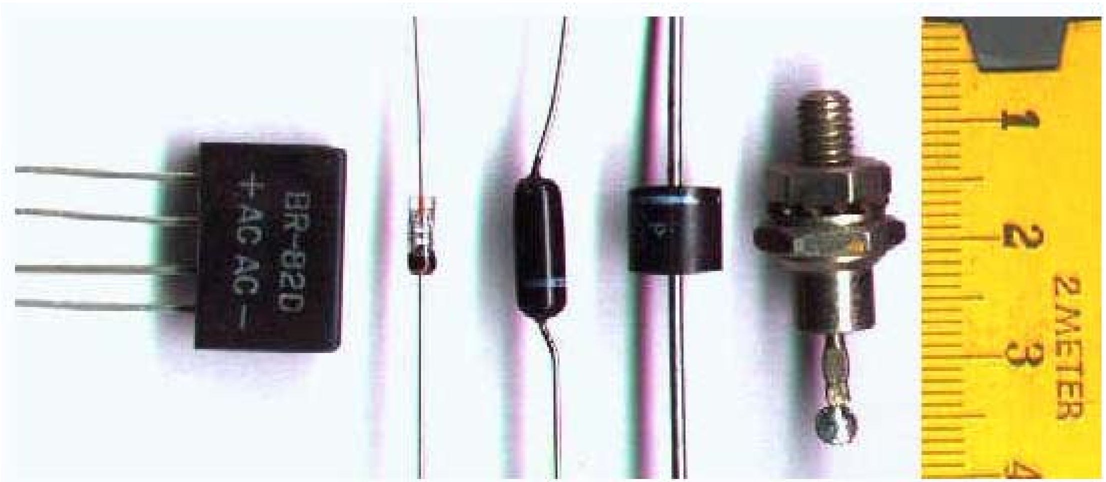
# *Week 2: PN Junction Diode*




types of diodes - from the German wikipedia

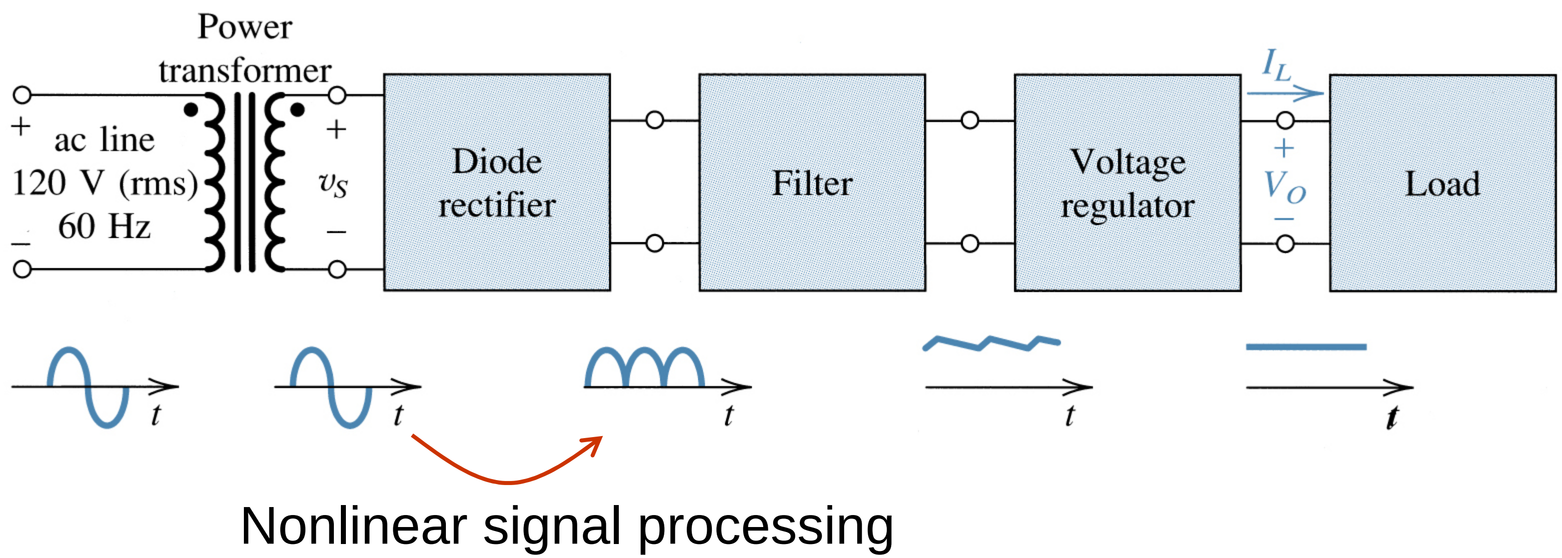
# Topics to cover…

- Physical operation of *pn* junction diode
- Terminal I-V characteristics
- Breakdown and junction capacitances
- Diode circuits analysis

- *Reading Assignment: Chap 3.1-3.7, 3.10-3.13 of Jaeger & Blalock*

# Why diodes?

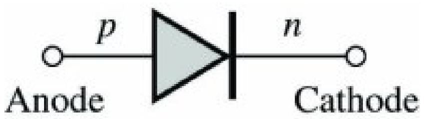
- R, C, L are linear circuit elements
- Many signal processing functions need non-linear elements, like signal rectification (to convert negative signal to be positive).




An AC-DC converter

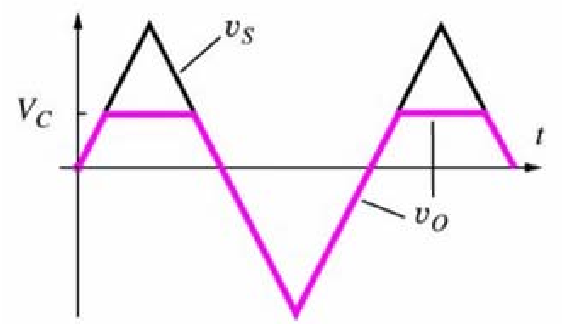
# Why diodes?

- Basic function of diode: to allow current to flow only in one direction.

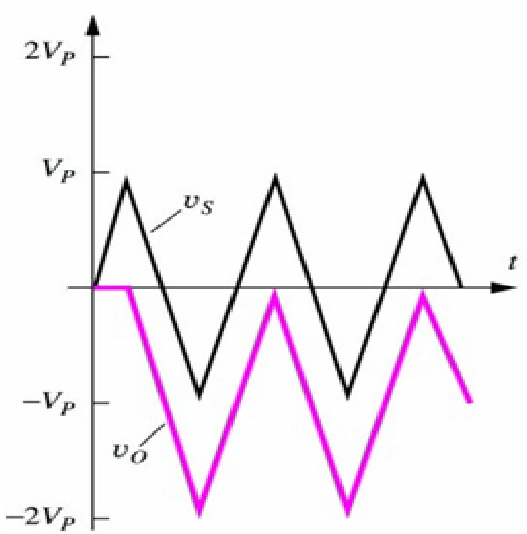



Diode symbol

- Applications:
  – Rectifiers
  – Waveform clipping and clamping circuits
  – DC-DC converters

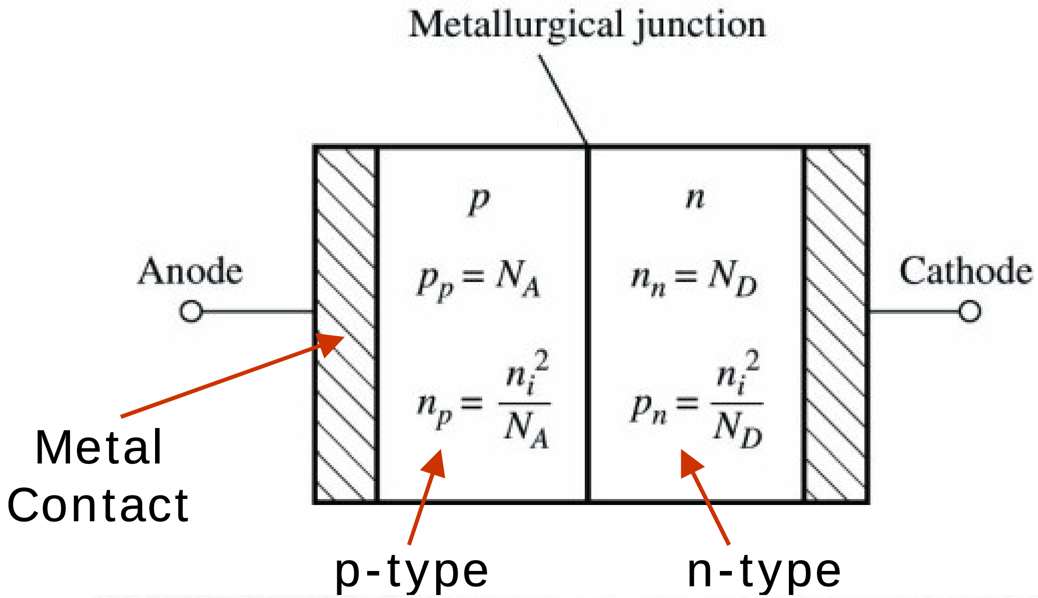


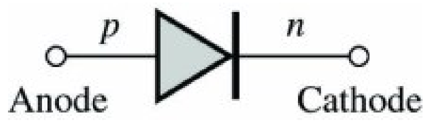

Waveform clipping



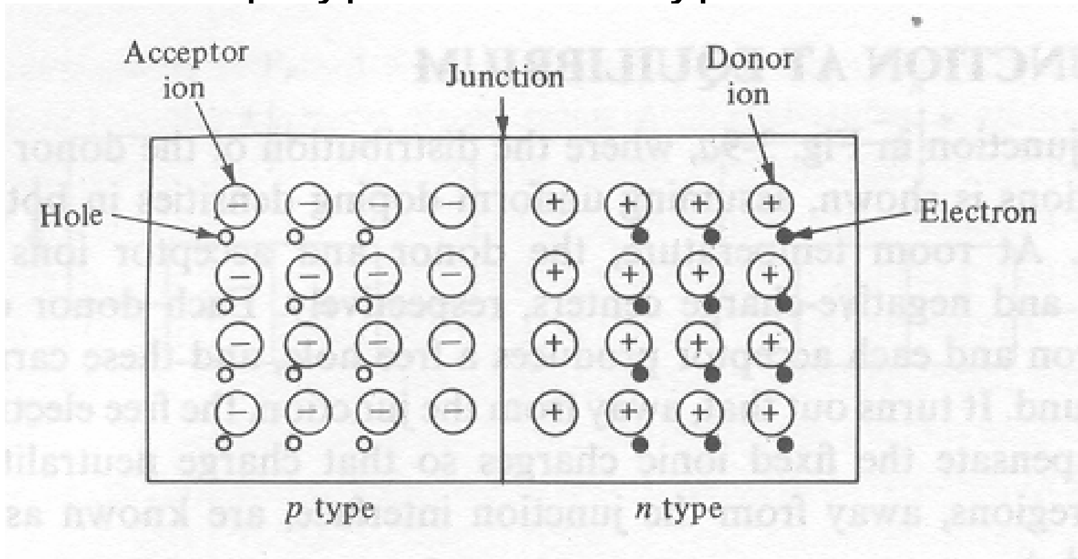

Waveform clamping

# Physical Structure of pn Junction Diode



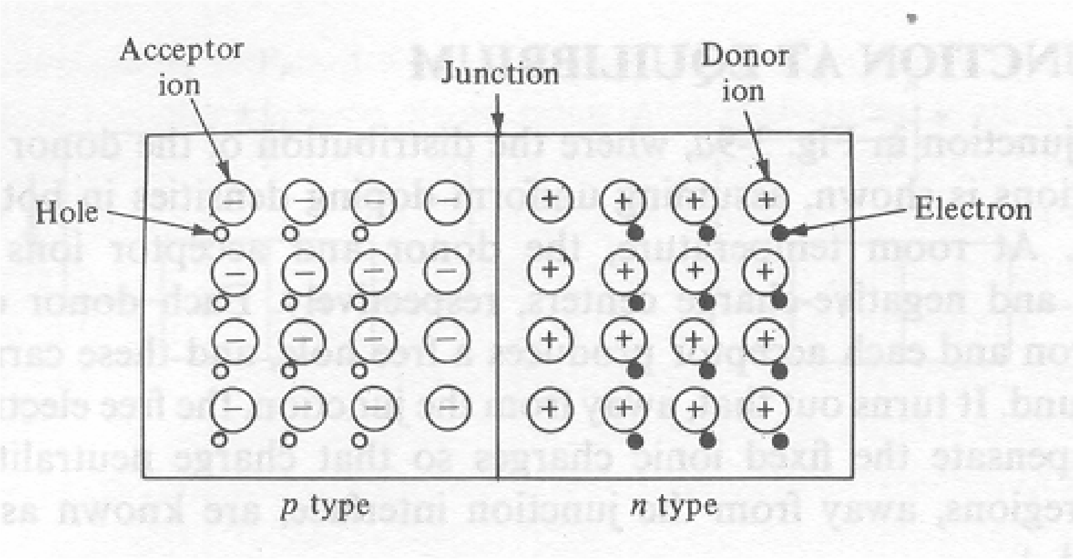

Diode symbol




Note:
both p and n regions are charge neutral.

