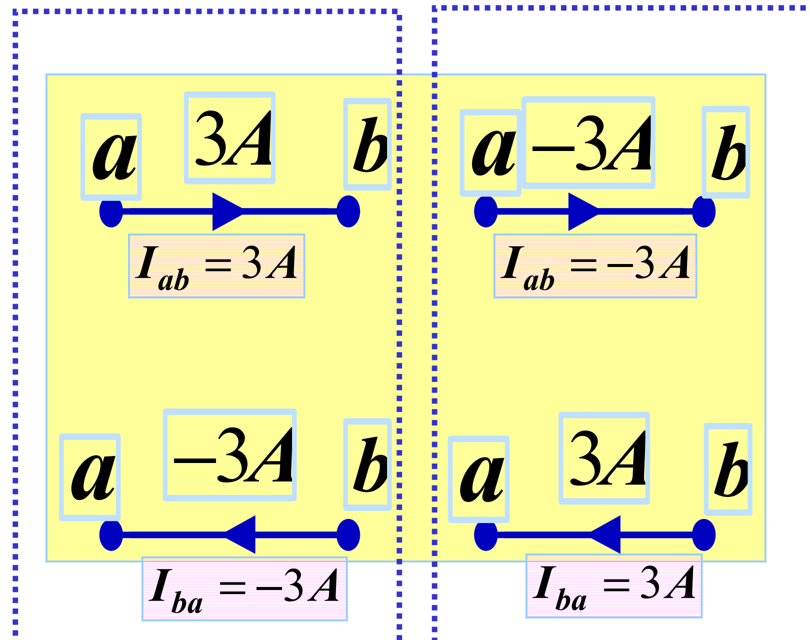
A NEGATIVE VALUE FOR THE CURRENT INDICATES FLOW IN THE OPPOSITE DIRECTION THAN THE REFERENCE DIRECTION

THE DOUBLE INDEX NOTATION

IF THE INITIAL AND TERMINAL NODE ARE LABELED ONE CAN INDICATE THEM AS SUBINDICES FOR THE CURRENT NAME



$$I_{ab} = 5A$$

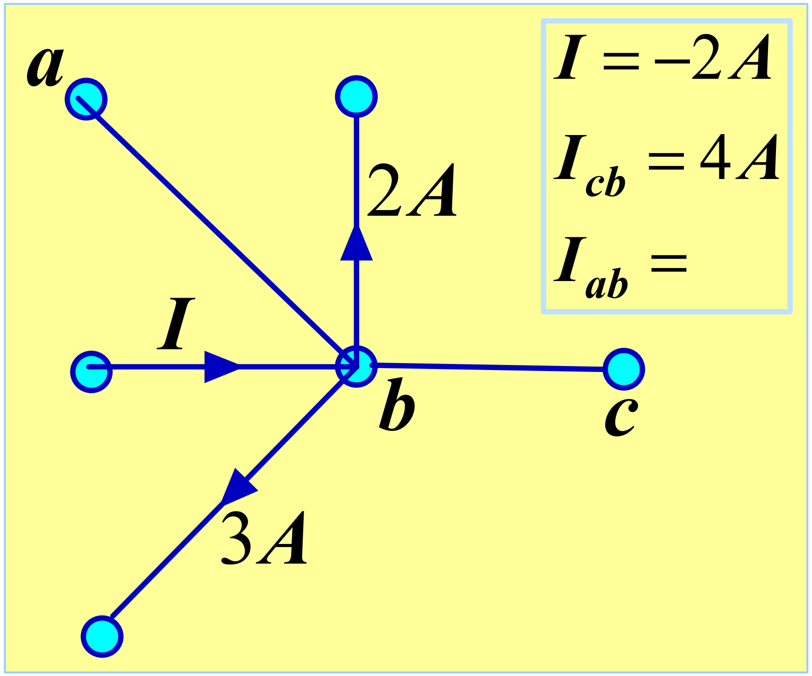


POSITIVE CHARGES FLOW LEFT-RIGHT

POSITIVE CHARGES FLOW RIGHT-LEFT

$$I_{ab} = -I_{ba}$$

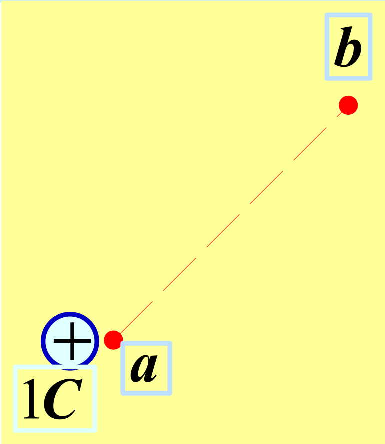
GEAUX



This example illustrates the various ways in which the current notation can be used

CONVENTIONS FOR VOLTAGES

ONE DEFINITION FOR VOLT
TWO POINTS HAVE A VOLTAGE DIFFERENTIAL OF ONE VOLT IF ONE COULOMB OF CHARGE GAINS (OR LOSES) ONE JOULE OF ENERGY WHEN IT MOVES FROM ONE POINT TO THE OTHER



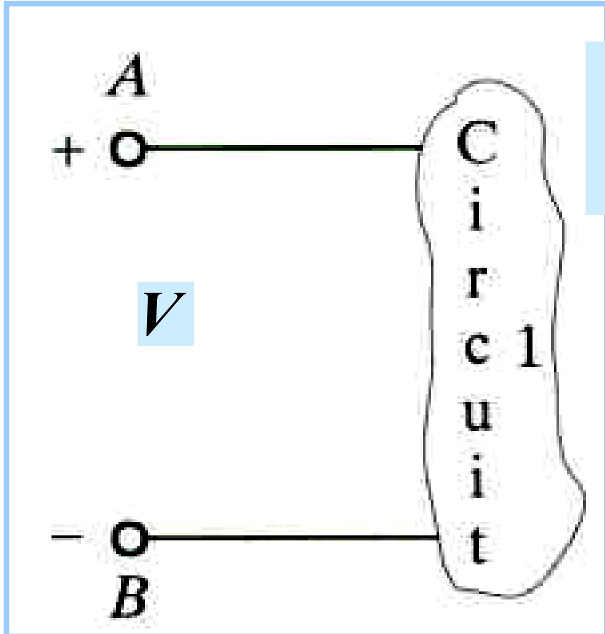
IF THE CHARGE GAINS ENERGY MOVING FROM a TO b THEN b HAS HIGHER VOLTAGE THAN a.
IF IT LOSES ENERGY THEN b HAS LOWER VOLTAGE THAN a

