EE126: Probability and Random Processes **SP'07**

Problem Set 4 — Due Feb, 15

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Problem 4.1. *Suppose three fair dice are rolled. What is the probability that at most one six appears?*

Problem 4.2. *Suppose we are given a coin for which the probability of heads is p $(0 < p < 1)$ and the probability of tails is $1 - p$. Consider a sequence of independent flips of the coin.*

1. *Let y $(y = 1, 2, ...)$ be the number of flips up to and including the flip on which the first head occurs. Determine the pmf $p_y(y_0)$ for all values of y_0.*
2. *Let x $(x = 0, 1)$ be the number of heads that occur on any particular flip.*
   (a) *Determine E(x).*
   (b) *Determine σ_x^2.*
3. *Let k $(k = 0, 1, ..., N)$ be the number of heads that occur on the first N flips of the coin. Determine*
   (a) *the pmf $p_k(k_0)$*
   (b) *$E(k)$ [Hint: Your results from part (b) may help you in determining $E(k)$ and σ_k^2.]*
   (c) *σ_k^2*
4. *Given that a total of exactly six heads resulted from the first nine flips, what is the conditional probability that both the first and seventh flips were tails?*
5. *Let h be the number of heads that occur on the first twenty flips. Let C be the event that a total of exactly ten heads resulted from the first eighteen flips. Find*
   (a) *$E[h|C]$*
   (b) *$\sigma_{h|C}^2$*

Problem 4.3. *Let X be a random variable with density function*

$$f(x) = \begin{cases} ce^{-2x}, & 0 < x < \infty \\ 0, x < 0 \end{cases}$$

1. *Find c.*

2. *What is* $P[X > 2]$*?*
3. *Compute* $E[X]$.

Problem 4.4. *A marksman takes 10 shots at a target and has probability 0.2 of hitting the target with each shot. Let X be the number of hits.*

1. *Calculate and plot the PMF of X.*
2. *What is the probability of scoring no hits?*
3. *What is the probability of scoring more hits than misses?*

Problem 4.5. *Your probability class has 250 undergraduate students and 50 graduate students. The probability of an undergraduate (or graduate) student getting an A is 1/3 (or 1/2, respectively). Let* X *be the number of students that get an A in your class.*

1. *Find the PMF of* X.
2. *Calculate* $E[X]$ *using the total expectation theorem, rather than the PMF of* X.
3. *Calculate* $E[X]$ *and var*(X) *by viewing* X *as a sum of random variables, whose statistics are easily calculated.*

Problem 4.6. *Consider the random variable* X *with PMF*

$$p_X(x) = \begin{cases} \frac{x^2}{a} & \text{if } x = -3, -2, -1, 0, 1, 2, 3, \\ 0 & \text{otherwise.} \end{cases}$$

1. *Find* a *and* $\mathbf{E}[X]$.
2. *What is the PMF of the random variable* $Z = (X - \mathbf{E}[X])^2$ *?*
3. *Using part (b) compute the variance of* X.

Problem 4.7. *Prove that*

$$E[X^2] \geq (E[X])^2 \,.$$

When do we have equality?

Problem 4.8. *A coin having probability* p *of coming up heads is successively flipped until the* r*th head appears. Argue that X, the number if flips required, will be* n, $n \geq r$, *with probabilty*

$$P[X = n] = \binom{n-1}{r-1} p^r (1-p)^{n-r}$$

Compute the expectation of X.