CHAPTER 11 - AC POWER ANALYSIS

List of topics for this chapter :
- Instantaneous and Average Power
- Maximum Average Power Transfer
- Effective or RMS Value
- Apparent Power and Power Factor
- Complex Power
- Conservation of AC Power
- Power Factor Correction
- Applications

INSTANTANEOUS AND AVERAGE POWER

Problem 11.1 *[11.3]* Refer to the circuit depicted in Figure 11.1. Find the average power absorbed by each element.

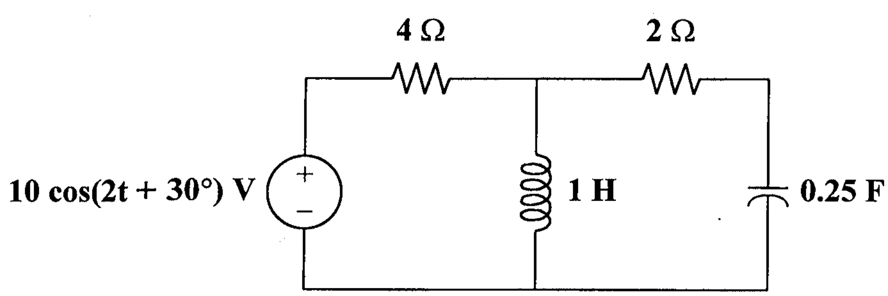
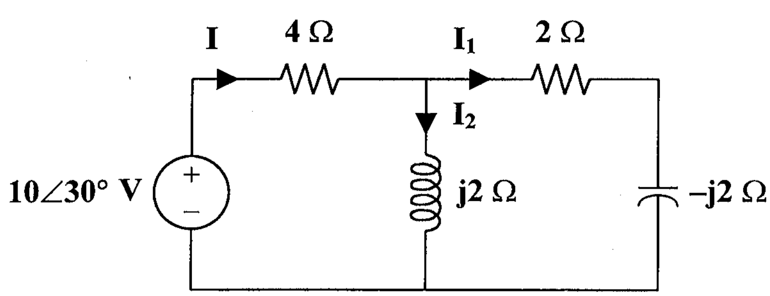


Figure 11.1

$$10\cos(2t + 30°) \longrightarrow 10\angle 30°, \qquad \omega = 2$$

$$1\text{ H} \longrightarrow j\omega L = j2$$

$$0.25\text{ F} \longrightarrow \frac{1}{j\omega C} = -j2$$



$$j2 \,||\, (2-j2) = \frac{(j2)(2-j2)}{2} = 2+j2$$

$$\mathbf{I} = \frac{10\angle 30°}{4+2+j2} = 1.581\angle 11.565°$$

$$\mathbf{I}_1 = \frac{j2}{2}\mathbf{I} = j\mathbf{I} = 1.581\angle 101.565°$$

$$\mathbf{I}_2 = \frac{2-j2}{2}\mathbf{I} = 2.236\angle 56.565°$$

For the source,

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = \frac{1}{2}(10\angle 30°)(1.581\angle -11.565°)$$

$$\mathbf{S} = 7.905\angle 18.43° = 7.5 + j2.5$$

The average power supplied by the source = 7.5 W

For the 4-Ω resistor, the average power absorbed is

$$P = \frac{1}{2}|\mathbf{I}|^2 R = \frac{1}{2}(1.581)^2(4) = 5 \text{ W}$$

For the inductor,

$$\mathbf{S} = \frac{1}{2}|\mathbf{I}_2|^2 \mathbf{Z}_L = \frac{1}{2}(2.236)^2(j2) = j5$$

The average power absorbed by the inductor = 0 W

For the 2-Ω resistor, the average power absorbed is

$$P = \frac{1}{2}|\mathbf{I}_1|^2 R = \frac{1}{2}(1.581)^2(2) = 2.5 \text{ W}$$

For the capacitor,

$$\mathbf{S} = \frac{1}{2}|\mathbf{I}_1|^2 \mathbf{Z}_c = \frac{1}{2}(1.581)^2(-j2) = -j2.5$$

