Alexander-Sadiku
Fundamentals of Electric Circuits

Chapter 8
Second-Order Circuits

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Second-Order Circuits
Chapter 8

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8.2 The source-free series RLC circuit
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8.4 Step response of a series RLC circuit
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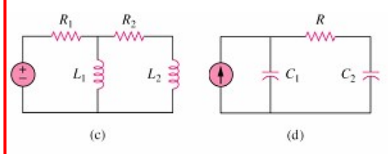
8.1 Examples of Second Order RLC circuits (1)

What is a 2nd order circuit?

A second-order circuit is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.



RL T-config RC Pi-config

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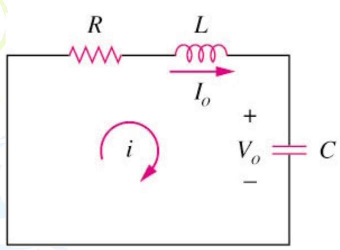
8.2 Source-Free Series RLC Circuits (1)



The solution of the source-free series RLC circuit is called as the natural response of the circuit.

The circuit is excited by the energy initially stored in the capacitor and inductor.

The 2nd order of expression

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

How to derive and how to solve?

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8.2 Source-Free Series RLC Circuits (2)

Method will be illustrated during the lecture

8.2 Source-Free Series RLC Circuits (3)

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

=> $$\frac{d^2 i}{dt^2} + 2\alpha\frac{di}{dt} + \omega_0^2 i = 0$$

General 2nd order Form

where $\alpha = \frac{R}{2L}$ and $\omega_0 = \sqrt{\frac{1}{LC}}$

The types of solutions for i(t) depend on the relative values of α and ω.

8.2 Source-Free Series RLC Circuits (4)

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2 i}{dt^2} + 2\alpha \frac{di}{dt} + \omega_0^2 i = 0$$

1. If $\alpha > \omega_o$, over-damped case

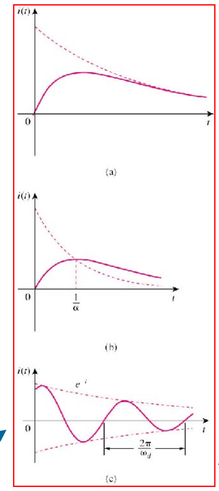
$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ **where** $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

2. If $\alpha = \omega_o$, critical damped case

$i(t) = (A_2 + A_1 t) e^{-\alpha t}$ **where** $s_{1,2} = -\alpha$

3. If $\alpha < \omega_o$, under-damped case

$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$ **where** $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$



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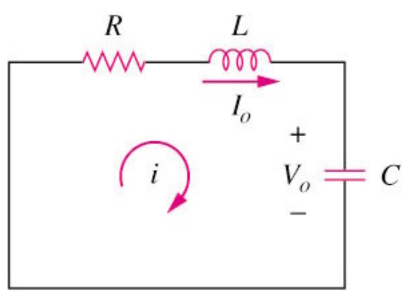
8.2 Source-Free Series RLC Circuits (5)

Example 1

If R = 10 Ω, L = 5 H, and C = 2 mF in 8.8, find α, ω0, s1 and s2.

What type of natural response will the circuit have?



Please refer to lecture or textbook for more detail elaboration.

Answer: *underdamped*

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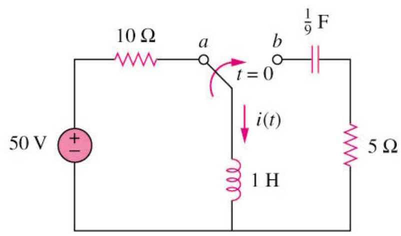
8.2 Source-Free Series RLC Circuits (6)

Example 2

The circuit shown below has reached steady state at t = 0-.

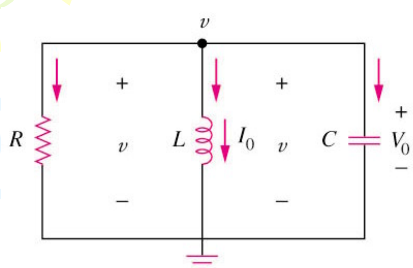


If the make-before-break switch moves to position b at t = 0, calculate i(t) for t > 0.

Please refer to lecture or textbook for more detail elaboration.

Answer: $i(t) = e^{2.5t}[5\cos 1.6583t \quad 7.538\sin 1.6583t]$ A

8.3 Source-Free Parallel RLC Circuits (1)



Let $i(0) = I_0 = \frac{1}{L}\int_{\infty}^{0} v(t)dt$

$v(0) = V_0$

Apply KCL to the top node:

$$\frac{v}{R} + \frac{1}{L}\int_{-\infty}^{t} v dt + C\frac{dv}{dt} = 0$$

Taking the derivative with respect to t and dividing by C

The 2nd order of expression

$$\frac{d^2 v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

8.3 Source-Free Parallel RLC Circuits (2)

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0 \quad \text{where} \quad \alpha = \frac{1}{2RC} \quad \text{and} \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

