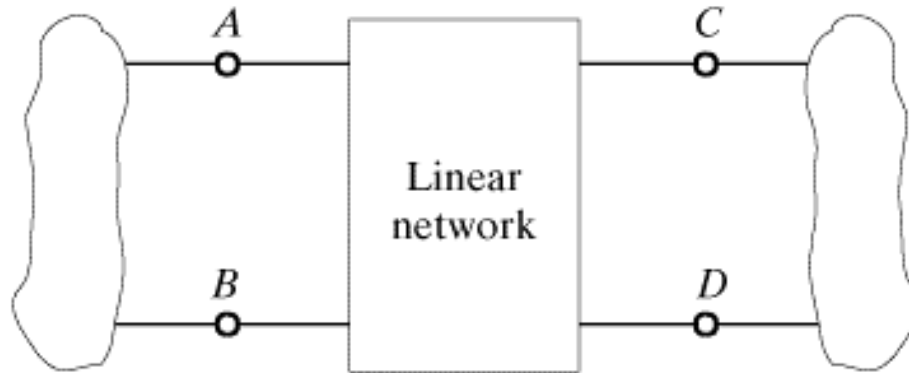


TWO-PORT NETWORKS



In many situations one is not interested in the internal organization of a network. A description relating input and output variables may be sufficient

A two-port model is a description of a network that relates voltages and currents at two pairs of terminals

LEARNING GOALS

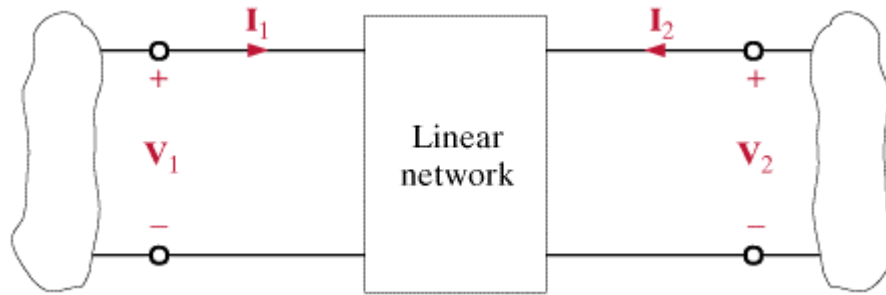
Study the basic types of two-port models

- Admittance parameters
- Impedance parameters
- Hybrid parameters
- Transmission parameters

Understand how to convert one model into another



ADMITTANCE PARAMETERS



The network contains NO independent sources

The admittance parameters describe the currents in terms of the voltages

y_{21} determines the current flowing into port 2 when the port is short-circuited and a voltage is applied to port 1

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

The first subindex identifies the output port. The second the input port.

The computation of the parameters follows directly from the definition

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \quad y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} \quad y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$

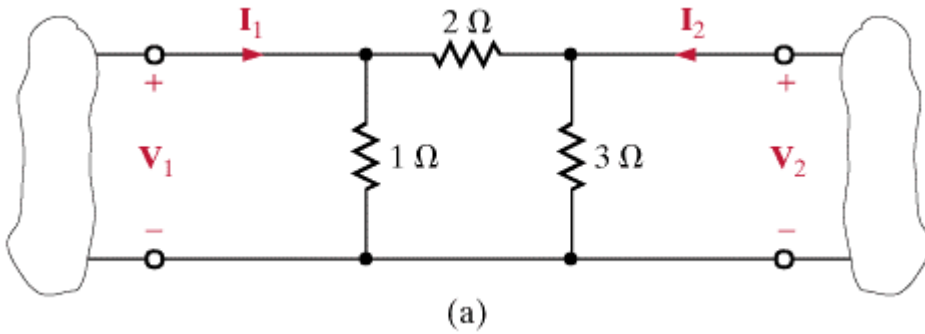


LEARNING EXAMPLE

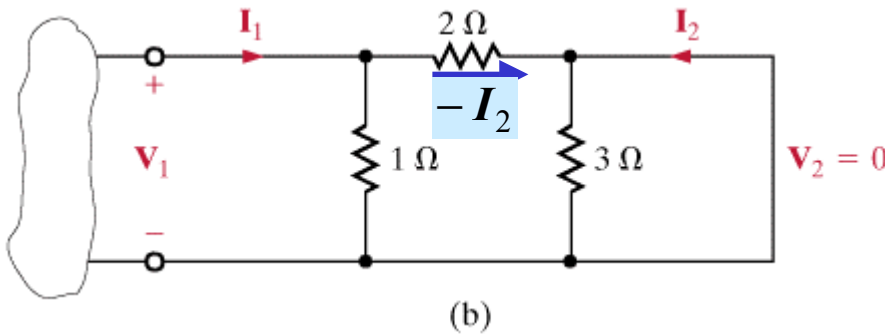
Find the admittance parameters for the network

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$



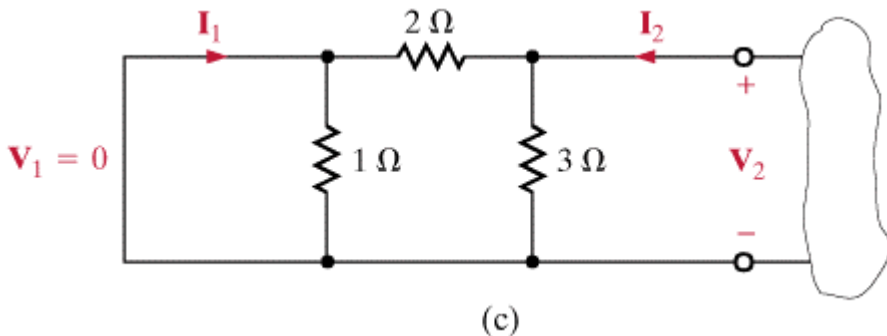
Circuit used to determine y_{11}, y_{21}



$$I_1 = \left(1 + \frac{1}{2}\right)V_1 \Rightarrow y_{11} = \frac{3}{2}[\mathcal{S}]$$

$$-I_2 = \frac{1}{1+2}I_1 \Rightarrow I_2 = -\frac{1}{2}V_1 \Rightarrow y_{21} = -\frac{1}{2}[\mathcal{S}]$$

Circuit used to determine y_{12}, y_{22}



$$I_2 = \left(\frac{1}{2} + \frac{1}{3}\right)V_2 \Rightarrow y_{22} = \frac{5}{6}[\mathcal{S}]$$

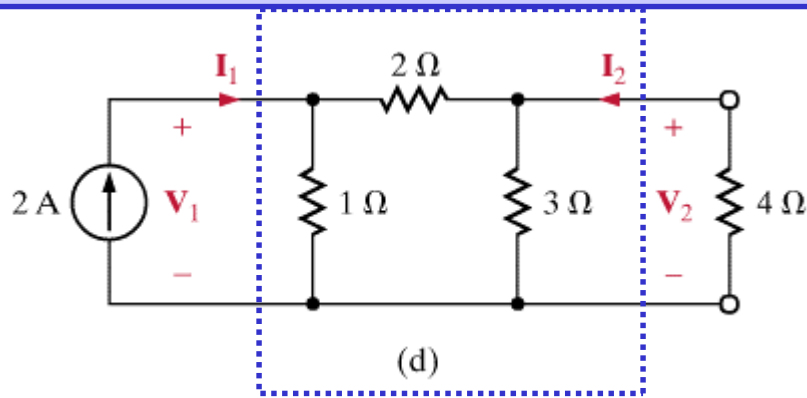
$$-I_1 = \frac{3}{2+3}I_2 = \frac{3 \times 5}{5 \times 6}V_2 \Rightarrow y_{12} = \frac{1}{2}[\mathcal{S}]$$

Next we show one use of this model



An application of the admittance parameters

Determine the current through the 4 Ohm resistor



$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$I_1 = \frac{3}{2}V_1 - \frac{1}{2}V_2$$

$$I_2 = -\frac{1}{2}V_1 + \frac{5}{6}V_2$$

$$I_1 = 2A, \quad V_2 = -4I_2 \quad I_2 = -\frac{1}{4}V_2$$

The model plus the conditions at the ports are sufficient to determine the other variables.