DIMENSIONALLY VOLT IS A DERIVED UNIT

$$\text{VOLT} = \frac{\text{JOULE}}{\text{COULOMB}} = \frac{N \bullet m}{A \bullet s}$$

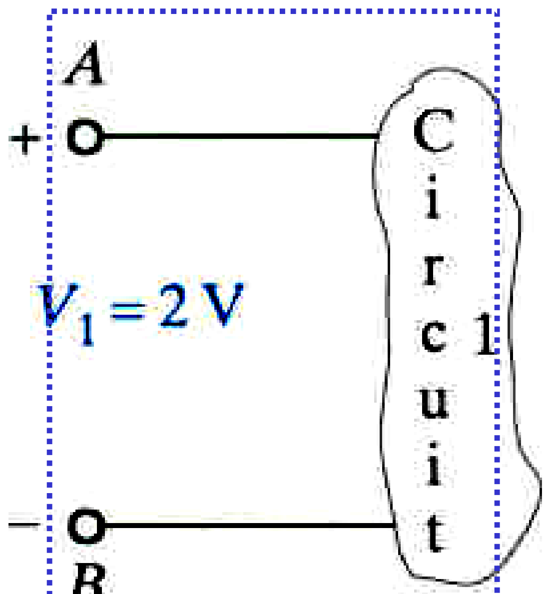
VOLTAGE IS ALWAYS MEASURED IN A RELATIVE FORM AS THE VOLTAGE DIFFERENCE BETWEEN TWO POINTS

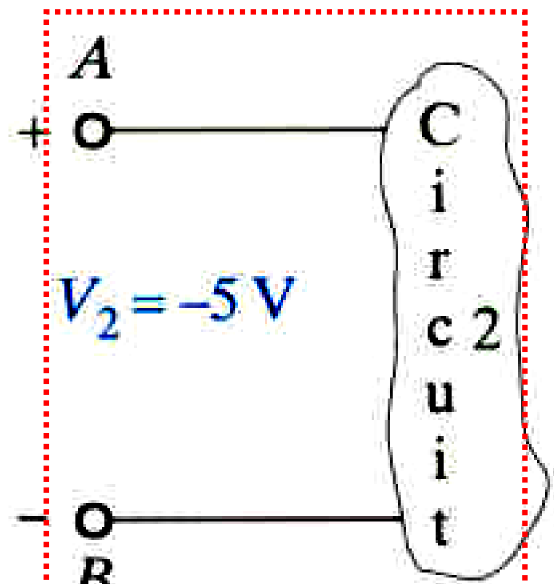
IT IS ESSENTIAL THAT OUR NOTATION ALLOWS US TO DETERMINE WHICH POINT HAS THE HIGHER VOLTAGE



THE + AND - SIGNS DEFINE THE REFERENCE POLARITY

IF THE NUMBER V IS POSITIVE POINT A HAS V VOLTS MORE THAN POINT B.
IF THE NUMBER V IS NEGATIVE POINT A HAS |V| LESS THAN POINT B





POINT A HAS 2V MORE THAN POINT B

POINT A HAS 5V LESS THAN POINT B

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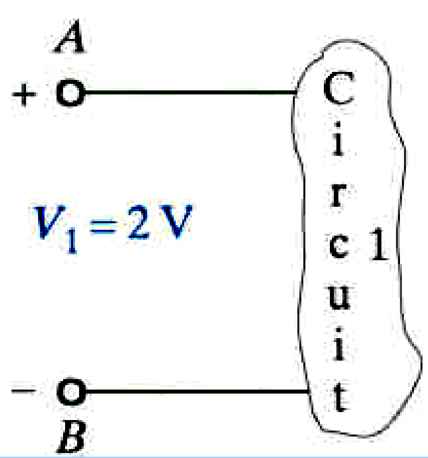
THE TWO-INDEX NOTATION FOR VOLTAGES

INSTEAD OF SHOWING THE REFERENCE POLARITY WE AGREE THAT THE FIRST SUBINDEX DENOTES THE POINT WITH POSITIVE REFERENCE POLARITY



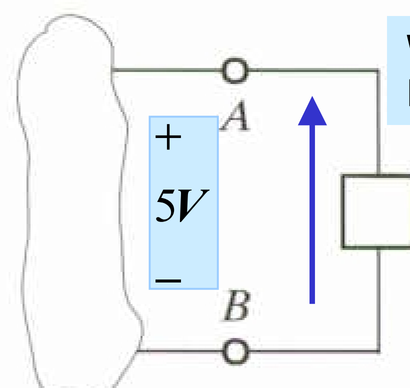
$$V_{AB} = 2V$$

WHAT ENERGY IS REQUIRED TO MOVE 120[C] FROM POINT B TO POINT A IN THE CIRCUIT?

$$V = \frac{W}{Q} \Rightarrow W = VQ = 240J$$

THE CHARGES MOVE TO A POINT WITH HIGHER VOLTAGE -THEY GAINED (OR ABSORBED) ENERGY

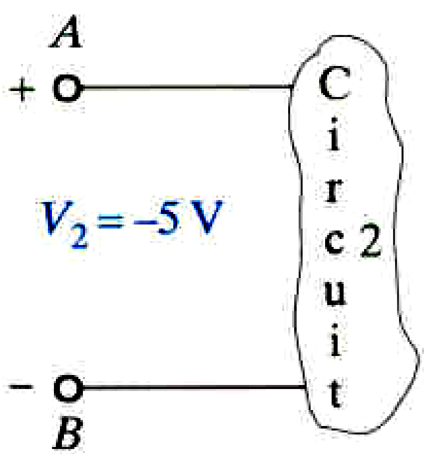
D1.3 Five joules of energy are absorbed by 1 C of charge when moved from point *B* to point *A* in 1 s. Find the voltage between points *A* and *B*.

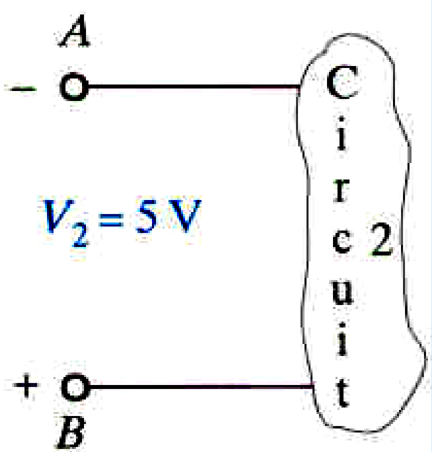


THE VOLTAGE DIFFERENCE IS 5V

WHICH POINT HAS THE HIGHER VOLTAGE?

$$V_{AB} = 5V$$





$$V_{AB} = -5V$$

$$V_{AB} = -V_{BA}$$

$$V_{BA} = 5V$$

EXAMPLE
A CAMCODER BATTERY PLATE CLAIMS THAT THE UNIT STORES 2700mAHr AT 7.2V.
WHAT IS THE TOTAL CHARGE AND ENERGY STORED?

