Alexander-Sadiku
Fundamentals of Electric Circuits

Chapter 14

Frequency Response

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Chapter 14

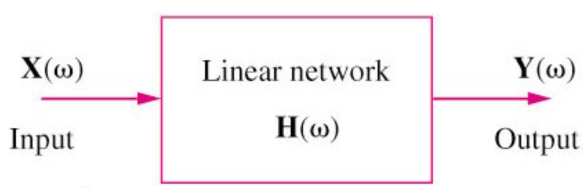
14.1 Introduction (1)

What is FrequencyResponse of a Circuit?

It is the variation in a circuit's behavior with change in signal frequency and may also be considered as the variation of the gain and phase with frequency.

14.2 Transfer Function (1)

The transfer function H(ω) of a circuit is the **frequency-dependent ratio** of a phasor output Y(ω) (an element voltage or current) to a phasor input X(ω) (source voltage or current).



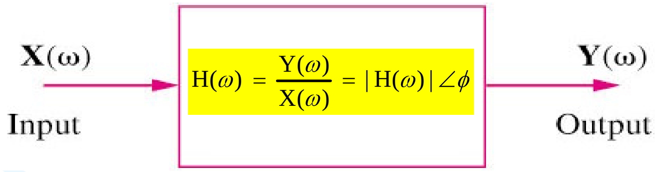
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = |H(\omega)| \angle \phi$$

14.2 Transfer Function (2)

Four possible transfer functions:

$$H(\omega) = \text{Voltage gain} = \frac{V_o(\omega)}{V_i(\omega)}$$

$$H(\omega) = \text{Transfer Impedance} = \frac{V_o(\omega)}{I_i(\omega)}$$



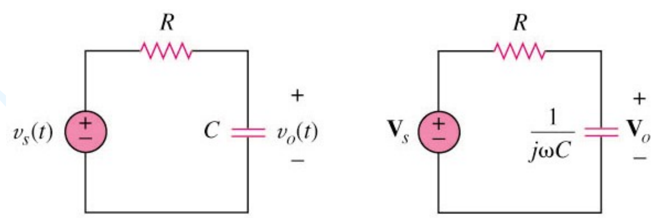
$$H(\omega) = \text{Current gain} = \frac{I_o(\omega)}{I_i(\omega)}$$

$$H(\omega) = \text{Transfer Admittance} = \frac{I_o(\omega)}{V_i(\omega)}$$

14.2 Transfer Function (3)

Example 1

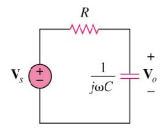
For the RC circuit shown below, obtain the transfer function Vo/Vs and its frequency response.
Let $v_s = V_m \cos\omega t$.



(a) (b)

14.2 Transfer Function (4)

Solution:



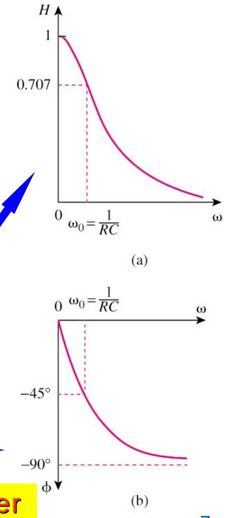
The transfer function is

$$H(\omega) = \frac{V_o}{V_s} = \frac{\frac{1}{j\omega C}}{R + 1/\,j\omega C} = \frac{1}{1 + j\omega RC}$$

The magnitude is $|H(\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_o)^2}}$

The phase is $\phi = -\tan^{-1}\frac{\omega}{\omega_o}$

$\omega_o = 1/RC$

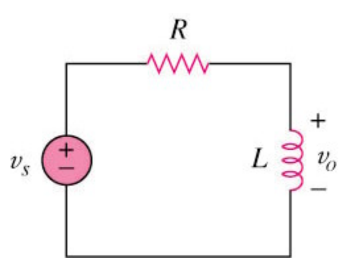


Low Pass Filter

14.2 Transfer Function (5)

Example 2

Obtain the transfer function Vo/Vs of the RL circuit shown below, assuming $v_s = V_m\cos\omega t$. Sketch its frequency response.