$$2 = \frac{3}{2}V_1 - \frac{1}{2}V_2$$

$$0 = -\frac{1}{2}V_1 + \left(\frac{5}{6} + \frac{1}{4}\right)V_2$$

$$V_1 = \frac{13}{6}V_2$$

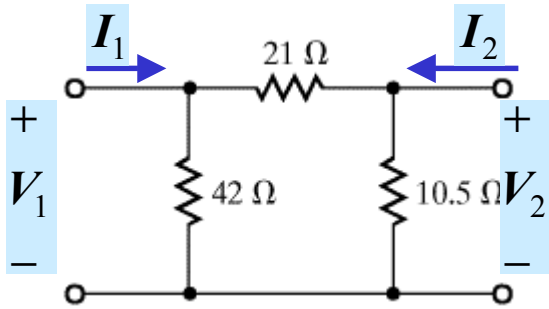
$$V_2 = \frac{8}{11}[V]$$

$$I_2 = -\frac{2}{11}[A]$$



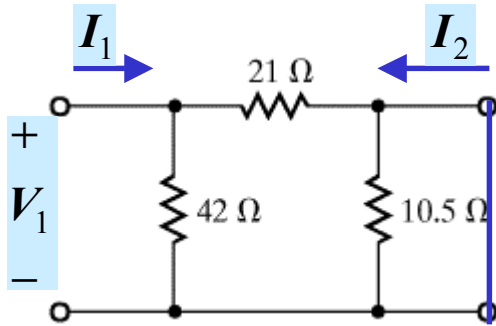
LEARNING EXTENSION

Find the admittance (Y) parameters



$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

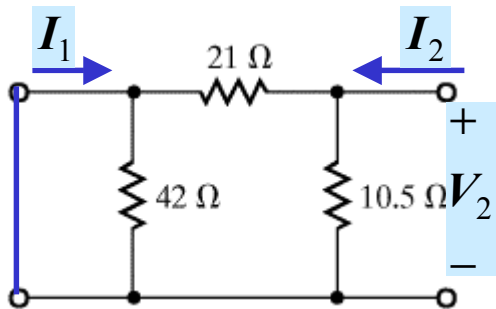


$$I_1 = \left(\frac{1}{21} + \frac{1}{42}\right)V_1 = \frac{3}{42}V_1$$

$$y_{11} = \frac{1}{14}[\mathcal{S}]$$

$$I_2 = -\frac{42}{21+42}I_1$$

$$y_{21} = -\frac{1}{21}[\mathcal{S}]$$



$$I_2 = \left(\frac{2}{21} + \frac{1}{21}\right)V_2$$

$$y_{22} = \frac{1}{7}[\mathcal{S}]$$

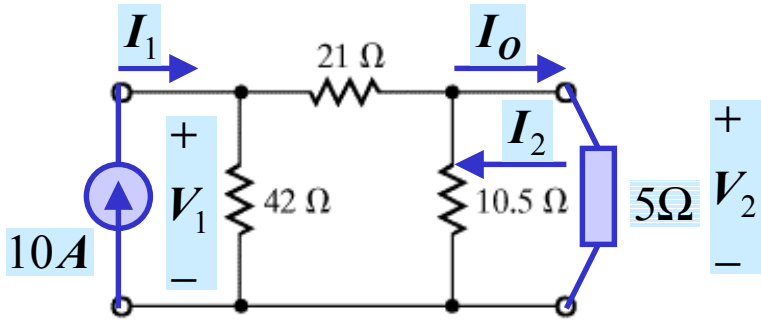
$$I_1 = -\frac{10.5}{21+10.5}I_2$$

$$y_{12} = -\frac{1}{21}[\mathcal{S}]$$



LEARNING EXTENSION

Use the admittance (Y) parameters to find the current I_o



$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$y_{11} = \frac{1}{14} [S]$$

$$y_{21} = -\frac{1}{21} [S]$$

$$y_{22} = \frac{1}{7} [S]$$

$$y_{12} = -\frac{1}{21} [S]$$

Conditions at I/O ports

$$I_1 = 10A$$

$$I_2 = -\frac{1}{5}V_2$$

$$I_o = -I_2$$

Replace in model

$$10 = \frac{1}{14}V_1 - \frac{1}{21}(5I_o)$$

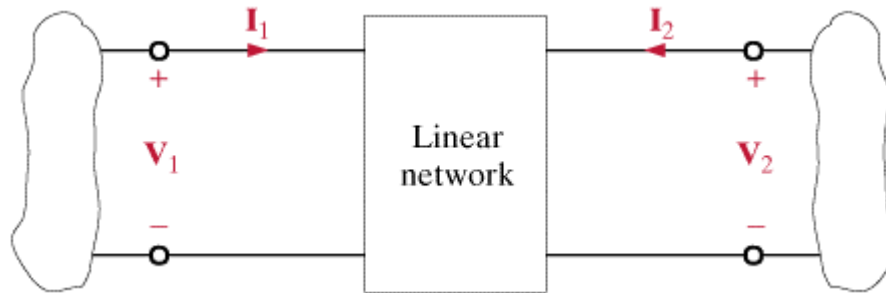
$$-I_o = -\frac{1}{21}V_1 + \frac{1}{7}(5I_o)$$

Solve for variable of interest

$$I_o = \frac{420}{98} [A]$$



IMPEDANCE PARAMETERS



The network contains **NO** independent sources

$$V_1 = z_{11}I_1 + z_{12}I_2$$

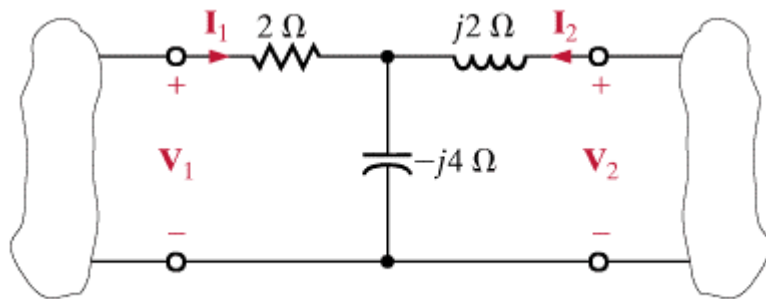
$$V_2 = z_{21}I_1 + z_{22}I_2$$

The '*z parameters*' can be derived in a manner similar to the Y parameters

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}$$

$$z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} \quad z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$



LEARNING EXAMPLE**Find the Z parameters**

(a)

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}$$

$$z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} \quad z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$

Write the loop equations

$$V_1 = 2I_1 - j4(I_1 + I_2)$$

$$V_2 = j2I_2 - j4(I_2 + I_1)$$

rearranging

$$V_1 = (2 - j4)I_1 - j4I_2$$

$$V_2 = -j4I_1 - j2I_2$$

$$\Rightarrow z_{11} = 2 - j4\Omega$$

$$z_{12} = -j4\Omega$$

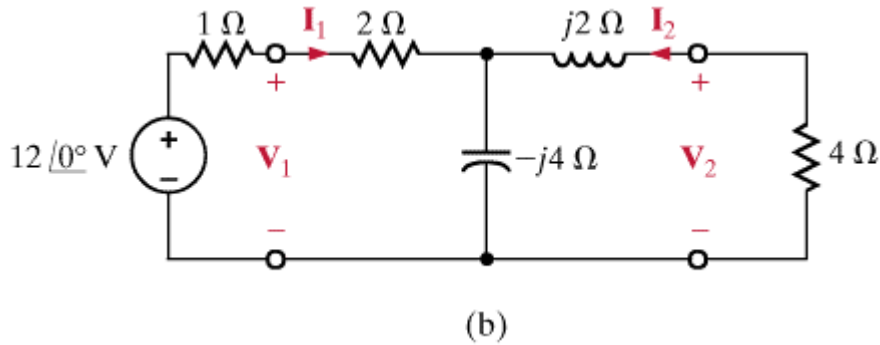
$$z_{21} = -j4\Omega$$

$$z_{22} = -j2\Omega$$



LEARNING EXAMPLE

Use the Z parameters to find the current through the 4 Ohm resistor



$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

Output port constraint

$$V_2 = -4I_2$$

Input port constraint

$$V_1 = 12\angle 0^\circ - (1)I_1$$

$$V_1 = (2 - j4)I_1 - j4I_2$$

$$V_2 = -j4I_1 - j2I_2$$

$$0 = -j4I_1 + (4 - j2)I_2 \quad \times (3 - j4)$$

$$12 = (3 - j4)I_1 - j4I_2 \quad \times j4$$

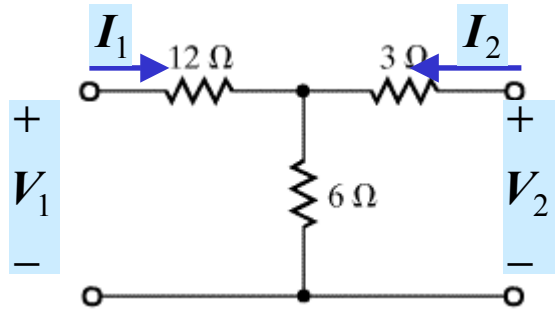
$$48j = (16 + (4 - j2)(3 - j4))I_2 \quad \Rightarrow I_2 = 1.61\angle 137.73^\circ$$



LEARNING EXTENSION

Find the Z parameters.

Find the current on a 4 Ohm load with a 24V input source

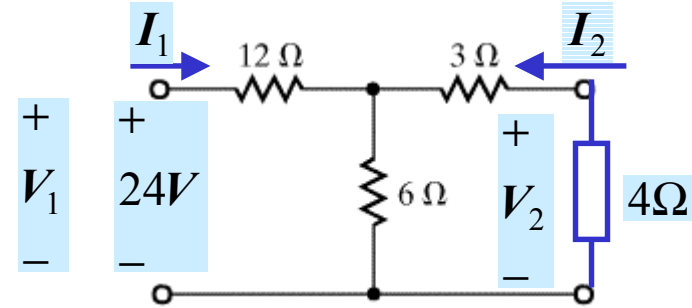


$$V_1 = 12I_1 + 6(I_1 + I_2)$$

$$V_2 = 3I_2 + 6(I_1 + I_2)$$

$$z_{11} = 18\Omega, \quad z_{12} = 6\Omega$$

$$z_{21} = 6\Omega, \quad z_{22} = 9\Omega$$



$$V_1 = 18I_1 + 6I_2$$

$$V_2 = 6I_1 + 9I_2$$

output port constraint : $V_2 = -4I_2$

input port constraint : $V_1 = 24[V]$

$$24 = 18I_1 + 6I_2$$

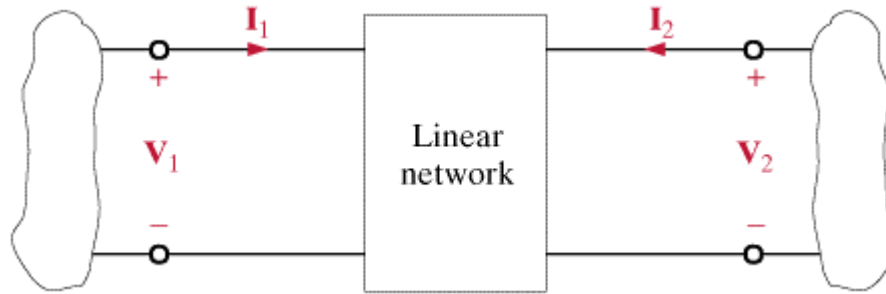
$$0 = 6I_1 + 9I_2 \quad \times(-3)$$

$$24 = (-39 + 6)I_2$$

$$I_2 = -\frac{24}{33}[A]$$



HYBRID PARAMETERS



The network contains **NO** independent sources

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} \quad h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0}$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} \quad h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0}$$

h_{11} = short - circuit input impedance

h_{12} = open - circuit reverse voltage gain

h_{21} = short - circuit forward current gain

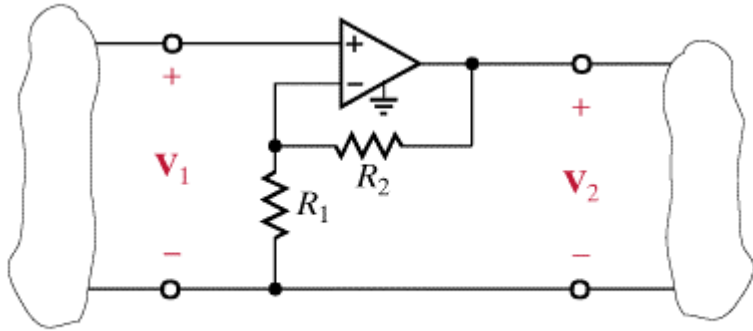
h_{22} = open - circuit output admittance

