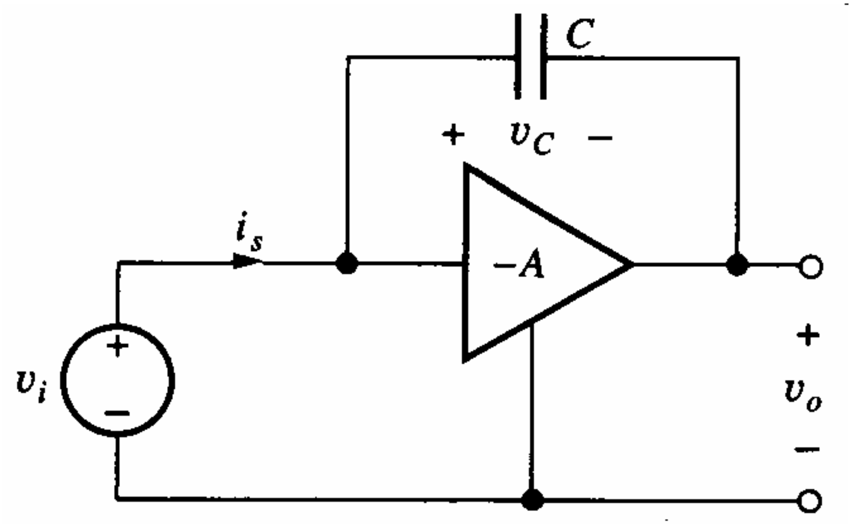
Week 13: Frequency Response, Feedback

Topics to cover …

- Miller Effect
- Bode Plot
- Negative Feedback
  - Properties
  - Topologies
  - Stability issue

Reading Assignment:
Chap 16.6-16.9, 17.1-17.2, 17.11 of Jaeger and Blalock

Miller's Theorem



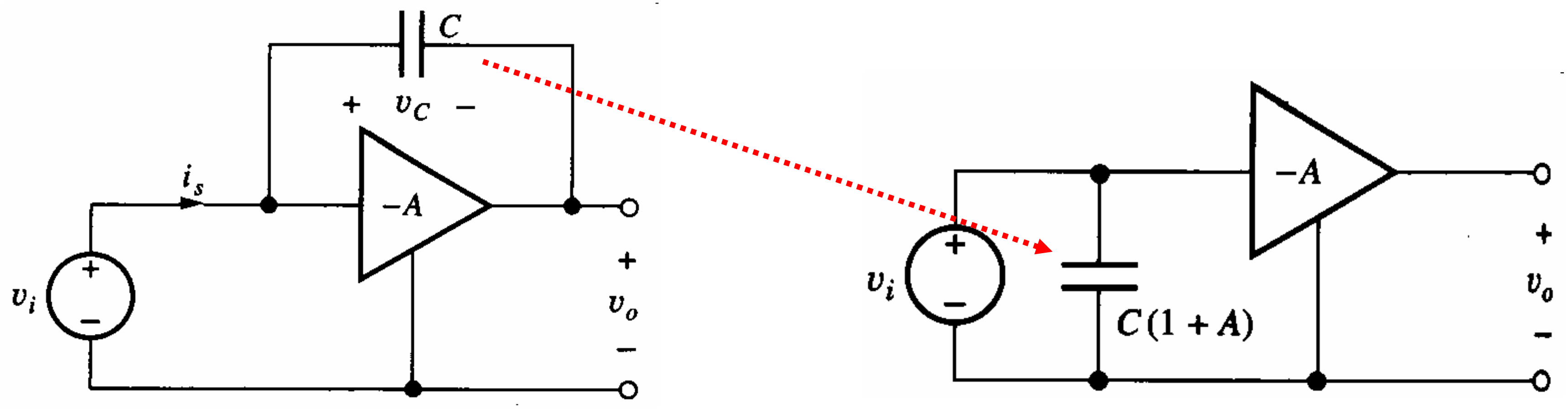
- Capacitor C is connected between the input and output of an inverting amplifier with gain (-A)
- Consider the input admittance (Y=1/Z) of the amplifier:

$$\mathbf{V}_o(s) = -A\mathbf{V}_i(s) \quad \text{and} \quad \mathbf{I}_s(s) = sC\left[\mathbf{V}_i(s) - \mathbf{V}_o(s)\right]$$

$$\rightarrow \quad Y(s) \equiv \frac{\mathbf{I}_s(s)}{\mathbf{V}_i(s)} = sC(1+A) \qquad \text{(Miller's theorem)}$$

- In general, the C can be any admittance element

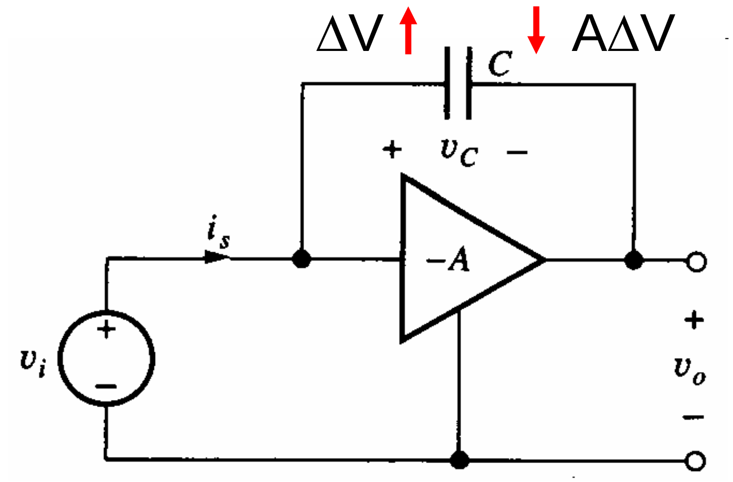
Miller Effect



$$Y(s) \equiv \frac{\mathbf{I}_s(s)}{\mathbf{V}_i(s)} = sC(1+A)$$

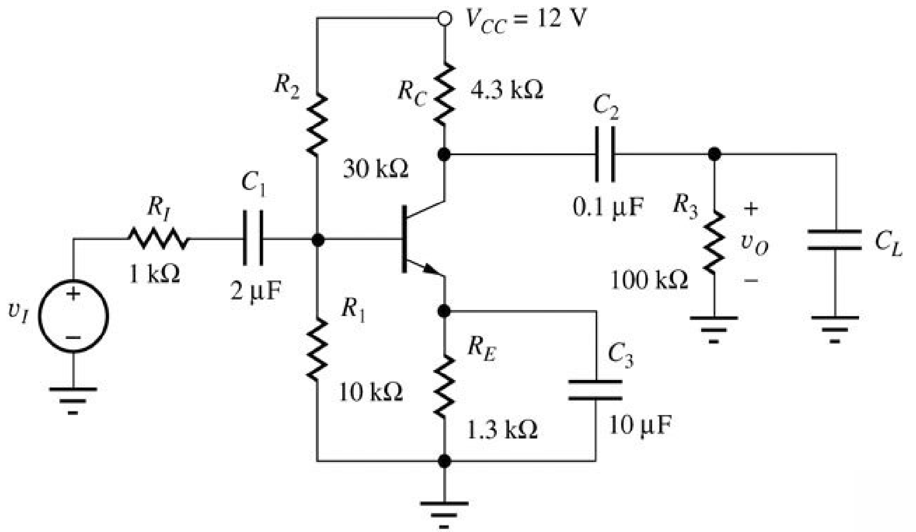
- The capacitor C can be represented by an equivalent capacitor connected between input of the amplifier and the ground
- The input equivalent capacitance = C x (1+A)
- This capacitance multiplication is referred to as ***Miller effect***

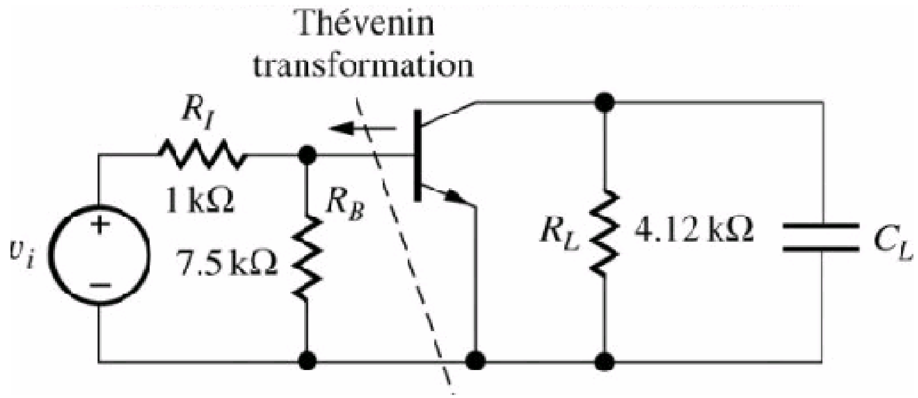
Understanding the Miller Effect

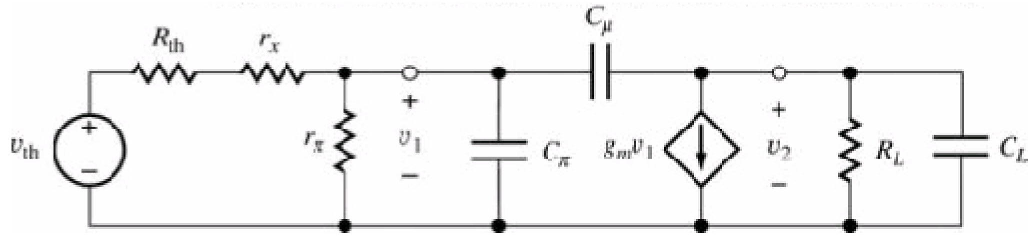


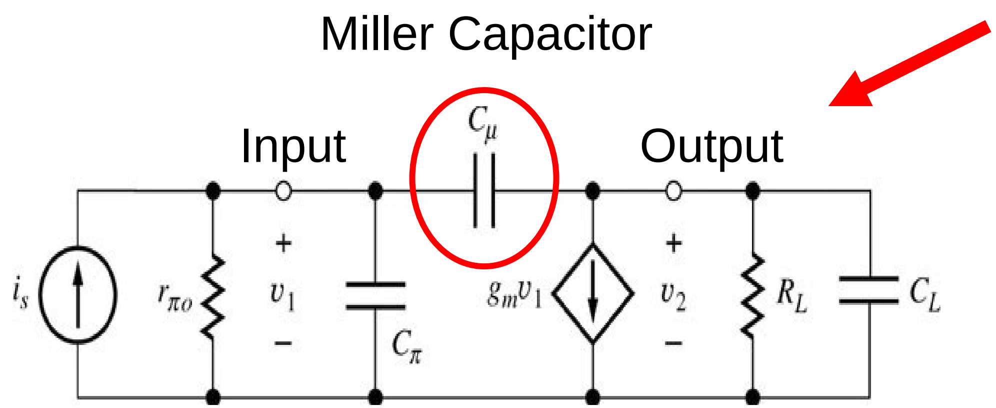
- If V_i changes by ΔV, V_o will change by –AΔV, so V_C will change by (1+A)ΔV → amount of charges supplied to C is Cx(1+A) ΔV
- If the capacitor were connecting between the input and ground, to absorb the same amount of charges, the equivalent capacitance must be C x (1+A)