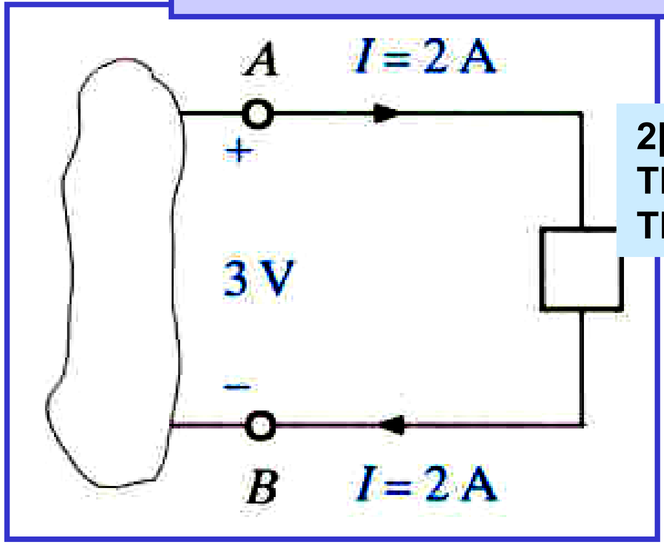
<u>CHARGE</u>
THE NOTATION 2700mAHr INDICATES THAT THE UNIT CAN DELIVER 2700mA FOR ONE FULL HOUR

$$Q = 2700 \times 10^{-3}\left[\frac{C}{S}\right] \times 3600\frac{s}{Hr} \times 1Hr$$
$$= 9.72 \times 10^{3}[C]$$

<u>TOTAL ENERGY STORED</u>
THE CHARGES ARE MOVED THROUGH A 7.2V VOLTAGE DIFFERENTIAL

$$W = Q[C] \times V\left[\frac{J}{C}\right] = 9.72 \times 10^{3} \times 7.2[J]$$
$$= 6.998 \times 10^{4}[J]$$

ENERGY AND POWER



2[C/s] PASS THROUGH THE ELEMENT

EACH COULOMB OF CHARGE LOSES 3[J] OR SUPPLIES 3[J] OF ENERGY TO THE ELEMENT

THE ELEMENT RECEIVES ENERGY AT A RATE OF 6[J/s]

THE ELECTRIC POWER RECEIVED BY THE ELEMENT IS 6[W]

IN GENERAL

$$P = VI$$

$$w(t_2, t_1) = \int_{t_1}^{t_2} p(x)dx$$

HOW DO WE RECOGNIZE IF AN ELEMENT SUPPLIES OR RECEIVES POWER?

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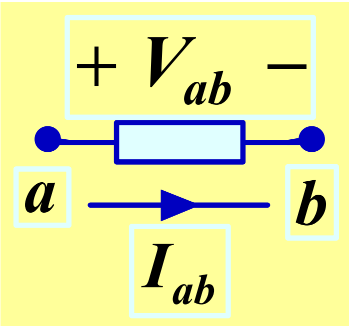
PASSIVE SIGN CONVENTION

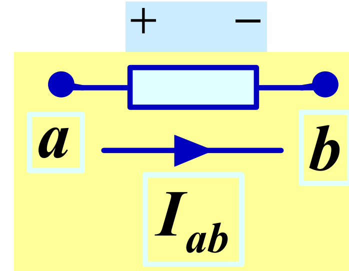
POWER RECEIVED IS POSITIVE WHILE POWER SUPPLIED IS CONSIDERED NEGATIVE



$$P = V_{ab} I_{ab}$$

IF VOLTAGE AND CURRENT ARE BOTH POSITIVE THE CHARGES MOVE FROM HIGH TO LOW VOLTAGE AND THE COMPONENT RECEIVES ENERGY --IT IS A PASSIVE ELEMENT

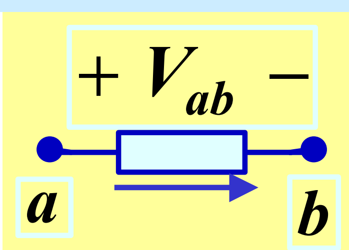
THIS IS THE REFERENCE FOR POLARITY



IF THE REFERENCE DIRECTION FOR CURRENT IS GIVEN

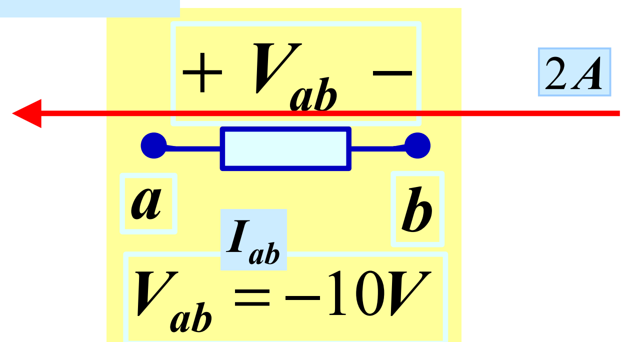
A CONSEQUENCE OF THIS CONVENTION IS THAT THE REFERENCE DIRECTIONS FOR CURRENT AND VOLTAGE ARE NOT INDEPENDENT -- IF WE ASSUME PASSIVE ELEMENTS

GIVEN THE REFERENCE POLARITY



REFERENCE DIRECTION FOR CURRENT

EXAMPLE



THE ELEMENT RECEIVES 20W OF POWER. WHAT IS THE CURRENT?