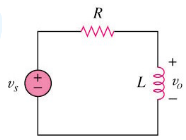


14.2 Transfer Function (6)

Solution:

The transfer function is

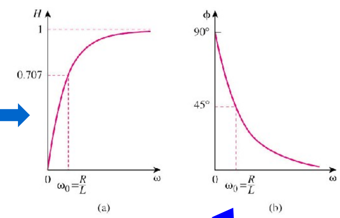
$$H(\omega) = \frac{V_o}{V_s} = \frac{j\omega L}{R + j\omega L} = \frac{1}{1 + \frac{R}{j\omega L}}$$



High Pass Filter

The magnitude is

$$H(\omega) = \frac{1}{\sqrt{1 + (\frac{\omega_o}{\omega})^2}}$$

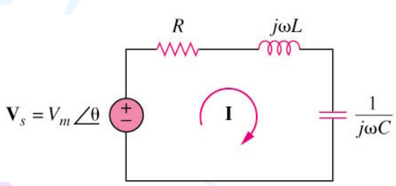


The phase is $\phi = \angle 90° - \tan^{-1} \frac{\omega}{\omega_o}$

$$\omega_o = R/L$$

14.3 Series Resonance (1)

Resonance is a condition in an RLC circuit in which the capacitive and inductive reactance are equal in magnitude, thereby resulting in purely resistive impedance.



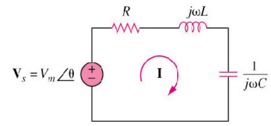
$$Z = R + j(\omega L - \frac{1}{\omega C})$$

Resonance frequency:

$$\omega_o = \frac{1}{\sqrt{LC}} \text{rad/s} \quad \textit{or}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \text{Hz}$$

14.3 Series Resonance (2)



The features of series resonance:

$$Z = R + j(\omega L - \frac{1}{\omega C})$$

The impedance is purely resistive, Z = R;

The supply voltage Vs and the current I are in phase, so cos θ = 1;

The magnitude of the transfer function H(ω) = Z(ω) is minimum;

The inductor voltage and capacitor voltage can be much more than the source voltage.

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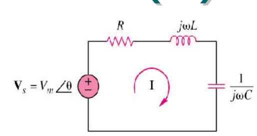
14.3 Series Resonance (3)

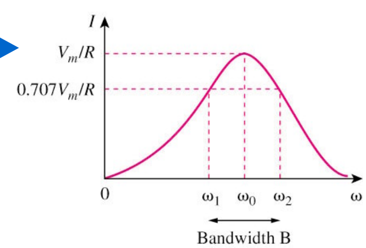
Bandwidth B



The frequency response of the resonance circuit current is

$$Z = R + j(\omega L - \frac{1}{\omega C})$$

$$I = |I| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$



The average power absorbed by the RLC circuit is

$$P(\omega) = \frac{1}{2} I^2 R$$

The highest power dissipated occurs at resonance:

$$P(\omega_o) = \frac{1}{2}\frac{V_m^2}{R}$$

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14 3 Series Resonance (4)

Half-power frequencies ω_1 and ω_2 are frequencies at which the dissipated power is half the maximum value:

$$P(\omega_1) = P(\omega_2) = \frac{1}{2}\frac{(V_m/\sqrt{2})^2}{R} = \frac{V_m^2}{4R}$$

The half-power frequencies can be obtained by setting Z equal to $\sqrt{2}$ R.

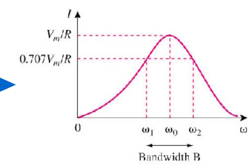
$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_o = \sqrt{\omega_1 \omega_2}$$

Bandwidth B

$$B = \omega_2 - \omega_1$$



14.3 Series Resonance (5)

Quality factor,

$$Q = \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

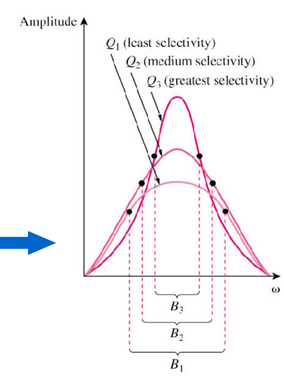
The relationship between the B, Q and ω_o:

$$B = \frac{R}{L} = \frac{\omega_o}{Q} = \omega_o^2 CR$$

The quality factor is the ratio of its resonant frequency to its bandwidth.

