Alexander-Sadiku
Fundamentals of Electric Circuits

Chapter 19

Two-Port Networks

Two Port Networks
Chapter 19

19.1 Introduction (1)

<u>What is a port?</u>

It is a pair of terminals through which a <u>current</u> may enter or leave a network.

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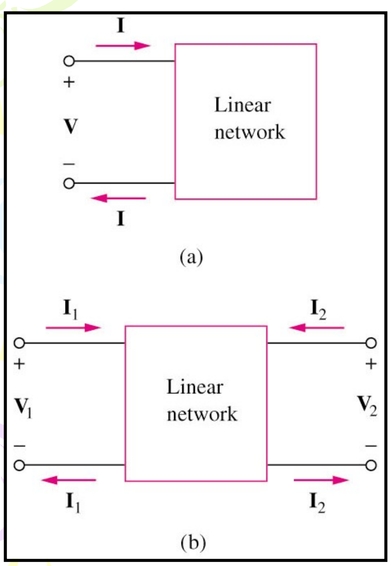
19.1 Introduction (2)



One port or two terminal circuit

Two port or four terminal circuit

It is an electrical network with two separate ports for input and output.

No independent sources.

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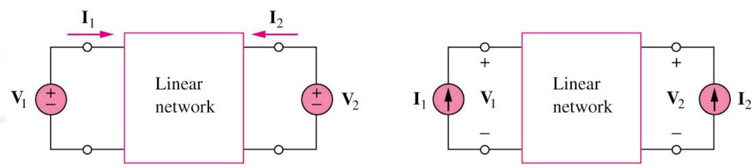
19.2 Impedance parameters (1)



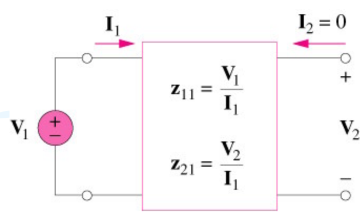
Assume no independent source in the network

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

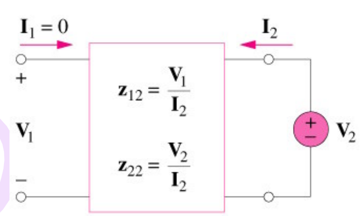
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where the **z** terms are called the impedance parameters, or simply z parameters, and have units of ohms.

19.2 Impedance parameters (2)



(a)



(b)

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

z_{11} = Open-circuit input impedance

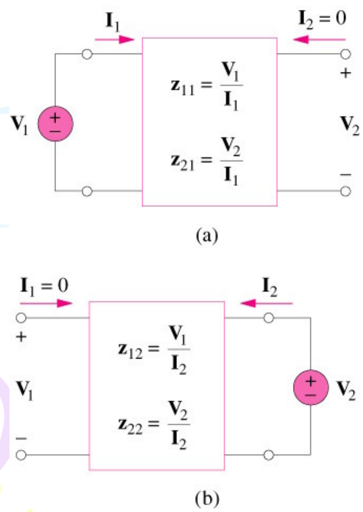
z_{21} = Open-circuit transfer impedance from port 1 to port 2

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

z_{12} = Open-circuit transfer impedance from port 2 to port 1

z_{22} = Open-circuit output impedance

19.2 Impedance parameters (2a)



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

When $z_{11} = z_{22}$, the two-port network is said to be **symmetrical**.

When the two-port network is **linear** and has **no dependent sources**, the transfer impedances are equal ($z_{12} = z_{21}$), and the two-port is said to be **reciprocal**.

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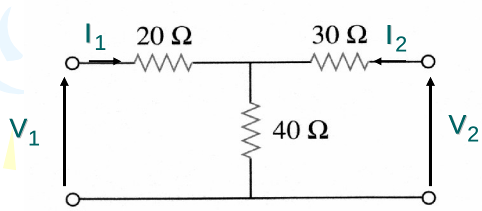
19.2 Impedance parameters (3)

Example 1

Determine the Z-parameters of the following circuit.



