EE 3025 S2005 Homework Set #2 Solutions
(We are grading Problems 1,3,4)

Solution to Problem 1.

part(a). The outcomes we are interested in are those elements of the sample space of form

$$(2or3or4, 1or3or4, 1or2or4, 1or2or3)$$

Listing them, we get the 9 outcomes

```
     2     1     4     3
     2     3     4     1
     2     4     1     3
     3     1     4     2
     3     4     1     2
     3     4     2     1
     4     1     2     3
     4     3     1     2
     4     3     2     1
```

Therefore, $P(E) = 9/24 = 0.375$.

part(b). First, I used Matlab to generate the sample space S:

```
S=[];
for a=1:4,for b=1:4, for c=1:4, for d=1:4
x=[a b c d];
y=sort(x);
z=[1 2 3 4];
if sum(abs(y-z))==0
S=[S;x];else end,end,end,end,end
S
S =

     1     2     3     4
     1     2     4     3
     1     3     2     4
     1     3     4     2
     1     4     2     3
     1     4     3     2
     2     1     3     4
     2     1     4     3
     2     3     1     4
     2     3     4     1
     2     4     1     3
     2     4     3     1
     3     1     2     4
     3     1     4     2
```

```
3    2    1    4
3    2    4    1
3    4    1    2
3    4    2    1
4    1    2    3
4    1    3    2
4    2    1    3
4    2    3    1
4    3    1    2
4    3    2    1
```

My Matlab script is a one liner which will randomly pick a row of S n times:

```
Results=S(ceil(24*rand(1,n)),:);
```

part(c). I run my script from (b) with $n = 5000$, and then check how many times I get something in E.

```
n=5000;
Results=S(ceil(24*rand(1,n)),:);
Compare=[ones(n,1) 2*ones(n,1) 3*ones(n,1) 4*ones(n,1)];
Q=(Results~=Compare);
PE_estimate=mean(sum(Q')==4)
```

I ran this 10 times and got the following estimates:

$$0.3786, 0.3712, 0.3794, 0.3748, 0.3834, 0.3698, 0.3824, 0.3786, 0.3762, 0.3750$$

The root mean square deviation is 0.0046.

part(d). I ran the following script 10 times:

```
n=50000;
Results=S(ceil(24*rand(1,n)),:);
Compare=[ones(n,1) 2*ones(n,1) 3*ones(n,1) 4*ones(n,1)];
Q=(Results~=Compare);
PE_estimate=mean(sum(Q')==4)
```

I got:

```
  0.3743    0.3721    0.3740    0.3722    0.3713    0.3784    0.3732

  0.3743    0.3781    0.3760
```

I got root mean square deviation of 0.0024, smaller than with only 5000 trials. So, the prob estimate is better for 50000 trials than it is for 5000 trials.

Solution to Problem 2. Verify the conditions in Definition 1.9:

$$P(E_1 \cap E_2 \cap E_3) = P(E_1)P(E_2)P(E_3) \tag{1}$$
$$P(E_1 \cap E_2) = P(E_1)P(E_2) \tag{2}$$
$$P(E_1 \cap E_3) = P(E_1)P(E_3) \tag{3}$$
$$P(E_2 \cap E_3) = P(E_2)P(E_3) \tag{4}$$

We have

$$\begin{aligned}
E_1 &= \{222, 323, 332, 433\} \\
E_2 &= \{222, 233, 332, 343\} \\
E_3 &= \{222, 233, 323, 334\} \\
E_1 \cap E_2 &= \{222, 332\} \\
E_1 \cap E_3 &= \{222, 323\} \\
E_2 \cap E_3 &= \{222, 233\} \\
E_1 \cap E_2 \cap E_3 &= \{222\}
\end{aligned}$$

It follows that the $P(E_i)$'s are all $1/2$, the $P(E_i \cap E_j)$'s are all $1/4$, and $P(E_1 \cap E_2 \cap E_3) = 1/8$. Eq(1) is true because both sides are $1/8$. Eq(2)-Eq(4) are each true because both sides are $1/4$. Conclusion: The three events are independent.

Solution to Problem 3. We have

$$S = \{HWB, HWW, HBB, HBW, TWB, TWW, TBB, TBW\}.$$

The model is

$$\begin{aligned}
P(HWB) &= (1/2)(2/5)(5/8) \\
P(HWW) &= (1/2)(2/5)(3/8) \\
P(HBB) &= (1/2)(3/5)(5/8) \\
P(HBW) &= (1/2)(3/5)(3/8) \\
P(TWB) &= (1/2)(3/8)(2/6) \\
P(TWW) &= (1/2)(3/8)(4/6) \\
P(TBB) &= (1/2)(5/8)(2/6) \\
P(TBW) &= (1/2)(5/8)(4/6)
\end{aligned}$$

part(a). The prob both balls are same color is computed as:

$$P(HWW) + P(HBB) + P(TWW) + P(TBB) = 59/120.$$

part(b). Define events E, F as:

$$\begin{aligned}
E &= \{\textit{both white}\} = \{HWW, TWW\} \\
F &= \{\textit{at least one white}\} = \{HWW, TWW, HBW, HWB, TBW, TWB\}
\end{aligned}$$

$$\begin{aligned}
P(E|F) &= \frac{P(E \cap F)}{P(F)} \\
&= \frac{P(HWW, TWW)}{P(HWW, TWW, HBW, HWB, TBW, TWB)} \\
&= \frac{P(HWW) + P(TWW)}{P(HWW) + P(TWW) + P(HBW) + P(HWB) + P(TBW) + P(TWB)} = 24/85
\end{aligned}$$

part(c). This is just a slight modification of (b). You get

$$\frac{P(HWB, HWW, HBW)}{P(HWW, TWW, HBW, HWB, TBW, TWB)} = 15/34.$$

Solution to Problem 4. The initial "forward cond prob matrix" is:

$$\begin{array}{r} \\ 0.40,\ Ind \\ 0.25,\ Rep \\ 0.35,\ Dem \end{array} \begin{array}{c} \begin{array}{cc} Kimball & Douglas \end{array} \\ \begin{pmatrix} 0.5 & 0.5 \\ 1 & 0 \\ .25 & .75 \end{pmatrix} \end{array}$$

(a). The joint prob matrix is

$$\begin{array}{r} \\ Ind \\ Rep \\ Dem \end{array} \begin{array}{c} \begin{array}{cc} Kimball & Douglas \end{array} \\ \begin{pmatrix} 0.2 & 0.2 \\ 0.25 & 0 \\ .0875 & .2625 \end{pmatrix} \end{array}$$

The sum of the right column is the prob a voter voted for Douglas. This is 0.4625.

(b). Divide the left column of the joint prob matrix by the col sum 0.5375. The bottom entry would then be the answer:

$$(0.0875/0.5375) = 0.1628.$$

Solution to Problem 5. The channel matrix is

$$\begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} 0.95 & 0.05 \\ 0.15 & 0.85 \end{pmatrix} \end{array}$$

(a). Multiply the channel matrix on the left by

$$(1/2 \;\; 1/2).$$

You get

$$(0.55 \;\; 0.45).$$

1 is therefore received with prob 0.45.

(b). The joint prob matrix is

$$\begin{array}{cc} p(0.95) & p(0.05) \\ (1-p)(0.15) & (1-p)(0.85) \end{array}$$

Sum up the 1st column, set equal to 1/2, and solve for p. You get $p = 7/16$.