Miller Effect in C-E Amplifier

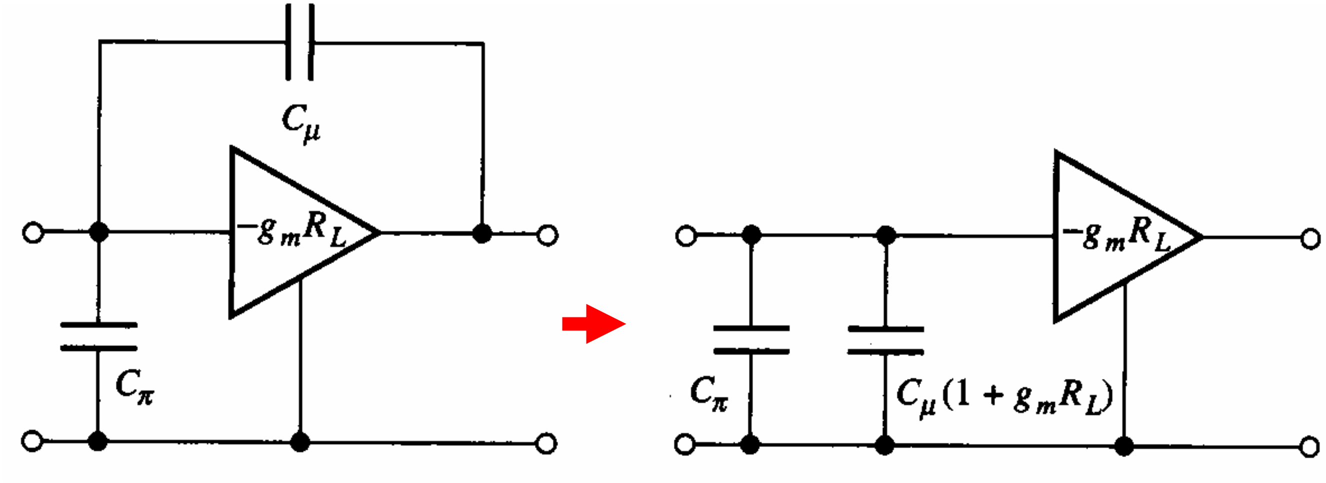








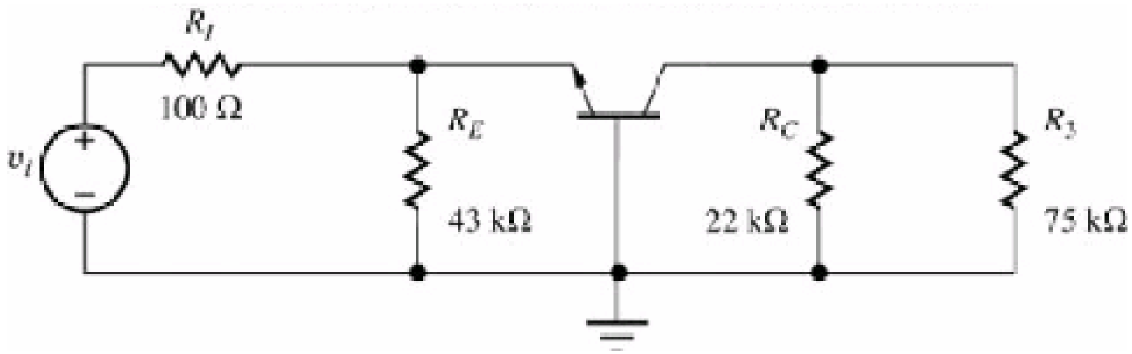
Miller Effect in CE amplifier

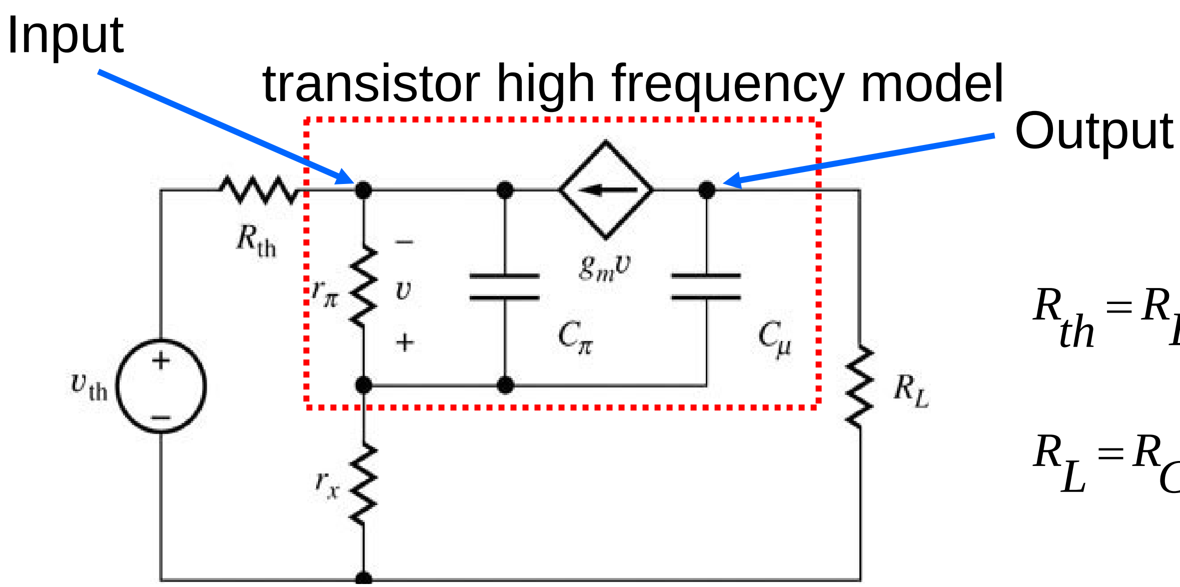


- C_μ is amplified by $(1+g_m R_L)$ at the input → Common emitter amplifiers generally have a low upper-cutoff frequency

Does Miller Effect exist in CB Amplifier?

CB Amplifier AC equivalent:



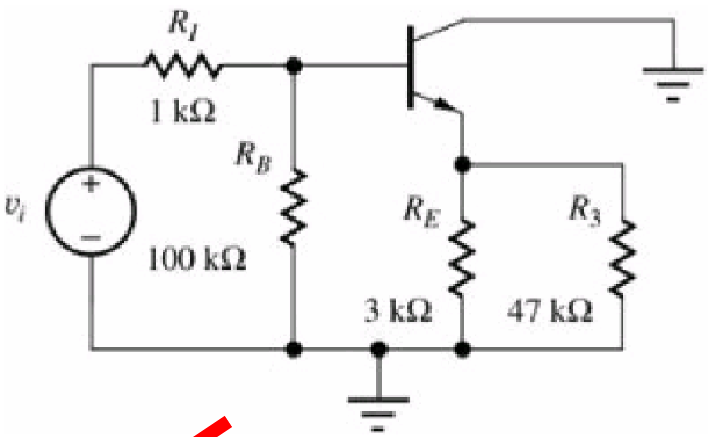


$$R_{th} = R_E \| R_I$$

$$R_L = R_C \| R_3$$

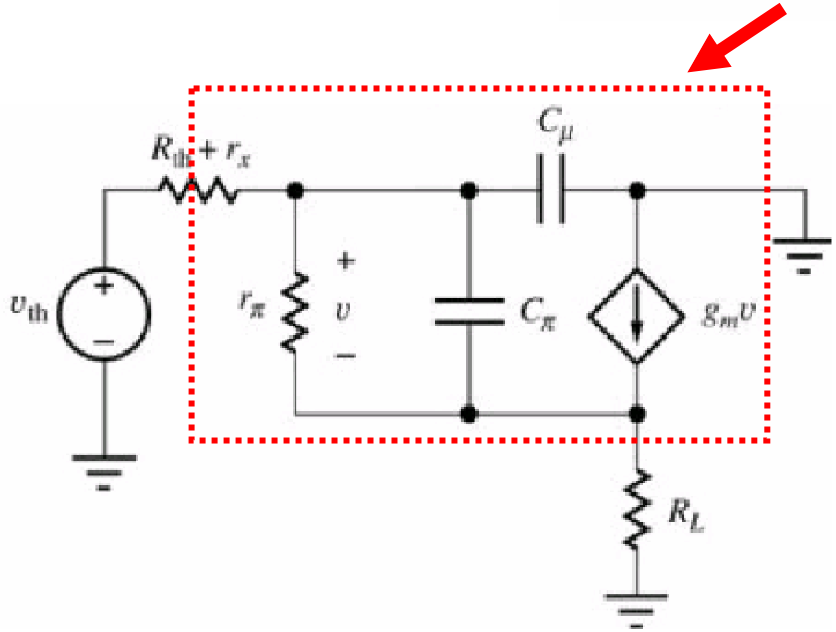
Does Miller Effect exist in Emitter Follower?

AC equivalent
of emitter follower:

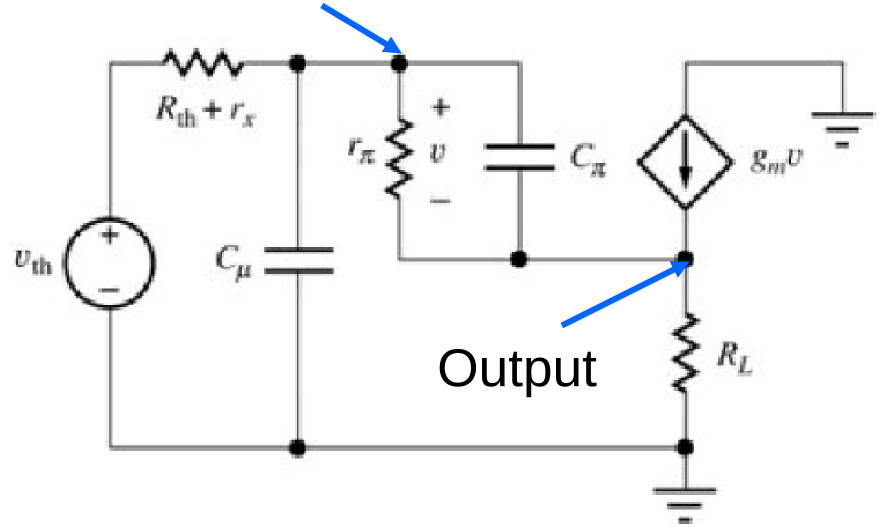


$$R_{th} = R_B \| R_I$$

$$R_L = R_E \| R_3$$



Input



Output

Re-arranged

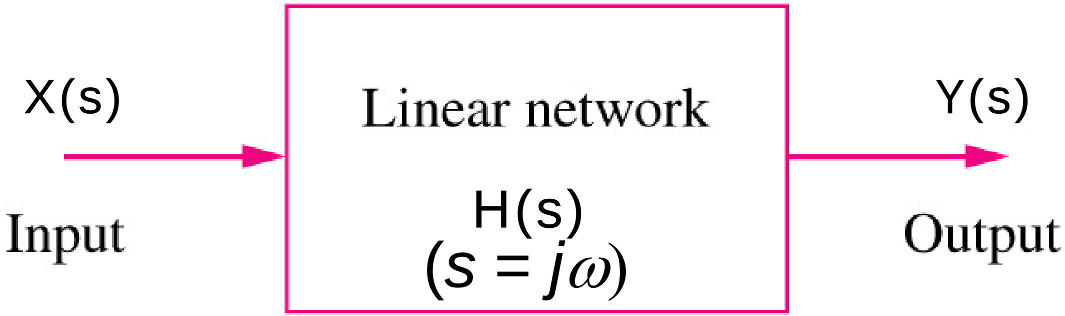
Frequency Response of Single Stage Amplifiers

- C-E/C-S amplifier has a narrow bandwidth due to the Miller effect
- C-B/C-G amplifier has a wide bandwidth
- Emitter/Source follower has a wide bandwidth

Topics to cover …

- Miller Effect
- Bode Plot
- Negative Feedback
  – Properties
  – Topologies
  – Stability issue

Transfer Function



The transfer function of a circuit is the s-domain ratio of an output (voltage or current) to an input (voltage or current) :

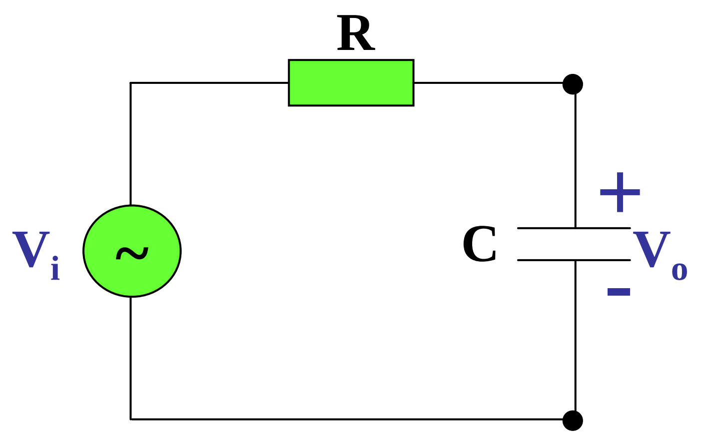
$$H(s) = \frac{Y(s)}{X(s)}$$

$$H(s) = \text{Voltage Gain} = \frac{V_0(s)}{V_i(s)} \qquad H(s) = \text{Transfer Impedance} = \frac{V_0(s)}{I_i(s)}$$

$$H(s) = \text{Current Gain} = \frac{I_0(s)}{I_i(s)} \qquad H(s) = \text{Transfer Admittance} = \frac{I_0(s)}{V_i(s)}$$

Transfer Function Example

- Simple RC circuit



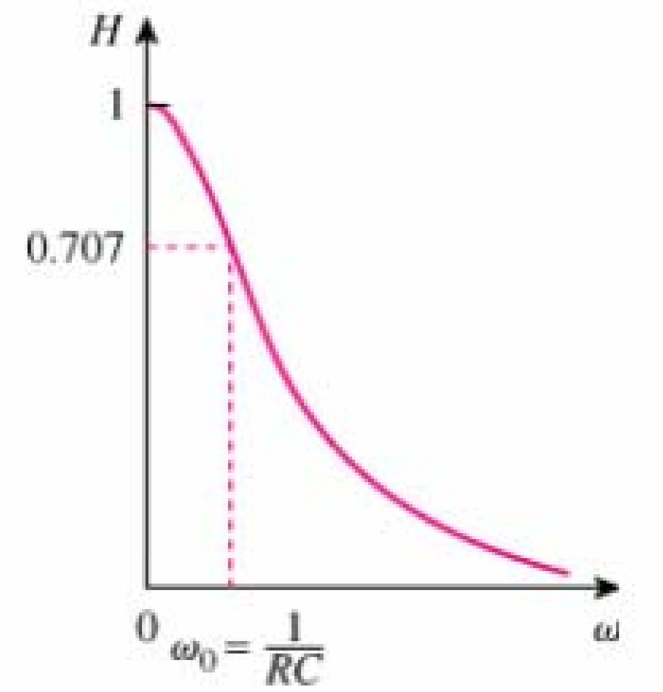
$$\mathbf{V}_o = \frac{1/\,j\omega C}{R + 1/\,j\omega C}\mathbf{V}_i$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1 + j\omega RC}$$