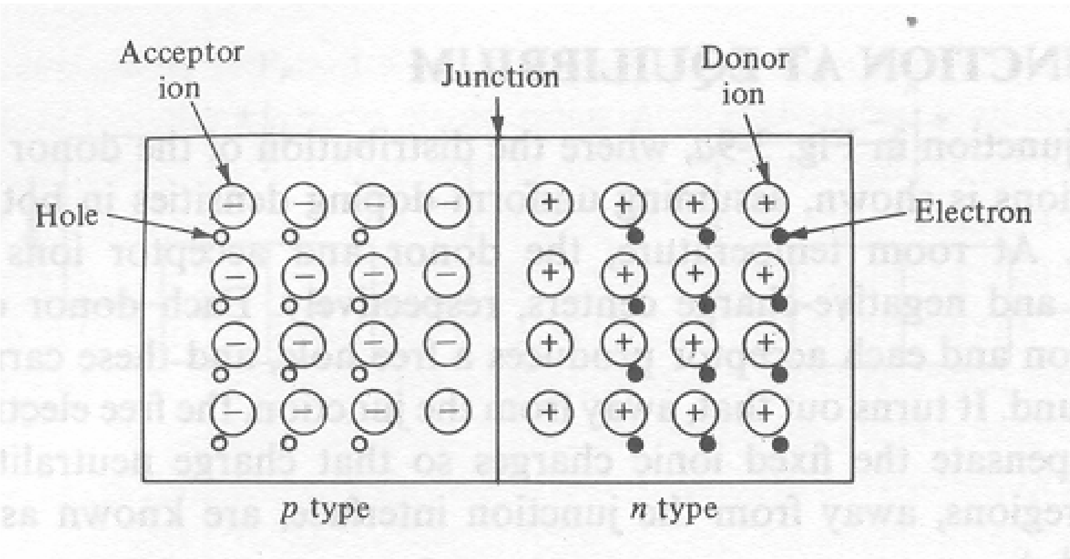
# Diffusion of Electrons and Holes

- Electrons tend to diffuse from n side to p side
- Holes tend to diffuse from p side to n side
- The resulted current is diffuse current. It is due to the movement of majority carriers.
- Q: What are the directions of electron and hole diffusion currents?



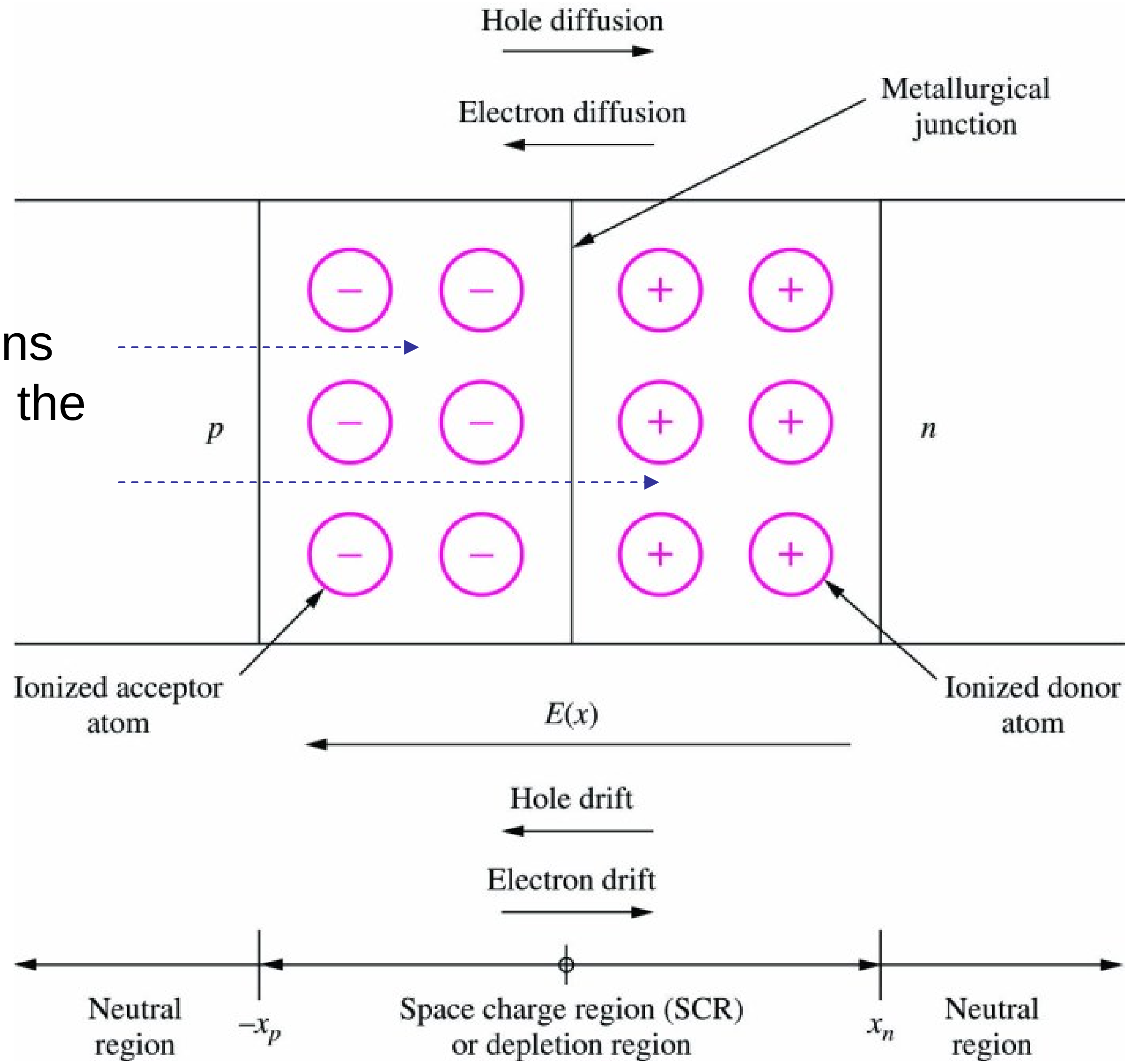
# Diffusion of Electrons and Holes

- If diffusion processes were to continue unabated, there would eventually be a uniform distribution of electrons and holes.
- But there is a counter force…



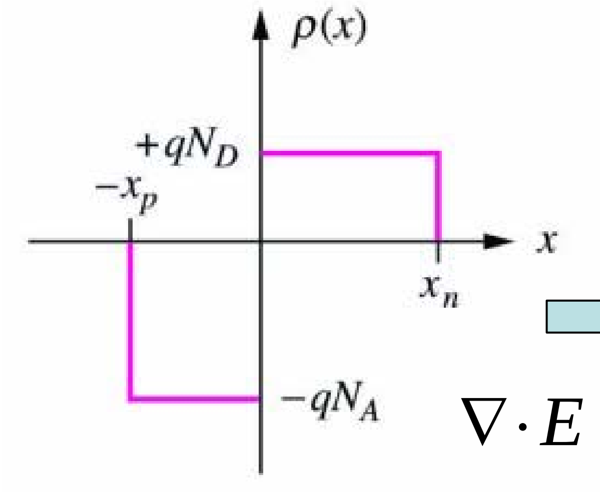
# Formation of Depletion Region

Holes and electrons are depleted from the dopant atoms

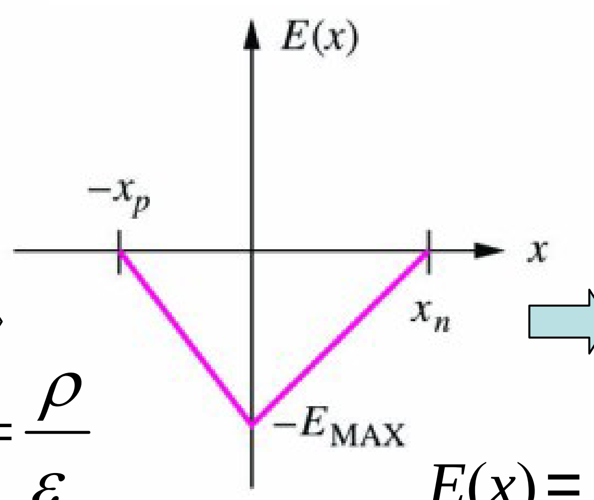


# Built-in Electric Field

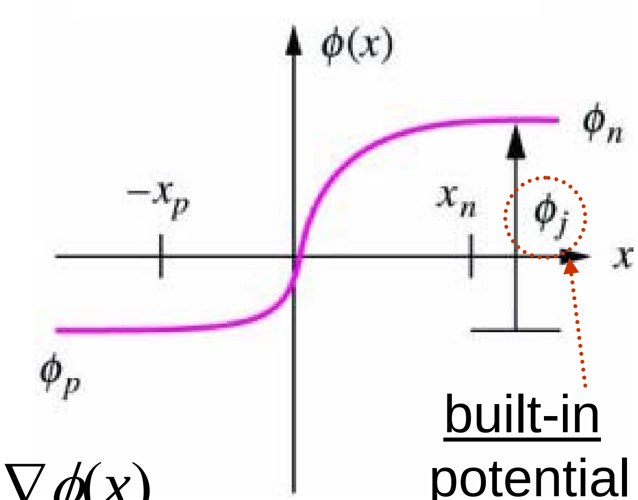
Charge Density




Electric Field




Potential




$$\nabla \cdot E = \frac{\rho}{\varepsilon_s}$$

(Gauss' Law)

$$E(x) = \frac{1}{\varepsilon_s} \int \rho(x) dx$$

(1 dimension)

$$E(x) = -\nabla \phi(x)$$

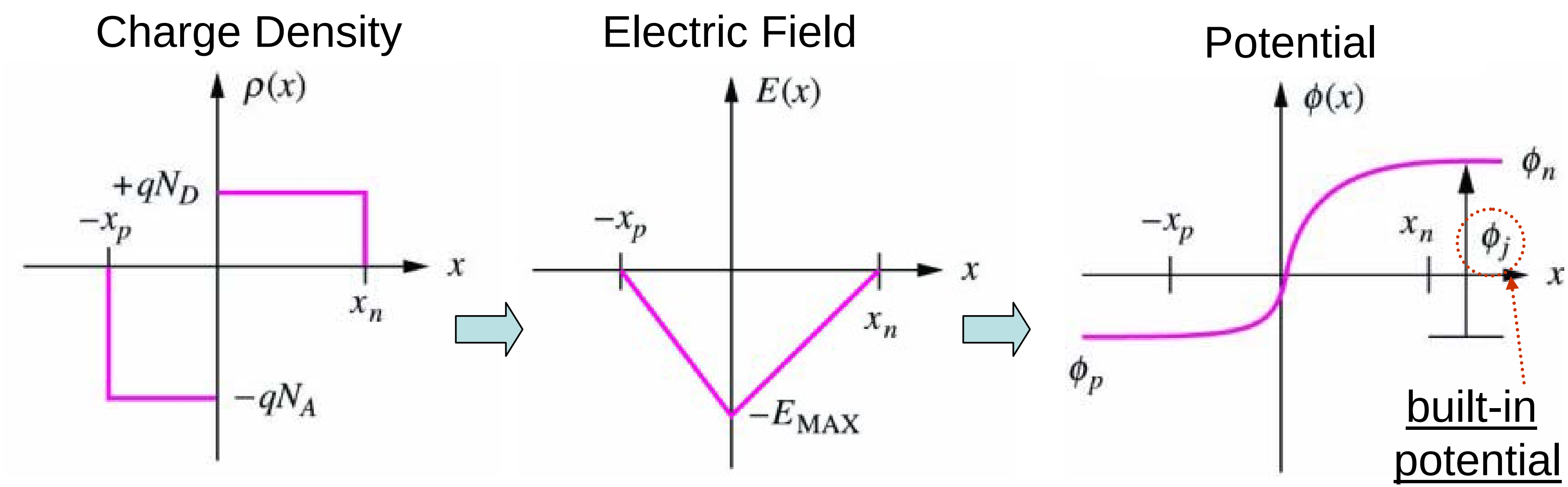
$$\phi_j = -\int E(x) dx$$

Solid-state physics tells us that:

$$\phi_j = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

The resulting potential gives rise to *drift currents*.

# Built-in Electric Field



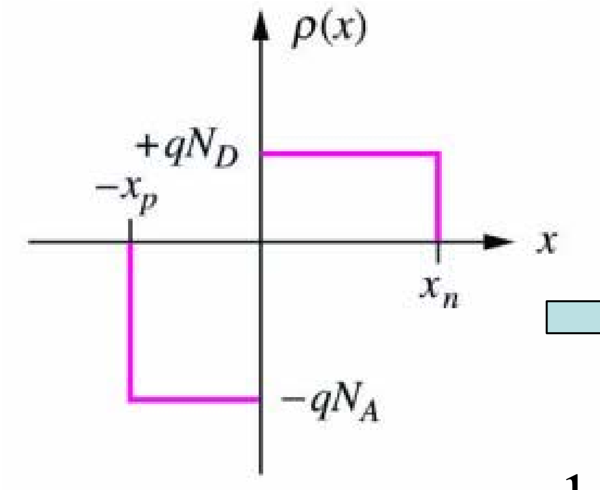

Direction of drift current:

Drift current is due to the movement of minorities.

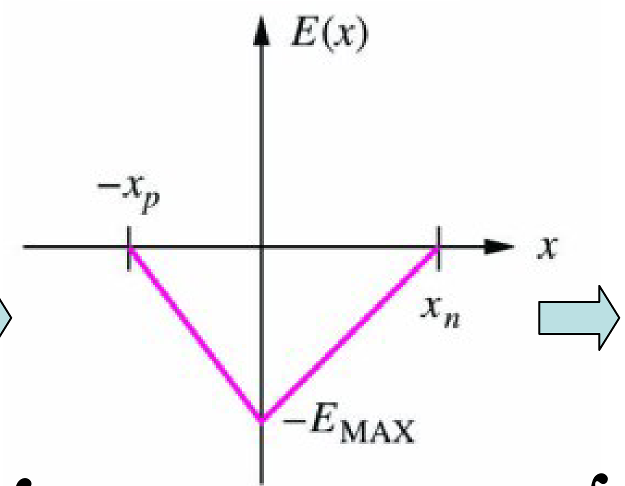
For a diode with no external connections, the total current through it must be zero. Therefore, under thermal equilibrium (no external voltage), drift and diffusion currents have the same magnitude but opposite direction.

