LECTURE 22

Markov Processes – III

Readings: Section 6.4

Lecture outline

- Review of steady-state behavior
- Probability of blocked phone calls
- Calculating absorption probabilities
- Calculating expected time to absorption

Review

- Assume a single class of recurrent states, aperiodic. Then,

$$\lim_{n\to\infty} r_{ij}(n) = \pi_j$$

where π_j does not depend on the initial conditions

$$\lim_{n\to\infty} \mathbf{P}(X_n = j \mid X_0) = \pi_j$$

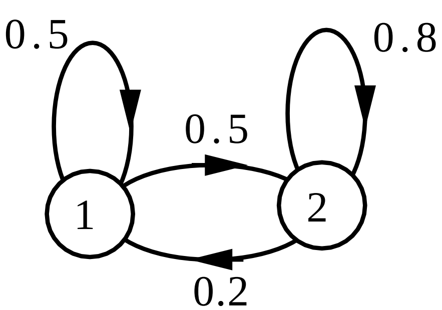
- $\pi_1, \ldots, \pi_m$ can be found as the unique solution of the balance equations

$$\pi_j = \sum_k \pi_k p_{kj}$$

together with

$$\sum_j \pi_j = 1$$

Example



$\pi_1 = 2/7$, $\pi_2 = 5/7$

- Assume process starts at state 1.

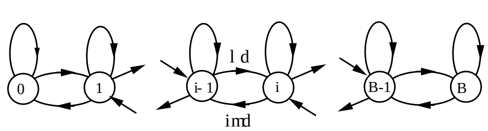
- $\mathbf{P}(X_1 = 1, \text{ and } X_{100} = 1) =$

- $\mathbf{P}[X_{100} = 1 \text{ and } X_{101} = 2]$

The phone company problem

- Calls originate as a Poisson process, rate λ
  - Each call duration is exponentially distributed (parameter μ)
  - B lines available

- Discrete time intervals of (small) length δ

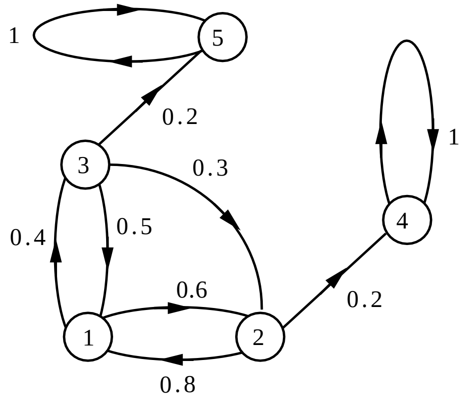


- Balance equations: $\lambda\pi_{i-1} = i\mu\pi_i$

- $\pi_i = \pi_0 \dfrac{\lambda^i}{\mu^i i!}$ $\qquad \pi_0 = 1 \Big/ \sum_{i=0}^{B} \dfrac{\lambda^i}{\mu^i i!}$

Calculating absorption probabilities

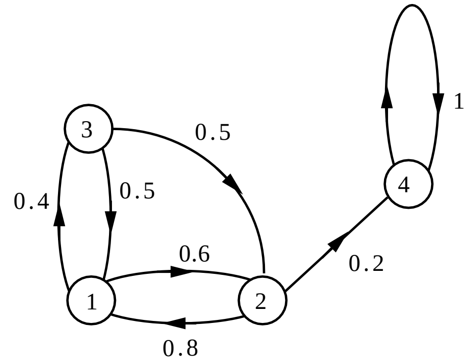
- What is the probability a_i that the process eventually settles in state 4, given that the initial state is i?



For $i = 4$, $a_i =$
For $i = 5$, $a_i =$

$$a_i = \sum_j p_{ij} a_j, \quad \text{for all other } i$$

Expected time to absorption



- What is the expected number of transitions μ_i until the process reaches the absorbing state, given that the initial state is i?

$\mu_i = 0$ for $i =$

For all other i: $\mu_i = 1 + \sum_j p_{ij} \mu_j$

Constructing Markov models

- Many processes are Markov provided the state is suitably defined
- Let times until next bus arrival be i.i.d., uniform on $\{1, 2, 3\}$
- Let $Y_n = A$ if arrival, $Y_n = N$ otherwise
  - Is Y_n Markov?
- Let X: time since last arrival