The average power absorbed by the capacitor = **0 W**

The average power supplied by the source = **7.5 W**

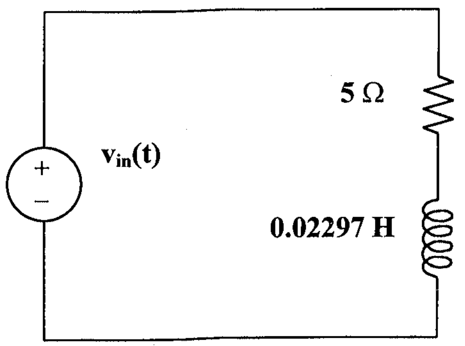
The average power absorbed by the 4-Ω resistor = **5 W**

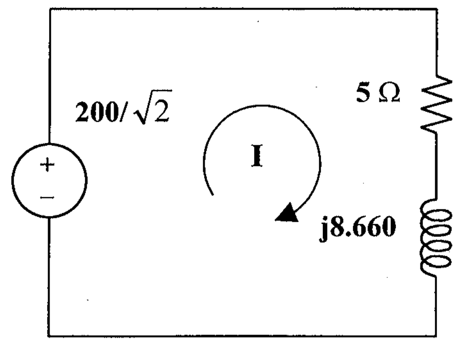
The average power absorbed by the inductor = **0 W**

The average power absorbed by the 2-Ω resistor = **2.5 W**

The average power absorbed by the capacitor = **0 W**

Problem 11.2 The load for the following circuit is given by the 5 ohm resistor and the 0.02297 Henry inductor. In addition, $v_{in}(t) = 200\sin(377t)$ volts. Determine the average power delivered to the load.





Using the frequency domain circuit on the right, we can solve for I.

$$-141.4 + (5 + j8.660)I = 0.$$ Which leads to,

$I = 141.4/(10\angle 60^\circ) = 14.14\angle{-60^\circ}$ amps. But power delivered to the load is equal to,

$$P_{avg} = |I|^2 R = (14.14)^2 \times 5 = 200\times 5 = \underline{\textbf{1000 watts}}.$$

MAXIMUM AVERAGE POWER TRANSFER

Problem 11.3 Given the circuit in Figure 11.2 and $v(t) = 200\sin(\omega t)$ volts, calculate the values of R_L and X for maximum power transfer to R_L.

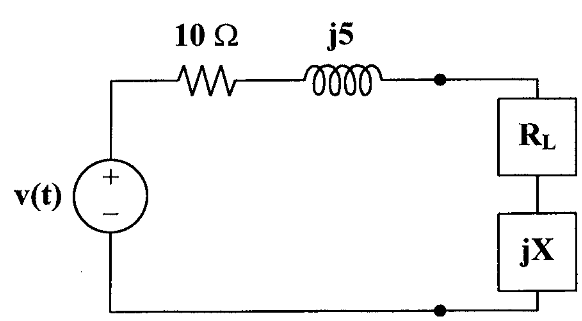


Figure 11.2

This is a straightforward classical maximum power transfer problem. If you remember that for maximum power transfer, $Z_L = Z_s^*$ or the value of the load is equal to the complex conjugate of the source impedance.

$R_L + jX = 10 - j5$ Thus the load resistor must be **10 Ω** and the reactance must be capacitive and equal to **[1/(5ω)] F**.

What if you do not remember the maximum power transfer theorem? Well, you can usually work it out just by looking at what you have. It seems reasonable to cancel whatever source reactance there is. Then if the source resistance is either zero or infinity, there is no power transfer. The only thing that makes sense is that the load resistance must equal the source resistance.

Problem 11.4 *[11.15]* Find the value of $\mathbf{Z}_L$ in the circuit of Figure 11.3 for maximum power transfer.