# Width of Depletion Region

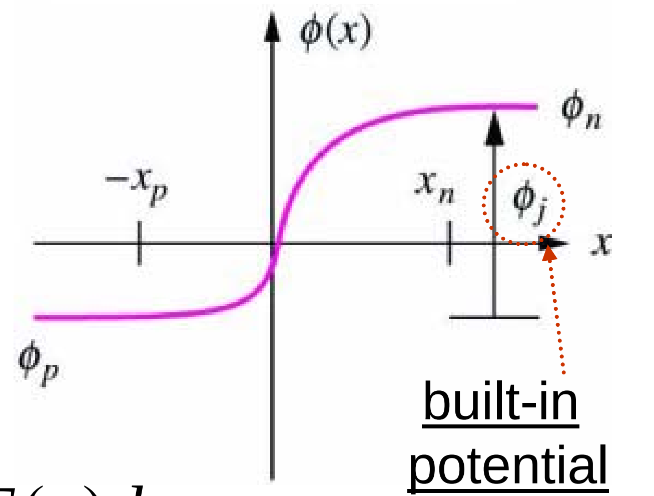
Charge Density




Electric Field




Potential




$$E(x) = \frac{1}{\varepsilon_s}\int \rho(x)dx$$

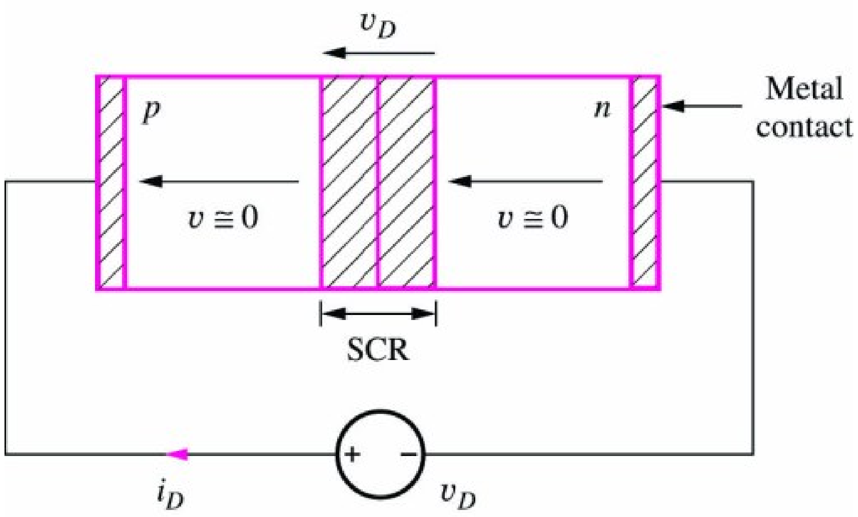
$$\phi_j = -\int E(x)dx$$

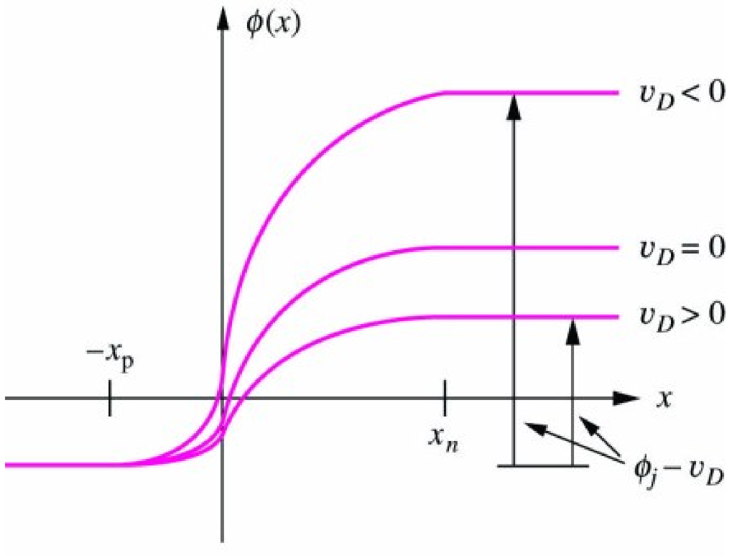
$$\phi_j = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Depletion width is an important device parameter.
It can be shown that:

$$w_{d0} = (x_n + x_p) = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\phi_j}$$

# Forward-Biased PN Junction



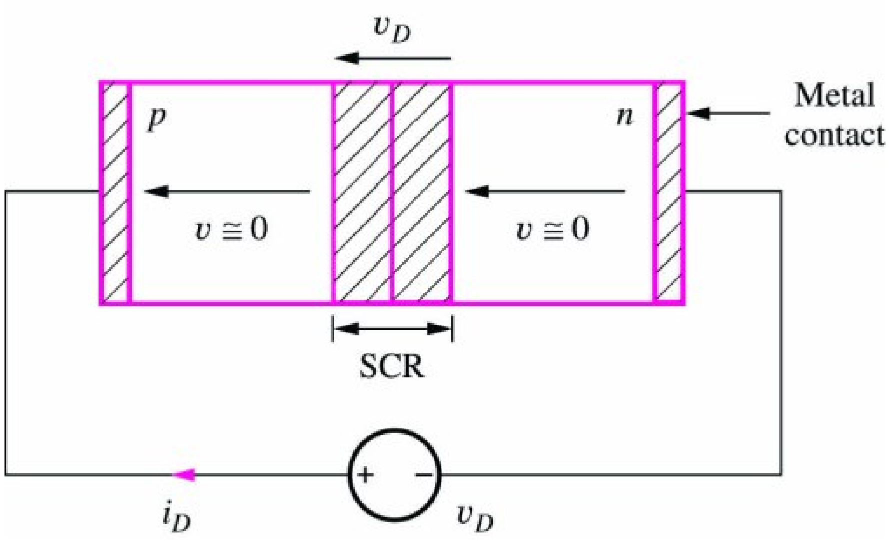

Forward biasing means applying an external voltage $v_D > 0$:

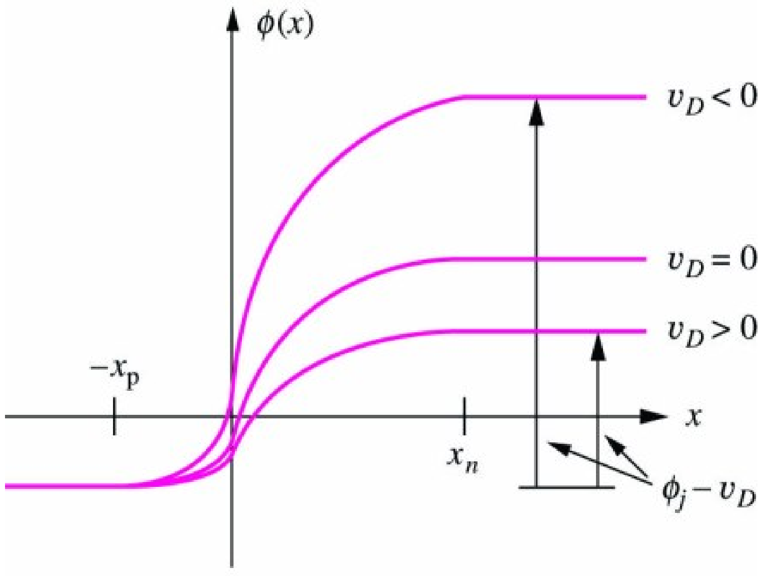
- Built-in potential drops (barrier lowers)
- Majority electrons and holes easily cross the junction
- Balance between diffusion and drift breaks
- Current appears at the terminals:

$$i_D = I_{diffusion} - I_{drift} > 0$$

In fact, $I_{diffusion}$ increases exponentially with $V_D$.

# Reverse-Biased PN Junction




Reverse biasing means applying an external voltage $v_D < 0$:

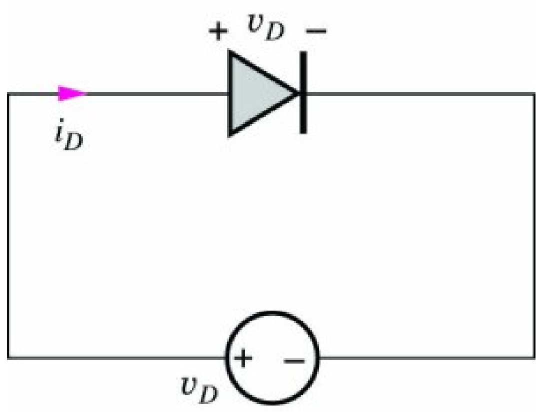
- Built-in potential increases/barrier increases;
- Majority electrons and holes can hardly cross the junction
  - Diffusion decreases
- Drift current almost unchanged as there are little supply of carriers (minorities).

  $\therefore\ i_D = I_{diffusion} - I_{drift} < 0$ and tends to $-I_{drift}$.

# Topics to cover…

- Physical operation of *pn* junction diode
- Terminal I-V characteristics
- Breakdown and junction capacitances
- Diode circuits analysis

# Diode Terminal I-V Characteristics




$$i_D = I_S\left[\exp\left(\frac{v_D}{V_T}\right) - 1\right]$$

where $I_S$ = reverse saturation current [A]

$v_D$ = voltage applied to diode [V]

$V_T$ = kT/q = thermal voltage [V] (25 mV at room temp.)

$q$ = electronic charge [$1.60 \times 10^{-19}$ C]

$k$ = Boltzmann's constant [$1.38 \times 10^{-23}$ J/K]

$T$ = absolute temperature [K]

# Diode Current for Reverse, Zero, and Forward Bias

- Reverse bias:

$$i_D = I_S\left[\exp\left(\frac{v_D}{V_T}\right)-1\right] \approx I_S[0-1] \approx -I_S \quad \text{for } v_D < -4V_T$$

- Zero bias:

$$i_D = I_S\left[\exp\left(\frac{v_D}{V_T}\right)-1\right] = I_S[1-1] = 0$$

- Forward bias:

$$i_D = I_S\left[\exp\left(\frac{v_D}{V_T}\right)-1\right] \approx I_S \exp\left(\frac{v_D}{V_T}\right) \quad \text{for } v_D > 4V_T$$

# Example 1

**Problem:** Find diode voltage for diode with given specifications

**Given data**: $I_S$=0.1 *f*A, $I_D$= 300 μA / 1mA

**Assumptions**: Room-temperature dc operation with $V_T$ =25mV

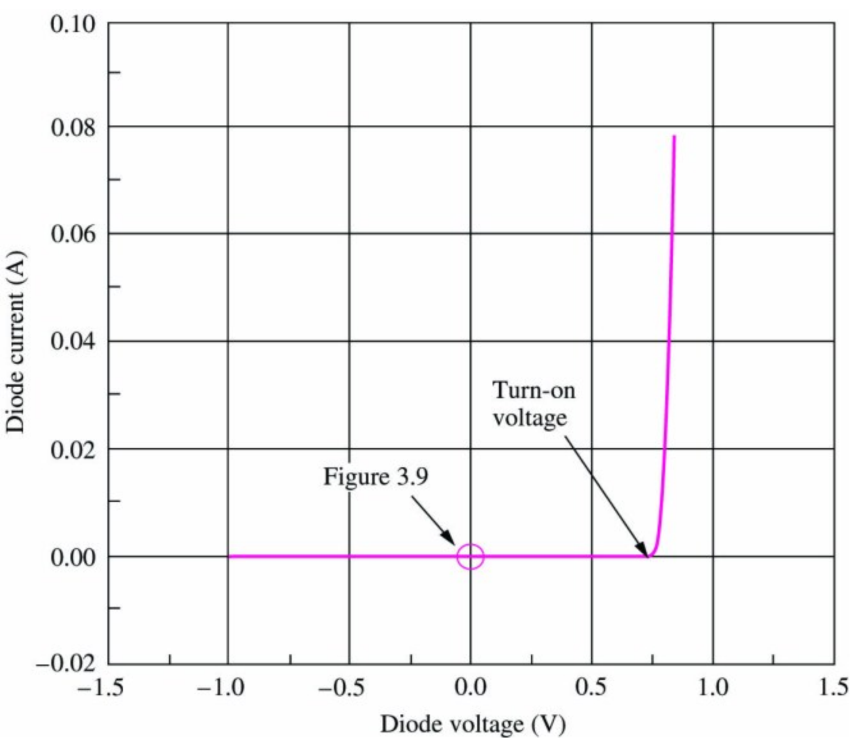
**Analysis:**

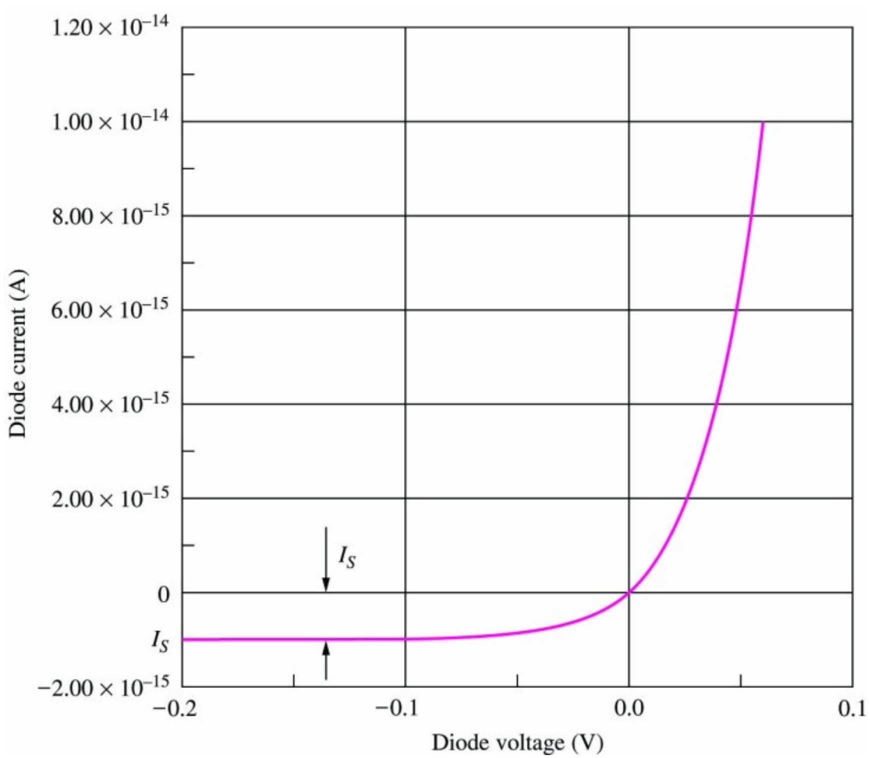
With $I_S$=0.1 fA, $I_D$= 300 μA

$$V_D = V_T \ln\left(1+\frac{I_D}{I_S}\right) = (0.025\text{V})\ln(1+\frac{3\times10^{-4}\text{A}}{10^{-16}\text{A}}) = 0.718\text{V}$$