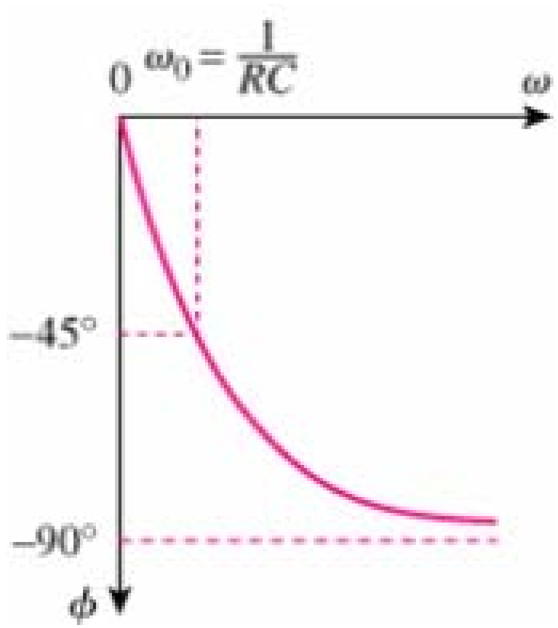
is maximum when ω = 0!

or $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{\sqrt{1 + (\omega RC)^2}}\angle\phi$$ **where** $$\phi = -\tan^{-1}(\omega RC)$$

Frequency response of RC circuit



Magnitude

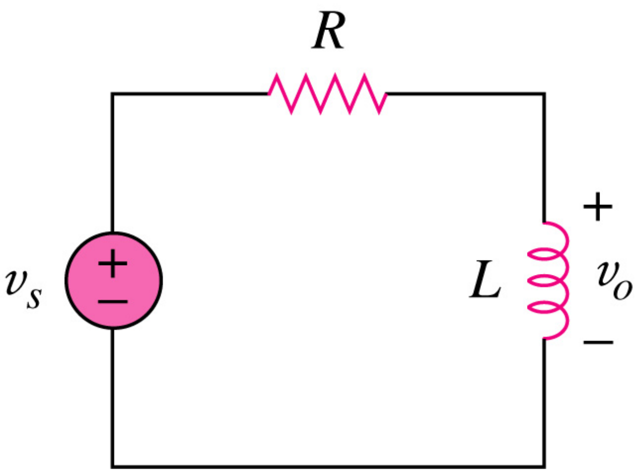


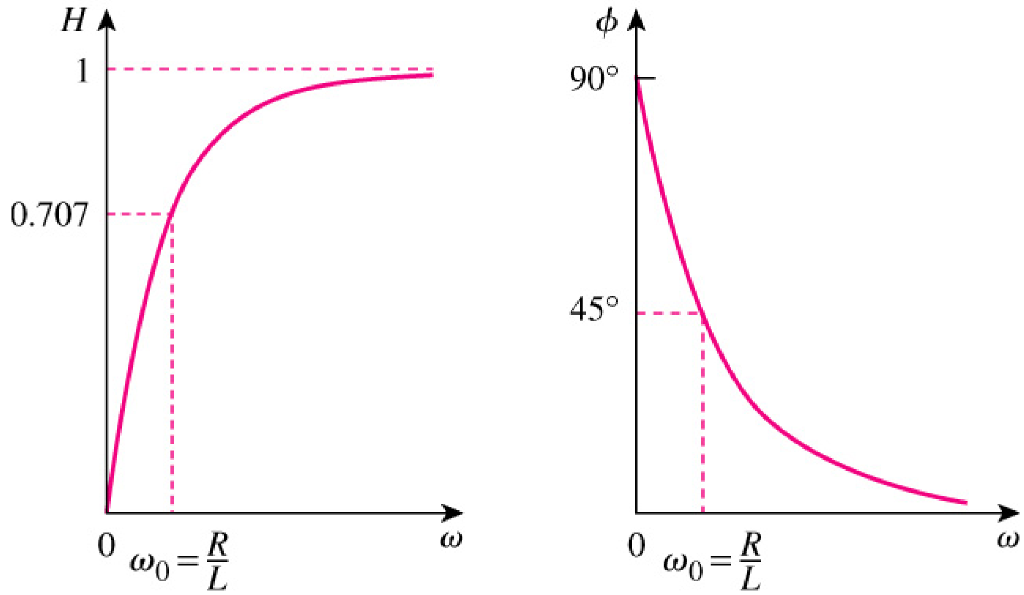
Phase

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{\sqrt{1+(\omega RC)^2}} \angle \phi$$

$$\phi = -\tan^{-1}(\omega RC)$$

Frequency Response Example





(a) (b)

Transfer function of RL circuit:

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{j\omega L}{R + j\omega L}$$

Magnitude response

Phase response

Pole and Zero in Transfer Functions

A circuit can have a general transfer function in the following form:

$$T(s) = \frac{K'(1+s/\omega_{z_1})(1+s/\omega_{z_2})\cdots(1+s/\omega_{z_m})}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})\cdots(1+s/\omega_{p_n})} \qquad s = j\omega$$

where $-\omega_{pk}$ and $-\omega_{zk}$ are called zeros and poles, respectively, of the transfer function

Logarithms

- Bode plot is a systematic way of plotting the magnitude and phase of a gain function as functions of frequency.

- Bode plots are based on logarithms. Some useful properties of logarithms are

$$1.\ \log P_1 \times P_2 = \log P_1 + \log P_2$$

$$2.\ \log P_1 / P_2 = \log P_1 - \log P_2$$

$$3.\ \log P^n = n \log P$$

$$4.\ \log 1 = 0$$

Decibel Scale

- In communications systems, gain is measured in *bels*. Historically, the bel is used to measure the ratio of two levels of power or power gain G:

$$G = \text{Number of bels} = \log_{10} P_2 / P_1$$

- The *decibel* (dB) is 1/10th of a bel:

$$G_{\text{dB}} = 10 \log_{10} \frac{P_2}{P_1}$$

- Alternatively, the gain can be expressed in terms of ratio of voltage or current amplitudes. For amplitude ratio, the decibel is defined as:

$$G_{\text{dB}} = 20 \log_{10} \frac{A_2}{A_1}$$

A_2 and A_1 are voltage or current amplitudes.

$(P=I^2R = V^2/R)$

So, 10dB voltage gain represents the same thing as 10dB power gain does

Magnitude Response

$$T(s) = \frac{K'(1+s/\omega_{z_1})(1+s/\omega_{z_2})\cdots(1+s/\omega_{z_m})}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})\cdots(1+s/\omega_{p_n})}$$

For frequency response, put s = jω

The magnitude of the transfer function, in dB scale, is given by:

$$\therefore 20\log_{10}\left|T(j\omega)\right| = 20\log_{10} K' + \sum 20\log_{10}\sqrt{1+(\omega/\omega_{z_k})^2} - \sum 20\log_{10}\sqrt{1+(\omega/\omega_{p_k})^2}$$

Bode Magnitude Plot

$$20\log_{10}|T(j\omega)| = 20\log_{10} K' + \sum 20\log_{10}\sqrt{1+(\omega/\omega_{z_k})^2} - \sum 20\log_{10}\sqrt{1+(\omega/\omega_{p_k})^2}$$

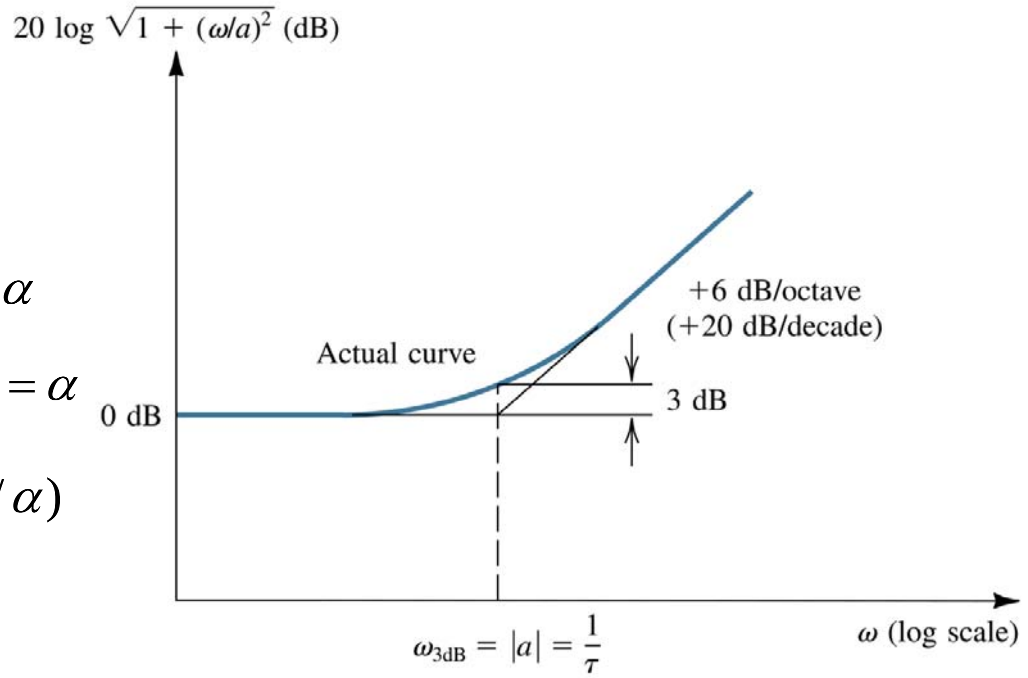
Consider the general term

$$20\log_{10}\sqrt{1+(\omega/\alpha)^2}$$

(i) $20\log_{10}\sqrt{1+(\omega/\alpha)^2} \approx 0$ for $\omega << \alpha$

(ii) $20\log_{10}\sqrt{1+(\omega/\alpha)^2} = 3dB$ for $\omega = \alpha$

(iii) $20\log_{10}\sqrt{1+(\omega/\alpha)^2} \approx 20\log_{10}(\omega/\alpha)$ for $\omega >> \alpha$



Bode Plot Example

Sketch the magnitude response of an amplifier with the transfer function of

$$T(s) = \frac{10s}{(1+s/10^2)(1+s/10^5)}$$

Curve 1: T(s) = s

A straight line with +20dB/decade, passing through the point (1,0).

Curve 2:

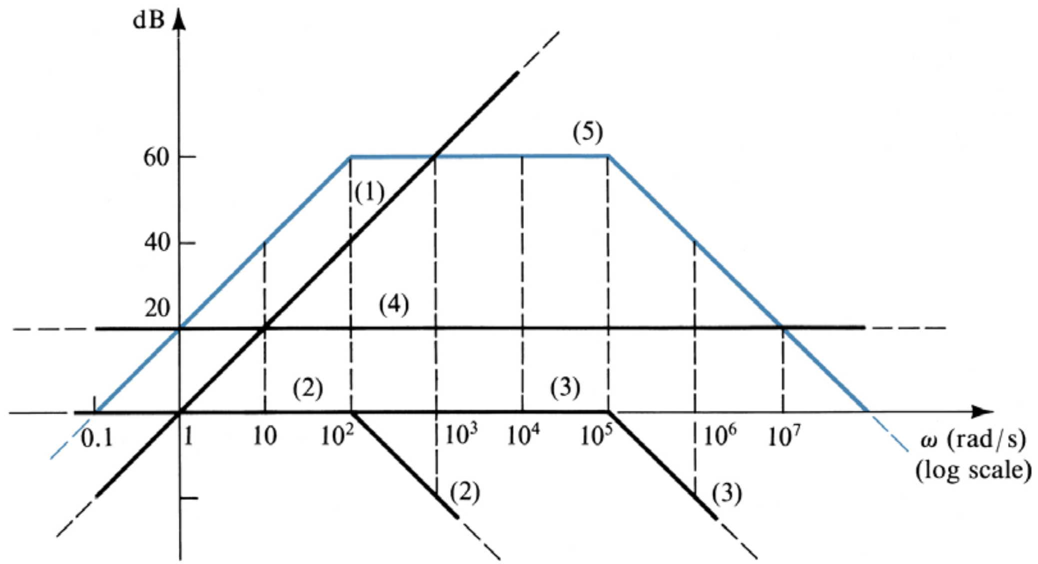
$$T(s) = \frac{1}{(1+s/10^2)}$$

Curve 3:

$$T(s) = \frac{1}{(1+s/10^5)}$$

Curve 4: T(s)=10

$$20\log_{10} 10 = 20$$



Adding four curves (1), (2), (3), (4), we have the overall magnitude response of curve (5