SELECT REFERENCE DIRECTION BASED ON PASSIVE SIGN CONVENTION

$$20[W] = V_{ab} I_{ab} = (-10V) I_{ab}$$

$$I_{ab} = -2[A]$$

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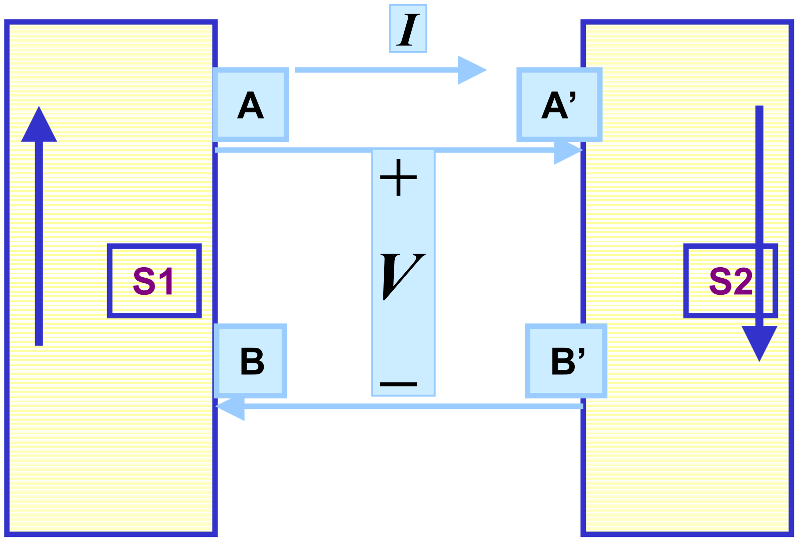
UNDERSTANDING PASSIVE SIGN CONVENTION

We must examine the voltage across the component and the current through it



$$P_{S1} = V_{AB} I_{AB}$$
$$P_{S2} = V_{A'B'} I_{A'B'}$$

Voltage(V)	Current A - A'	S1	S2
positive	positive	supplies	receives
positive	negative	receives	supplies
negative	positive	receives	supplies
negative	negative	supplies	receives

