# LECTURE 19

## Poisson process review and examples

- **Readings:** Finish Section 5.2.

- Review of Poisson process
- Examples
- Random incidence

## Review

- Numbers of arrivals in disjoint time intervals are independent

$$P(k,\delta) \approx \begin{cases} 1-\lambda\delta & \text{if } k=0 \\ \lambda\delta & \text{if } k=1 \\ 0 & \text{if } k>1 \end{cases}$$

$$P(k,\tau) = \frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!}, \qquad k=0,1,\ldots$$

$\mathbf{E}[N] = \lambda\tau$

- Interarrival times ($k=1$): exponential

$$f_{T_1}(t) = \lambda e^{-\lambda t}, \quad t \geq 0, \qquad \mathbf{E}[T_1] = 1/\lambda$$

- Time $Y_k$ to $k$th arrival:

$$f_{Y_k}(y) = \frac{\lambda^k y^{k-1} e^{-\lambda y}}{(k-1)!}, \qquad y \geq 0$$

## Poisson catches

- Assume: Poisson, $\lambda = 0.6$/hour
  Fish for two hours; if no catch, continue until first catch.

**A.** **P**(fish for more than two hours)=

**B.** **P**(fish for more than two and less than five hours)=

**C.** **P**(catch at least two fish)=

**D.** **E**[number of fish]=

**E.** **E**[future fishing time | fished for four hours]=

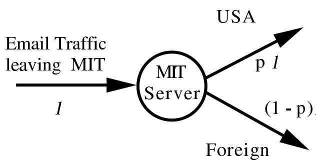
**F.** **E**[total fishing time]=

## Light bulb example

- Each light bulb has independent, exponentially distributed lifetime $\lambda e^{-\lambda t}$, $t \geq 0$
- Install three light bulbs.
  Find expected time until last light bulb dies out

## Splitting of Poisson processes

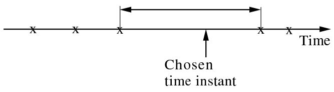
- Each message is routed along the first stream with probability $p$
  - Routings of different messages are independent




- Each output stream is Poisson

## Random incidence for Poisson

- Poisson process that has been running forever
- Show up at some "random time" ("random incidence")




- What is the distribution of the length of the chosen interarrival interval?

## Renewal processes

- Series of successive arrivals
  - i.i.d. interarrival times (but not necessarily exponential)
- **Example:** Bus interarrival times are equally likely to be 5 or 10 minutes
- If you arrive at a "random time":
  - what is the probability that you selected a 5 minute interarrival interval?
  - what is the expected time to next arrival?