Phase Response

$$T(s) = \frac{K'(1+s/\omega_{z_1})(1+s/\omega_{z_2})\cdots(1+s/\omega_{z_m})}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})\cdots(1+s/\omega_{p_n})}$$

s = jω

The phase of the transfer function is given by:

$$\angle T(j\omega) = \angle(1+j\omega/\omega_{z1}) + \angle(1+j\omega/\omega_{z2}) + \ldots + \angle(1+j\omega/\omega_{zm})$$
$$- \angle(1+j\omega/\omega_{p1}) - \angle(1+j\omega/\omega_{p2}) - \ldots - \angle(1+j\omega/\omega_{pn})$$

Phase Response of a First Order Term

Consider the general term $T(s) = \left.\frac{1}{1+s/\alpha}\right|_{s=j\omega} = \frac{1}{1+j\omega/\alpha}$

$$\theta = \angle T = -\tan^{-1}\frac{\omega}{\alpha}$$

(i) $\omega = 0.1\alpha \Rightarrow \theta = -\tan^{-1} 0.1 = -5.7^\circ$

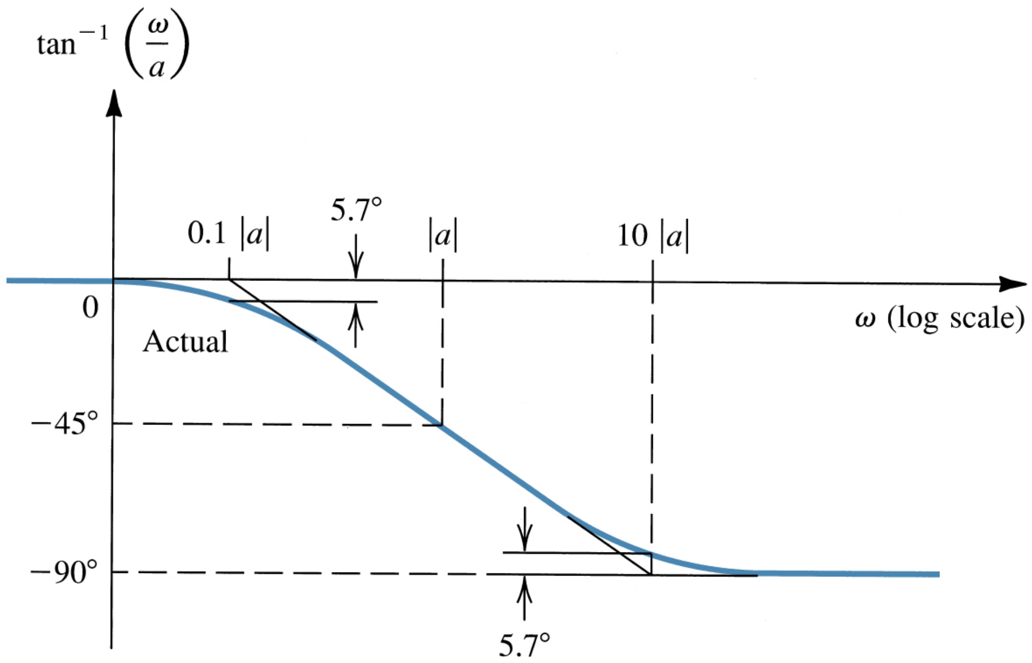
(ii) $\omega = \alpha \Rightarrow \theta = -\tan^{-1} 1 = -45^\circ$

(iii) $\omega = 10\alpha$

$$\Rightarrow \theta = -\tan^{-1} 10$$

$$= -(90^\circ - \tan^{-1} 0.1)$$

$$= -(90^\circ - 5.7^\circ)$$



Example

Find the Bode plot for the phase response of $T(s) = \dfrac{10s}{(1+s/10^2)(1+s/10^5)}$

Curve 1: For $T(s)=s$, $s=j\omega$,

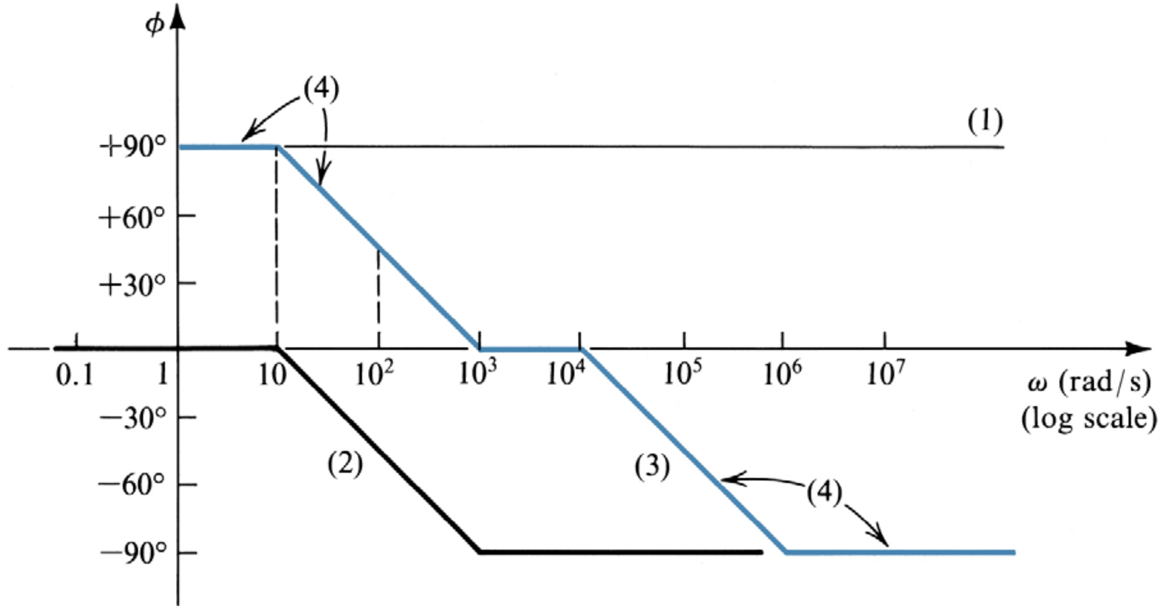
$$\theta = 90^\circ$$

Curve 2:

$$T(s) = \frac{1}{(1+s/10^2)}$$

Curve 3:

$$T(s) = \frac{1}{(1+s/10^5)}$$



Adding curves (1), (2), and (3), we have curve (4), the Bode plot for the phase response.

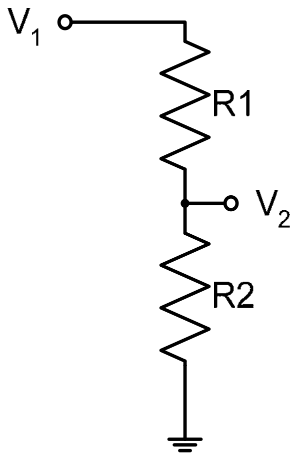
Topics to cover …

- Miller Effect
- Bode Plot
- Negative Feedback
  - Properties
  - Topologies
  - Stability issue

Background

Passive circuits

- Gain < 1
- "Gain" can be very accurate
  - E.g., in 2-resistor string:



$$A = \frac{V_2}{V_1} = \frac{R_2}{R_1 + R_2}$$

A depends only on resistance ratio, which is independent on temperature, etc.

Active circuits

- Can provide large gain >>1
- But gain is inaccurate
  - E.g., in CS amplifier:

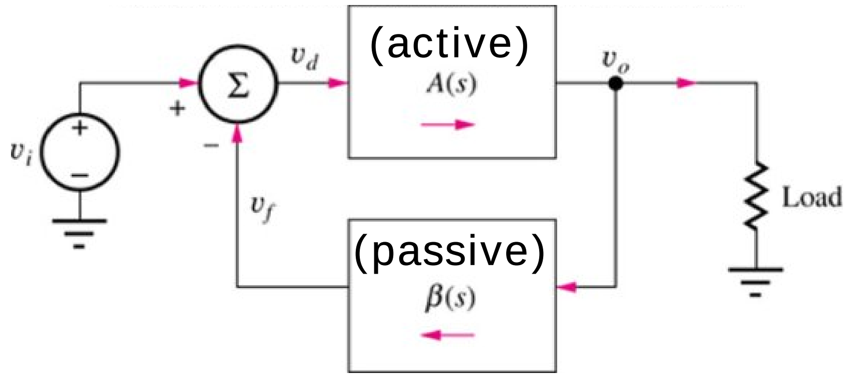
$$|A| = g_m R_L$$

$$g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D}$$

- g_m depends on mobility (which in term depends on Temp, doping level), W/L and I_D. All have large tolerance except W/L
- |A| can vary by ±50%

Feedback circuits: combine the advantages of both!

Classical Feedback System



$$V_o(s) = V_d(s)A(s)$$

$$V_f(s) = V_o(s)\beta(s)$$

$$V_d(s) = V_i(s) - V_f(s)$$

- *A(s)* = transfer function of open-loop amplifier, or **open-loop gain**
- *β(s)* = transfer function of feedback network

Assumptions:

1. Feedback network and amplifier do not load each other
2. Signal flows are unidirectional

Closed-loop gain:

$$A_f(s) = \frac{V_o(s)}{V_i(s)} = \frac{A(s)}{1 + A(s)\beta(s)} = \frac{A(s)}{1 + T(s)}$$

T(s) = A(s)β(s) is called **loop gain**

For negative feedback: *T(s)* > 0
For positive feedback: *T(s)* < 0

A_f is smaller than A by 1/[1+A(s)β(s)]

Closed-Loop Gain

- For $A\beta >> 1$:

$$A_f = \frac{A}{1 + A\beta} \cong \frac{1}{\beta}$$

- β is realized by accurate passive network
- Since $\beta < 1$, we have $A_f > 1$

- The closed-loop gain >1 and is accurate

Feedback combines the advantages of both passive and active circuits!

Topics to cover …

- Miller Effect
- Bode Plot
- Negative Feedback
  - Properties
  - Topologies
  - Stability issue

Desensitize the gain

$$A_f = \frac{A}{1 + A\beta}$$

Assume that β is constant. Taking differentials of both sides of the closed-loop gain with respect to A results in

$$\frac{dA_f}{dA} = \frac{1}{(1 + A\beta)} - \frac{A\beta}{(1 + A\beta)^2} = \frac{1}{(1 + A\beta)^2}$$

$$\Rightarrow \quad \frac{dA_f}{A_f} = \frac{1}{(1 + A\beta)} \frac{dA}{A}$$

which says that the percentage change in A_f (due to variations in some circuit parameter) is smaller than the percentage change in A by the amount of feedback.

Extend the Bandwidth

Consider an amplifier whose high-freq response is characterized by a single pole:

$$A(s) = \frac{A_M}{1 + s/\omega_H}$$

Therefore,

$$A_f(s) = \frac{A(s)}{1 + A(s)\beta} = \frac{\dfrac{A_M}{1 + s/\omega_H}}{1 + \dfrac{\beta A_M}{1 + s/\omega_H}} = \frac{A_M}{1 + \beta A_M + s/\omega_H}$$

$$= \frac{A_M \,/(1 + A_M \beta)}{1 + s/\omega_H (1 + A_M \beta)}$$

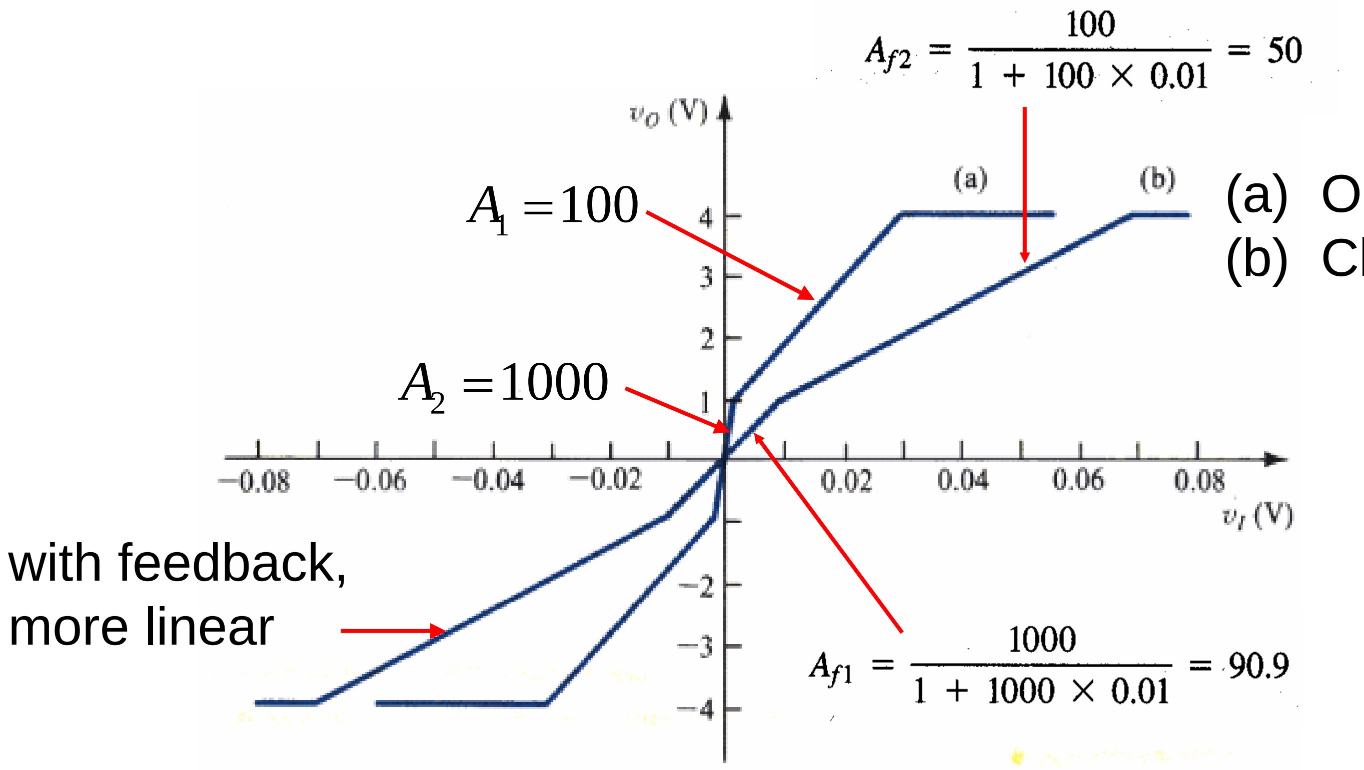
The upper-cutoff frequency ω_{Hf} of the closed loop amplifier is given by:

$$\omega_{Hf} = \omega_H (1 + A_M \beta)$$

Similarly, it can be proven that the lower cutoff frequency ω_{Lf} is given by

$$\omega_{Lf} = \omega_L \,/(1 + A_M \beta)$$

Reduce Nonlinear Distortion



(a) Open loop gain
(b) Closed loop gain

$\beta = 0.01$

Effects of Negative Feedback

- ***Gain Sensitivity***
  - Feedback reduces sensitivity of gain to variations in values of transistor parameters and circuit elements.
- ***Bandwidth***
  - Bandwidth of amplifier can be extended using feedback.
- ***Nonlinear Distortion***
  - Feedback reduces effects of nonlinear distortion.

In short, the basic idea of negative feedback is to trade off gain for other desirable properties.

Topics to cover …

- Miller Effect
- Bode Plot
- Negative Feedback
  - Properties
  - Topologies
  - Stability issue

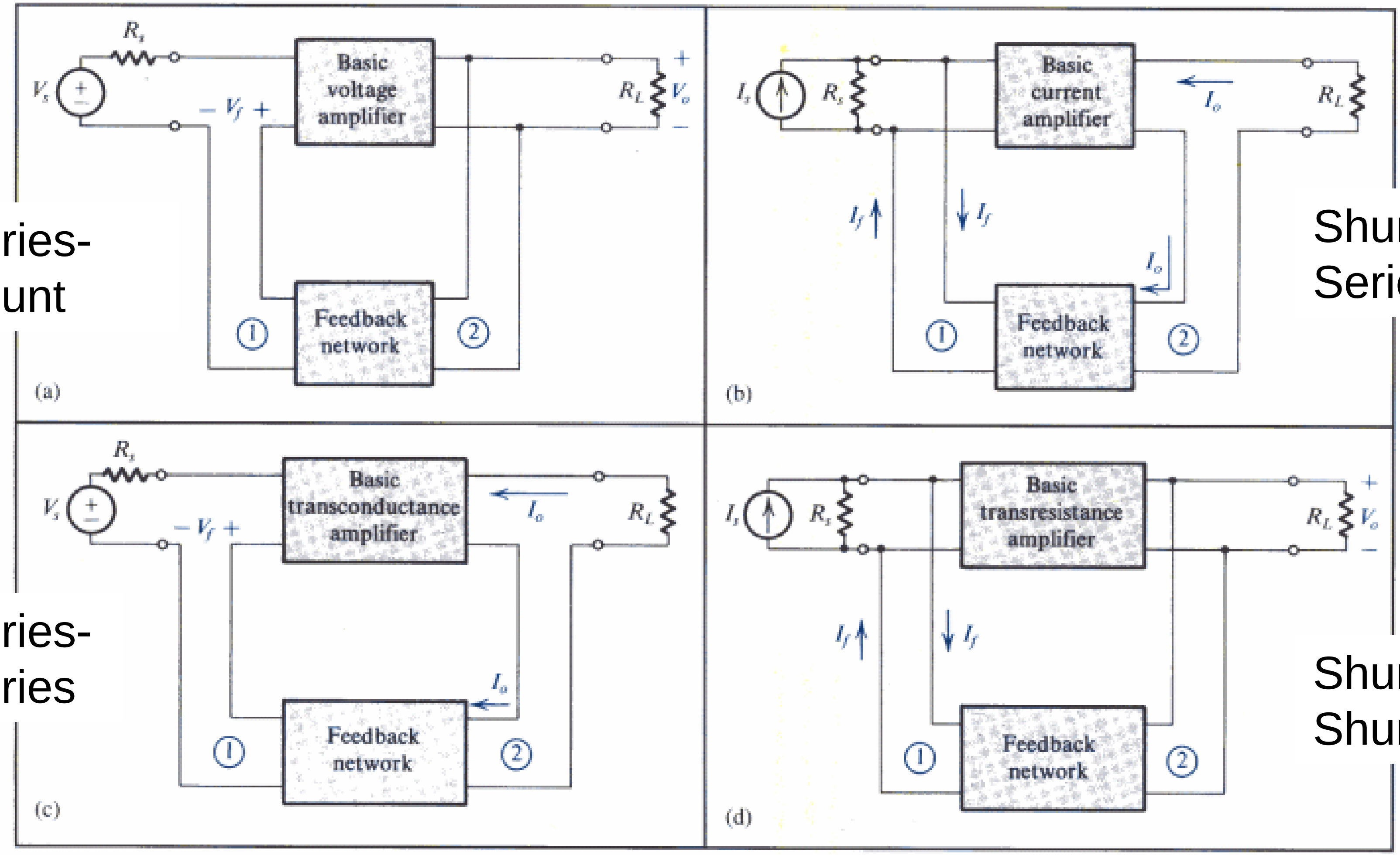
Feedback Topologies

Series-Shunt

Shunt-Series

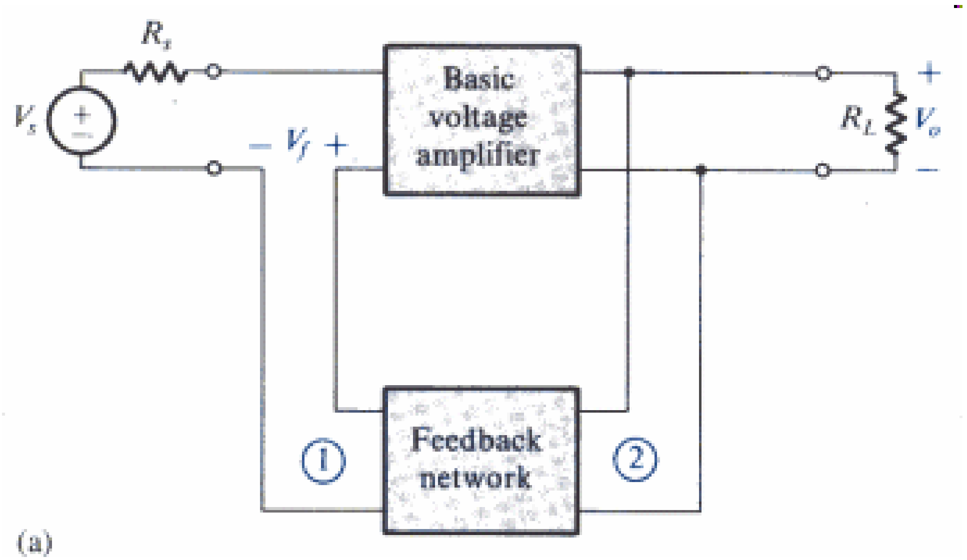
Series-Series

Shunt-Shunt



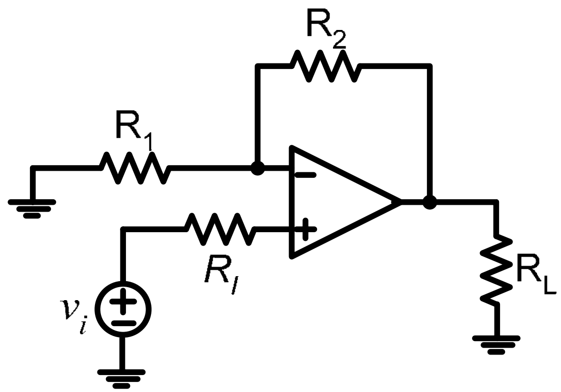
Shunt = Parallel

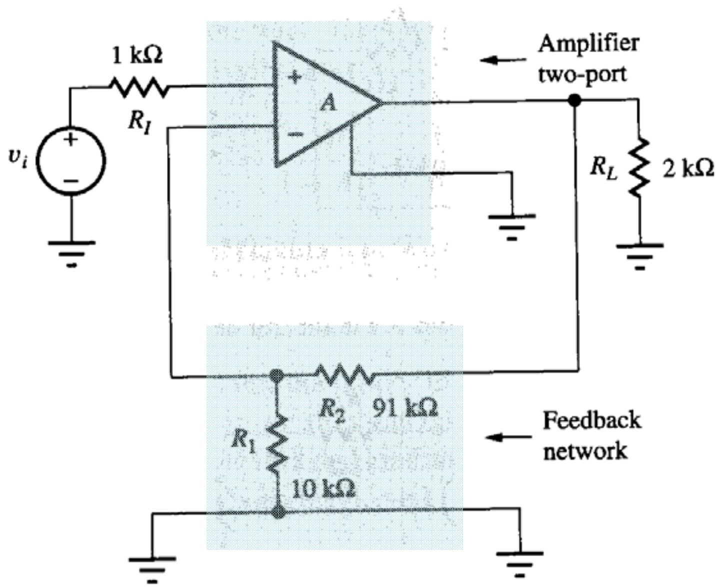
Series-Shunt Topology



- Measure the output voltage, subtract a voltage quantity from input
- Input resistance ↑ due to series connection
- Output resistance ↓ due to shunt connection
- Used in voltage amplifier

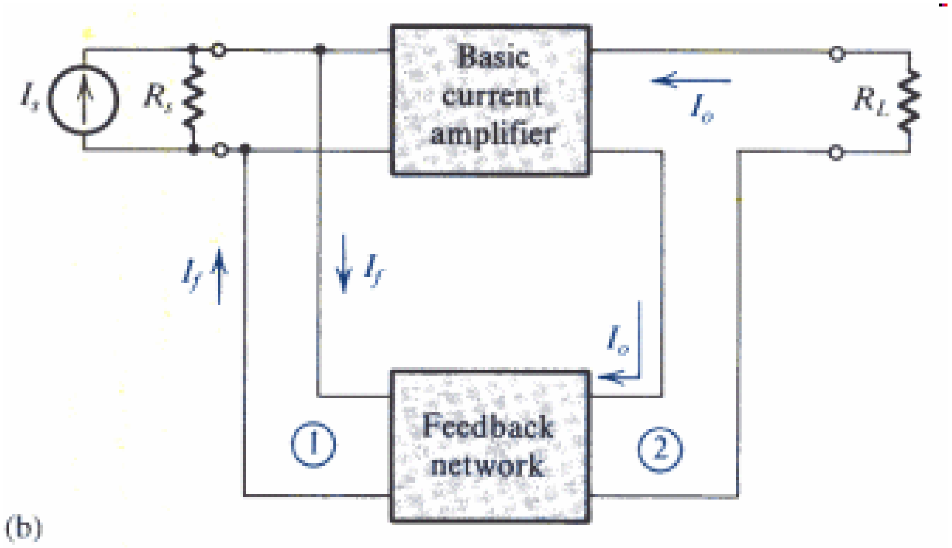
Series-Shunt Example





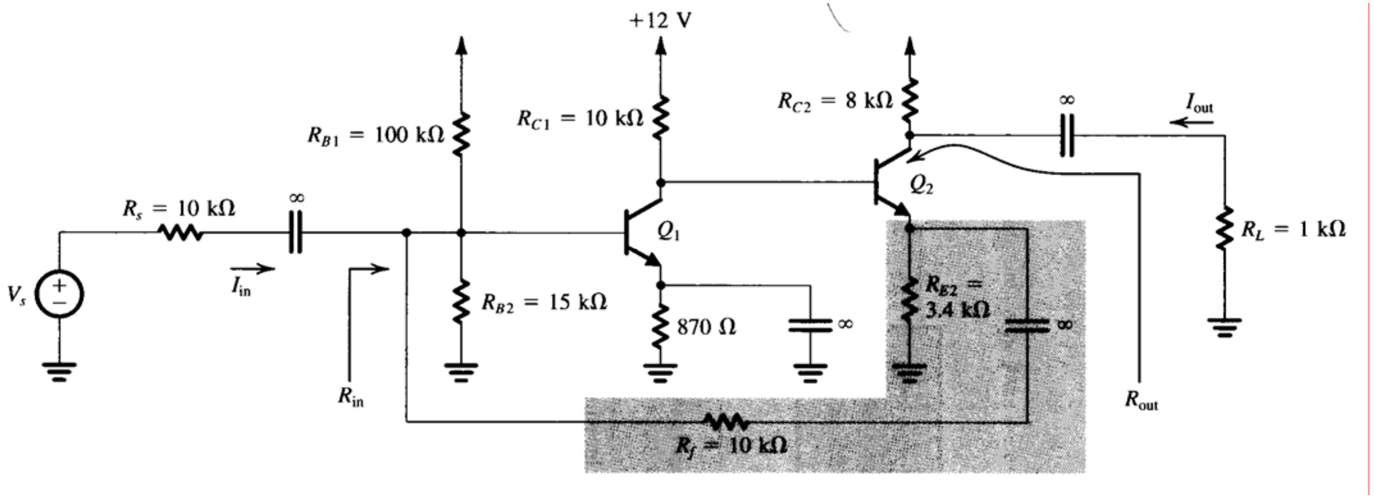
With two ports identified:

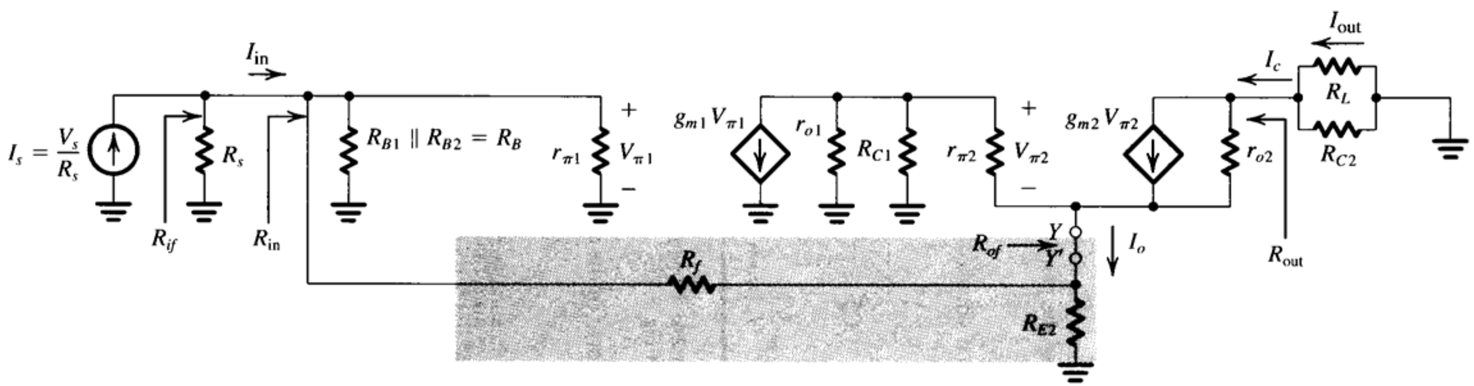
Shunt-Series Topology



- Measure the output current, subtract a current quantity from input
- Input resistance ↓
- Output resistance ↑
- Used in current amplifier

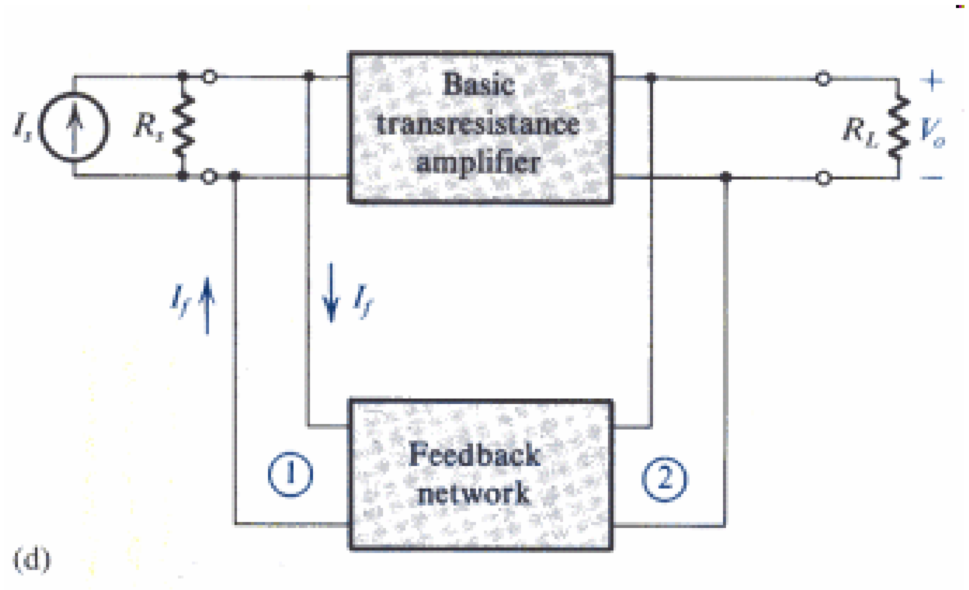
Shunt-Series Example





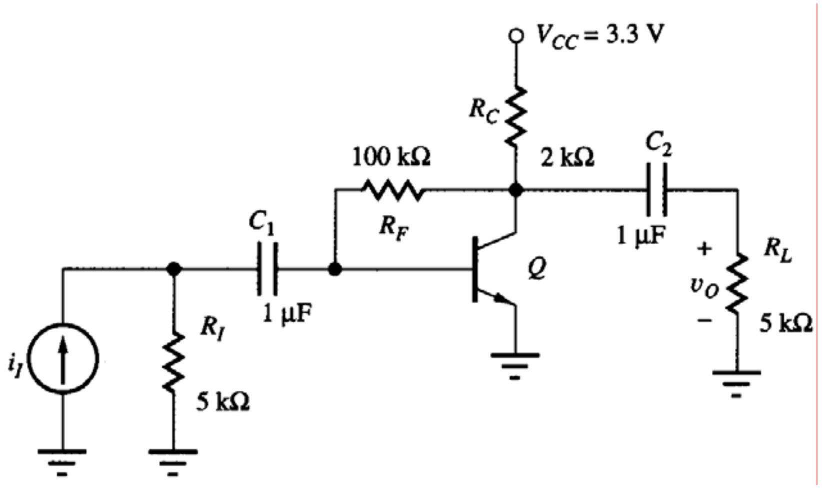
Small signal equivalent

Shunt-Shunt Topology

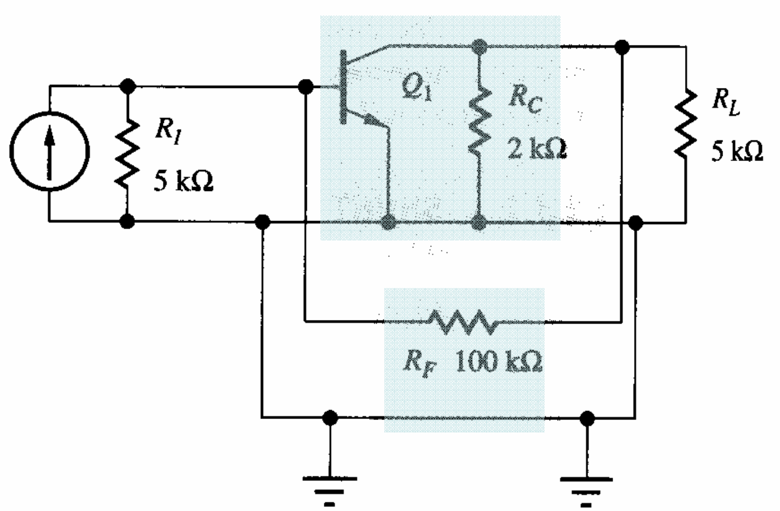


- Measure the output voltage, subtract a current quantity from input
- Input resistance ↓
- Output resistance ↓
- Used in trans-resistance (or more general, trans-impedance) amplifier

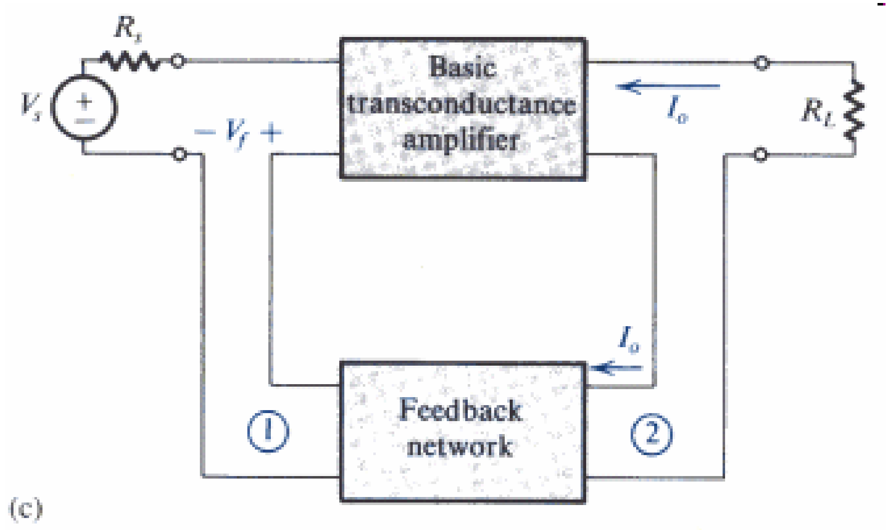
Shunt-Shunt Example



Two-port representation:

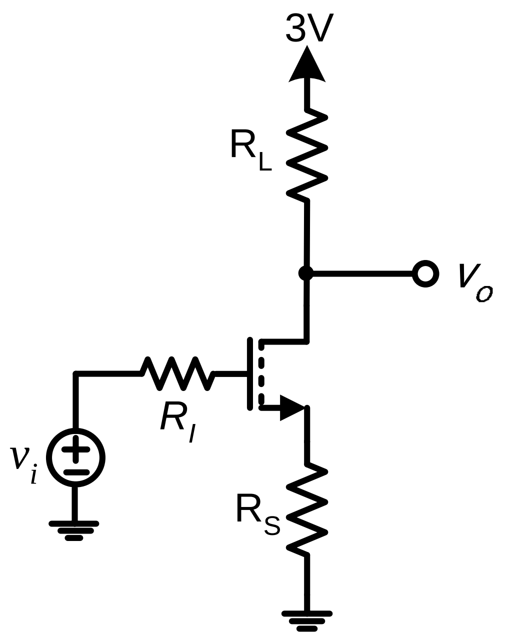


Series-Series Topology

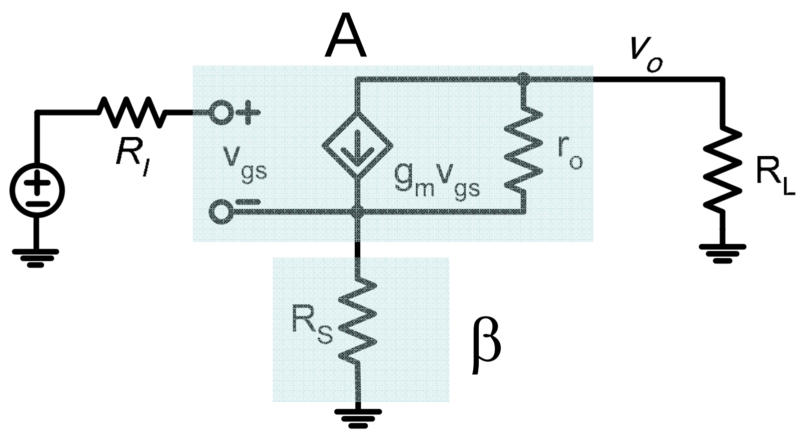


- Measure the output current, subtract a voltage quantity from input
- Input resistance ↑
- Output resistance ↑
- Used in transconductance amplifier

Series-Series Example



Source-degenerated common-source amplifier



Small signal equivalent

Topics to cover …

- Miller Effect
- Bode Plot
- Negative Feedback
  - Properties
  - Topologies
  - Stability issue

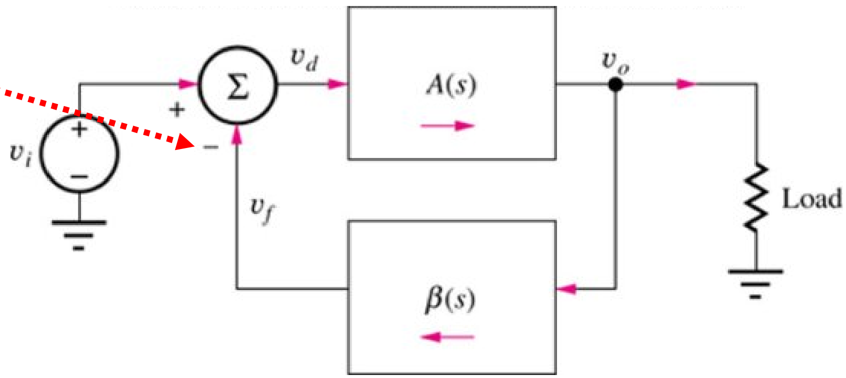
Stability of Feedback Amplifiers

Closed-loop gain (s=jω): $A_f(j\omega) = \dfrac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)}$

Loop gain: $T(j\omega) \equiv A(j\omega)\beta(j\omega) = |A(j\omega)\beta(j\omega)|\, e^{j\Phi(\omega)}$

At the frequency where $\Phi(\omega)$ becomes 180°:

Subtraction becomes addition; Feedback becomes effectively positive.