ON S_1

$V_{AB} > 0, I_{AB} < 0$

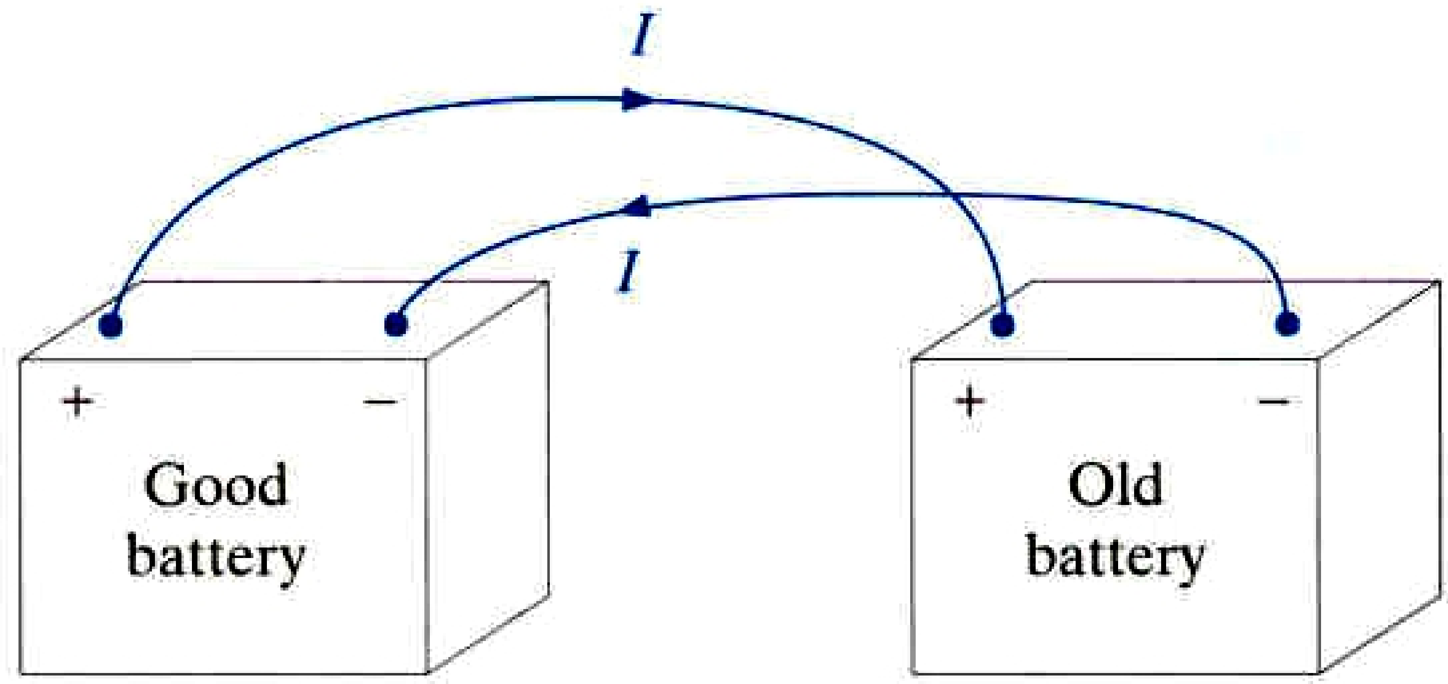
ON S_2

$V_{A'B'} > 0, I_{A'B'} > 0$

ON S2

$V_{A'B'} < 0, I_{A'B'} > 0$

GEAUX

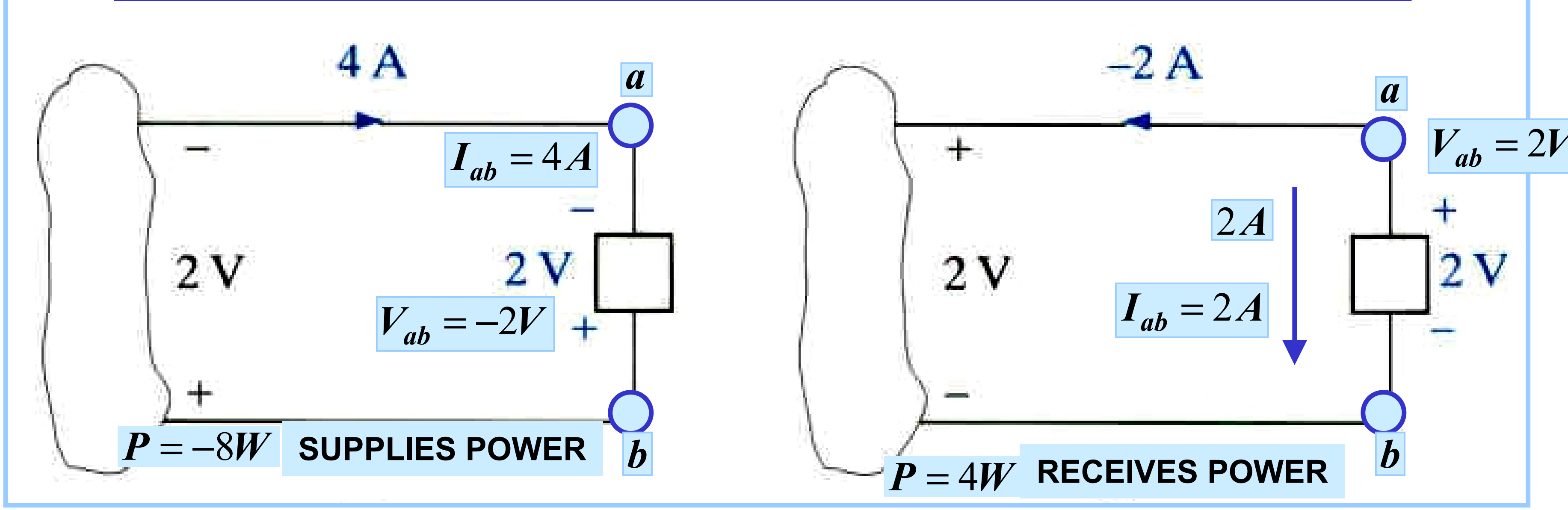


CHARGES RECEIVE ENERGY.
THIS BATTERY SUPPLIES ENERGY

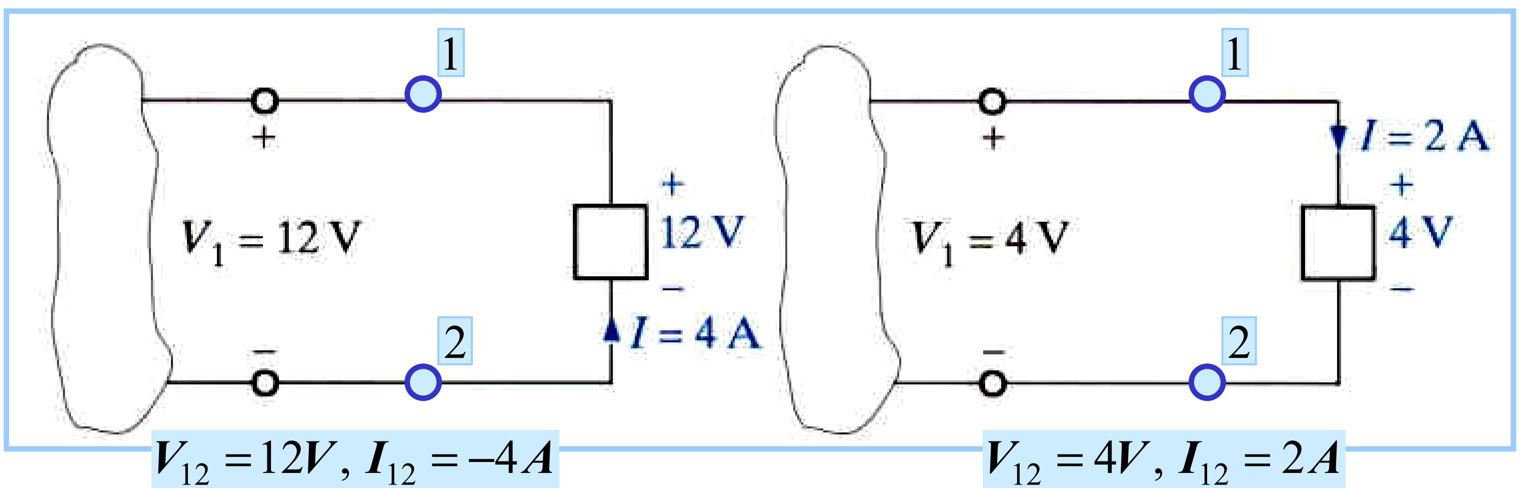
CHARGES LOSE ENERGY.
THIS BATTERY RECEIVES THE ENERGY

WHAT WOULD HAPPEN IF THE CONNECTIONS ARE REVERSED IN ONE OF THE BATTERIES?