1. If $\alpha > \omega_o$, over-damped case

$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ **where** $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

2. If $\alpha = \omega_o$, critical damped case

$v(t) = (A_2 + A_1 t)e^{-\alpha t}$ **where** $s_{1,2} = -\alpha$

3. If $\alpha < \omega_o$, under-damped case

$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$ **where** $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

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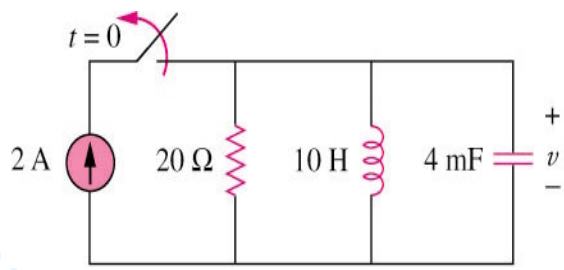
8.3 Source-Free Parallel RLC Circuits (3)

Example 3

Refer to the circuit shown below.

Find v(t) for t > 0.



Please refer to lecture or textbook for more detail elaboration.

Answer: $v(t) = 66.67(e^{10t} \quad e^{2.5t})$ V

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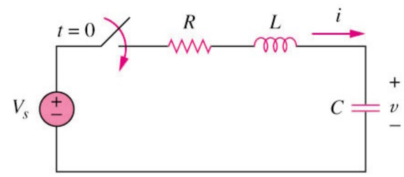
8.4 Step-Response Series RLC Circuits (1)

The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{v}{LC}=\frac{v_s}{LC}$$

The above equation has the same form as the equation for source-free series RLC circuit.

The same coefficients (important in determining the frequency parameters).

Different circuit variable in the equation.

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8.4 Step-Response Series RLC Circuits (2)

The solution of the equation should have two components: the transient response $v_t(t)$ & the steady-state response $v_{ss}(t)$:

$$v(t) = v_t(t) + v_{ss}(t)$$

- The transient response v_t is the same as that for source-free case

$v_t(t)=A_1e^{s_1t}+A_2e^{s_2t}$ (over-damped)

$v_t(t)=(A_1+A_2t)e^{-\alpha t}$ (critically damped)

$v_t(t)=e^{-\alpha t}(A_1\cos\omega_d t+A_2\sin\omega_d t)$ (under-damped)

- The steady-state response is the final value of v(t).
  - $v_{ss}(t) = v(\infty)$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - v(0) and dv(0)/dt.

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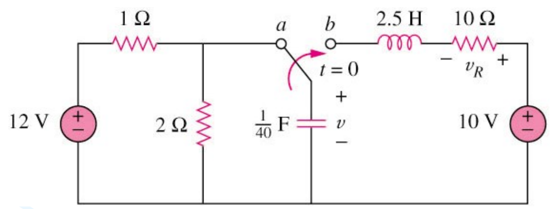
8.4 Step-Response Series RLC Circuits (3)

Example 4

Having been in position for a long time, the switch in the circuit below is moved to position b at t = 0. Find v(t) and $v_R(t)$ for t > 0.



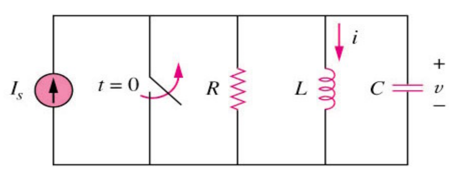
Please refer to lecture or textbook for more detail elaboration.

Answer: v(t) = {10 + [(2cos3.464t   1.1547sin3.464t)e 2t]} V

$v_R(t)$ = [2.31sin3.464t]e ^{2t} V

8.5 Step-Response Parallel RLC Circuits (1)

The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

It has the same form as the equation for source-free parallel RLC circuit.

- The same coefficients (important in determining the frequency parameters).
- Different circuit variable in the equation.

8.5 Step-Response Parallel RLC Circuits (2)

The solution of the equation should have two components: the transient response $v_t(t)$ & the steady-state response $v_{ss}(t)$:

$$i(t) = i_t(t) + i_{ss}(t)$$

- The transient response i_t is the same as that for source-free case

$i_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (over-damped)

$i_t(t) = (A_1 + A_2 t) e^{-\alpha t}$ (critical damped)

$i_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$ (under-damped)

- The steady-state response is the final value of i(t).
  - $i_{ss}(t) = i(\infty) = I_s$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - i(0) and di(0)/dt.

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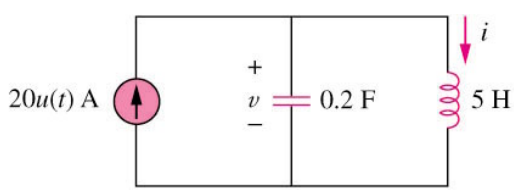
8.5 Step-Response Parallel RLC Circuits (3)

Example 5

Find i(t) and v(t) for t > 0 in the circuit shown in circuit shown below:



Please refer to lecture or textbook for more detail elaboration.

Answer: *v(t) = Ldi/dt = 5x20sint = 100sint V*

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