These parameters are very common in modeling transistors

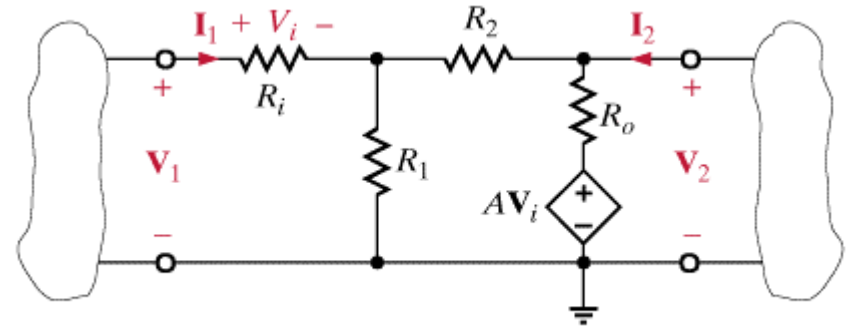


LEARNING EXAMPLE

Find the hybrid parameters for this circuit



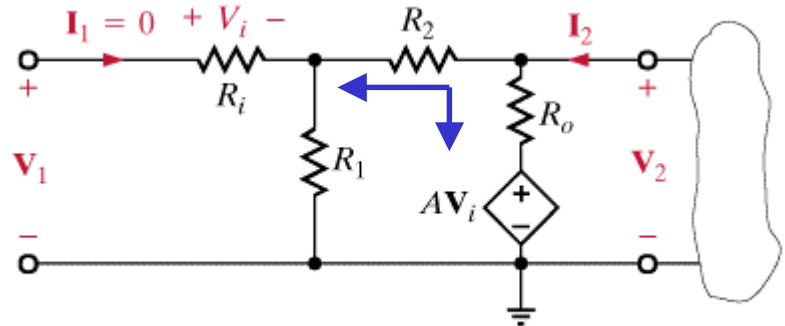
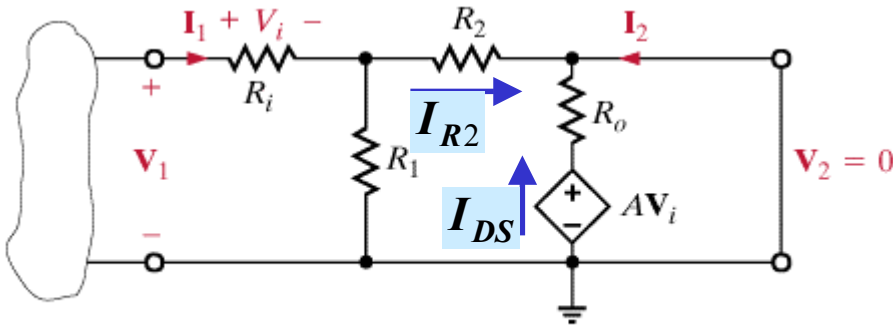
Non-inverting amplifier



Equivalent linear circuit

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$



$$V_1 = (R_i + R_1 \parallel R_2) I_1 \Rightarrow h_{11} = R_i + \frac{R_1 R_2}{R_1 + R_2}$$

$$V_1 = \frac{R_1}{R_1 + R_2} V_2 \Rightarrow h_{12} = \frac{R_1}{R_1 + R_2}$$

$$I_2 = -I_{R2} - I_{DS} = -\frac{R_1}{R_1 + R_2} I_1 - \frac{AR_i I_1}{R_o}$$

$$V_i = 0 \Rightarrow I_2 = \frac{V_2}{R_o \parallel (R_1 + R_2)}$$

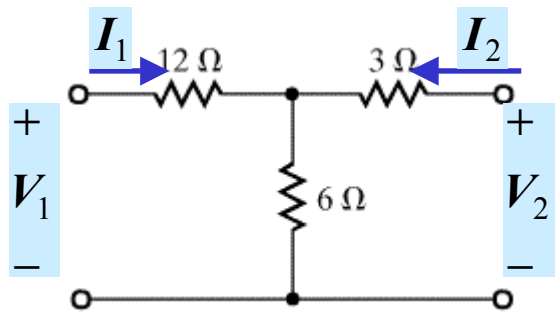
$$h_{21} = -\left(\frac{AR_i}{R_o} + \frac{R_1}{R_1 + R_2} \right)$$

$$h_{22} = \frac{R_o + R_1 + R_2}{R_o (R_1 + R_2)}$$



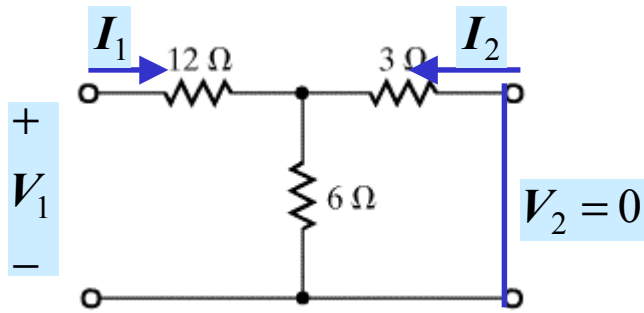
LEARNING EXTENSION

Find the hybrid parameters for the network



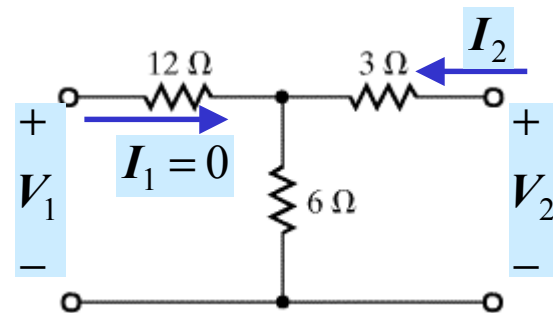
$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$



$$V_1 = (12 + (6 \parallel 3)) I_1 \Rightarrow h_{11} = 14 \Omega$$

$$I_2 = -\frac{6}{3+6} I_1 \Rightarrow h_{21} = -\frac{2}{3}$$



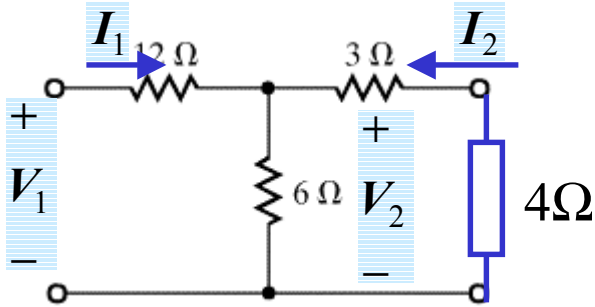
$$V_1 = \frac{6}{3+6} V_2 \Rightarrow h_{12} = \frac{2}{3}$$

$$I_2 = \frac{V_2}{9} \Rightarrow h_{22} = \frac{1}{9} [S]$$



LEARNING EXTENSION

Determine the input impedance of the two-port



$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$R_{in} = \frac{V_1}{I_1}$$

output port constraint : $V_2 = -4I_2$

$$V_1 = h_{11} I_1 + h_{12} (-4I_2)$$

$$I_2 = h_{21} I_1 + h_{22} (-4I_2)$$

$$\Rightarrow I_2 = \frac{h_{21}}{1 + 4h_{22}} I_1$$

$$h_{11} = 14\Omega, \quad h_{12} = \frac{2}{3}$$

$$h_{21} = -\frac{2}{3} \quad h_{22} = \frac{1}{9} [S]$$

$$V_1 = \left(h_{11} - \frac{4h_{12}h_{21}}{1 + 4h_{22}} \right) I_1$$

$$R_{in} = 14 - \frac{4(2/3)(-2/3)}{1 + 4(1/9)} = 14 + \frac{16}{13} = 15.23\Omega$$

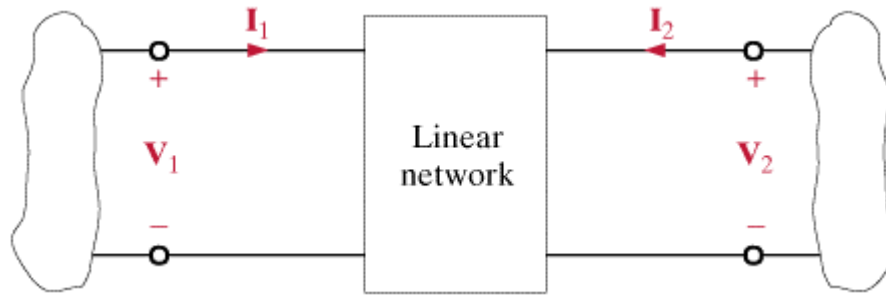
Verification

$$R_{in} = 12 + 6 \parallel 7 = 12 + \frac{42}{13} \Omega$$



TRANSMISSION PARAMETERS

ABCD parameters



The network contains **NO** independent sources

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \quad C = \left. \frac{I_1}{V_2} \right|_{I_2=0}$$

$$B = -\left. \frac{V_1}{I_2} \right|_{V_2=0} \quad D = -\left. \frac{I_1}{I_2} \right|_{V_2=0}$$