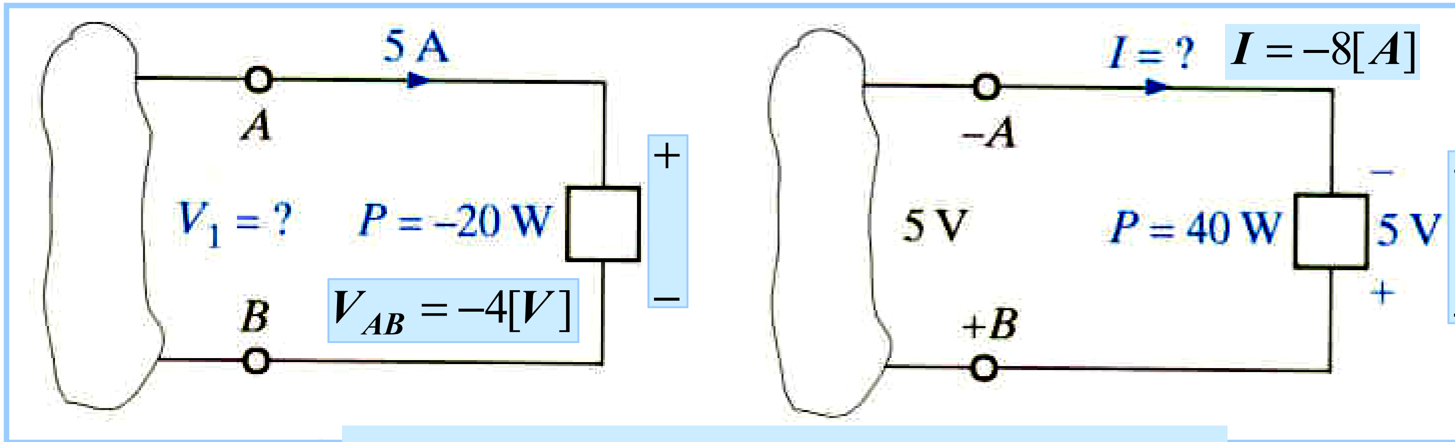
DETERMINE WHETHER THE ELEMENTS ARE SUPPLYING OR RECEIVING POWER AND HOW MUCH



WHEN IN DOUBT LABEL THE TERMINALS OF THE COMPONENT



GEAUX

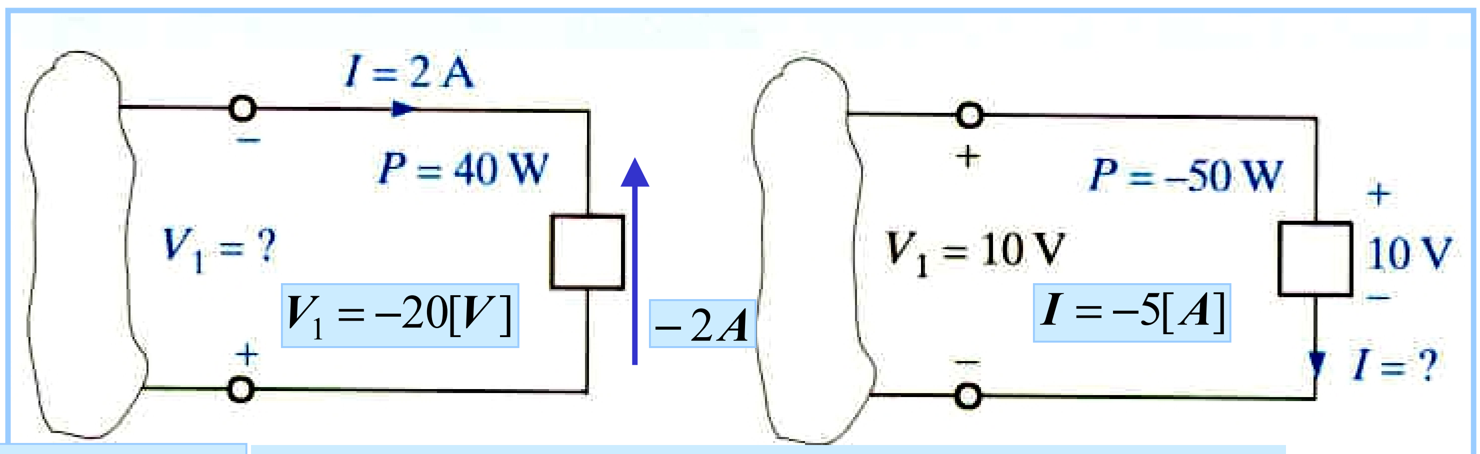


$-20[W] = V_{AB} \times (5A)$

SELECT VOLTAGE REFERENCE POLARITY BASED ON CURRENT REFERENCE DIRECTION

$40[W] = (-5V) \times I$

WHICH TERMINAL HAS HIGHER VOLTAGE AND WHICH IS THE CURRENT FLOW DIRECTION



$40[W] = V_1 \times (-2A)$

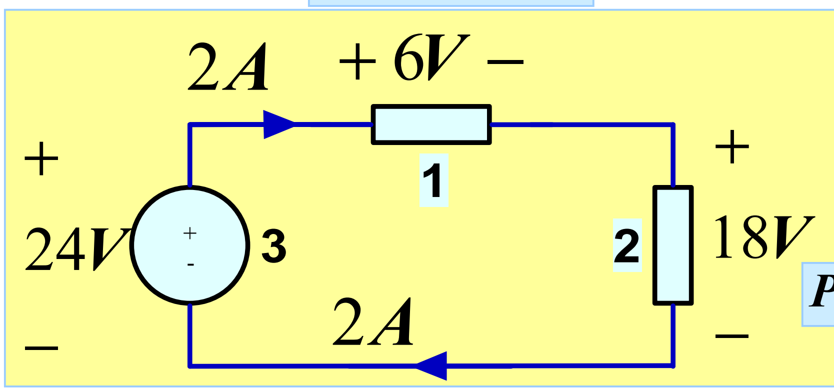
SELECT HERE THE CURRENT REFERENCE DIRECTION BASED ON VOLTAGE REFERENCE POLARITY