With $I_D$= 1 mA, $I_S$=0.1 fA $\qquad V_D = 0.748\text{V}$

# Diode *i-v* Characteristics

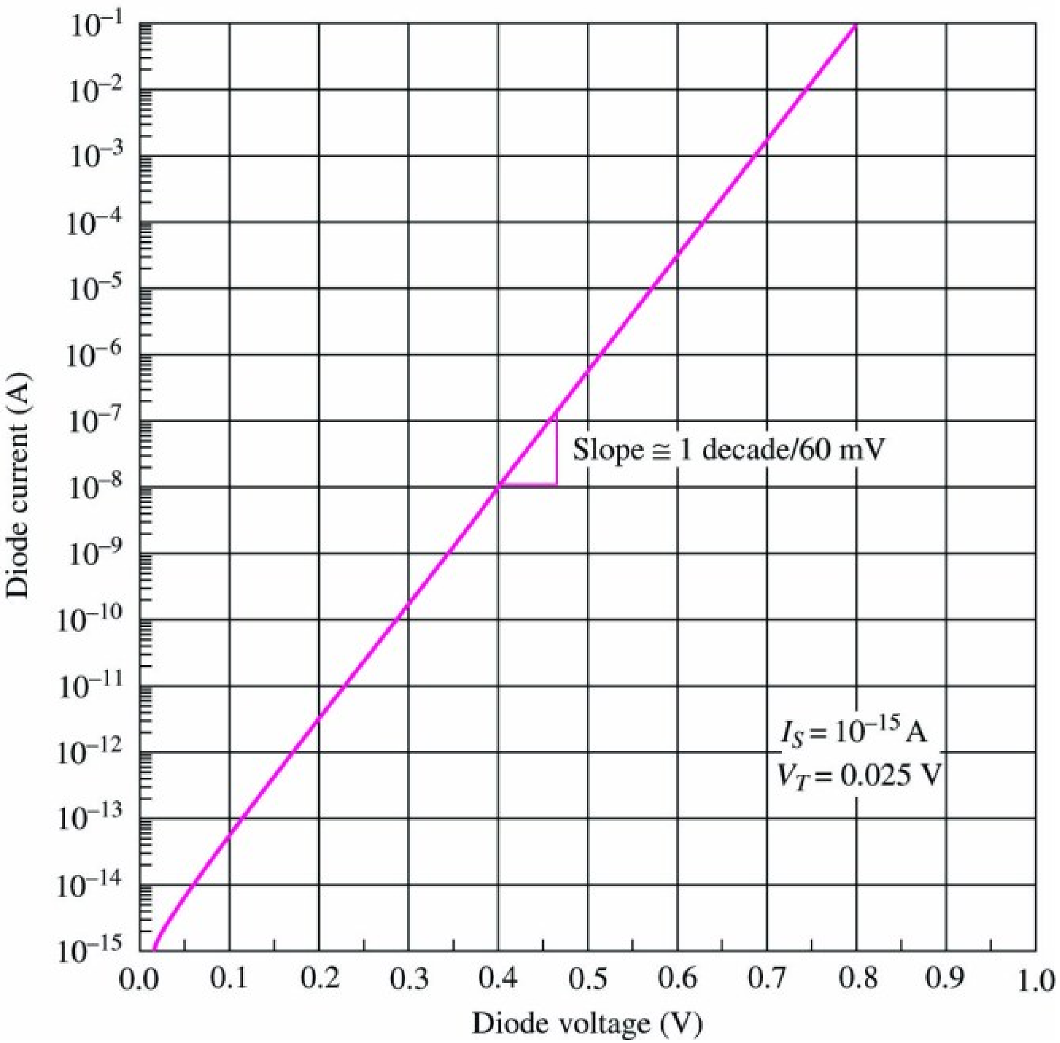



Turn-on voltage marks point of significant current flow.

$I_s$ : reverse saturation current

# Semi-log Plot of Diode Current

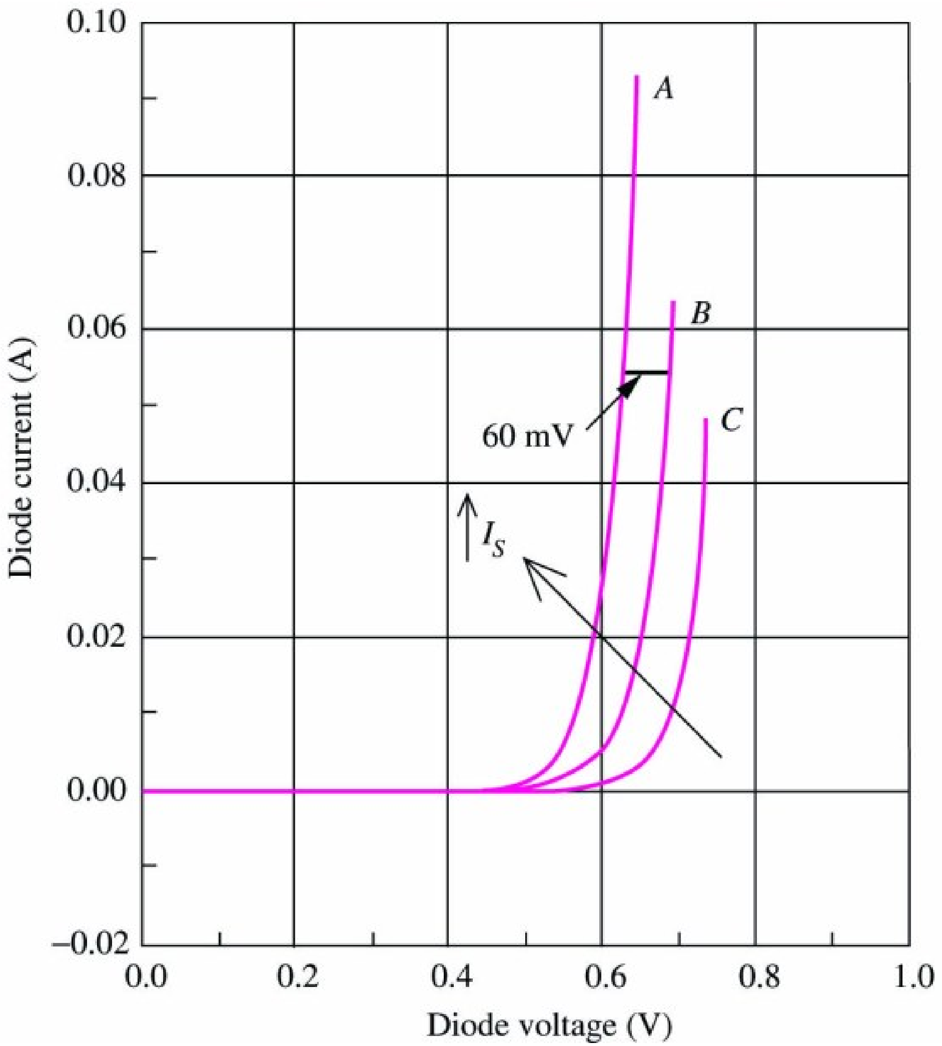



$$i_D = I_S \exp\left(\frac{v_D}{V_T}\right)$$

$$\text{Let } \frac{i_{D1}}{i_{D2}} = \exp\left(\frac{v_{D1} - v_{D2}}{V_T}\right) = 10$$

$$\begin{aligned} v_{D1} - v_{D2} &= V_T \ln 10 \\ &= 2.3 V_T \\ &\approx 60\,\text{mV} \end{aligned}$$

# Current for Three Different Values of $I_S$




$$I_{SA} = 10 I_{SB} = 100 I_{SC}$$

$$i_D = I_S \exp\left(\frac{v_D}{V_T}\right)$$

Let $i_{DA} = i_{DB}$

$$\frac{i_{SA}}{i_{SB}} = \exp\left(\frac{v_{DB} - v_{DA}}{V_T}\right) = 10$$

$$v_{DB} - v_{DA} = V_T \ln 10 \approx 60\,\text{mV}$$

# Topics to cover…

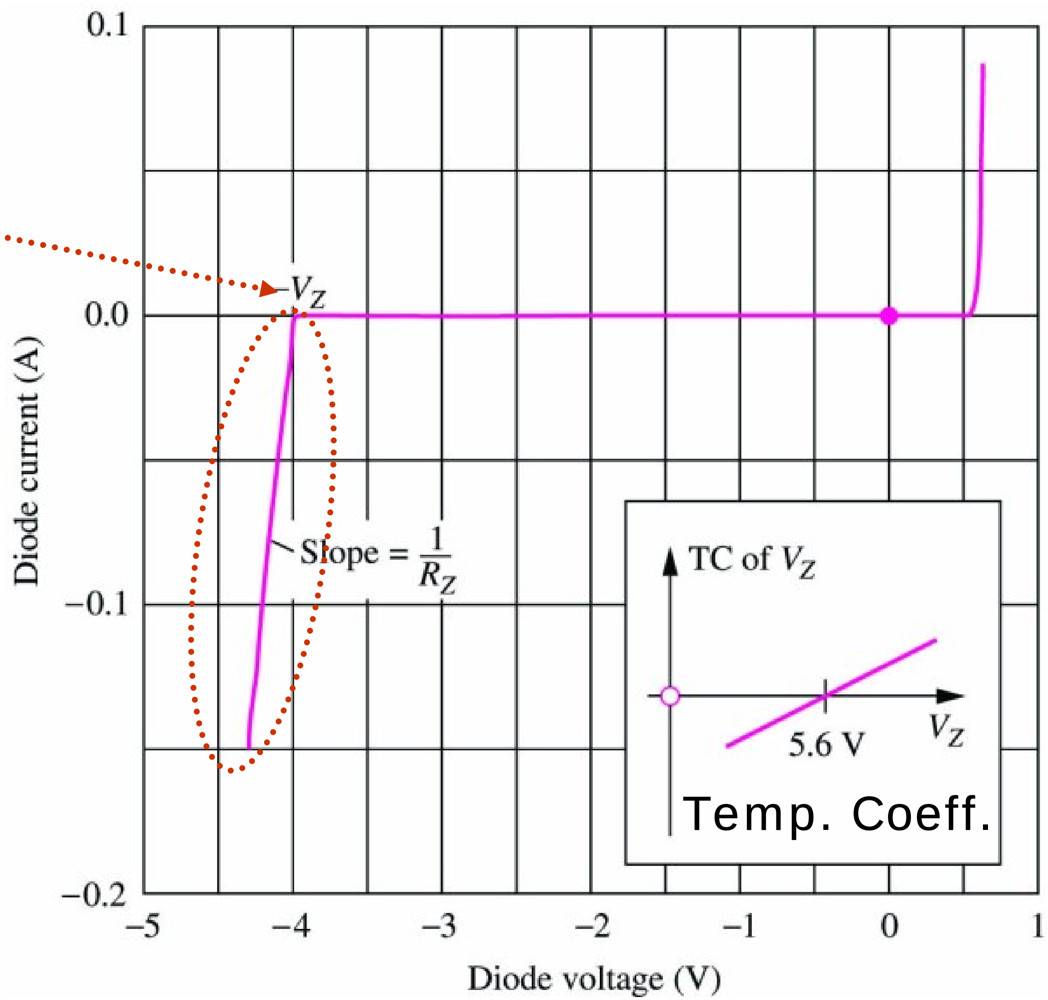
- Physical operation of *pn* junction diode
- Terminal I-V characteristics
- Breakdown and junction capacitances
- Diode circuits analysis

# Reverse Breakdown

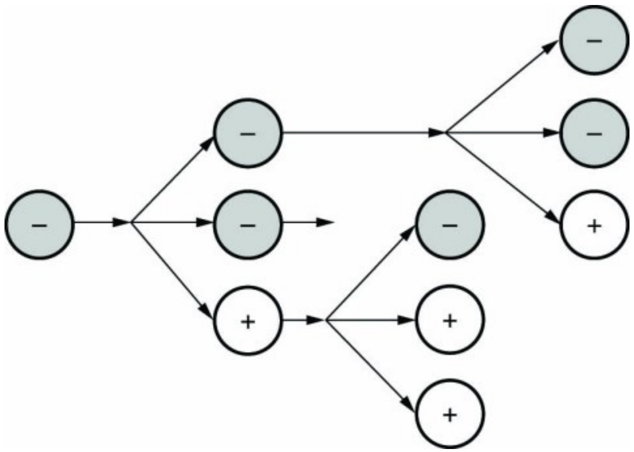
**Breakdown region:**

Rapid increase in $I_D$ when reverse bias voltage exceeds a **break down voltage** $V_Z$

Typical value of $V_z$:
$2\ V \leq V_Z \leq 2000\ V$



# Breakdown Mechanism 1: Avalanche Breakdown



- Carriers in the SCR are accelerated by the internal electric field and collide with fixed atoms
- As the reverse bias increases, the energy of the accelerated carriers increases, eventually breaking covalent bonds and generating electron-hole pairs
- The newly created carriers also accelerate, collide with atoms, and generate other electron-hole pairs. This process feeds on itself and leads to *avalanche breakdown*
- Si diodes with $V_Z > 5.6$ V enter breakdown through this mechanism

# Breakdown Mechanism 2: Zener Breakdown

- *Zener breakdown:* occurs when the electric field in the depletion region increases to the point where it can break covalent bonds and generate electron-hole pairs.