A = open circuit voltage ratio

B = negative short-circuit transfer impedance

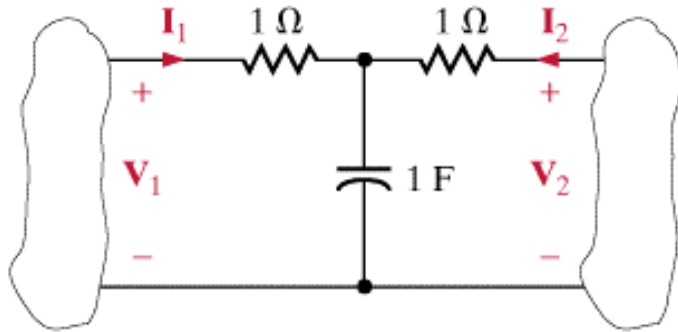
C = open-circuit transfer admittance

D = negative short-circuit current ratio



LEARNING EXAMPLE

Determine the transmission parameters



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0}$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0}$$

$$B = - \left. \frac{V_1}{I_2} \right|_{V_2=0}$$

$$D = - \left. \frac{I_1}{I_2} \right|_{V_2=0}$$

when $I_2 = 0$

$$V_2 = \frac{1}{1 + \frac{1}{j\omega}} V_1 \Rightarrow A = 1 + j\omega$$

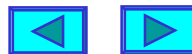
$$V_2 = \frac{1}{j\omega} I_1 \Rightarrow \frac{I_1}{V_2} = j\omega$$

when $V_2 = 0$

$$I_2 = - \frac{1}{1 + \frac{1}{j\omega}} I_1 = - \frac{1}{1 + j\omega} I_1 \Rightarrow D = 1 + j\omega$$

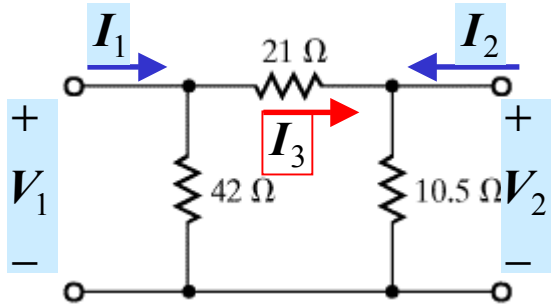
$$V_1 = \left[1 + \left(1 \parallel \frac{1}{j\omega} \right) \right] I_1 = \left[\frac{2 + j\omega}{1 + j\omega} \right] [-(1 + j\omega)] I_2$$

$$B = 2 + j\omega$$



LEARNING EXTENSION

Determine the transmission parameters



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \quad C = \left. \frac{I_1}{V_2} \right|_{I_2=0}$$

$$B = \left. -\frac{V_1}{I_2} \right|_{V_2=0} \quad D = \left. -\frac{I_1}{I_2} \right|_{V_2=0}$$

when $I_2 = 0$

$$V_2 = \frac{10.5}{10.5 + 21} V_1 \Rightarrow A = 3$$

$$I_3 = \frac{42}{42 + 21} I_1 = \frac{V_2}{10.5} \Rightarrow C = \frac{I_1}{V_2} = \frac{1}{7} [\text{S}]$$

When $V_2 = 0$

$$I_2 = -\frac{42}{42 + 21} I_1 \Rightarrow D = \frac{3}{2}$$

$$V_1 = (42 \parallel 21) I_1 = 14 I_1 = 14 \times \left(-\frac{3}{2} I_2\right)$$

$$B = 21 \Omega$$



PARAMETER CONVERSIONS

If all parameters exist, they can be related by conventional algebraic manipulations. As an example consider the relationship between Z and Y parameters

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta_Z} \begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

with $\Delta_Z = z_{11}z_{22} - z_{21}z_{12}$

In the following conversion table, the symbol Δ stands for the determinant of the corresponding matrix

$$\Delta_Z = \begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}, \Delta_Y = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}, \Delta_H = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix}, \Delta_T = \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$



Table 15.1 Two-port parameter conversion formulas

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$\begin{bmatrix} \frac{Y_{22}}{\Delta_Y} & -\frac{Y_{12}}{\Delta_Y} \\ -\frac{Y_{21}}{\Delta_Y} & \frac{Y_{11}}{\Delta_Y} \end{bmatrix}$$

$$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ 1 & \frac{D}{C} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\Delta_H}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{Z_{22}}{\Delta_Z} & -\frac{Z_{12}}{\Delta_Z} \\ -\frac{Z_{21}}{\Delta_Z} & \frac{Z_{11}}{\Delta_Z} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$\begin{bmatrix} \frac{D}{B} & -\frac{\Delta_T}{B} \\ 1 & \frac{A}{B} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{h_{12}}{h_{11}} \\ \frac{h_{11}}{h_{21}} & \frac{\Delta_H}{h_{11}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta_Z}{Z_{21}} \\ 1 & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

$$\begin{bmatrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ \frac{Y_{21}}{-\Delta_Y} & \frac{-Y_{11}}{Y_{21}} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\begin{bmatrix} -\frac{\Delta_H}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ \frac{h_{21}}{-h_{22}} & -\frac{1}{h_{21}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\Delta_Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{11}}{Y_{21}} & \frac{\Delta_Y}{Y_{11}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ 1 & \frac{C}{D} \end{bmatrix}$$

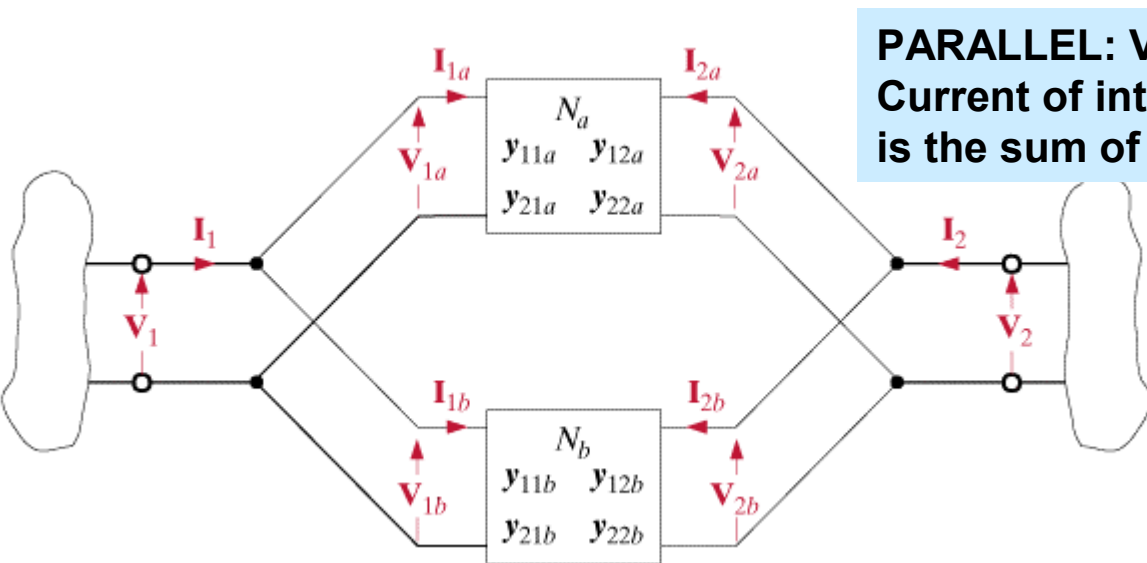
$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$



INTERCONNECTION OF TWO-PORTS

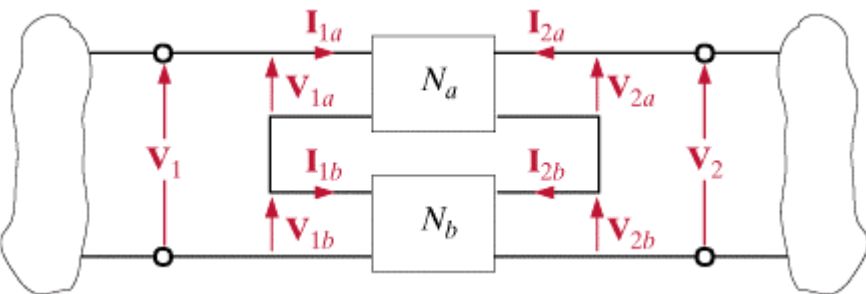
Interconnections permit the description of complex systems in terms of simpler components or subsystems

The basic interconnections to be considered are: *parallel*, *series* and *cascade*

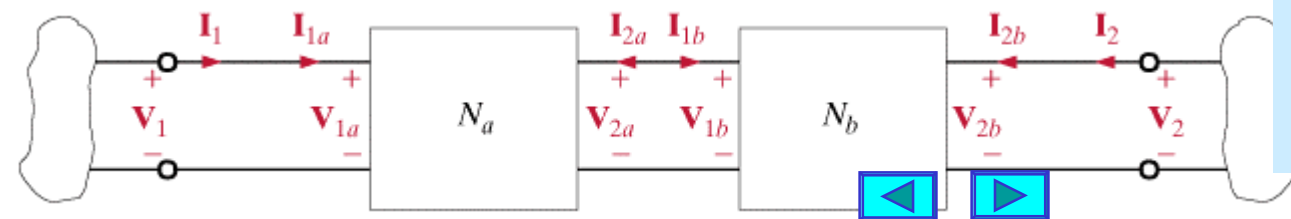


PARALLEL: Voltages are the same.
Current of interconnection
is the sum of currents

The rules used to derive models for interconnection assume that each subsystem behaves in the same manner before and after the interconnection

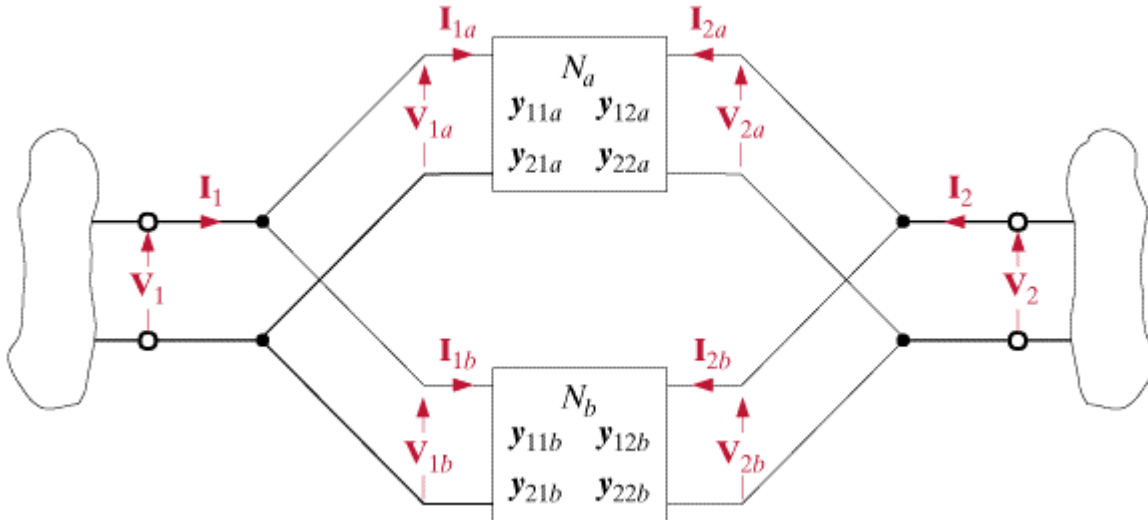


SERIES: Currents are the same.
Voltage of interconnection
is the sum of voltages



CASCADE:
Output of first subsystem
acts as input for the
second

Parallel connection. Description using Y parameters



Interconnection
description

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I = YV$$

$$I_a = \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}, V_a = \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}, Y_a = \begin{bmatrix} y_{11a} & y_{12a} \\ y_{21a} & y_{22a} \end{bmatrix} \Rightarrow I_a = Y_a V_a$$