$-50[W] = (10[V]) \times I$

GEAUX

COMPUTE POWER ABDORBED OR SUPPLIED BY EACH ELEMENT

$$P_1 = (6V)(2A)$$



$$P_2 = (18V)(2A)$$

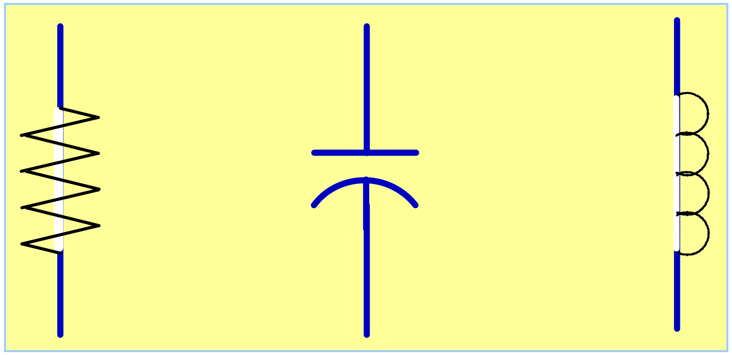
$$P_3 = (24V)(-2A) = (-24V)(2A)$$

P1 = 12W
P2 = 36W
P3 = -48W

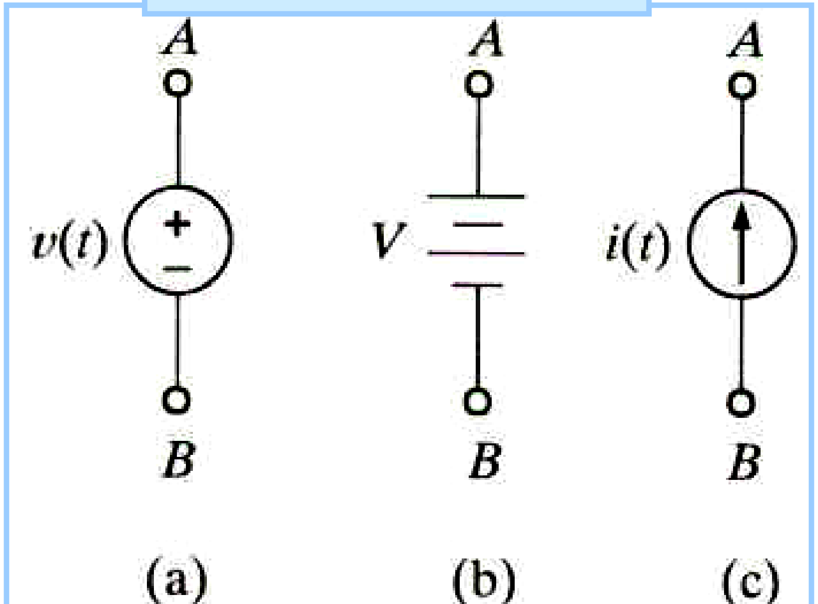
IMPORTANT: NOTICE THE POWER BALANCE IN THE CIRCUIT

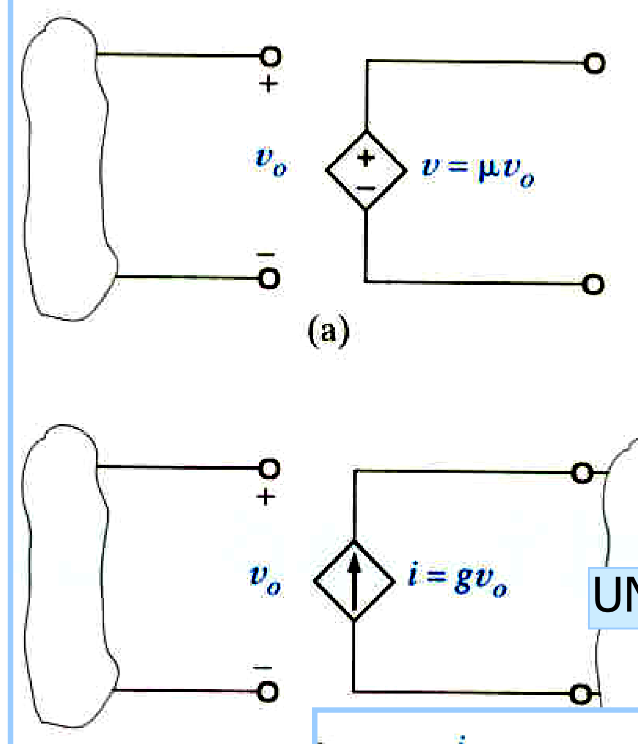
CIRCUIT ELEMENTS

PASSIVE ELEMENTS



INDEPENDENT SOURCES

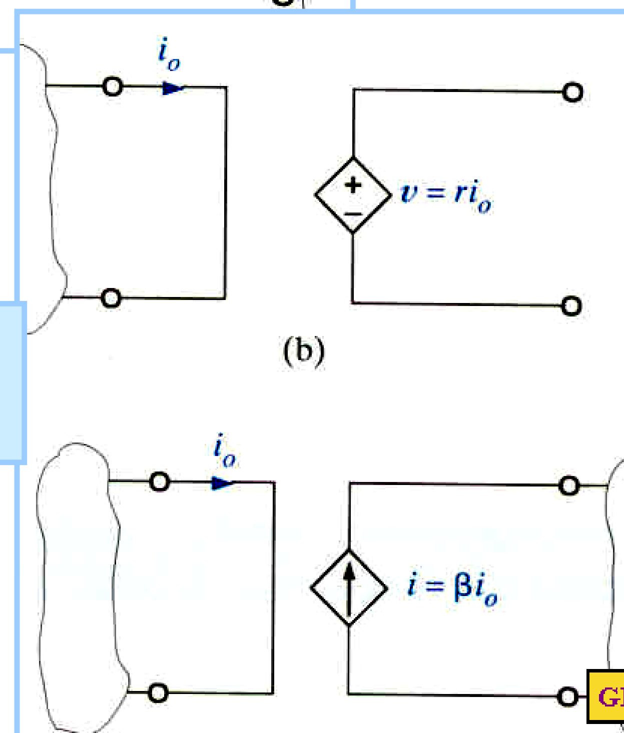




VOLTAGE DEPENDENT SOURCES

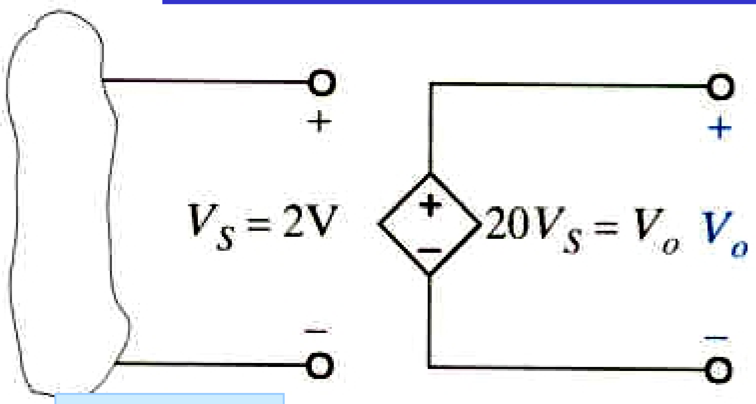
UNITS FOR μ, g, r, β?