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \Omega$$

Answer: $z = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix} \Omega$

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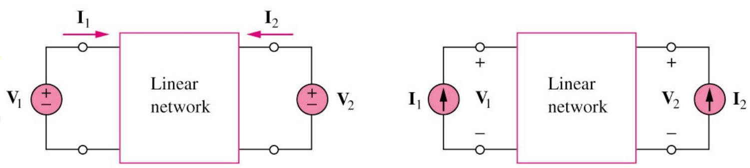
19.3 Admittance parameters (1)



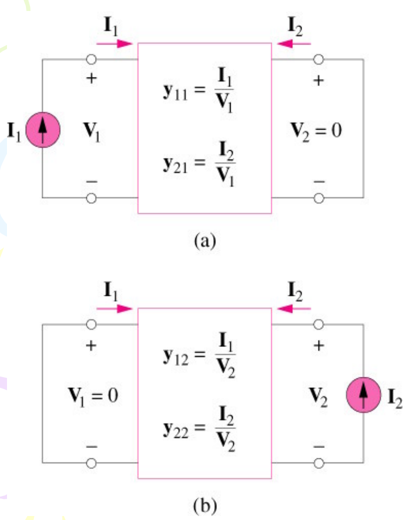
Assume no independent source in the network

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where the **y** terms are called the admittance parameters, or simply y parameters, and they have units of Siemens.

19.3 Admittance parameters (2)



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

y_{11} = Short-circuit input admittance

y_{21} = Short-circuit transfer admittance from port 1 to port 2

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

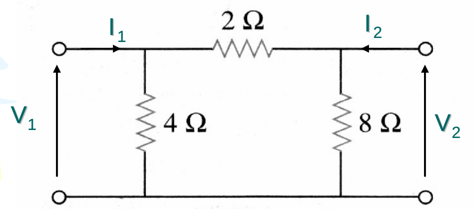
y_{12} = Short-circuit transfer admittance from port 2 to port 1

y_{22} = Short-circuit output admittance

19.3 Admittance parameters (3)

Example 2

Determine the y-parameters of the following circuit.



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} S$$

Answer: $y = \begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.625 \end{bmatrix} S$

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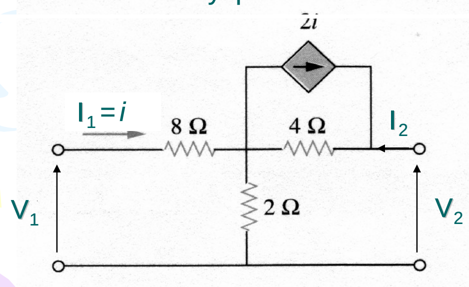
19.3 Admittance parameters (4)

Example 3

Determine the y-parameters of the following circuit.



$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

Apply KVL

$$V_1 = 8I_1 + 2(I_1 + I_2)$$
$$V_2 = 4(2i + I_2) + 2(I_1 + I_2)$$

$$I_1 = 0.15V_1 - 0.05V_2$$
$$I_2 = -0.25V_1 + 0.25V_2$$

Answer: $y = \begin{bmatrix} 0.15 & -0.05 \\ -0.25 & 0.25 \end{bmatrix} S$

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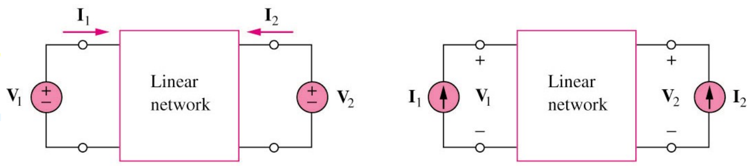
19.4 Hybrid parameters (1)



Assume no independent source in the network

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [h] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

where the **h** terms are called the impedance parameters, or simply h parameters, and each parameter has different units, refer above.