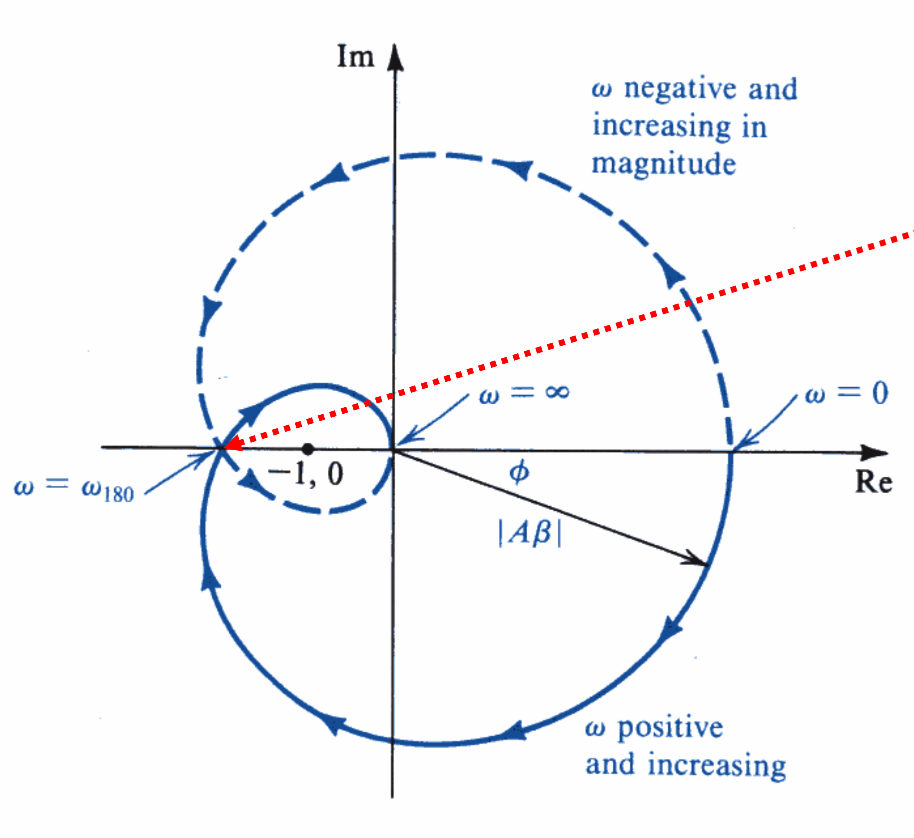


Signal adds to itself in-phase every time it travels the loop.

Case 1: |T| < 1, feedback amplifier will be stable.

Case 2: |T| >= 1, feedback amplifier will become unstable.

Determine Stability: Nyquist plot



- If the intersection occurs to the left of the point (-1,0), the amplifier is unstable
- Otherwise, the amplifier is stable

Plot the value of T(jω) in the complex plane for ω increasing from 0 to infinity

Stability and Pole Location

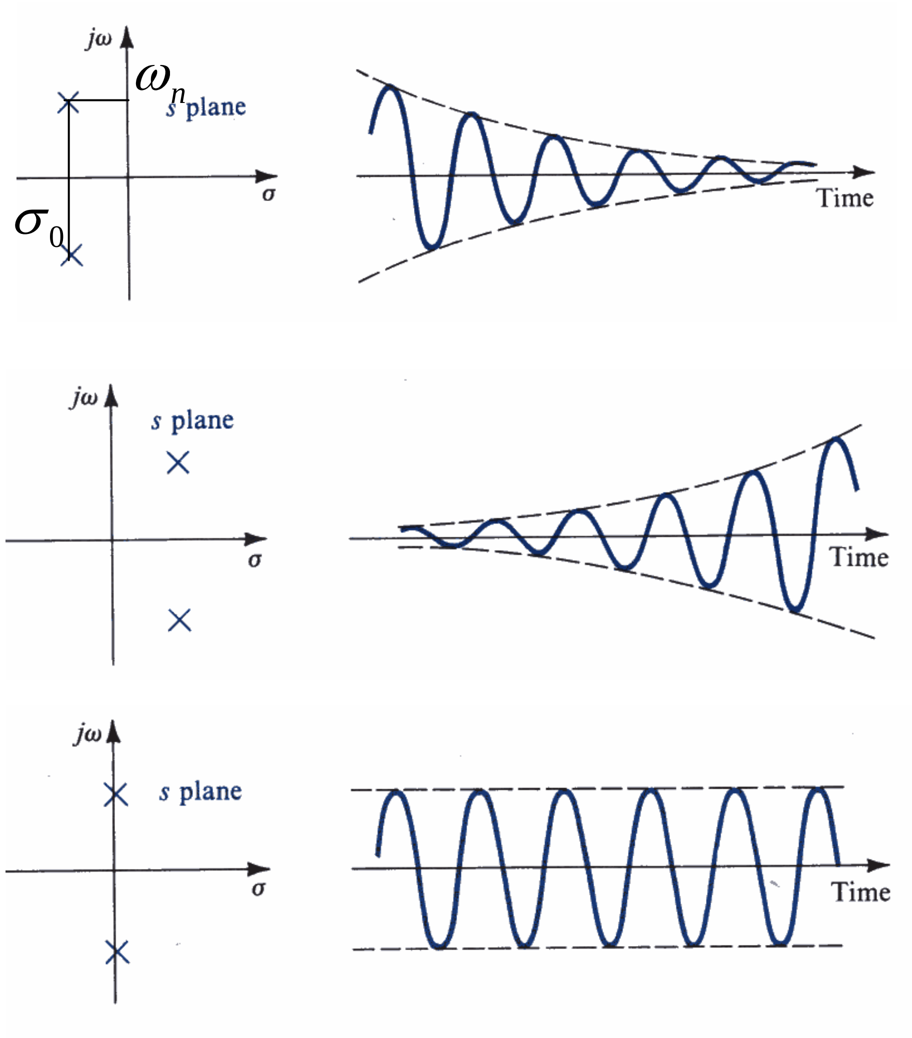
The impulse response for a transfer function with poles at $\sigma_o \pm j\omega_n$ will contain the term:

$$v(t) = e^{\sigma_0 t}[e^{+j\omega_n t} + e^{-j\omega_n t}] = 2e^{\sigma_0 t}\cos(\omega_n t)$$

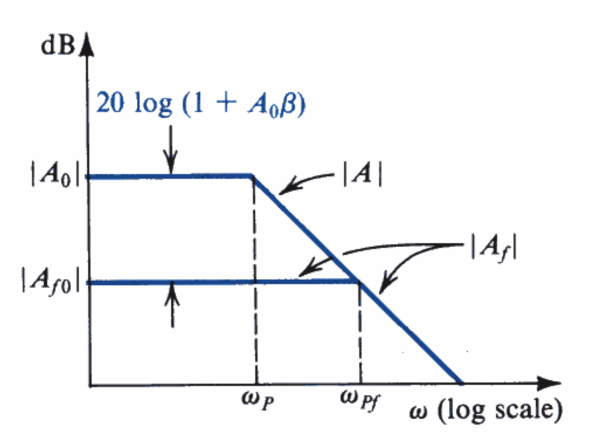
- Poles lie in the left half of the s-plane:
  Stable, Decaying Oscillations

- Poles lie in the right half of the s-plane:
  Unstable, Growing Oscillations

- A pair of complex conjugate poles on the $j\omega$ axis:
  Sustained Oscillations



Determine Stability: Root Locus



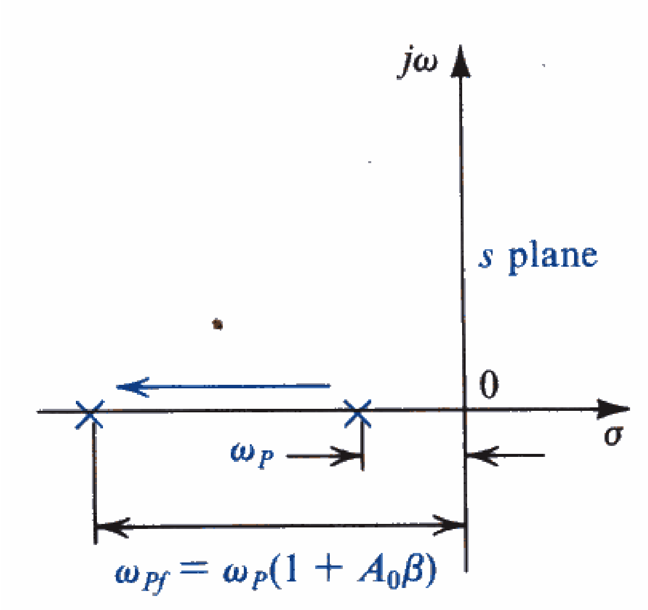
Considering an amplifier with single pole: $$A(s) = \frac{A_0}{1 + s/\omega_P}$$

Pole location: ω_P

The closed-loop gain is: $$A_f(s) = \frac{A(s)}{1 + A(s)\beta} = \frac{A_0/(1 + A_0\beta)}{1 + s/\omega_P(1 + A_0\beta)}$$

Pole location: $$\omega_{Pf} = \omega_P(1 + A_0\beta)$$

Determine Stability: Root Locus



Root locus: pole location of $A_f(s)$ while increasing $A_0\beta$

Single-pole amplifier is ***unconditionally stable*** as pole location is always in LHS. This can be understood because the phase lag associated with a single-pole response can never be greater than 90^0.

Root Locus: Two-pole Amplifier

For an amplifier with two poles

$$A(s) = \frac{A_0}{(1 + s/\omega_{P1})(1 + s/\omega_{P2})}$$

Set $1 + A(s)\beta = 0$ to find the closed-loop poles:

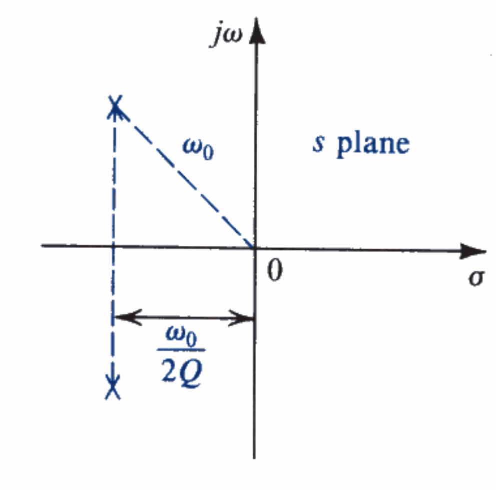
$$s^2 + s(\omega_{P1} + \omega_{P2}) + (1 + A_0\beta)\omega_{P1}\omega_{P2} = 0$$

The standard form is:

$$s^2 + s\frac{\omega_o}{Q} + \omega_o^2 = 0 \qquad Q = \frac{\sqrt{(1 + A_0\beta)\omega_{P1}\omega_{P2}}}{\omega_{P1} + \omega_{P2}}$$

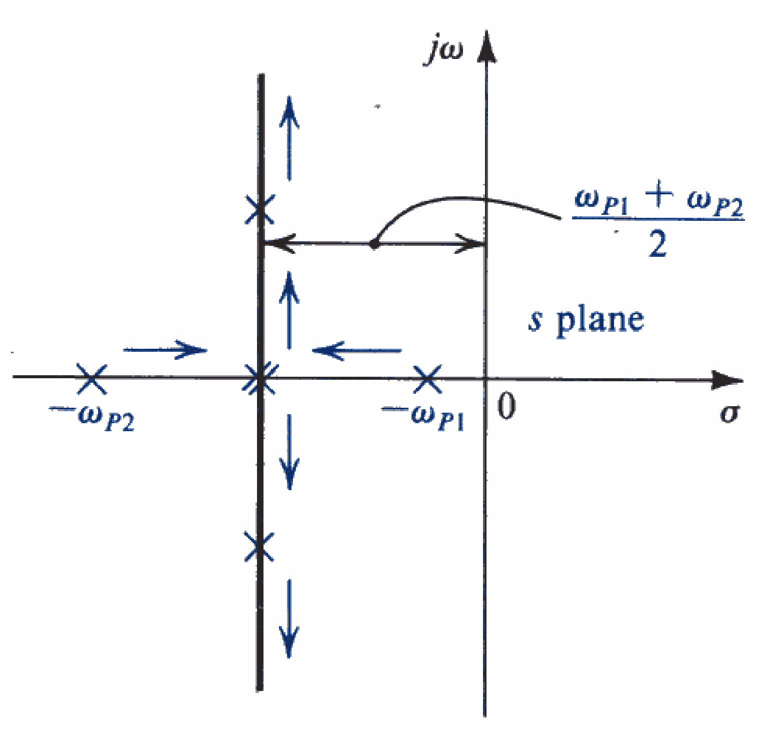
The closed-loop poles are:

$$s = -\frac{1}{2}(\omega_{P1} + \omega_{P2}) \pm \frac{1}{2}\sqrt{(\omega_{P1} + \omega_{P2})^2 - 4(1 + A_0\beta)\omega_{P1}\omega_{P20}}$$



