EE 3025 S2005 Homework Set #4

(due 10:10 AM Friday, February 18, 2005)

Directions: Work all 5 problems. We will grade Problem 1 and will randomly choose two of the other problems for grading.

1. In this Matlab problem, you simulate 100000 values of RV's and use these values to plot an estimated CDF. Before attempting this problem, you might want to read Experiment 1 of Recitation 3.

(a) Let X be the "shotput" RV in Example 7.7 of the Lecture 7 Notes. The PDF is

$$f_X(x) = (0.1)\delta(x-10) + (0.9)(1/10)[u(x-60) - u(x-70)].$$

Write some Matlab code and then run it to create in Matlab memory a vector `x` consisting of 100000 entries simulating values of X over 100000 independent trials. Once having created `x`, then run the following script from Exp. 1 of Rec. 3:

```
n=100000;
N=1000;
A=0; B=80;
Delta=(B-A)/N;
t=A-Delta/2+[1:N]*Delta;
p=hist(x,t)/n;
CDF=cumsum(p);
plot(t,CDF)
axis([0 80 -.1 1.1])
```

You will obtain a Matlab plot of the estimated CDF $F_X(x)$. Print out the code you used to create `x` and print out your plot and turn them in.

(b) Let U be the standard uniform RV (i.e., Uniform(0,1)). Let X be the RV

$$X = \sin((0.5)\pi U).$$

Write one line of Matlab code and run it to create in Matlab memory a vector `x` consisting of 100000 entries simulating values of X over 100000 independent trials. Once having created `x`, then run the following script from Exp. 1 of Rec. 3:

```
n=100000;
N=1000;
A=-0.5; B=1.5;
Delta=(B-A)/N;
t=A-Delta/2+[1:N]*Delta;
p=hist(x,t)/n;
CDF=cumsum(p);
plot(t,CDF)
axis([-0.5 1.5 -.1 1.1])
```

You will obtain a Matlab plot of the estimated CDF $F_X(x)$. Print out the code you used to create `x` and print out your plot and turn them in. Use your plot to estimate the probability $P(X \leq 0.5)$. Then see if you can compute the exact value of $P(X \leq 0.5)$ to four decimal places.

2. In this problem, you consider linear changes of variable of a RV.

(a) Let X be the continuous RV with PDF $f_X(x)$ as follows:

$$f_X(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 0, & elsewhere \end{cases}$$

Let Y be the RV

$$Y = -2X + 7.$$

Compute μ_Y, σ_Y^2 from μ_X, σ_X^2 which were found in Lecture 10 Notes. Sketch the plots of $f_Y(y)$ and $F_Y(y)$.

(b) Let X be standard uniform (that is, Uniform(0,1)). Find constants a, b so that the RV

$$Y = aX + b$$

will have mean 20 and variance 5.

3. You have a loaded die which comes up 6 with probability 0.20 and comes up $1, 2, 3, 4, 5$ with probability 0.16 each. You flip this die one time; let RV X be the number which comes up.

(a) Work out the moment generating function $\phi_X(s)$ of X.

(b) Use $\phi_X(s)$ to compute μ_X.

(c) Use $\phi_X(s)$ to compute σ_X^2.

4. Let discrete RV X take values $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ according to the following PMF:

$$\begin{aligned} P^X(x) &= 1/15, \quad x = 1, 2, 3, 4, 5 \\ P^X(x) &= 2/15, \quad x = 6, 7, 8, 9, 10 \end{aligned}$$

(a) Plot the conditional PMF for X given $X \leq 5.5$, and plot the conditional PMF for X given $X > 5.5$.

(b) Compute $P(3 \leq X \leq 7|X \leq 5.5)$ and $P(3 \leq X \leq 7|X > 5.5)$ using the conditional PMF's you found in (a). Then compute $P(3 \leq X \leq 7)$ directly from the PMF of X and verify that the following formula is true:

$$P(3 \leq X \leq 7) = P(3 \leq X \leq 7|X \leq 5.5)P(X \leq 5.5) + P(3 \leq X \leq 7|X > 5.5)P(X > 5.5).$$

(This is another form of the law on total probability).

(c) Compute $E(X|X \le 5.5)$ and $E(X|X > 5.5)$ using the conditional PMF's you found in (a). Then compute $E(X)$ directly from the PMF of X and verify that the following formula is true:

$$E(X) = E(X|X \le 5.5)P(X \le 5.5) + E(X|X > 5.5)P(X > 5.5).$$

(This is the law on total expectation.)

5. Let X be the continuous RV in Problem 5(b) of Homework Set 3. In this problem, it is OK to use the values for μ_X and σ_X^2 found in the Solutions to Homework Set 3.

(a) Determine the conditional density of X given $X \le 2.5$ and also determine the conditional density of X given $X > 2.5$.

(b) Use the conditional densities from (a) to compute $E(X|X \le 2.5)$ and $E(X|X > 2.5)$. Then verify the formula

$$E(X) = E(X|X \le 2.5)P(X \le 2.5) + E(X|X > 2.5)P(X > 2.5).$$

(c) Compute $Var(X|X \le 2.5)$ and $Var(X|X > 2.5)$ using the conditional densities from (a). Then determine whether or not the following formula holds:

$$Var(X) = Var(X|X \le 2.5)P(X \le 2.5) + Var(X|X > 2.5)P(X > 2.5).$$

Supplementary Problems: (not to hand in) From the textbook, you can try Problems 2.7.3, 2.8.6, 2.9.2, 3.6.8, 3.8.4; you can also consult the Solved Problems on Chapters 2-3 on the Web