If the bandwidth is narrow, the quality factor of the resonant circuit must be high.

If the band of frequencies is wide, the quality factor must be low.



14.3 Series Resonance (6)

Example 3

A series-connected circuit has R = 4 Ω and L = 25 mH.

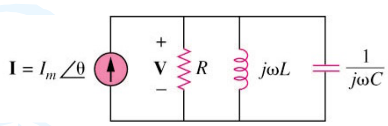
a. Calculate the value of C that will produce a quality factor of 50.

b. Find ω_1 and ω_2, and B.

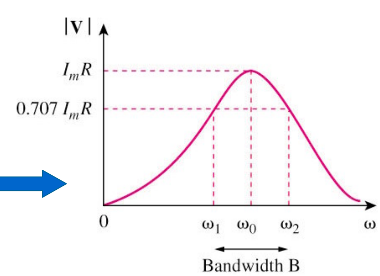
c. Determine the average power dissipated at $\omega = \omega_o, \omega_1, \omega_2$. Take $V_m = 100$V.

14.4 Parallel Resonance (1)

It occurs when imaginary part of Y is zero



$$Y = \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$



Resonance frequency:

$$\omega_o = \frac{1}{\sqrt{LC}} \text{ rad/s} \quad \text{or} \quad f_o = \frac{1}{2\pi\sqrt{LC}} Hz$$

14.4 Parallel Resonance (2)

Summary of series and parallel resonance circuits:

characteristic	Series circuit	Parallel circuit
ω_o	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Q	$\frac{\omega_o L}{R}$ or $\frac{1}{\omega_o RC}$	$\frac{R}{\omega_o L}$ or $\omega_o RC$
B	$\frac{\omega_o}{Q}$	$\frac{\omega_o}{Q}$
$\omega_1,\ \omega_2$	$\omega_o\sqrt{1+(\frac{1}{2Q})^2} \pm \frac{\omega_o}{2Q}$	$\omega_o\sqrt{1+(\frac{1}{2Q})^2} \pm \frac{\omega_o}{2Q}$
$Q \geq 10,\ \omega_1,\ \omega_2$	$\omega_o \pm \frac{B}{2}$	$\omega_o \pm \frac{B}{2}$

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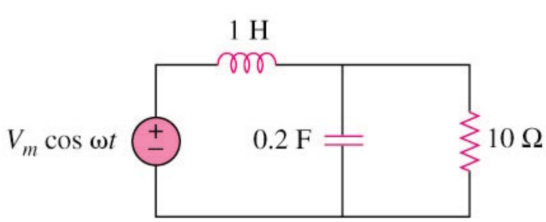
14.4 Parallel Resonance (3)

Example 4

Calculate the resonant frequency of the circuit in the figure shown below.



Answer: $\omega = \frac{\sqrt{19}}{2} = 2.179$ rad/s

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14.5 Passive Filters (1)

A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

Passive filter consists of only passive element R, L and C.

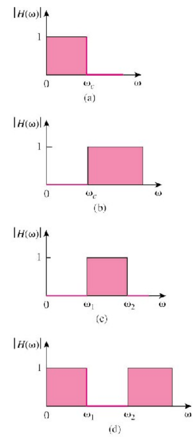
There are four types of filters.

Low Pass

High Pass

Band Pass

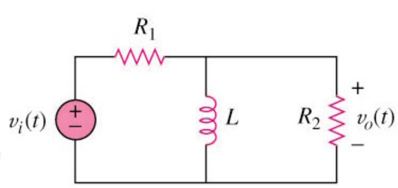
Band Stop



14.5 Passive Filters (2)

Example 5

For the circuit in the figure below, obtain the transfer function Vo(ω)/Vi(ω). Identify the type of filter the circuit represents and determine the corner frequency. Take R1=100Ω =R2 and L =2mH.



Answer: $\omega = 25$ krad/s