In a similar manner

$$I_b = Y_b V_b$$

Interconnection constraints:

$$I_1 = I_{1a} + I_{1b}, \quad I_2 = I_{2a} + I_{2b}$$

$$V_1 = V_{1a} = V_{1b}, \quad V_2 = V_{2a} = V_{2b}$$

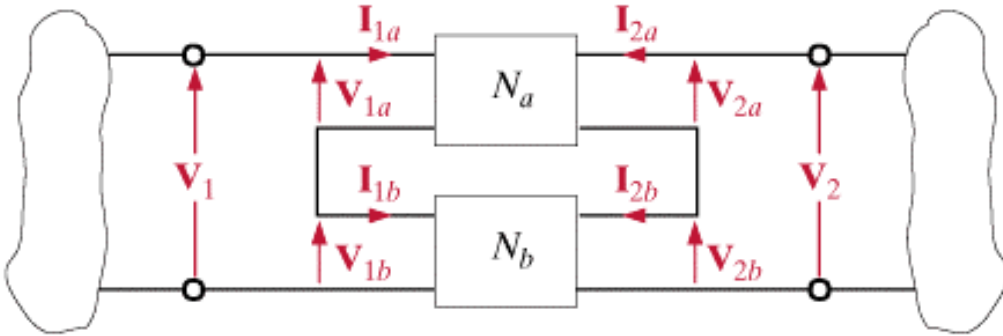
$$\Rightarrow \begin{cases} I = I_a + I_b \\ V = V_a = V_b \end{cases}$$

$$\Rightarrow I = Y_a V_a + Y_b V_b = (Y_a + Y_b) V$$

$$Y = Y_a + Y_b$$



Series interconnection using Z parameters



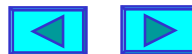
Interconnection constraints

$$\begin{aligned} I_a &= I_b = I \\ V &= V_a + V_b \end{aligned} \Rightarrow V = Z_a I + Z_b I = (Z_a + Z_b) I$$

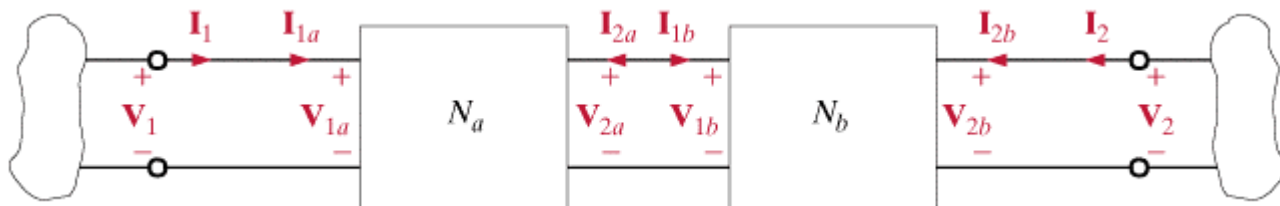
Description of each subsystem

$$V_a = Z_a I_a, \quad V_b = Z_b I_b$$

$$\mathbf{Z} = \mathbf{Z}_a + \mathbf{Z}_b$$



Cascade connection using transmission parameters

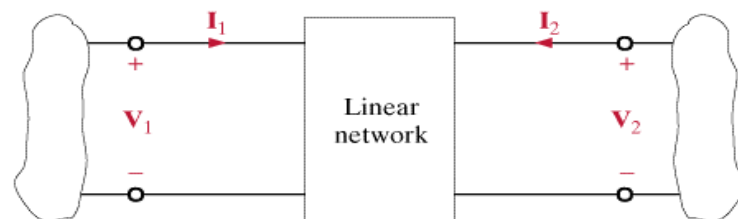


Interconnection constraints:

$$I_{2a} = -I_{1b} \quad V_{2a} = V_{1b}$$

$$V_1 = V_{1a} \quad V_2 = V_{2b}$$

$$I_1 = I_{1a} \quad I_2 = I_{2b}$$



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Matrix multiplication does not commute.
Order of the interconnection is important

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

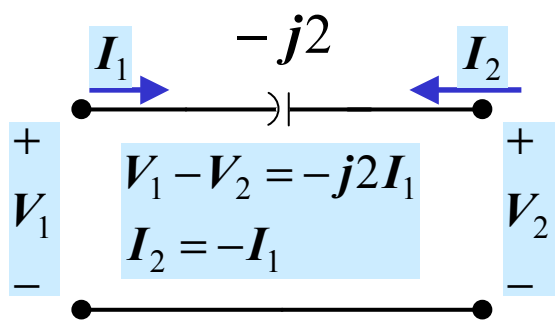
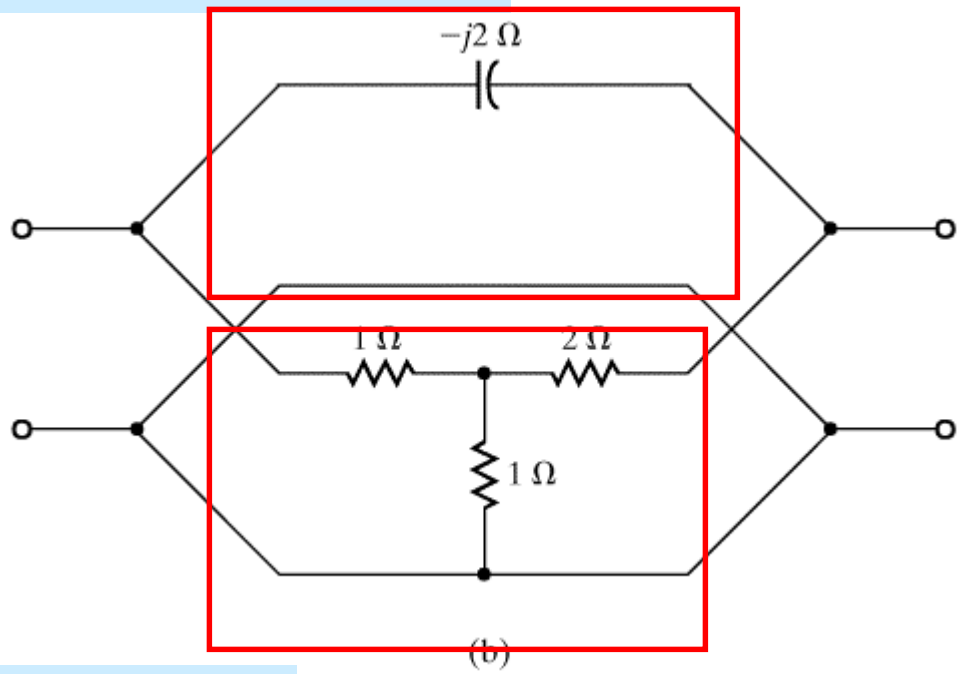
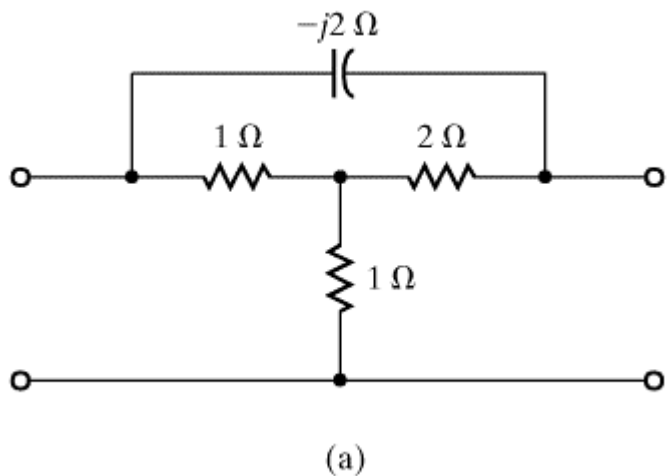
$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



LEARNING EXAMPLE

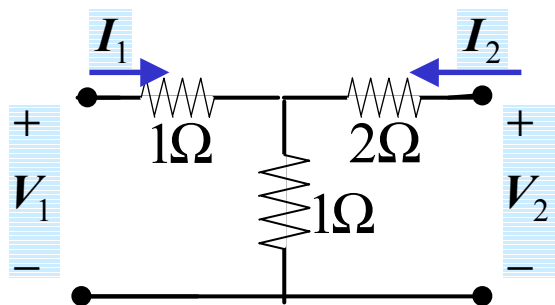
Find the Y parameters for the network



$$y_{11a} = -\frac{1}{2j} = j\frac{1}{2} \text{ S}, \quad y_{12a} = -j\frac{1}{2}$$

$$y_{21a} = -j\frac{1}{2} \text{ S}; \quad y_{22a} = j\frac{1}{2} \text{ S}$$

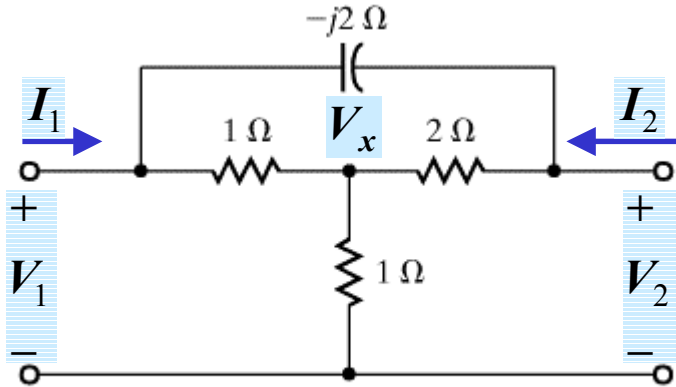
$$Y = \begin{bmatrix} \frac{3}{5} + j\frac{1}{2} & -\left(\frac{1}{5} + j\frac{1}{2}\right) \\ -\left(\frac{1}{5} + j\frac{1}{2}\right) & \frac{2}{5} + j\frac{1}{2} \end{bmatrix} [\text{S}]$$



$$Y_b = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}^{-1} = \frac{1}{5} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$