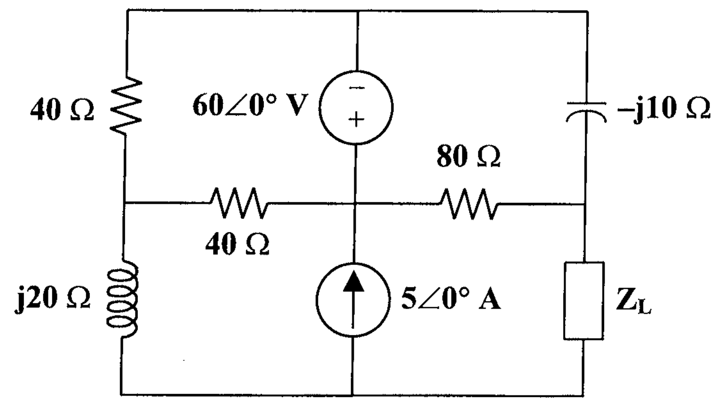
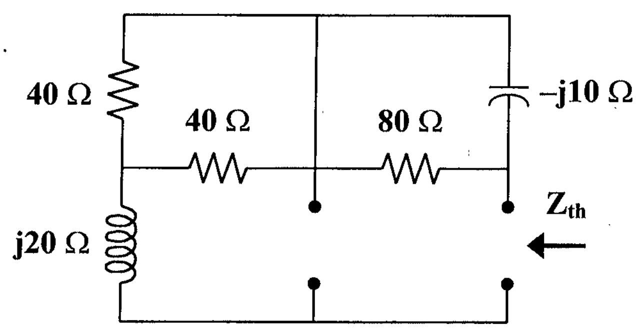


Figure 11.3

We find $\mathbf{Z}_{Th}$ at terminals a-b as shown in the figure below.



$$\mathbf{Z}_{Th} = j20 + 40 \,||\, 40 + 80 \,||\, (\text{-}j10) = j20 + 20 + \frac{(80)(\text{-}j10)}{80 - j10}$$

$$\mathbf{Z}_{Th} = 21.23 + j10.154$$

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = \underline{\mathbf{21.23 - j10.15\ \Omega}}$$

EFFECTIVE OR RMS VALUE

Problem 11.5 Calculate the RMS value of the signal shown in Figure 11.4. The curve can be represented by the function $V_P \sin(\omega t) + V_P$.

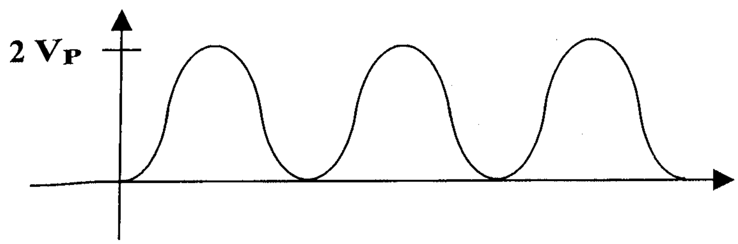


Figure 11.4

Since the value of the voltage, v(t), can be expressed as $V_p[\sin(\omega t) + 1]$, we can ignore the value of V_p in our calculations. The true value of the rms voltage will be what we obtain times V_p. Also, since the value of the rms voltage is independent of ω, we will let T = 1, which means that ω = 2π.

$$\frac{V_{rms}^2}{V_p^2} = \frac{1}{T}\int_0^T v(t)^2 dt = \int_0^T [\sin(2\pi t) + 1]^2 dt = \int_0^T [\sin^2(2\pi t) + 2\sin(2\pi t) + 1]dt\ \pi$$

$$= \int_0^1 \frac{1 - \cos(4\pi t)}{2} dt + 0 + \int_0^1 dt = \left[\frac{t}{2} - \frac{\sin(4\pi t)}{8\pi}\right]_0^1 + t\Big|_0^1 = \frac{1}{2} + 1 = 1.5$$

$$V_{rms} = \sqrt{1.5} = \underline{\mathbf{1.2247V_p}}$$

Problem 11.6 Calculate the RMS value of the signal shown in Figure 11.5. The curve can be represented by the function $V_P \sin(\omega t)$, (please note, this is often referred to as the full wave, rectified sine wave).