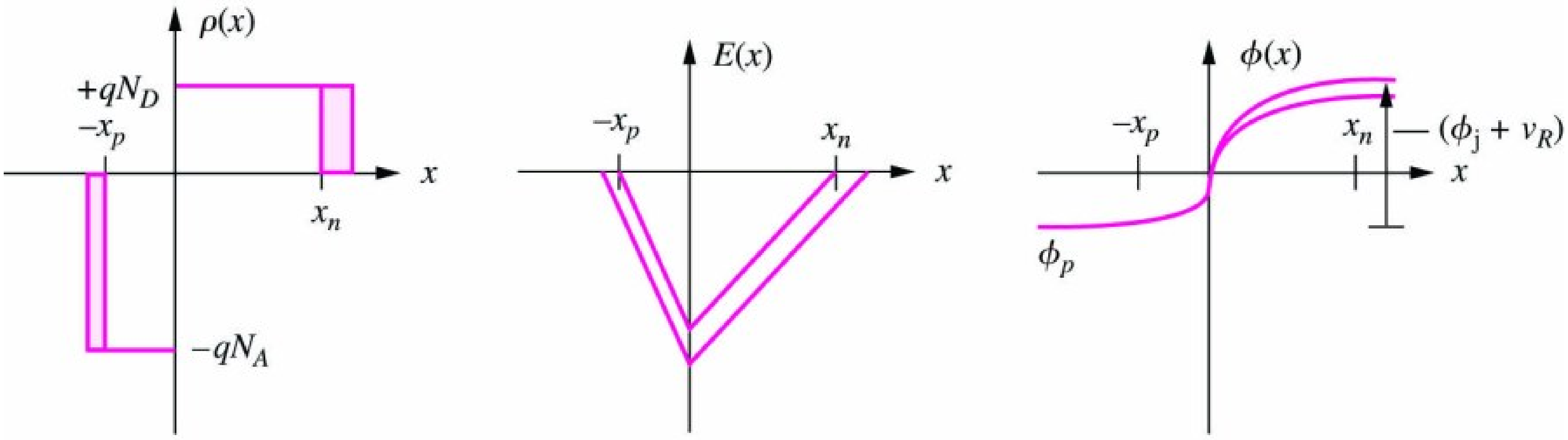
- $V_Z < 5.6$ V for diodes entering breakdown through this mechanism

- In avalanche breakdown, $V_Z$ increases with temperature
- In Zener breakdown, $V_Z$ decreases with temperature

- Breakdown is not a destructive process provided that the maximum specified power dissipation is not exceeded

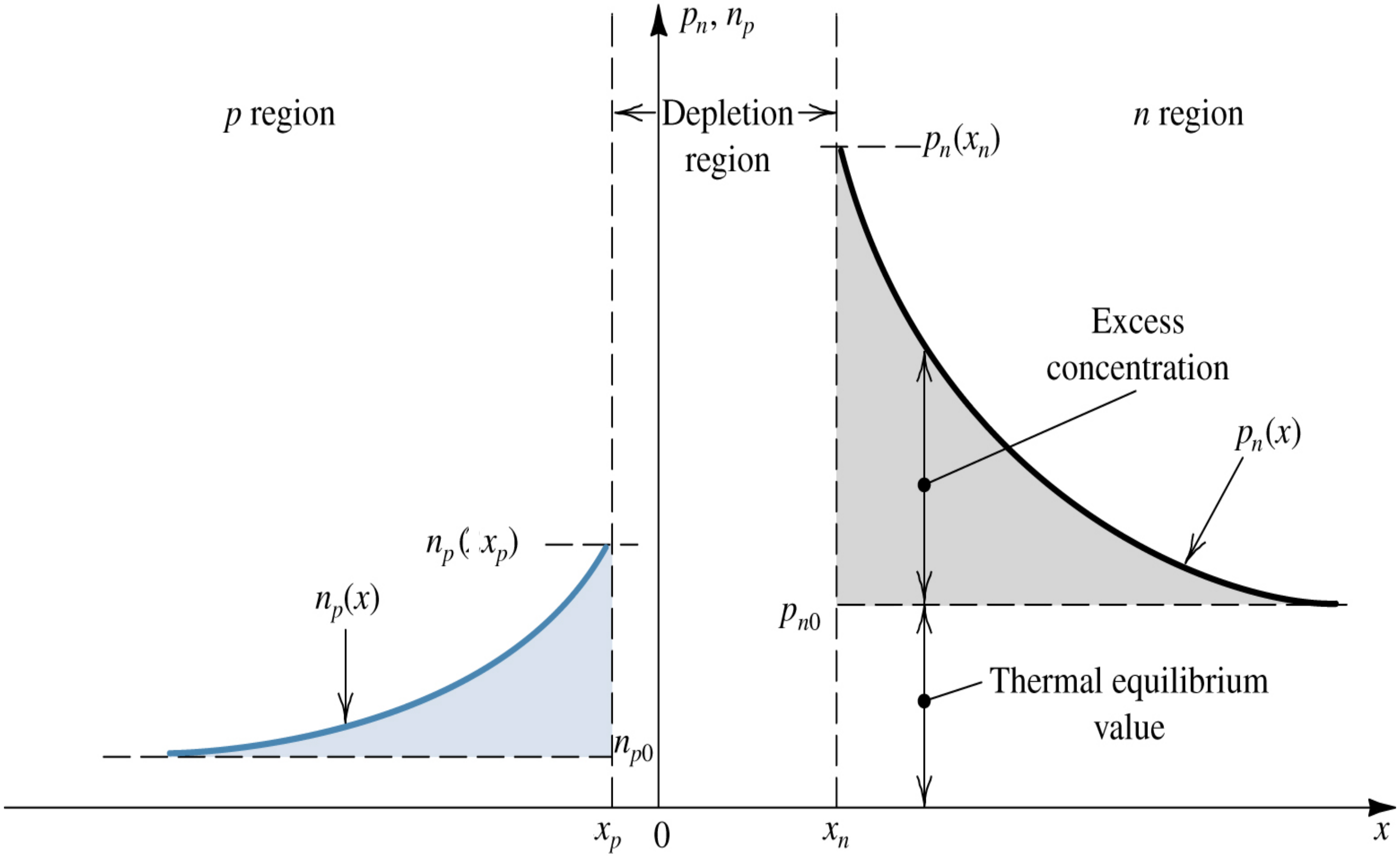
# *pn* Junction Capacitance: Reverse Bias

- External reverse bias leads to:
  - Increases of the built-in potential
  - Increases of the SCR, and thus fixed space charges
- This behaviors like a capacitor, and is referred to as *depletion capacitance*.




(a) Space charge density (b) Electric field (c) Electrostatic potential

# *pn* Junction Capacitance: Forward Bias




Forward bias minority distribution in neutral regions

# *pn* Junction Capacitance: Forward Bias

Excess charge stored in neutral region near edges of space charge region is

$$Q_D = i_D \tau_T \ \text{Coulombs}$$

$t_T$ is called diode *transit time* and depends on size and type of diode.

The Q depends on $i_D$ and in turn on $v_D$. This capacitance behavior is referred to as *diffusion capacitance* and given by:

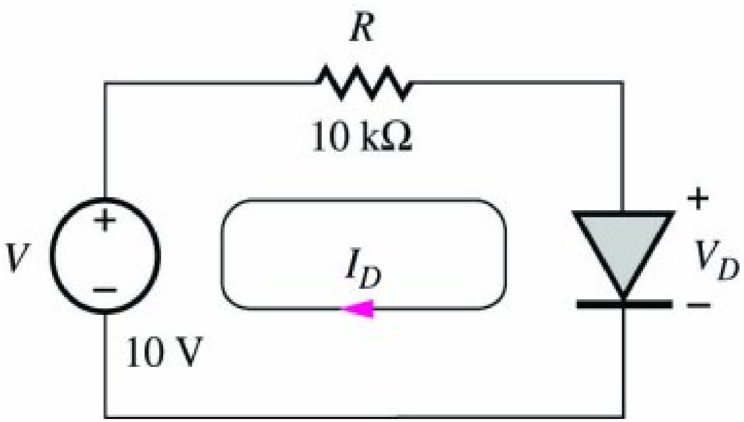
$$C_j = \frac{dQ_D}{dv_D} = \frac{dQ_D}{di_D}\frac{di_D}{dv_D} \cong \frac{i_D \tau_T}{V_T} \ \text{[F]}$$

$$\left| i_D = I_S \left[ \exp\left(\frac{v_D}{V_T}\right) - 1 \right] \approx I_S \exp\left(\frac{v_D}{V_T}\right) \right.$$

Diffusion capacitance is proportional to current and becomes quite large at high currents.

# Diode Circuit Analysis: Basics

<u>Objective</u>: to find the DC current ($I_D$) and voltage ($V_D$), or **quiescent operating point (Q-point)** of the diode.




V and R may represent Thevenin equivalent of a more complex 2-terminal network.

Full set of equations:

1. Diode equation:

$$\Rightarrow \quad I_D = I_S\left[\exp\left(\frac{V_D}{V_T}\right) - 1\right]$$

nonlinear

2. KVL along the loop:

$$\Rightarrow \quad V = I_D R + V_D$$

This is also called the **load line** equation of the diode, as if the V-source and R were the load of the diode.

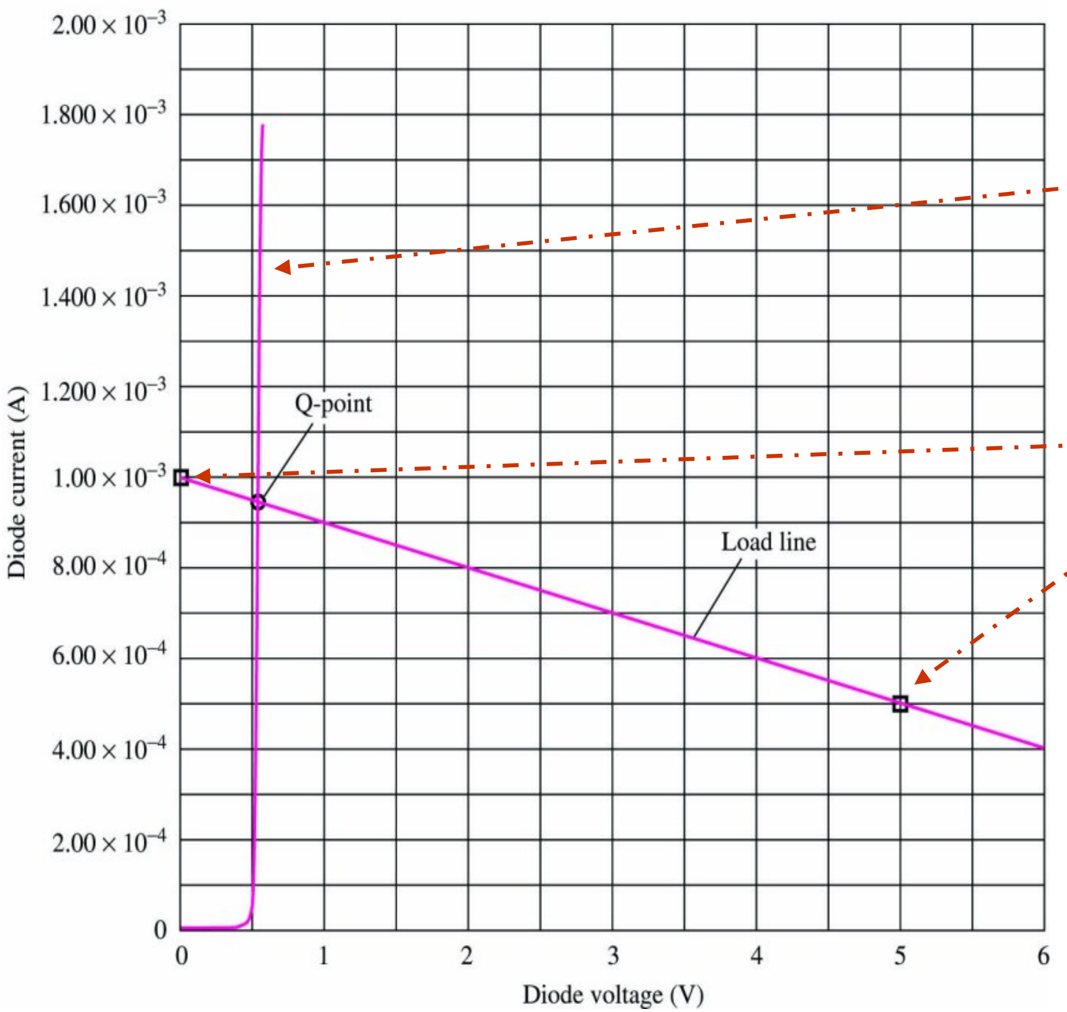
# Topics to cover…

- Physical operation of *pn* junction diode
- Terminal I-V characteristics
- Breakdown and junction capacitances
- Diode circuits analysis

# Diode Circuit Analysis Approaches

- Graphical analysis (load-line method)
- Numerical Analysis with diode’s mathematical model
- Simplified analysis with ideal diode model.
- Simplified analysis using constant voltage drop model.