Find the Y parameters for the network using a direct approach



$$\frac{V_x}{1} + \frac{V_x - V_1}{1} + \frac{V_x - V_2}{2} = 0 \Rightarrow V_x = \frac{2V_1 + V_2}{5}$$

$$I_1 = \frac{V_1 - V_x}{1} + \frac{V_1 - V_2}{-j2}$$

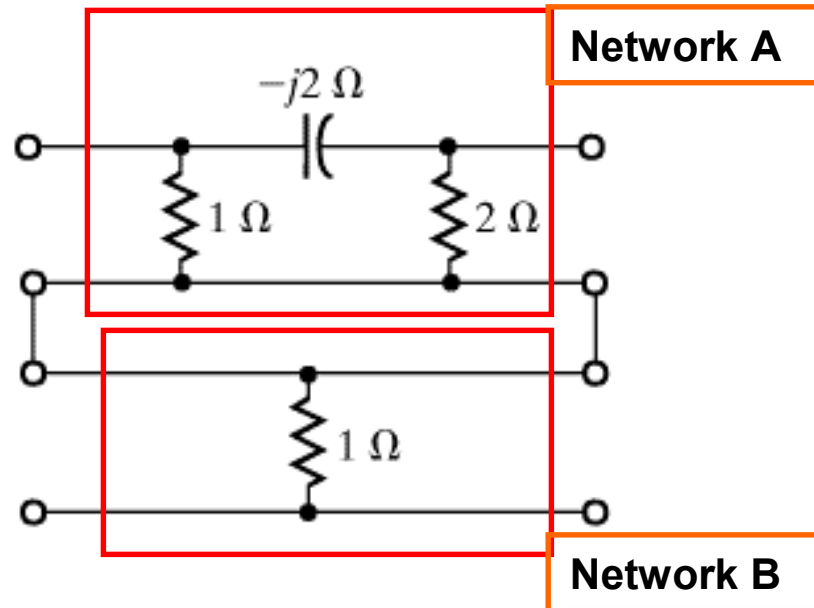
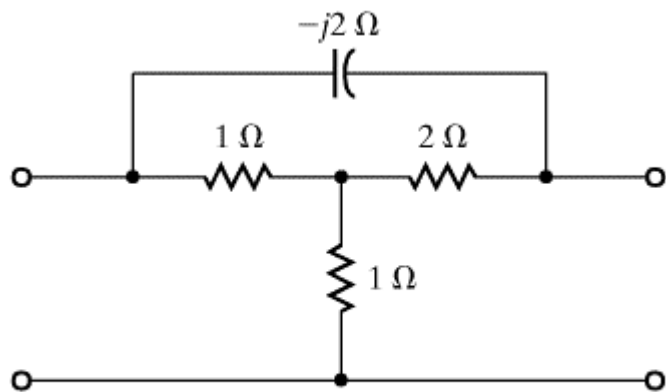
$$I_2 = \frac{V_2 - V_x}{2} + \frac{V_2 - V_1}{-j2}$$

Replace V_x and rearrange ...



LEARNING EXAMPLE

Find the Z parameters of the network



Network A

Network B

Use direct method,
or given the Y parameters transform to Z
... or decompose the network in a series
connection of simpler networks

$$\mathbf{Z}_a = \begin{bmatrix} \frac{2-2j}{3-2j} & \frac{2}{3-2j} \\ \frac{2}{3-2j} & \frac{2-4j}{3-2j} \end{bmatrix}$$

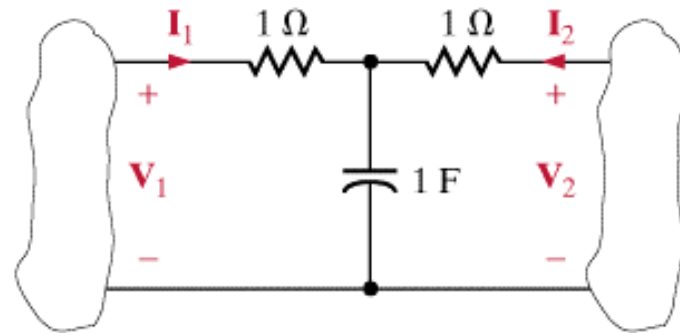
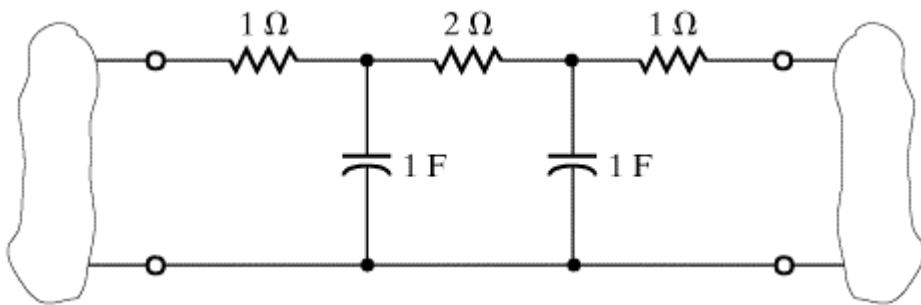
$$\mathbf{Z}_b = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\mathbf{Z} = \mathbf{Z}_a + \mathbf{Z}_b = \begin{bmatrix} \frac{5-4j}{3-2j} & \frac{5-2j}{3-2j} \\ \frac{5-2j}{3-2j} & \frac{5-6j}{3-2j} \end{bmatrix}$$



LEARNING EXAMPLE

Find the transmission parameters



By splitting the 2-Ohm resistor, the network can be viewed as the cascade connection of two identical networks

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+j\omega & 2+j\omega \\ j\omega & 1+j\omega \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+j\omega & 2+j\omega \\ j\omega & 1+j\omega \end{bmatrix} \begin{bmatrix} 1+j\omega & 2+j\omega \\ j\omega & 1+j\omega \end{bmatrix}$$

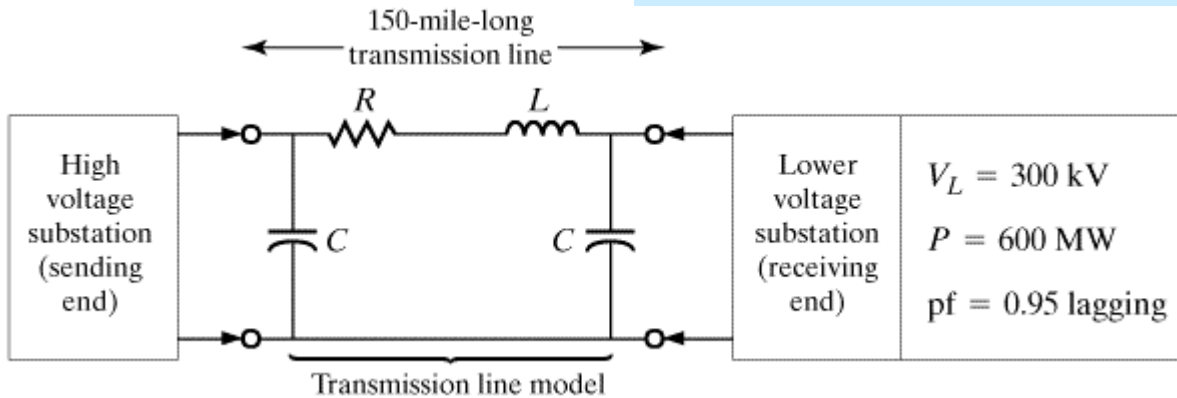
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} (1+j\omega)^2 + (2+j\omega)j\omega & (1+j\omega)(2+j\omega) + (2+j\omega)(1+j\omega) \\ j\omega(1+j\omega) + (1+j\omega)(j\omega) & j\omega(2+j\omega) + (1+j\omega)^2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+4j\omega-2\omega^2 & 4+6j\omega-2\omega^2 \\ 2j\omega-2\omega^2 & 1+4j\omega-2\omega^2 \end{bmatrix}$$



LEARNING by APPLICATION

Given the demand at the receiving end, determine the conditions on the sending end



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Transmission parameters are best suited for this application

In the next slide we show how to determine the transmission parameters for the line. Here we assume them known and use them for analysis

$$V_2 = |V_L| = 300 \text{ kV (line voltage)} \Rightarrow V_2 = |V_L| \angle 0^\circ$$

$$P = \sqrt{3} |V_L| |I_L| \times \text{pf} \Rightarrow |I_L| = |I_2| = \frac{P}{\sqrt{3} V_L \text{pf}}$$

$$\therefore I_2 = \frac{P}{\sqrt{3} V_L \text{pf}} \angle -\cos^{-1} \text{pf}$$

$$P_{\text{total}} = \sqrt{3} |V_{\text{line}}| |I_{\text{line}}| \cos \theta_f$$