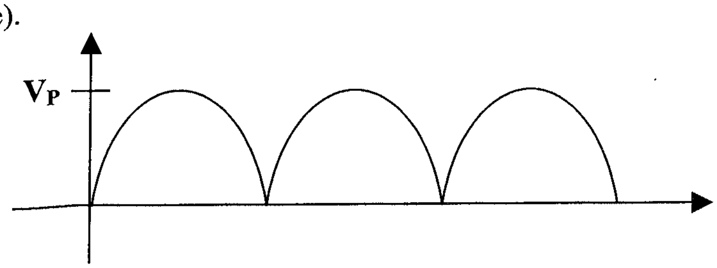


Figure 11.5

We can use the same approach that we used in problem 11.5. In this case, ω stays the same but now T = 0.5 sec.

$$\frac{V_{rms}^2}{V_p^2} = \frac{1}{0.5}\int_0^{0.5} \sin^2(2\pi t)\,dt = 2\int_0^{0.5}\frac{1-\cos(4\pi t)}{2}dt = 2\left[\frac{t}{2} - \frac{\sin(4\pi t)}{8\pi}\right]_0^{0.5}$$

$$= 2\left[\frac{0.5}{2} + 0\right] = 0.5$$ or V_{rms} = $\underline{\mathbf{0.707V_p}}$ which is to be expected since the rms value is taken by squaring the value of a signal. Squaring either wave produces the same result.

Problem 11.7 Calculate the RMS value of the signal shown in Figure 11.6.

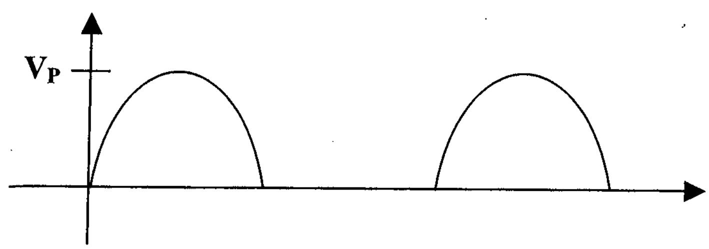


Figure 11.6

Again, the same approach is used.

$$\frac{V_{rms}^2}{V_p^2} = \frac{1}{1}\left[\int_0^{0.5} \sin^2(2\pi t)\,dt + \int_{0.5}^{1} 0\,dt\right] = \int_0^{0.5}\frac{1-\cos(4\pi t)}{2}dt = \left[\frac{t}{2} - \frac{\sin(4\pi t)}{8\pi}\right]_0^{0.5}$$

= (0.5)/2 – 0 = 0.25 thus, $V_{rms} = \sqrt{0.25}\,V_p$ = $\underline{\mathbf{0.5V_p}}$

Problem 11.8 *[11.21]* Find the effective value of the voltage waveform in Figure 11.7.

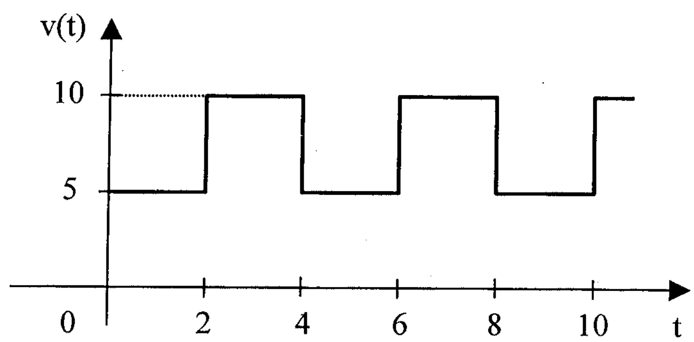


Figure 11.7

$$T = 4, \qquad v(t) = \begin{cases} 5 & 0 < t < 2 \\ 10 & 2 < t < 4 \end{cases}$$

$$V_{rms}^2 = \frac{1}{4}\left[\int_0^2 5^2\,dt + \int_2^4 (10)^2\,dt\right] = \frac{1}{4}[50 + 200] = 62.5$$

$$V_{rms} = \underline{\mathbf{7.906\ V}}$$

APPARENT POWER AND POWER FACTOR

Problem 11.9 Given the circuit in Figure 11.8, $V_{in} = 100\angle 30°$ and $I = 10\angle 60°$,

(a) calculate the average power assuming that V_{in} and I are already rms values
(b) calculate the apparent power
(c) calculate the reactive power