# Load-Line Analysis (Example 2)




**Problem:** Find Q-point

**Given data:** $V$=10 V, $R$=10kΩ. Diode I-V curve.

**Analysis:**

$$10 = I_D 10^4 + V_D$$

To define the load line we use,

$$V_D = 0, \quad I_D = (10\text{V} / 10\text{k}\Omega) = 1\,\text{mA}$$

$$V_D = 5, \quad I_D = 0.5\,\text{mA}$$

These points and the resulting load line are plotted. Q-point is given by **intersection** of load line and diode characteristic:

**Q-point = (0.95 mA, 0.6 V)**

# Numerical Analysis (Example 3)

**Problem:** Find Q-point for given diode characteristics.
**Given data:** $I_S = 10^{-13}$A, $V_T$ =25 mV, $V$ = 10 V, $R$ = 10kΩ.
**Analysis:**

$$\begin{cases} I_D = I_S\left[\exp\left(V_D / V_T\right) - 1\right] = 10^{-13}\left[\exp\left(40V_D\right) - 1\right] \\ 10 = I_D 10^4 + V_D \end{cases}$$

$$\Rightarrow 10 = 10^4 10^{-13}\left[\exp\left(40V_D\right) - 1\right] + V_D$$

This is a *transcendental equation* and does not have analytical solutions. A numerical answer can be found by using *Newton's iterative* method.

Let: $$f = 10 - 10^{-9}\left[\exp\left(40V_D\right) - 1\right] - V_D$$

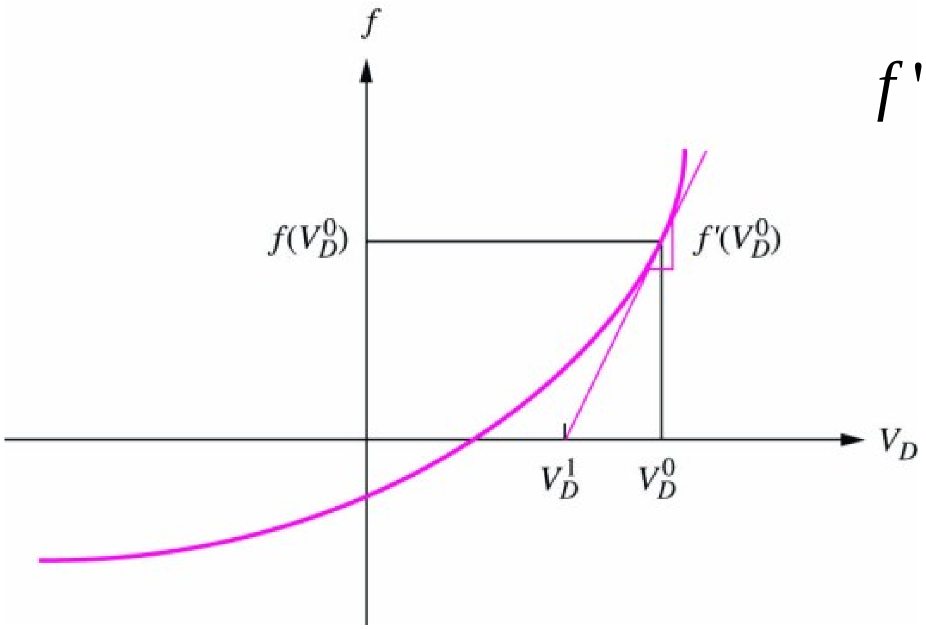
We desire to find $V_D$ for which $f$=0.

# Newton's Iteration Method for $f(v_D) = 0$

1. Make initial guess $V_D^{\,0}$
2. Evaluate $f$ and its derivative $f\,'$ for this value of $V_D$
3. Calculate new guess for $V_D$ using

$$V_D^{\,1} = V_D^{\,0} - \frac{f\left(V_D^{\,0}\right)}{f\,'(V_D^{\,0})}$$

4. Repeat steps 2 and 3 till convergence



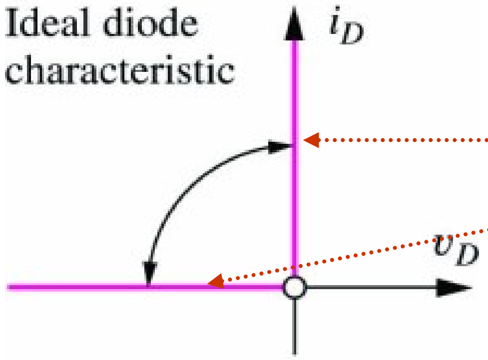

$$f\,'\left(V_D^{\,0}\right) = \frac{f\left(V_D^{\,0}\right) - 0}{V_D^{\,0} - V_D^{\,1}}$$

# Solution from a Spreadsheet

| Iteration # | *V_D* [V] | *f* | *f'* | *I_D* [A] |
|---|---|---|---|---|
| 0 | 0.8000 | -7.895E+04 | -3.159E+06 | 7.896E+00 |
| 1 | 0.7750 | -2.904E+04 | -1.162E+06 | 2.905E+00 |
| 2 | 0.7500 | -1.068E+04 | -4.276E+05 | 1.069E+00 |
| 3 | 0.7250 | -3.927E+03 | -1.575E+05 | 3.936E-01 |
| 4 | 0.7001 | -1.442E+03 | -5.806E+04 | 1.452E-01 |
| 5 | 0.6753 | -5.281E+02 | -2.150E+04 | 5.374E-02 |
| 6 | 0.6507 | -1.918E+02 | -8.048E+03 | 2.012E-02 |
| 7 | 0.6269 | -6.817E+01 | -3.103E+03 | 7.754E-03 |
| 8 | 0.6049 | -2.281E+01 | -1.289E+03 | 3.220E-03 |
| 9 | 0.5872 | -6.455E+00 | -6.357E+02 | 1.587E-03 |
| 10 | 0.5770 | -1.148E+00 | -4.238E+02 | 1.057E-03 |
| 11 | 0.5743 | -5.989E-02 | -3.804E+02 | 9.486E-04 |
| 12 | 0.5742 | -1.876E-04 | -3.780E+02 | 9.426E-04 |
| 13 | **0.5742** | -1.858E-09 | -3.780E+02 | **9.426E-04** |

← **Q-point**

# Analysis using Ideal Diode Model

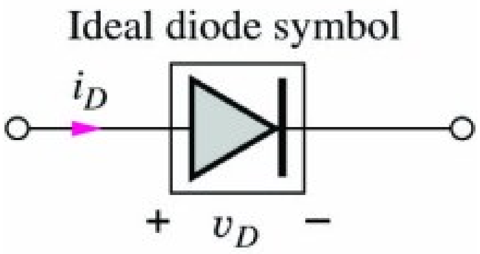


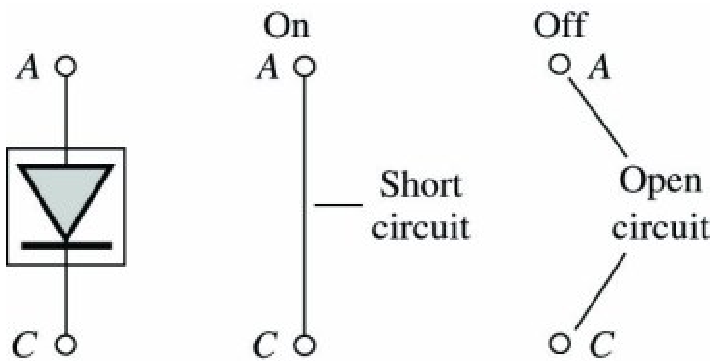

Ideal diode model:

**Forward-biased**: voltage = zero, $i_D > 0$
**Reverse-biased**: current = zero, $v_D < 0$

Diode is assumed to be either on or off.

I-V curve is **linear** in both regions.

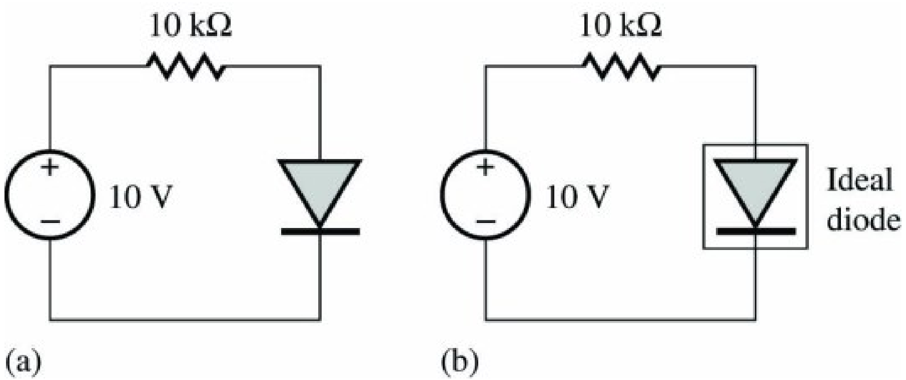


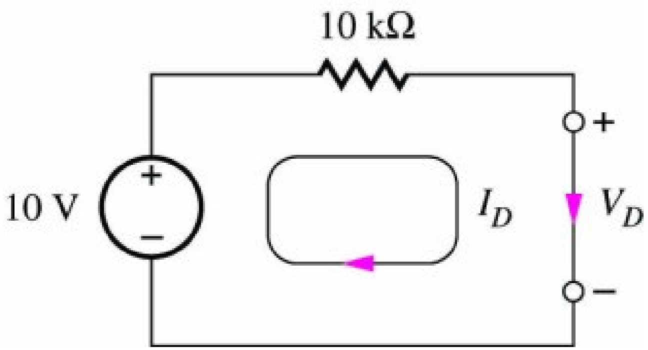

Analysis steps:

1. Identify anode and cathode of diode and label $v_D$ and $i_D$.
2. Guess diode's region of operation from circuit.
3. Analyze circuit using diode model appropriate for assumed operation region.
4. Check results to check consistency with assumptions.

# Example 4




Since source is forcing positive current through diode, assume diode is **on**. So $V_D$=0.

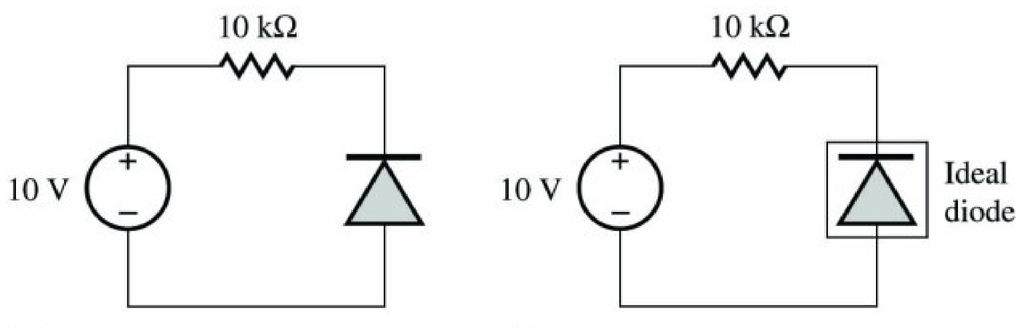



$$I_D = \frac{(10-0)\mathrm{V}}{10\mathrm{k}\Omega} = 1\,\mathrm{m}A$$

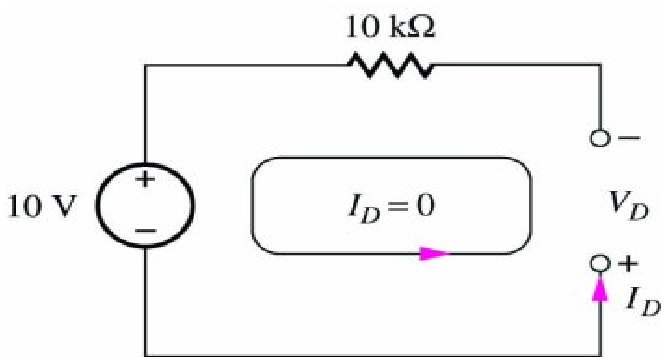
Since $I_D$ > 0, our assumption is correct.

**Q-point is (1 mA, 0V).**

# Example 5




Since source is forcing current backward through diode, assume diode is off. Hence $I_D$ =0.



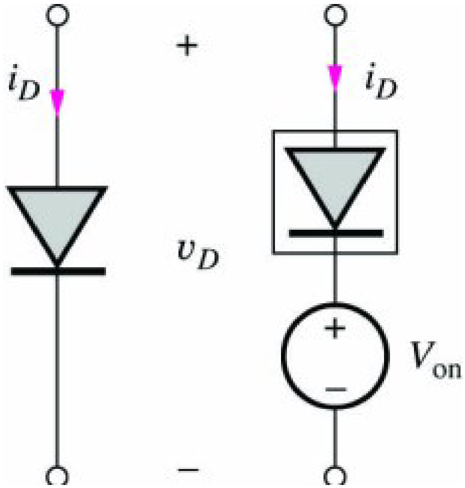

$$10 + V_D + 10^4 I_D = 0$$

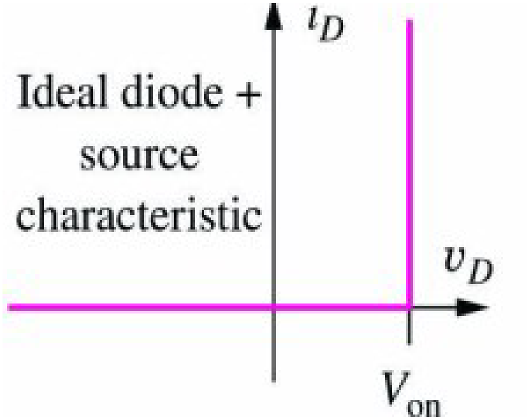
$$\therefore V_D = -10\text{V}$$

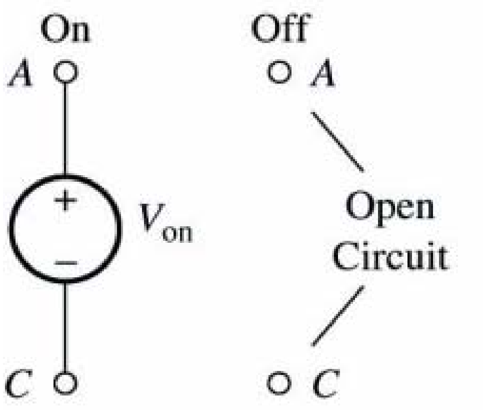
Since $V_D < 0$, our assumption is right.

