# Important formulae and device models for you to prepare for ELE2110A test 2 

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BJT collector current in active mode: $i_{C}=I_{s} e^{v_{S E} / V_{T}}$
BJT collector current in active mode (including Early effect): $i_{C}=I_{s}{ }^{v_{\text {vE }} / V_{T}}\left(1+\frac{v_{C E}}{V_{A}}\right)$
Relationship between base and collector currents for BJT in active mode: $i_{B}=\frac{i_{c}}{\beta}$
Relationship between emitter and collector currents for BJT in active mode: $i_{C}=\alpha i_{E}$
Relationship between $\alpha$ and $\beta$ : $\alpha=\frac{\beta}{1+\beta}$
Single stage BJT amplifier properties:

|  | $\mathrm{C}-\mathrm{E}$ <br> $\left(\mathrm{R}_{\mathrm{E}}=0\right)$ | Emitter Degenerated <br> $\mathrm{C}-\mathrm{E}$ | $\mathrm{C}-\mathrm{C}$ | C-B |
| :---: | :---: | :---: | :---: | :---: |
| Terminal <br> Voltage Gain | Inverting <br> \& large |  <br> moderate | 1 |  <br> Large |
| Input <br> Resistance | Moderate | Large | Large | Low |
| Output <br> Resistance | Moderate | Large | Low | Large |
| Input Voltage <br> Range | Small | Moderate | Large | Moderate |
| Terminal <br> Current Gain | Inverting <br> \& Large | Inverting \& Large | Non- <br>  <br> Large | 1 |

Simplified DC model for $n p n$ transistor in active mode:

( $\beta_{\mathrm{F}}$ is another notation for $\beta$ ).
Simplified DC model for npn transistor in saturation mode:


Simplified DC model for $p n p$ transistor in active mode:


Simplified DC model for pnp transistor in saturation mode:


$$
\begin{aligned}
& V_{E B} \cong 0.7 \mathrm{~V} \\
& V_{E C} \cong 0.2 \mathrm{~V}
\end{aligned}
$$

Small signal AC model for BJT (both npn and pnp) in active mode (Hybrid- $\pi$ model):


Constraint on $\mathrm{V}_{\mathrm{be}}$ for BJT small signal models to be valid: $\mathrm{V}_{\mathrm{be}} \ll \mathrm{V}_{\mathrm{T}}$ (thermal voltage)

## Hybrid- $\boldsymbol{\pi}$ model including $\mathbf{r}_{\mathbf{0}}$ :



$$
\begin{aligned}
& g_{m}=\frac{I_{C}}{V_{T}}, \\
& r_{\pi}=\frac{V_{T}}{I_{B}}, \\
& r_{o}=\frac{V_{A}+V_{C B}}{I_{C}} \approx \frac{V_{A}}{I_{C}}
\end{aligned}
$$

Small signal AC model for BJT (both $n p n$ and $p n p$ ) in active mode (T-model):

n-channel MOSFET I-V equations in different modes:

| Region | Cutoff | Triode | Saturation |
| :--- | :---: | :---: | :---: |
| Conditions | $v_{G S}<V_{t}$ | $v_{G S} \geq V_{t}$ |  |
|  |  | $v_{D S}<v_{G S}-V_{t}$ | $v_{D S} \geq v_{G S}-V_{t}$ |
| I-V relation | $i_{D}=0$ | $i_{D}=K^{\prime}{ }_{n} \frac{W}{L}\left[\left(v_{G S}-V_{t}\right) v_{D S}-\frac{1}{2} v_{D S}^{2}\right]$ | $i_{D}=\frac{1}{2} K_{n}^{\prime} \frac{W}{L}\left(v_{G S}-V_{t}\right)^{2}$ |

where $K_{n}{ }^{\prime}=\mu_{n} C_{o x}, \mathrm{~V}_{\mathrm{t}}$ is the threshold voltage (sometimes denoted as $\mathrm{V}_{\mathrm{TN}}$ for nmos).
Saturation mode equation including the channel length modulation effect:
$i_{D}=\frac{K_{n}}{2} \frac{W}{L}\left(v_{G S}-V_{t}\right)^{2}\left(1+\lambda v_{D S}\right)$
p-channel MOSFET I-V equations in different modes:

| Cutoff | Triode/Linear | Saturation |
| :---: | :---: | :---: |
| $i_{D}=0$ | $i_{D}=K_{p}\left[\left(v_{G S}-V_{t}\right) v_{D S}-\frac{1}{2} v_{D S}^{2}\right]$ | $i_{D}=\frac{1}{2} K_{p}\left(v_{G S}-V_{t}\right)^{2}$ |

where $K_{p}=K_{p}{ }^{\prime} \frac{W}{L}, \quad K_{p}{ }^{\prime}=\mu_{p} C_{o x}$
$\mathrm{V}_{\mathrm{t}}, \mathrm{v}_{\mathrm{GS}}$ and $\mathrm{v}_{\mathrm{DS}}$ are negative for pmos.

Charts helping you to judge the operational mode of nmos (left) and pmos (right):


MOSFET small signal model (for both nmos and pmos):

$g_{m}=k_{n}^{\prime} \frac{W}{L}\left(V_{G S}-V_{t}\right), \quad g_{m}=\sqrt{2 k_{n}^{\prime}} \sqrt{\frac{W}{L}} \sqrt{I_{D}}, \quad$ or $g_{m}=\frac{I_{D}}{\left(V_{G S}-V_{t}\right) / 2}$
$r_{0}=\frac{1+\lambda V_{D S}}{\lambda I_{D}} \cong \frac{1}{\lambda I_{D}}$
Constraint on $\mathrm{V}_{\mathrm{gs}}$ for the small signal model to be valid: $v_{g s} \ll 2\left(V_{G S}-V_{t}\right)$, or $\mathrm{V}_{\mathrm{gs}}<0.2\left(\mathrm{~V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{t}}\right)$.