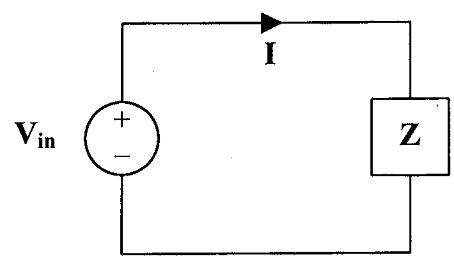


Figure 11.8

(a) power = 100x10cos(30°– 60°) = 1000cos(-30°) = **<u>866 watts</u>**

(b) apparent power = 100x10 = 1000 = **<u>1 kVA</u>**

(c) reactive power = 100x10sin(30°– 60°) = 1000sin(-30°) = **<u>–500 VARS</u>**

Problem 11.10 *[11.29]* A relay coil is connected to a 210-V, 50-Hz supply. If it has a resistance of 30 Ω and an inductance of 0.5 H, calculate the apparent power and the power factor.

$$0.5\ H \longrightarrow j\omega L = j(2\pi)(50)(0.5) = j157.08$$

$$\mathbf{Z} = R + jX_L = 30 + j157.08$$

$$\mathbf{S} = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(210)^2}{30 - j157.08}$$

$$\text{Apparent power} = |\mathbf{S}| = \frac{(210)^2}{160} = \underline{\mathbf{275.6\ VA}}$$

$$\text{pf} = \cos\theta = \cos\left(\tan^{-1}\left(\frac{157.08}{36}\right)\right) = \cos(79.19°)$$

$$\text{pf} = \underline{\mathbf{0.1876 \quad (lagging)}}$$

COMPLEX POWER

Problem 11.11 *[11.35]* Determine the complex power for the following cases:

(a) $P = 269\ \text{W}$, $Q = 150\ \text{VAR}$ (capacitive)

(b) $Q = 2000\ \text{VAR}$, $\text{pf} = 0.9$ (leading)

(c) $S = 600\ \text{VA}$, $Q = 450\ \text{VAR}$ (inductive)

(d) $V_{rms} = 220\ \text{V}$, $P = 1\ \text{kW}$, $|\mathbf{Z}| = 40\ \Omega$ (inductive)

(a) $\mathbf{S} = P - jQ = \underline{\mathbf{269 - j150\ VA}}$

(b) $\text{pf} = \cos\theta = 0.9 \longrightarrow \theta = 25.84°$

$$Q = S\sin\theta \longrightarrow S = \frac{Q}{\sin\theta} = \frac{2000}{\sin(25.84°)} = 4588.31$$

$$P = S\cos\theta = 4129.48$$

$$\mathbf{S} = \underline{\mathbf{4129 - j2000\ VA}}$$

(c) $Q = S\sin\theta \longrightarrow \sin\theta = \frac{Q}{S} = \frac{450}{600} = 0.75$

$\theta = 48.59$, $\quad \text{pf} = 0.6614$

$$P = S\cos\theta = (600)(06614) = 396.86$$

$$\mathbf{S} = \underline{\mathbf{396.9 + j450\ VA}}$$

(d) $S = \frac{|\mathbf{V}|^2}{|\mathbf{Z}|} = \frac{(220)^2}{40} = 1210$

$P = S\cos\theta \longrightarrow \cos\theta = \frac{P}{S} = \frac{1000}{1210} = 0.8264$

$\theta = 34.26°$

$Q = S\sin\theta = 681.25$

$\underline{\mathbf{S = 1000 + j681.2\ VA}}$

CONSERVATION OF AC POWER

Problem 11.12 *[11.43]* Obtain the power delivered to the 10-kΩ resistor in the circuit of Figure 11.9.