CURRENT DEPENDENT SOURCES



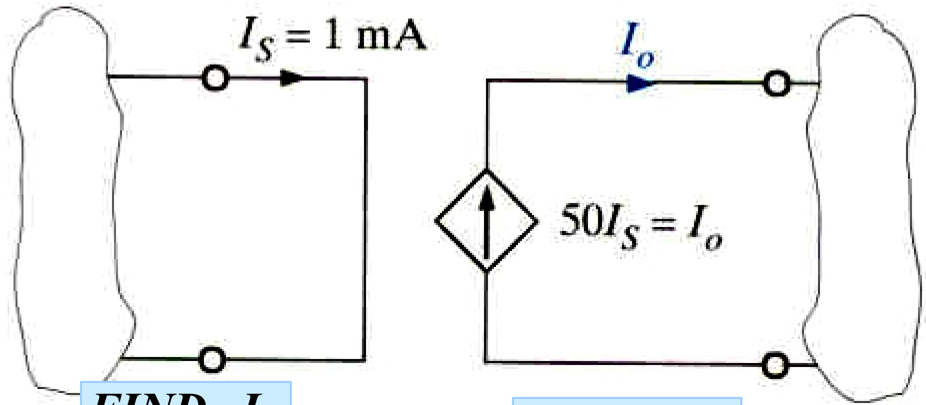
GEAUX

EXERCISES WITH DEPENDENT SOURCES



FIND V_o

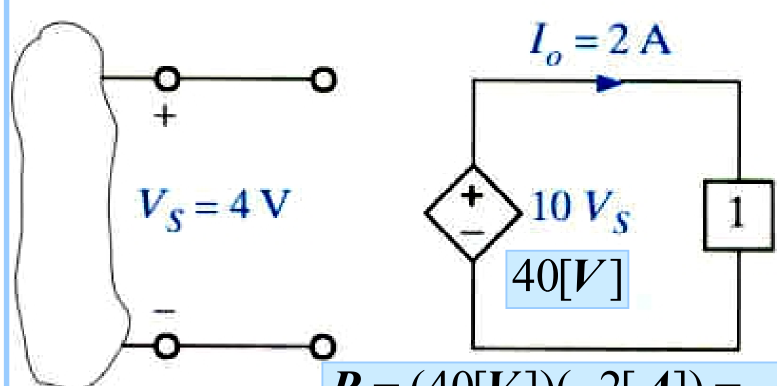
$V_o = 40[V]$



FIND I_o

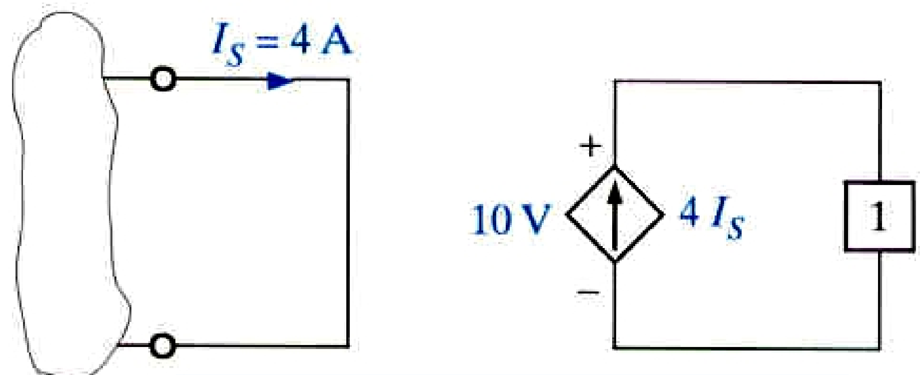
$I_o = 50mA$

DETERMINE THE POWER SUPPLIED BY THE DEPENDENT SOURCES



$40[V]$

$P = (40[V])(-2[A]) = -80[W]$



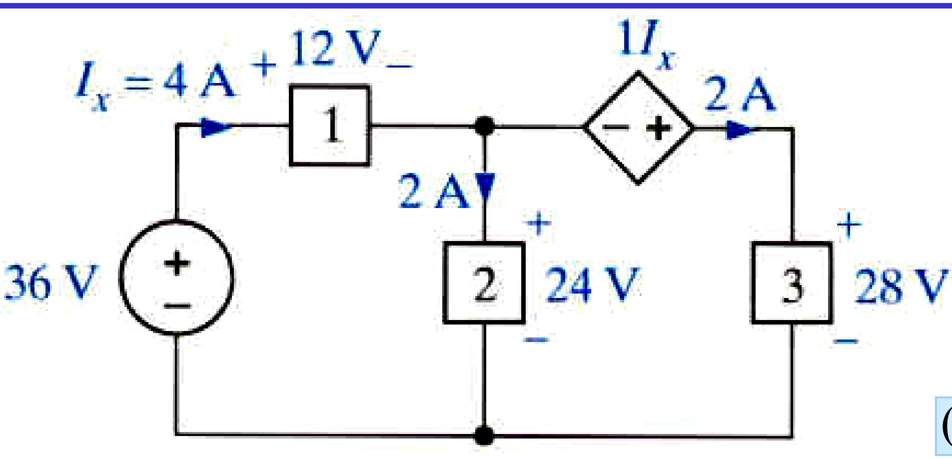
$P = (-10[V])(4 \times 4[A]) = -160[W]$

TAKE VOLTAGE POLARITY REFERENCE

TAKE CURRENT REFERENCE DIRECTION

GEAUX

POWER ABSORBED OR SUPPLIED BY EACH ELEMENT



$$P_1 = (12V)(4A) = 48[W]$$

$$P_2 = (24V)(2A) = 48[W]$$

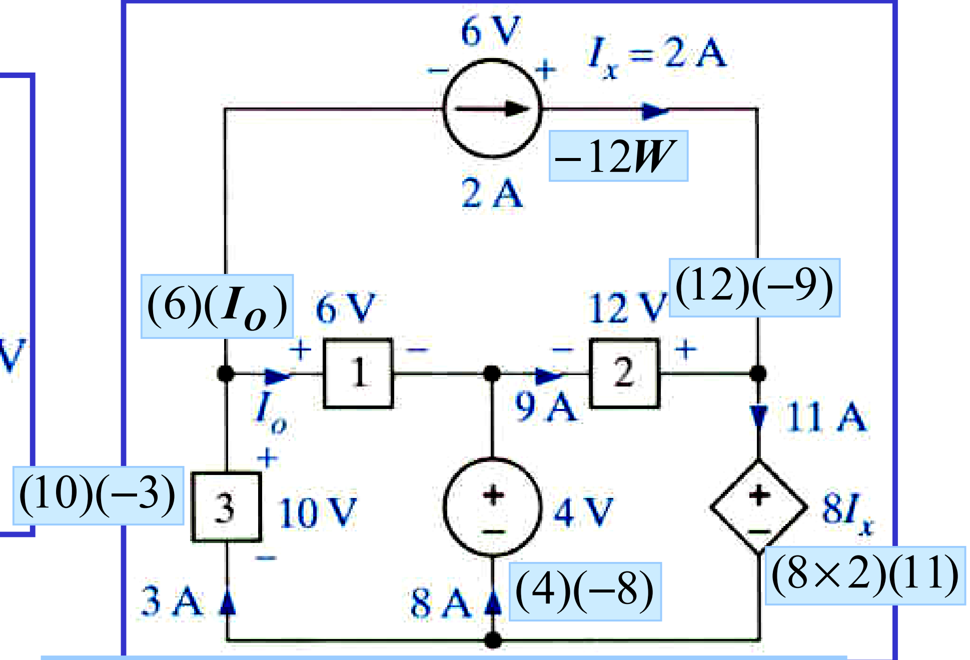
$$P_3 = (28V)(2A) = 56[W]$$

$$P_{DS} = (1I_x)(-2A) = (4V)(-2A) = -8[W]$$

$$P_{36V} = (36V)(-4A) = -144[W]$$

NOTICE THE POWER BALANCE

USE POWER BALANCE TO COMPUTE Io



$$P_{2\text{ A}} = (6)(-2) = -12\text{ W}$$

$$P_1 = (6)(I_o) = 6I_o\text{ W}$$

$$P_2 = (12)(-9) = -108\text{ W}$$

$$P_3 = (10)(-3) = -30\text{ W}$$

$$P_{4\text{ V}} = (4)(-8) = -32\text{ W}$$

$$P_{DS} = (8I_x)(11) = (16)(11) = 176\text{ W}$$

POWER BALANCE

$$-12 + 6I_o - 108 - 30 - 32 + 176 = 0$$

$$I_O = 1[A]$$

STOP