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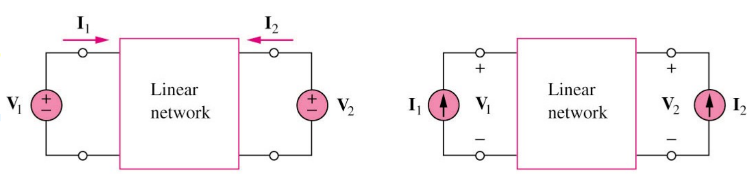
19.4 Hybrid parameters (2)



Assume no independent source in the network

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

h_{11} = short-circuit input impedance (Ω)

H_2 = short-circuit forward current gain

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$
$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

H_{12} = open-circuit reverse voltage-gain

H_{22} = open-circuit output admittance (S)

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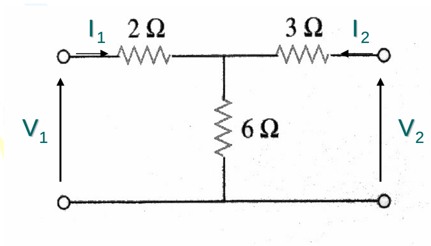
19.4 Hybrid parameters (3)

Example 4

Determine the h-parameters of the following circuit.



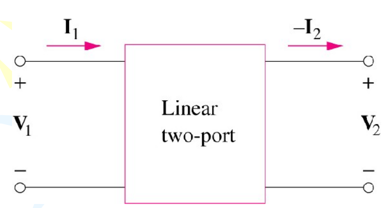
$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \text{and} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{and} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$h = \begin{bmatrix} h_{11}\Omega & h_{12} \\ h_{21} & h_{22}S \end{bmatrix}$$

Answer: $h = \begin{bmatrix} 4\Omega & -\frac{2}{3} \\ \frac{2}{3} & \frac{1}{9}S \end{bmatrix}$

19.5 Transmission parameters (1)



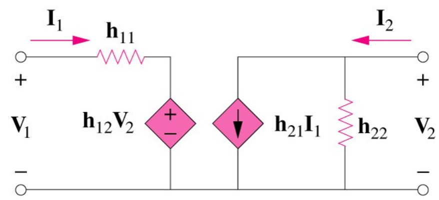
Assume no independent source in the network

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where the **T** terms are called the transmission parameters, or simply T or ABCD parameters, and each parameter has different units.

19.5 Transmission parameters (2)



$A = \left. \frac{V_1}{V_2} \right|_{I_2 = 0}$

A=open-circuit voltage ratio

$C = \left. \frac{I_1}{V_2} \right|_{I_2 = 0}$

C= open-circuit transfer admittance (S)

$B = -\left. \frac{V_1}{I_2} \right|_{V_2 = 0}$

B= negative short-circuit transfer impedance (Ω)

$D = -\left. \frac{I_1}{I_2} \right|_{V_2 = 0}$

D=negative short-circuit current ratio

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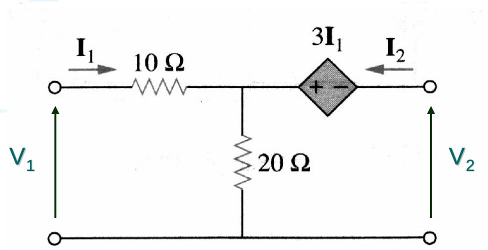
19.5 Transmission parameters (3)

Example 5

Determine the T-parameters of the following circuit.



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Apply KVL

$$V_1 = 10I_1 + 20(I_1 + I_2)$$
$$V_2 = -3I_1 + 20(I_1 + I_2)$$

$$V_1 = \frac{30}{17}V_2 - \frac{260}{17}I_2$$
$$I_1 = \frac{1}{17}V_2 - \frac{20}{17}I_2$$

Answer: $T = \begin{bmatrix} 1.765 & 15.294\Omega \\ 0.059S & 1.176 \end{bmatrix}$

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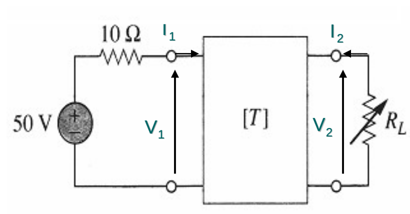
19.5 Transmission parameters (4)

Example 6

The ABCD parameters of the two-port network below are

$$T = \begin{bmatrix} 4 & 20\ \Omega \\ 0.1\text{S} & 2 \end{bmatrix}$$

The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.



Answer: V_{TH} = 10V V; R_L = 8Ω; Pm = 3.125W.

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