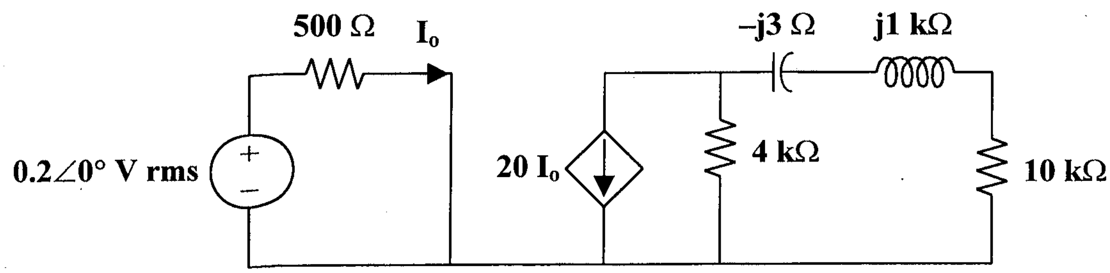
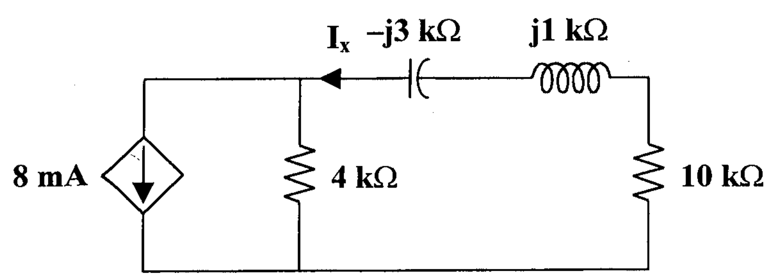


Figure 11.9



From the left portion of the circuit,

$$\mathbf{I}_o = \frac{0.2}{500} = 0.4\ \text{mA}$$

$$20\mathbf{I}_o = 8\ \text{mA}$$

From the right portion of the circuit,

$$\mathbf{I}_x = \frac{4}{4+10+j-j3}(8\text{ mA}) = \frac{16}{7-j}\text{ mA}$$

$$\mathbf{P} = |\mathbf{I}_x|^2 \mathbf{R} = \frac{(\mathbf{16\times10^{-3}})^2}{\mathbf{50}}\cdot(\mathbf{10\times10^3})$$

$$\mathbf{P} = \underline{\mathbf{51.2\ mW}}$$

POWER FACTOR CORRECTION

Problem 11.13 A small industry operates from 220 volts supplied by a utility. The small industry represents a load to the utility that represents 22,000 watts and a power factor of 0.8. Develop an equivalent circuit for the load. Determine the value of a capacitor to correct the circuit to unity power factor.

power = VI cosθ = 220xIx0.8 = 22,000 or I = 125 A

Thus, |Z| = 220/125 = 1.76 and cosθ = 0.8 leads to θ = 36.87°

Although it was not specified, most industries, if not all, represent an inductive load, thus the power factor is lagging and θ is positive.

$$Z = 1.76\angle 36.87° = \underline{(1.408 + j1.056)\ \Omega}$$

Since this represents a resistor in series with an inductor, we place a capacitor in parallel with the combination in order to correct to unity power factor. The easiest way to do this is to just cancel the reactive power with the parallel capacitor.

Q = VIsinθ = 220x125xxin(36.87°) = 16881 = $220^2/X_C$

X_C = 220^2/16881 = 2.867 = 1/(ωC) with ω = 377 the C = **<u>925 μF</u>**

Problem 11.14 For the network in Figure 11.10, determine the value of C that corrects the power factor to 0.8, 0.85, 0.9, 0.95, 1.0.

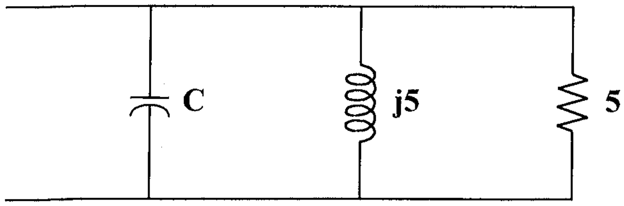


Figure 11.10

$$\text{pf} = \text{power/VA} = \frac{P}{S} = \frac{V^2/R}{V^2/|Z|} = \frac{1/5}{\sqrt{\frac{1}{X^2}+\frac{1}{5^2}}} \quad \text{where } X = \frac{5\frac{1}{\omega C}}{\frac{1}{\omega C}-5}$$

$$\text{or } X = \frac{5}{1-5\omega C}$$

$$\text{pf}^2 = \frac{1/25}{\frac{1}{X^2}+\frac{1}{25}} \longrightarrow \frac{1}{X^2}+\frac{1}{25} = \frac{1}{25\text{pf}^2} \longrightarrow \frac{1}{X^2} = \frac{1}{25}\left(\frac{1}{\text{pf}^2}-1\right)$$

$$X^2 = \frac{25}{\frac{1}{\text{pf}^2}-1} \quad \text{If we let a} = X, \text{ then } a = \frac{5}{\sqrt{\frac{1}{\text{pf}^2}-1}} \quad \text{and} \quad \frac{5}{1-5\omega C} = a$$