**Q-point is (0, -10 V)**

# Constant Voltage Drop Model



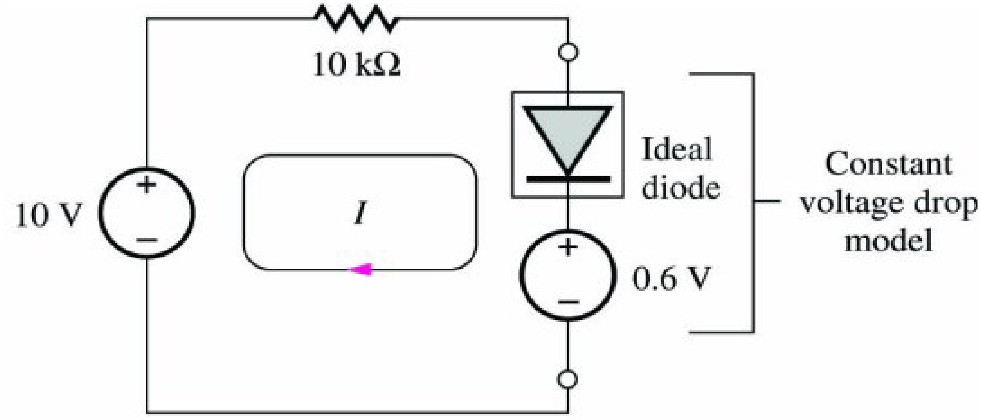



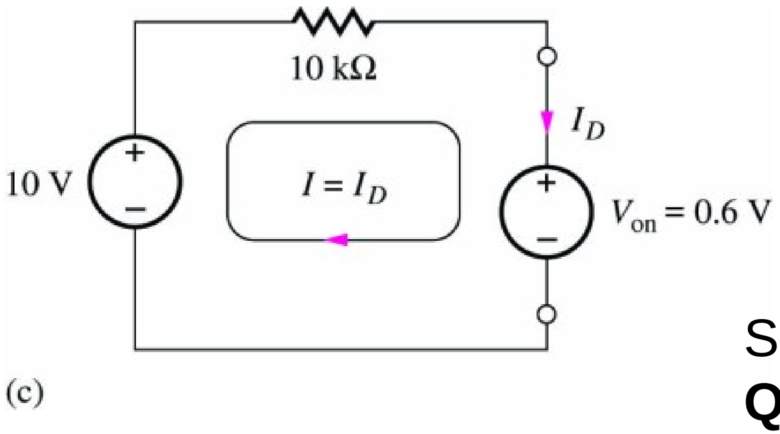

Forward-bias: $v_D = V_{on}$, $i_D > 0$

Reverse-bias: $i_D = 0$, $v_D < V_{on}$

# Example 6




**Analysis:** Since 10V source is forcing positive current through diode, we assume diode is on.



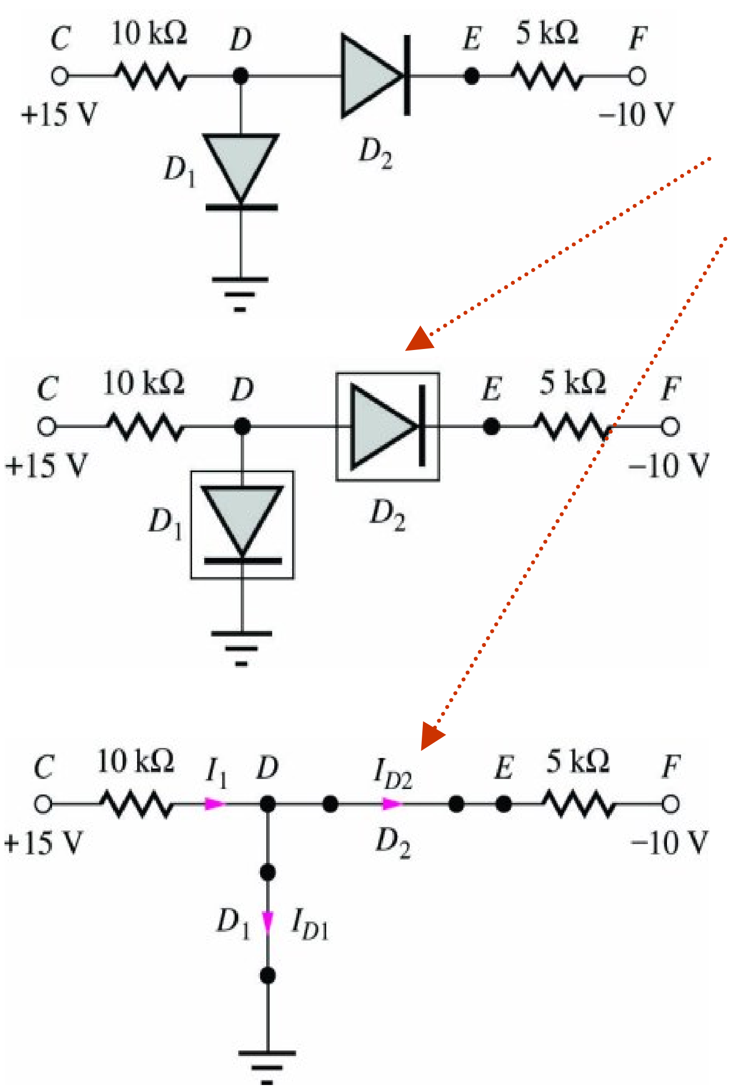

$$I_D = \frac{(10 - V_{on})\text{V}}{10\text{k}\Omega}$$

$$= \frac{(10 - 0.6)\text{V}}{10\text{k}\Omega} = 0.94\text{mA}$$

Since $I_D > 0$, our assumption is correct.

**Q-point is (0.94 mA, 0.6V).**

# Two-Diode Circuit Analysis (Example 7)




**Analysis:**

Choose ideal diode model.

Assume both diodes are on.

Since $V_{D1}=0$,

$$I_1 = \frac{(15-0)\text{V}}{10\text{k}\Omega} = 1.50\text{mA}$$

$$I_{D2} = \frac{0-(-10)\text{ V}}{5\text{ k}\Omega} = 2.00\text{ mA}$$

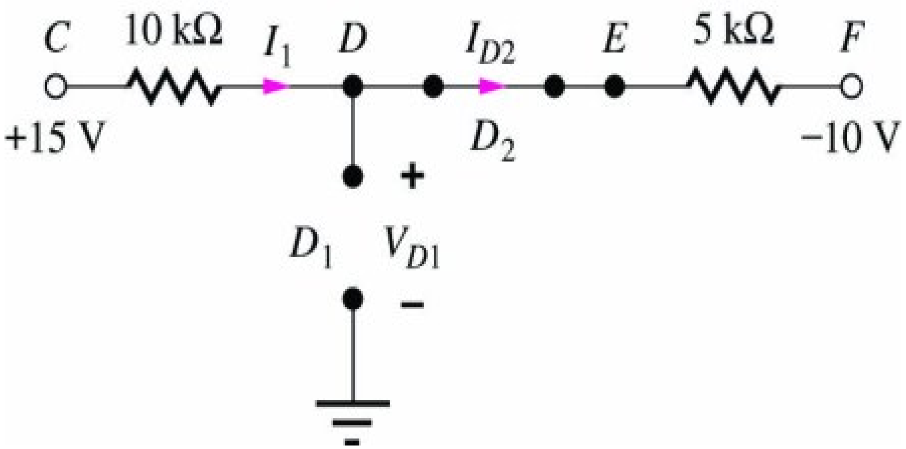
$$I_{D1} = I_1 - I_{D2} = 1.5 - 2 = -0.5\text{ mA}$$

Q-points are (-0.5mA, 0V) and (2.0mA, 0V)

But, $I_{D1}$ <0 is not consistent with the assumption of D1 is ON, so try again.

# Example 7 (Cont.)

The second guess is $D_1$ off and $D_2$ on:




Since $I_{D2} = I_1$,

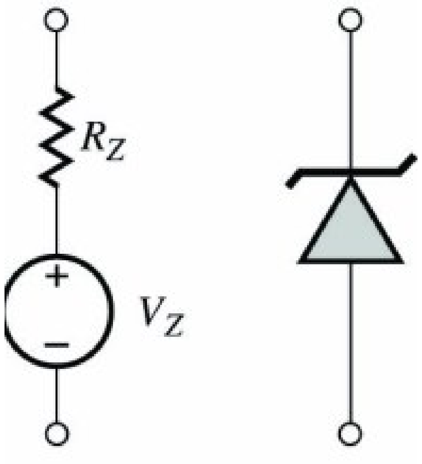
$$I_1 = \frac{15-(-10)}{10\text{k}\Omega + 5\text{k}\Omega} = 1.67\text{mA} \qquad V_{D1} = 15 - 10{,}000 I_1 = -1.67\text{V}$$

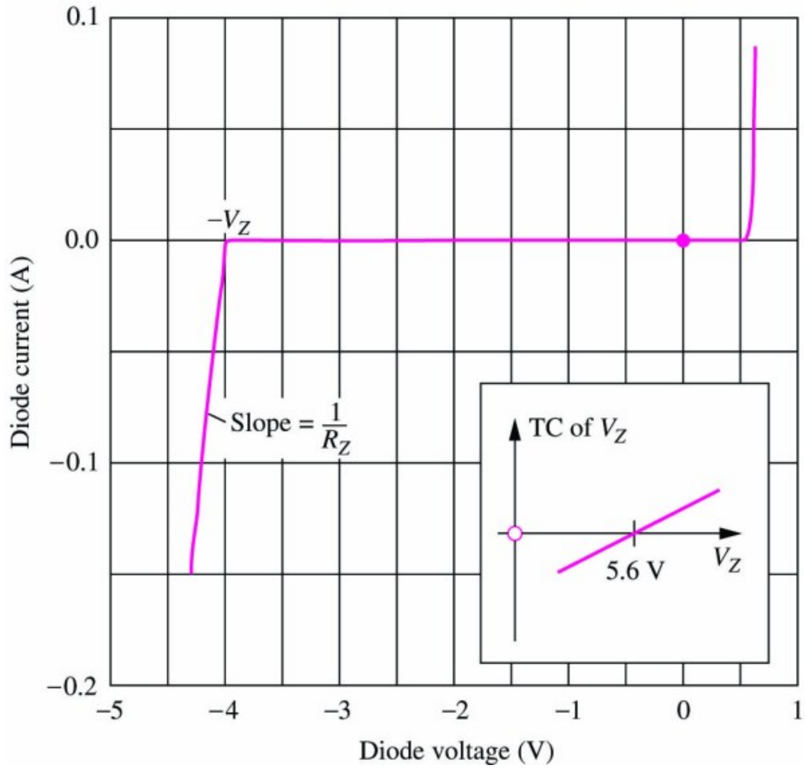
**Q-points are $D_1$ : (0 mA, -1.67 V): off**

**$D_2$ : (1.67 mA, 0 V) : on**

This is consistent with our assumptions and analysis is finished.

# Breakdown Region Diode Model

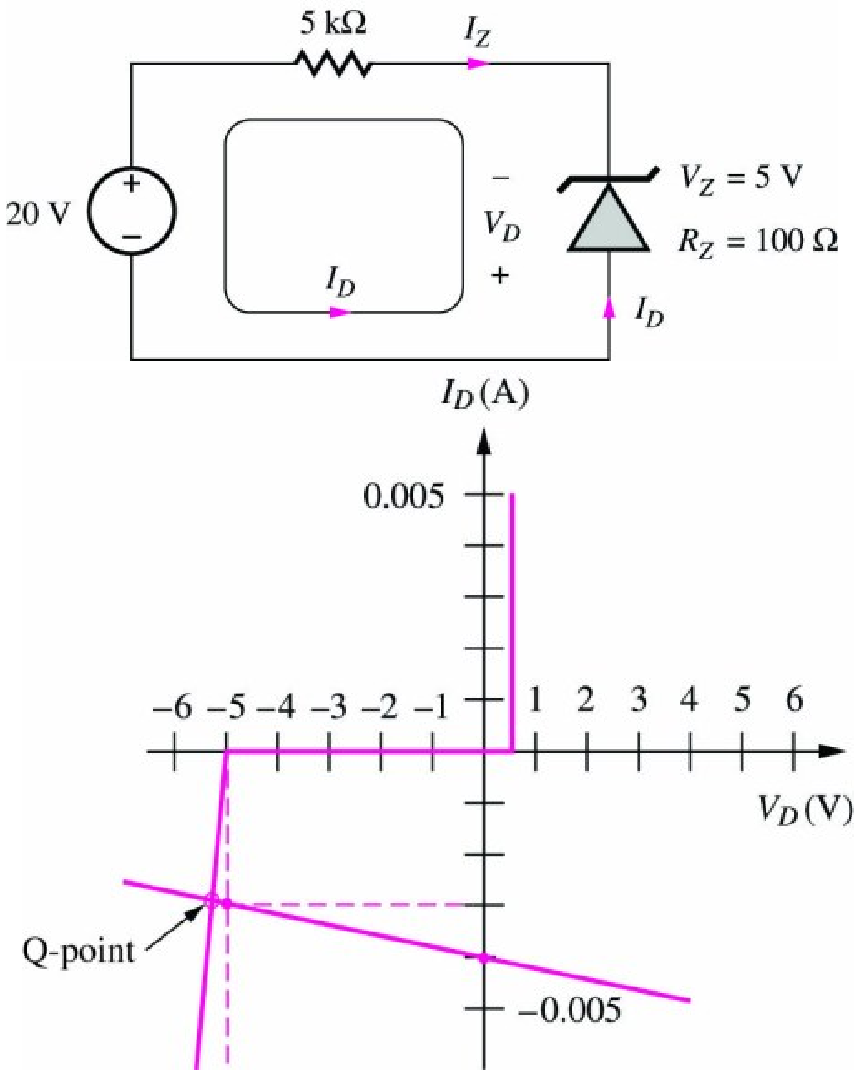



In breakdown, the diode is modeled with a DC voltage source, $V_Z$, and a series resistance, $R_Z$. $R_Z$ models the slope of the *i-v* characteristic.