Pole location of A(s)

Root Locus: Two-pole Amplifier

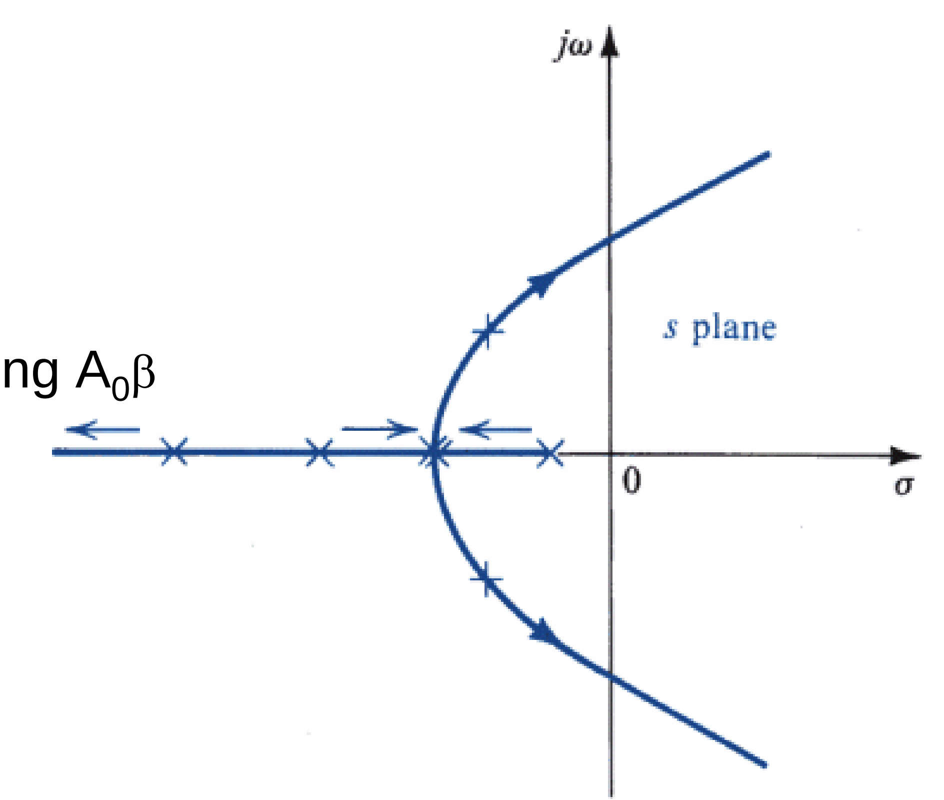


Pole location of $A_f(s)$ while increasing $A_0\beta$

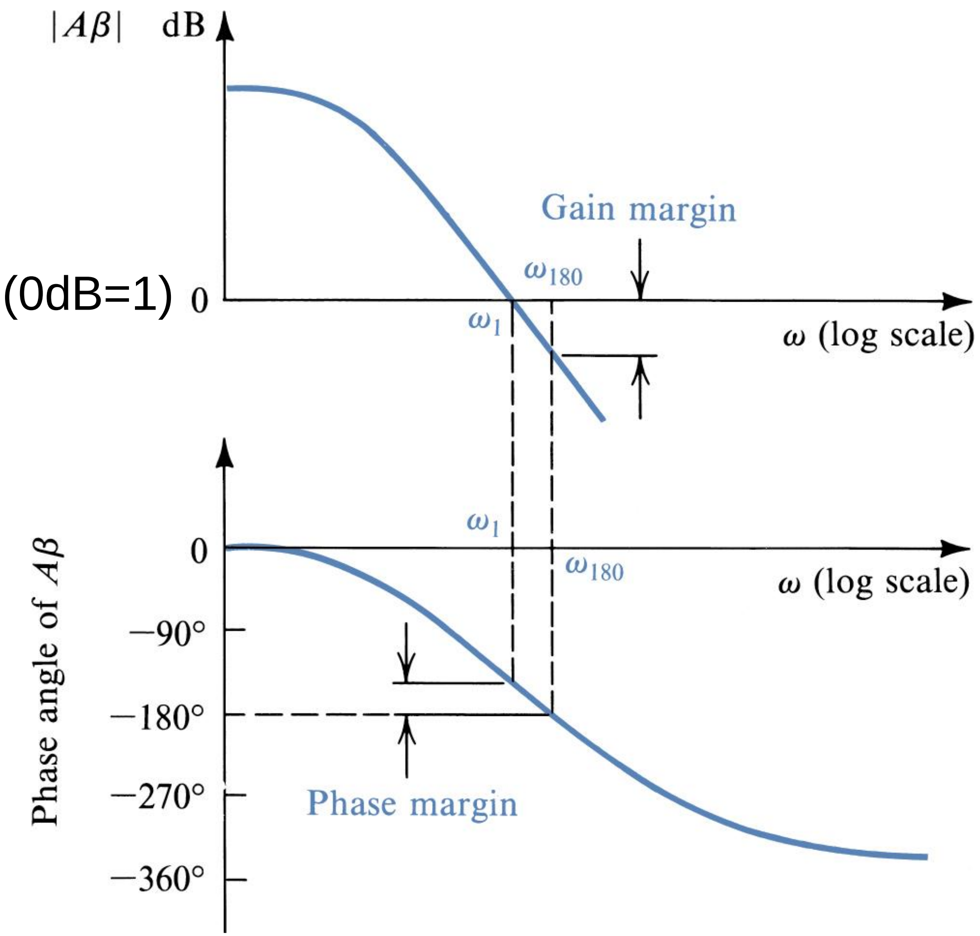
The two-pole feedback amplifier is also ***unconditionally stable***.

Root Locus: Three-pole Amplifier

Increasing $A_0\beta$



Determine Stability from Bode Plot: Gain and Phase margins



- Gain margin = the difference between the value of $|A\beta|$ at ω_{180} and unity
  - ω_{180}: the frequency at which $\angle A\beta = 180^o$
- Phase margin = the difference between the phase angle at ω_1 and 180^o
  - ω_1: the frequency at which $|A\beta| = 1$
- Feedback amplifiers are normally designed with a phase margin of at least 45^o

Example

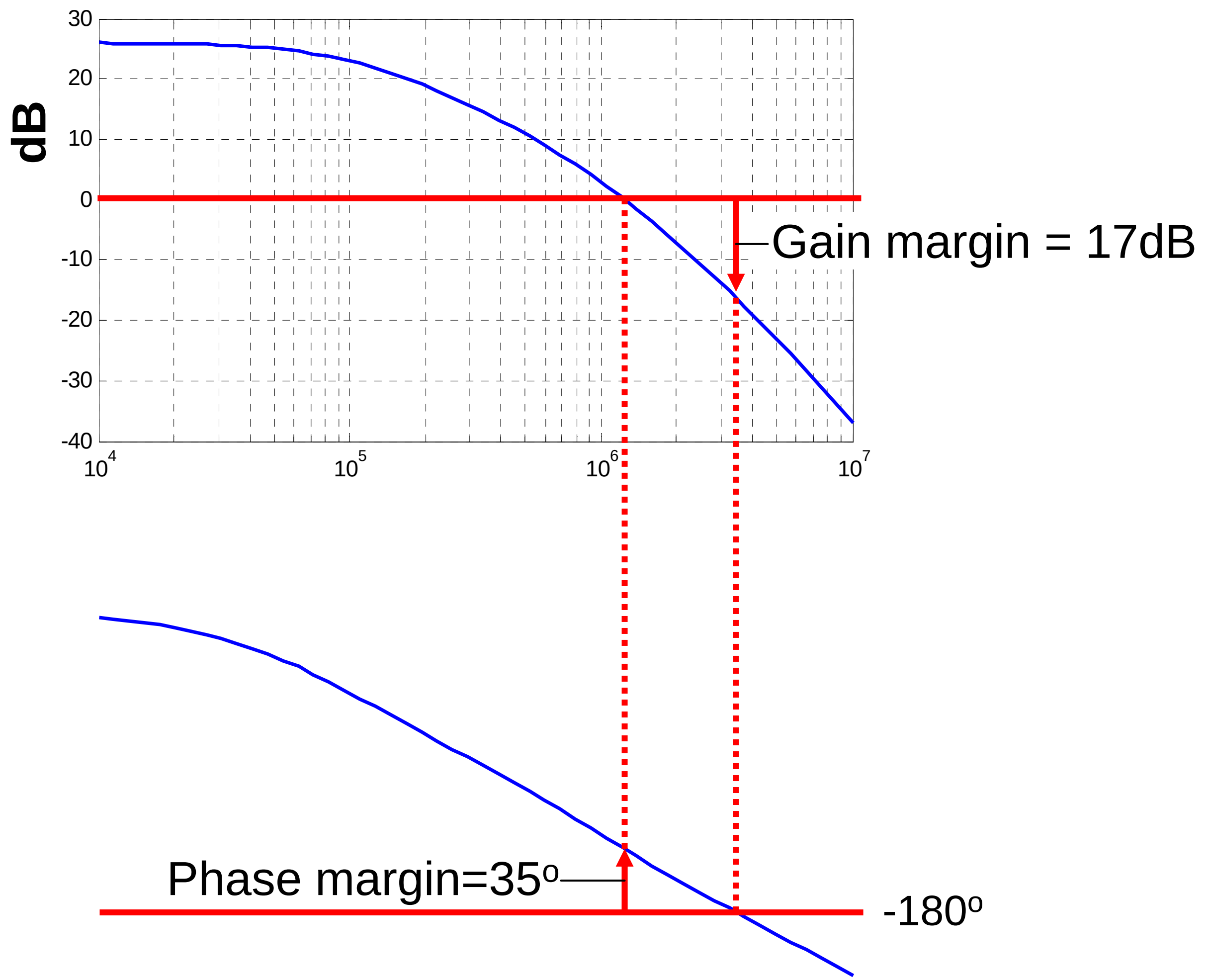
- Evaluate the phase and gain margin of a feedback amplifier with the following loop gain:

$$A\beta = \frac{2\times10^{19}}{\left(s+10^{5}\right)\left(s+10^{6}\right)\left(s+10^{7}\right)}$$

- A Matlab™ script is given in the next slide to generate the gain and phase response of the above loop gain

```
% Phase margin example for ELE2110A
% 17 April 2007, by KP Pun/EE/CUHK, free for distribution

clear all
 % build loop gain function in zero-pole-gain form
Abeta_zpk=zpk([],[-1e5 -1e6 -1e7],2e19) % no zero
Abeta_tf=tf(Abeta_zpk)

w=logspace(4,7); % a vector storing the angular frequencies

% evaluate loop gain for s=j*w
Abeta_val=polyval(Abeta_tf.num{1},j*w)./polyval(Abeta_tf.den{1},j*w);

% evaluate the magnitude and phase of the loop gain
Abeta_db=20*log10(abs(Abeta_val));
Abeta_angle=unwrap(angle(Abeta_val))*180/pi;

% create the magnitude and phase plots
subplot(2,1,1);
semilogx(w,Abeta_db,'LineWidth',2);  % plot the magnitude response
grid on; xlabel('Frequency (rad/s)','FontSize',14)
ylabel('Loop-gain A\beta (rad/s)','FontSize',14)
subplot(2,1,2);
semilogx(w,Abeta_angle,'LineWidth',2); % plot the phase response
xlabel('Frequency (rad/s)','FontSize',14);
ylabel('Phase (degree)','FontSize',14); grid on;
```

Phase and Gain Margins Example



Phase Margin and Closed-Loop Gain

- Express loop gain in terms of open-loop gain and closed-loop gain:

$$20\log|A\beta| = 20\log|A| - 20\log\left|\frac{1}{\beta}\right|$$

  – A: open loop gain
  – $1/\beta$ = closed loop gain for $|A\beta| >> 1$ $\quad A_f = \frac{A}{1+A\beta} \cong \frac{1}{\beta}$

- Can plot open loop gain and closed loop gain separately
- The frequency at which these two curves intersect is the point $|A\beta| = 1$, or 0dB → Phase margin can be obtained

Phase Margin and Closed-Loop Gain

Example:

$$A = \frac{2 \times 10^{24}}{(s+10^5)(s+10^6)(s+10^7)}$$

- Consider $1/\beta$ = 80dB, 50dB and 0dB
- From the intersects we can find the corresponding phase margins