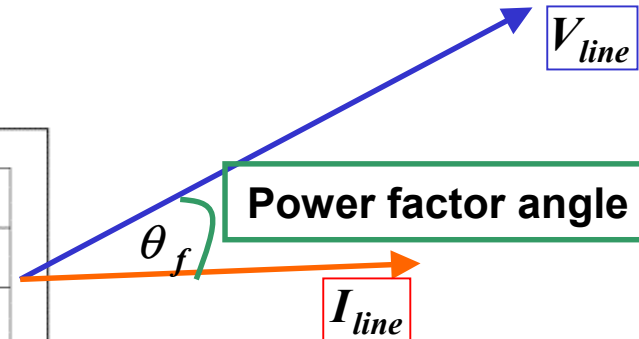
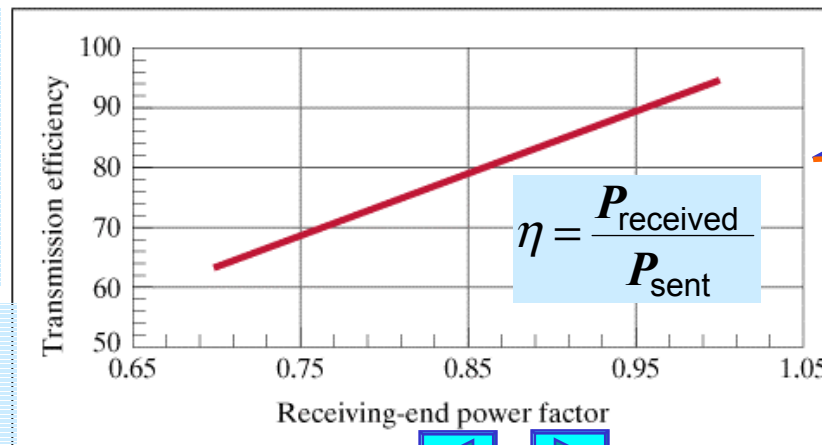
$$Q_{\text{total}} = \sqrt{3} |V_{\text{line}}| |I_{\text{line}}| \sin \theta_f$$

Conditions at the sending end

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

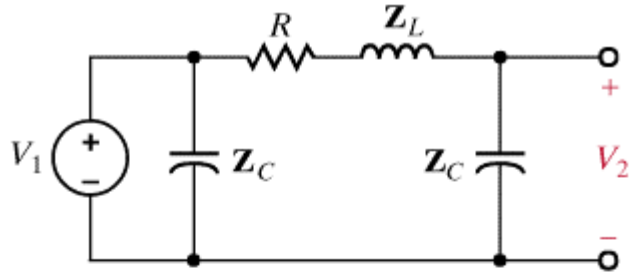
$$\left. \begin{matrix} V_1 \\ I_1 \end{matrix} \right\} \text{pf}_{\text{sending}} \Rightarrow P_{\text{sending}}$$



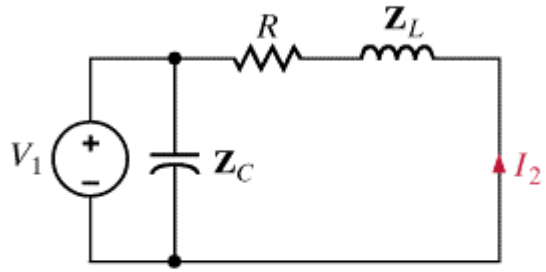
Determining the transmission parameters for the line

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

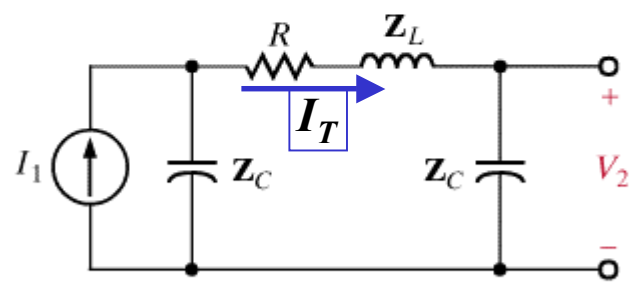


$$I_2 = 0 \Rightarrow A = \frac{V_1}{V_2} = \frac{R + Z_L + Z_C}{Z_C} = 0.9590 \angle 0.27^\circ$$



$$V_2 = 0 \Rightarrow B = -\frac{V_1}{I_2}$$

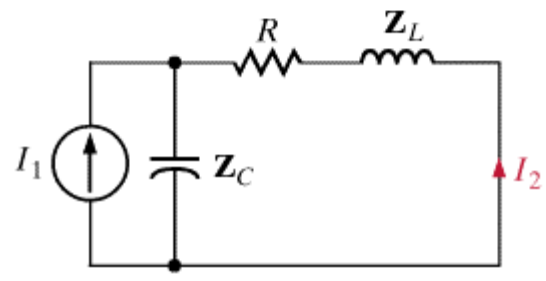
$$V_1 = -(R + Z_L)I_2 \Rightarrow B = R + Z_L = 100.00 \angle 84.84^\circ \Omega$$



$$I_T = \frac{Z_C}{R + Z_L + 2Z_C} I_1 = \frac{V_2}{Z_C}$$

$$I_2 = 0 \Rightarrow C = \frac{I_1}{V_2}$$

$$C = \frac{R + Z_L + 2Z_C}{Z_C^2} = 975.10 \angle 90.13 \mu S$$



$$V_2 = 0 \Rightarrow D = -\frac{I_1}{I_2} = \frac{R + Z_L + Z_C}{Z_C} = 0.9590 \angle 0.27^\circ$$

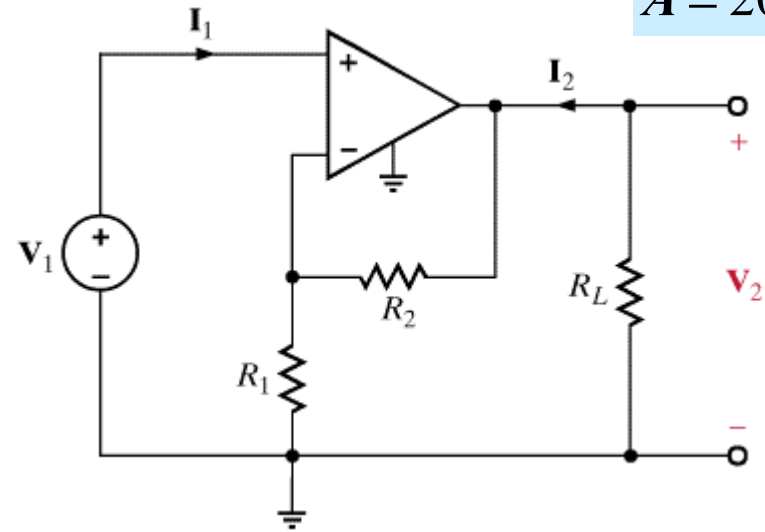


LEARNING EXAMPLE

Determine the effect of the load on the voltage gain

$$A = 20,000, R_i = 1M\Omega, R_o = 500\Omega, R_1 = 1k\Omega, R_2 = 49k\Omega$$

Hybrid parameters are computed in next slide



$$\text{Ideal gain} = 1 + \frac{R_2}{R_1} = 50$$

$$G = \frac{-h_{21}}{h_{11}h_{22} - h_{12}h_{21} + \frac{h_{11}}{R_L}} = \frac{49.88}{1 + \frac{1.247}{R_L}}$$

Using the hybrid parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Eliminating I_1 and solving for V_2

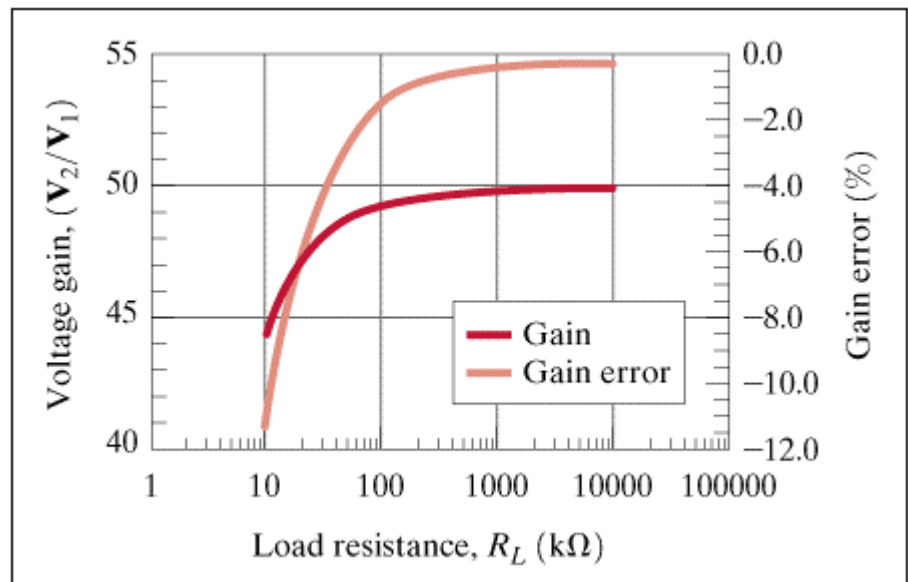
$$V_2 = \frac{-h_{21}V_1 + h_{11}I_2}{h_{11}h_{22} - h_{12}h_{21}}$$

Constraint at output port : $V_2 = -R_L I_2$

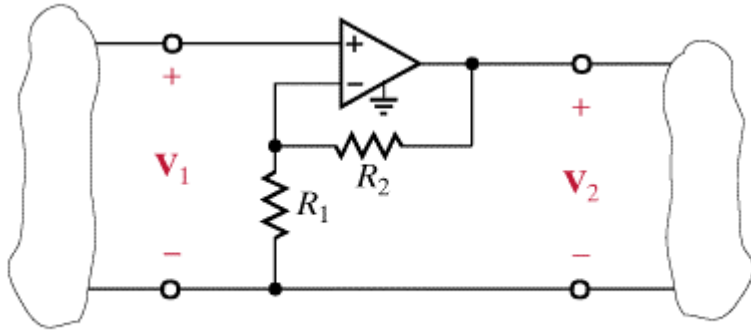
Solving for V_2

$$V_2 = \frac{-h_{21}}{h_{11}h_{22} - h_{12}h_{21} + \frac{h_{11}}{R_L}} V_1$$

Effect of load resistance



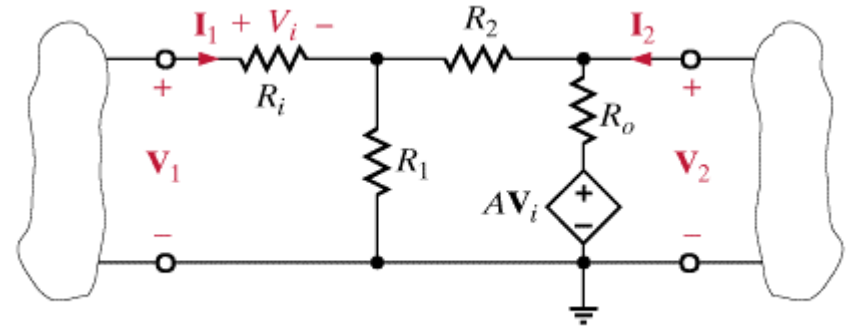
Computing the hybrid parameters for non-inverting amplifier (repeat earlier example)