Solving this for C, 5/a = 1 – 5ωC or C = [1 – 5/a]/(377x5) = [1 – 5/a]/1885

pf	a	C
0.707	5	0
0.8	6.667	132.65 μF
0.85	8.068	201.7 μF
0.9	10.324	273.6 μF
0.95	15.212	356.1 μF
1.0	∞	530.5 μF

Problem 11.15 Referring to the results of Problem 11.14, what can you say about the relative costs of power factor correction?

To correct to 0.8 pf requires a 132.65 μF capacitor. (Clearly one would purchase the closest value commercially available for the desired use. Taking into account energy requirements, this might be a 150 μF capacitor.)

To correct to 0.85 requires only a 200 μF capacitor. However, to go to 0.95 requires almost two of these. This would mean that the cost of correcting to 0.95 is twice as much as correcting to 0.85. To go to unity costs even more. It would require 4 times the number of capacitors to correct to unity as to correct to 0.8. Fortunately, utilities gain little from corrections from 0.85 to unity, which can save a lot since these compensating capacitors are very expensive.

Problem 11.16 *[11.53]* Refer to the circuit shown in Figure 11.11.

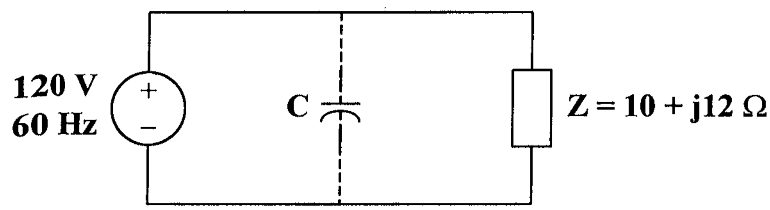


Figure 11.11

(a) What is the power factor?
(b) What is the average power dissipated?
(c) What is the value of capacitance that will give unity power factor when connected to the load?

(a) Given that $\mathbf{Z} = 10 + j12$

$$\tan\theta = \frac{12}{10} \longrightarrow \theta = 50.19°$$

$$\text{pf} = \cos\theta = \underline{\mathbf{0.6402}}$$

(b) $$\mathbf{S} = \frac{|\mathbf{V}|^2}{2\mathbf{Z}^*} = \frac{(120)^2}{(2)(10 - j12)} = 295.12 + j354.09$$

The average power absorbed $= P = \text{Re}(\mathbf{S}) = \underline{\mathbf{295.1\ W}}$

(c) For unity power factor, $\theta_1 = 0°$, which implies that the reactive power due to the capacitor is $Q_c = 354.09$

But $$Q_c = \frac{V^2}{2X_c} = \frac{1}{2}\omega C V^2$$

$$C = \frac{2Q_c}{\omega V^2} = \frac{(2)(354.09)}{(2\pi)(60)(120)^2} = \underline{\mathbf{130.4\ \mu F}}$$

APPLICATIONS

Problem 11.17 *[11.63]* The kilowatt-hour-meter of a home is read once a month. For a particular month, the previous and present readings are as follows:

Previous reading:	3246 kWh
Present reading:	4017 kWh

Calculate the electricity bill for that month based on the following residential rate schedule:

Base monthly charge: $12.00
First 100 kWh per month at 16 cents/kWh
Next 200 kWh per month at 10 cents/kWh
Over 300 kWh per month at 6 cents/kWh

$$\text{kWh consumed} = 4017 - 3246 = 771 \text{ kWh}$$

The electricity bill is calculated as follows :

(a)	Base charge	= $12
(b)	First 100 kWh at $0.16 per kWh	= $16
(c)	Next 200 kWh at $0.10 per kWh	= $20
(d)	The remaining energy (771 – 300) = 471 kWh at $0.06 per kWh	= $28.26.

Adding (a) to (d) gives a total of **$76.26**