EE 3025 S2005 Homework Set #7

(due 10:10 AM Friday, March 25, 2005)

Directions: Work all 5 problems. We will grade Problem 1 and will randomly choose two of the other problems for grading.

1. You might want to try the last experiment of Recitation 7 before trying this problem.

(a) Let Z_1, Z_2, Z_3 be independent Uniform(0,1) RV's, and define X_1, X_2, X_3 to be the RV's

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ -1 & 2 & 1 \\ 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} \tag{1}$$

Use Matlab to generate vectors `x1,x2,x3` of 50000 samples each of X_1, X_2, X_3. Use these three vectors to estimate the 3×3 covariance matrix Σ_X of the RV's X_1, X_2, X_3, given by

$$\Sigma_X = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \sigma_{1,3} \\ \sigma_{2,1} & \sigma_{2,2} & \sigma_{2,3} \\ \sigma_{3,1} & \sigma_{3,2} & \sigma_{3,3} \end{bmatrix},$$

where $\sigma_{i,j} \triangleq Cov(X_i, X_j)$. Turn in printout of your Matlab code used to find the estimated 3×3 covariance matrix and also print out the estimated 3×3 covariance matrix that Matlab gives you. (For help, you can look at Step 3 on page 11 of Recitation 7 and extend the estimate from two variables to three variables.)

(b) Find by hand the exact 3×3 covariance matrix Σ_Z of the RV's Z_1, Z_2, Z_3. (This is easy to do, using the independence of the Z_i's.) Let A be the 3×3 coefficient matrix on the left side of (1). Use Matlab to compute the matrix triple product

$$A * \Sigma_Z * A^T$$

and compare this answer with your estimated 3×3 matrix from (a). Are the answers about the same? Are you surprised?

(c) Repeat parts (a),(b) assuming that Z_1, Z_2, Z_3 are independent Gaussian(0,1) RV's.

2. Random variables X, Y are each discrete and the set S of allowable (X, Y) pairs consists of all (i, j) in which $i \leq j$ and i and j are integers between 1 and 30, inclusively. The joint PMF is of the form

$$P^{X,Y}(i, j) = Cij, \quad (i, j) \in S \ (zero\ elsewhere)$$

The computations in this problem are kind of messy, so you can use Matlab to do them if you want.

(a) Let B be the event that $X + Y > 20$. Compute the conditional PMF

$$P(X = i|B), \quad i = 1, 2, \cdots, 30$$

and put the results as two columns of a table of the form

i	$P(X = i \mid B)$
col of i values	col of $P(X = i \mid B)$ values

(b) Use your conditional PMF from (a) to compute each of the following:

$$P(X \geq 20 \mid B), \quad E(X \mid B), \quad Var(X \mid B).$$

3. Random variables X, Y are jointly continuously distributed with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} Cxy, & (x,y) \in R \\ 0, & \text{elsewhere} \end{cases}$$

where R is the triangular region $R = \{(x,y) : 0 \leq y \leq x,\ 0 \leq x \leq 2\}$.

(a) Plot the conditional density of Y given $X = 3/4$. Use this conditional density to compute $P[Y \geq 3/8 \mid X = 3/4]$ and $E[Y \mid X = 3/4]$.

(b) Plot the conditional density of X given $Y = 1$. Compute $P[X \leq 1.5 \mid Y = 1]$ and $\text{Var}[X \mid Y = 1]$.

4. You are to work this problem using the law of iterated expectation:

$$E[\phi(X)\psi(Y)] = E[\phi(X)E[\psi(Y) \mid X]].$$

You are not allowed to use the joint density of (X, Y). In this problem, X is continuous with density

$$f_X(x) = Cx, \quad 0 \leq x \leq 3 \ (\textit{zero elsewhere})$$

Given $X = x$, Y is conditionally uniformly distributed between 0 and $3 - x$.

(a) Compute $E[Y]$ by the law of iterated expectation.

(b) Compute the correlation $E[XY]$ by the law of iterated expectation.

(c) Compute $E[Y^2]$ by the law of iterated expectation. Then use this answer and the answer to part(a) to compute $Var(Y)$.

5. Let T_1, T_2, T_3 be independent RV's each exponentially distributed with mean 1. Compute each of the following:

(a) $P[T_1 + 2T_2 + 3T_3 > 4]$

(b) $P[\min(T_1, 2T_2, 3T_3) > 0.2]$

(c) $P[\max(T_1, 2T_2, 3T_3) > 2]$

Supplementary Problems: (not to hand in) From the textbook, you can try Problems 4.9.7, 4.9.9, 4.9.11, 4.9.12, 4.10.12