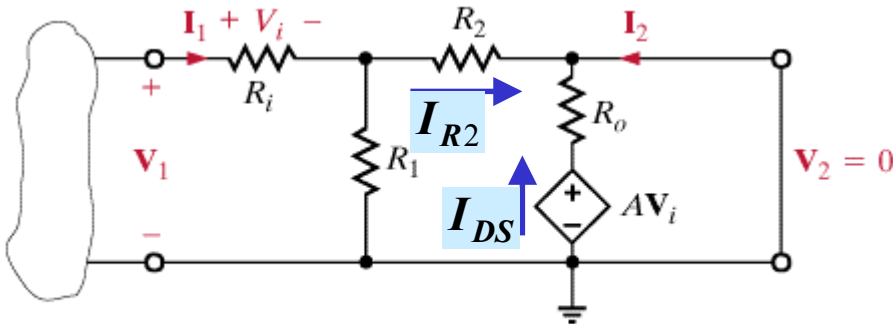
Non-inverting amplifier

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$



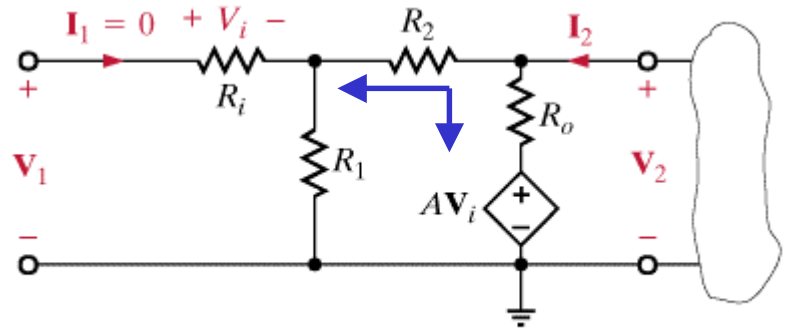
Equivalent linear circuit



$$V_1 = (R_i + R_1 \parallel R_2) I_1 \Rightarrow h_{11} = R_i + \frac{R_1 R_2}{R_1 + R_2}$$

$$I_2 = -I_{R2} - I_{DS} = -\frac{R_1}{R_1 + R_2} I_1 - \frac{A R_i I_1}{R_o}$$

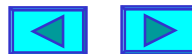
$$h_{21} = -\left(\frac{A R_i}{R_o} + \frac{R_1}{R_1 + R_2} \right)$$



$$V_1 = \frac{R_1}{R_1 + R_2} V_2 \Rightarrow h_{12} = \frac{R_1}{R_1 + R_2}$$

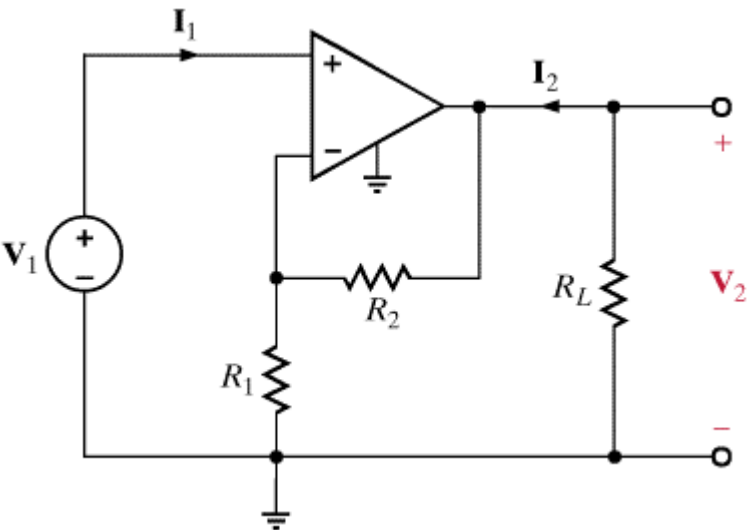
$$V_i = 0 \Rightarrow I_2 = \frac{V_2}{R_o \parallel (R_1 + R_2)}$$

$$h_{22} = \frac{R_o + R_1 + R_2}{R_o (R_1 + R_2)}$$



LEARNING BY DESIGN

Gain required = 10,000 on a load of 1kOhm



Amplifier parameters

$$A = 20,000$$

$$R_i = 1M\Omega$$

$$R_o = 500\Omega$$

$$\text{Ideal gain} = 1 + \frac{R_2}{R_1} = 10,000$$

$$R_2 = 9.999M\Omega$$

$$R_1 = 1k\Omega$$

$$h_{11} = 1.001M\Omega, h_{12} = 1.0 \times 10^{-4}$$

$$h_{21} = -4.0 \times 10^7, h_{22} = 1.0mS$$

For the final solution we will need to cascade amplifiers. Hence the transmission parameters will prove very useful

From the conversion table

$$A = \frac{-\Delta_H}{h_{21}} = \frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}} \quad B = -\frac{h_{11}}{h_{21}}$$

$$C = -\frac{h_{22}}{h_{21}} \quad D = -\frac{1}{h_{21}}$$

$$V_1 = AV_2 - BI_2$$

$$V_2 = -R_L I_2$$

$$I_1 = CV_2 - DI_2$$

eliminating I_1

$$\frac{V_2}{V_1} = \frac{1}{A + \frac{B}{R_L}} = \frac{6667}{1 + \frac{166.7}{R_L}}$$

Analysis of solution:

-Even with infinite load the maximum gain is only 6,667

Likely causes:

- R2 is higher than input resistance Ri
- Desired gain is comparable to the maximum gain, A, of the Op-Amp

Proposed solution:

- Cascade two stages, each with ideal gain of 100. This also lowers R2 to 99kOhm



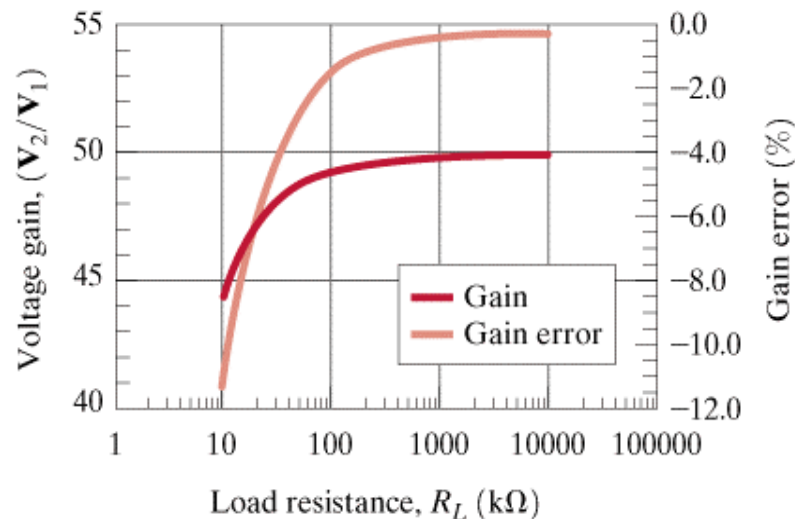
Analysis of proposed solution

Since the two stages will be cascaded, the transmission parameters of the proposed solution will be

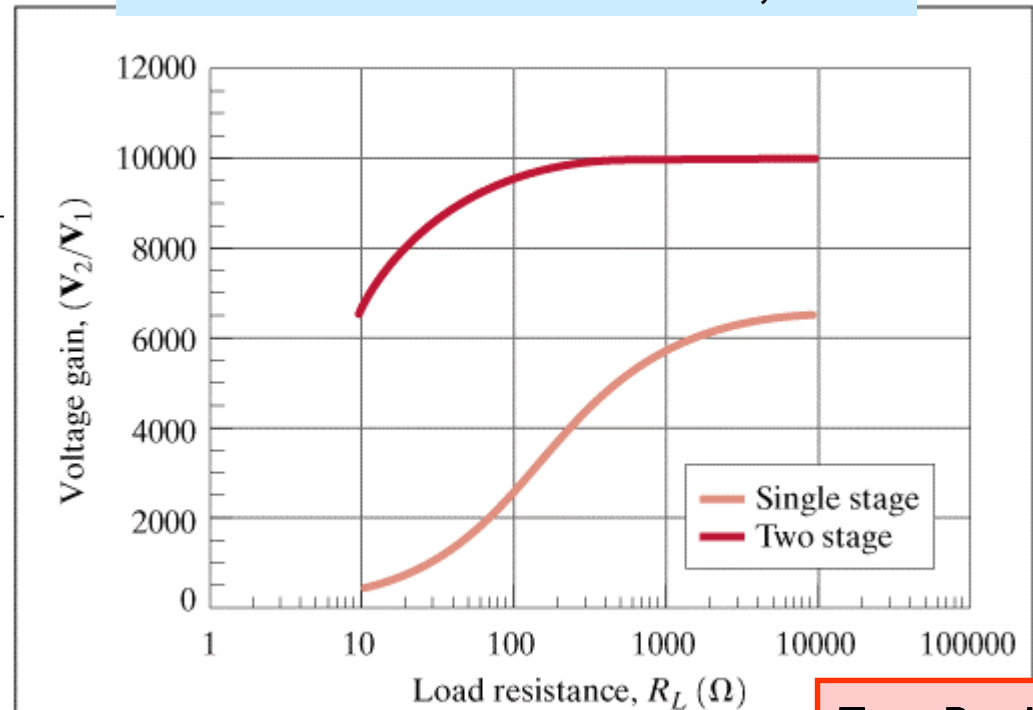
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & -B_b \\ C_b & -D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{aligned} A &= A_a A_b - B_a C_b \\ B &= A_a B_b - B_a D_b \end{aligned} \Rightarrow \frac{V_2}{V_1} = \frac{1}{A + \frac{B}{R_L}}$$

Effect of load resistance. G=50



Effect of load resistance. G=10,000



Two-Ports