Diodes designed to operate in reverse breakdown are called **Zener diodes** and use the indicated symbol

# Zener Diode Circuit Analysis (Example 8)




Load-line equation :
(KVL along the loop)

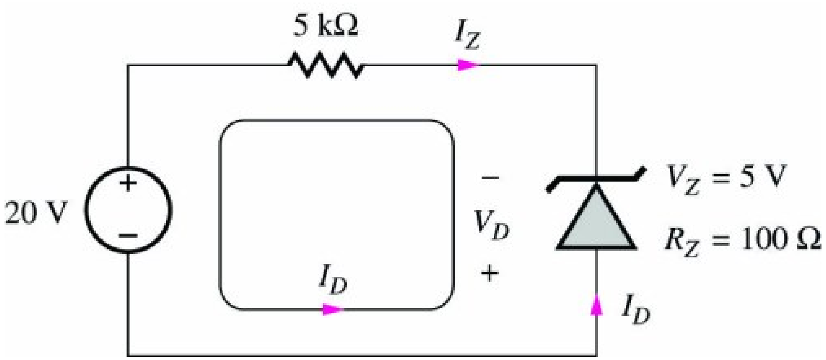
$$V_D + 5000 I_D + 20 = 0$$

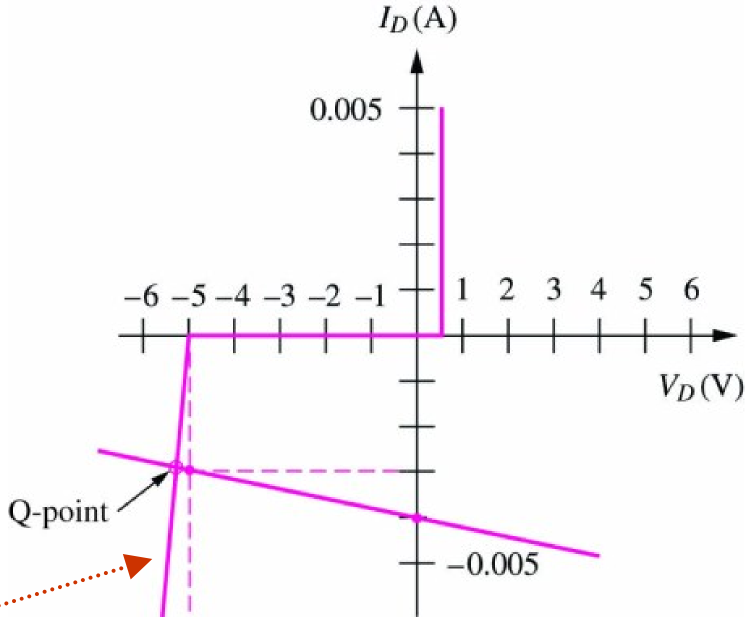
**Using graphic analysis:**

Choose 2 points (0V, -4mA) and (-5V, -3mA) to draw the load line.

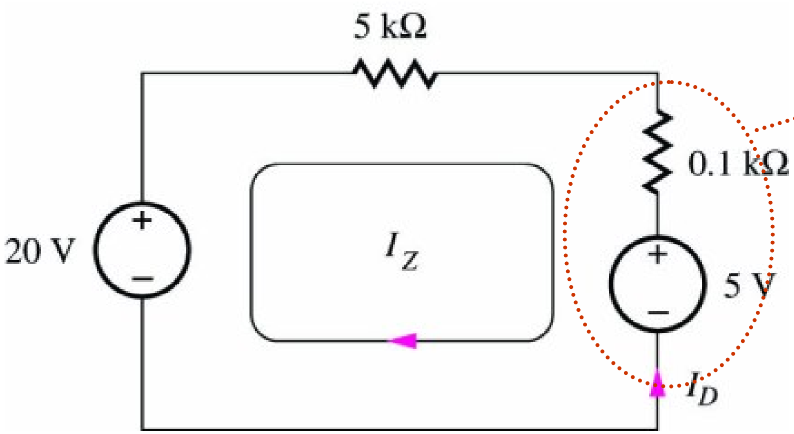
It intersects with *i-v* characteristic at **Q-point (-2.9 mA, -5.2 V)**.

# Zener Diode Circuit Analysis (Example 9)




**Using piecewise linear model**:




$$I_Z = -I_D > 0$$

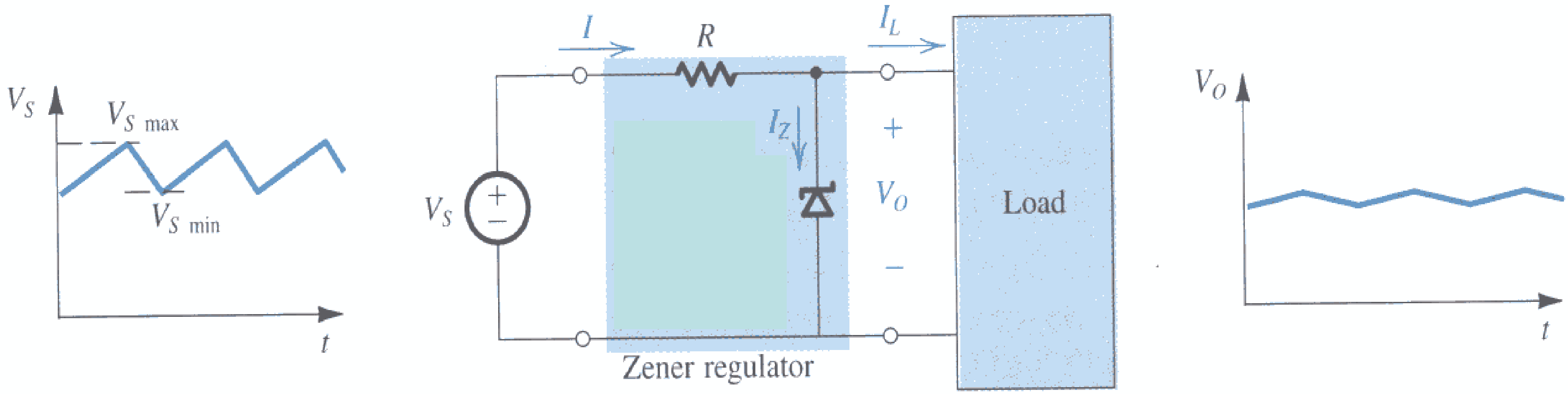
Assume in breakdown region

$$-20 + 5100 I_Z + 5 = 0$$

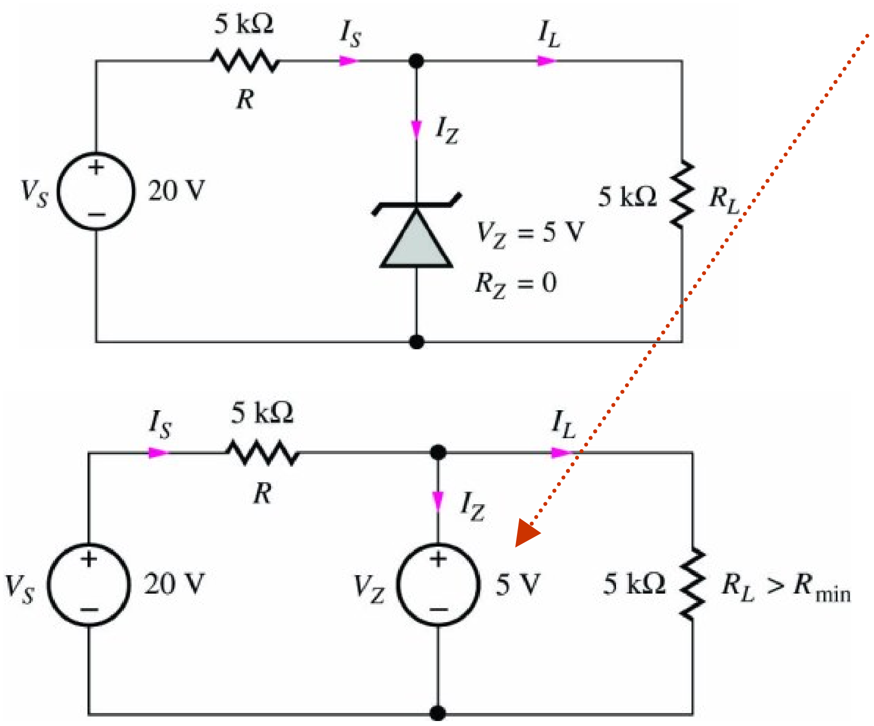
$$I_Z = \frac{(20-5)\text{V}}{5100\Omega} = 2.94\text{mA}$$

Since $I_Z$ >0 ($I_D$ <0), solution is consistent with Zener breakdown assumption.

# Zener Diode Applications: Voltage Regulator



# Zener Diode Applications: Voltage Regulator




Use constant voltage drop model:

$$I_S = \frac{V_S - V_Z}{R} = \frac{(20-5)\text{V}}{5\text{k}\Omega} = 3\text{mA}$$

$$I_L = \frac{V_Z}{R_L} = \frac{5\text{V}}{5\text{k}\Omega} = 1\text{mA}$$

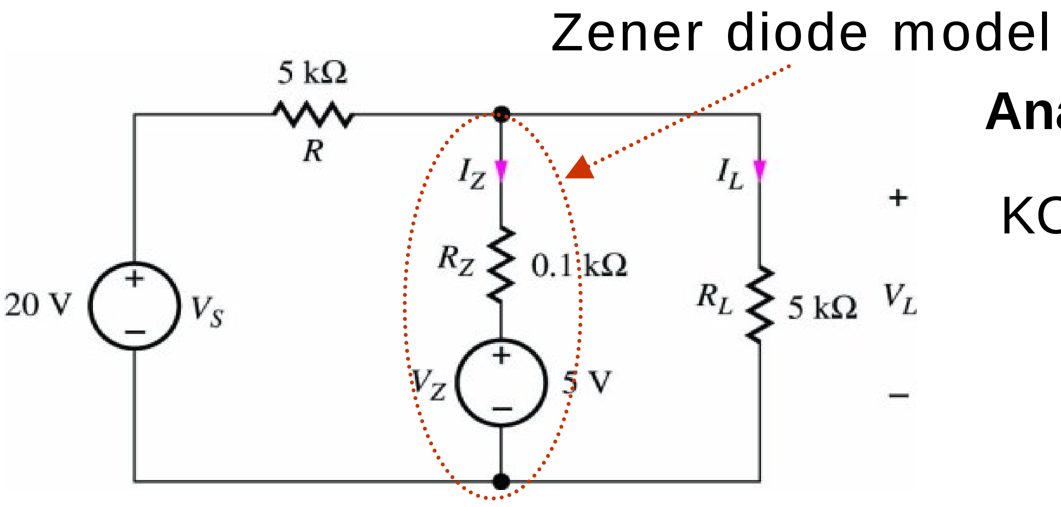
$$I_Z = I_S - I_L = 2\text{mA}$$

For proper regulation, $I_Z > 0$ . If $I_Z < 0$, Zener diode no longer controls voltage across load resistor and the regulator is said to have "dropped out of regulation".

Zener diode keeps voltage across load resistor constant

$$\therefore I_Z = \frac{V_S}{R} - V_Z\left(\frac{1}{R} + \frac{1}{R_L}\right) > 0 \qquad R_L > \frac{R}{\left(\frac{V_S}{V_Z} - 1\right)} = R_{min}$$

# Example 10




**Problem:** Find $V_L$ and $I_Z$ using the piece-wise linear model.

**Given data:**

$V_S$=20 V, $R$=5 kΩ,

$R_Z$= 0.1 kΩ, $V_Z$=5 V

**Analysis:**

KCL at output node:

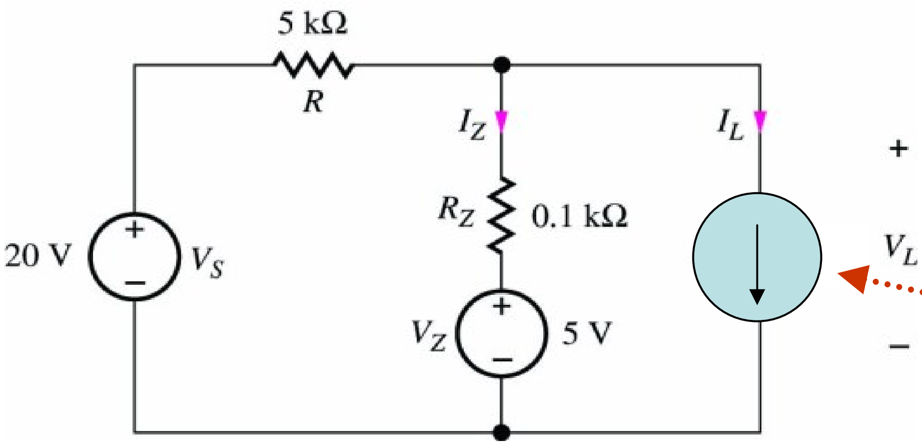
$$\frac{V_L - 20}{5000} + \frac{V_L - 5}{100} + \frac{V_L}{5000} = 0$$

$$\therefore V_L = 5.19\text{V}$$

$$I_Z = \frac{V_L - 5}{100} = \frac{5.19 - 5}{100} = 1.9\text{mA} > 0$$

Diode is actually operating in breakdown region as assumed.

# Line and Load Regulation




Load is modeled as an I-source

By superposition: $$V_L = \frac{R}{R+R_z}V_Z + \frac{R_z}{R+R_z}V_S - (R_z // R)I_L$$

$$Line\ \ regulation \equiv \frac{\partial V_L}{\partial V_S} = \frac{R_z}{R+R_z}$$

(characterizes how sensitive output voltage is to input voltage changes)

$$Load\ \ regulation \equiv \frac{\partial V_L}{\partial I_L} = -(R_z // R)\,[\Omega]$$

(characterizes how sensitive output voltage is to load current changes)