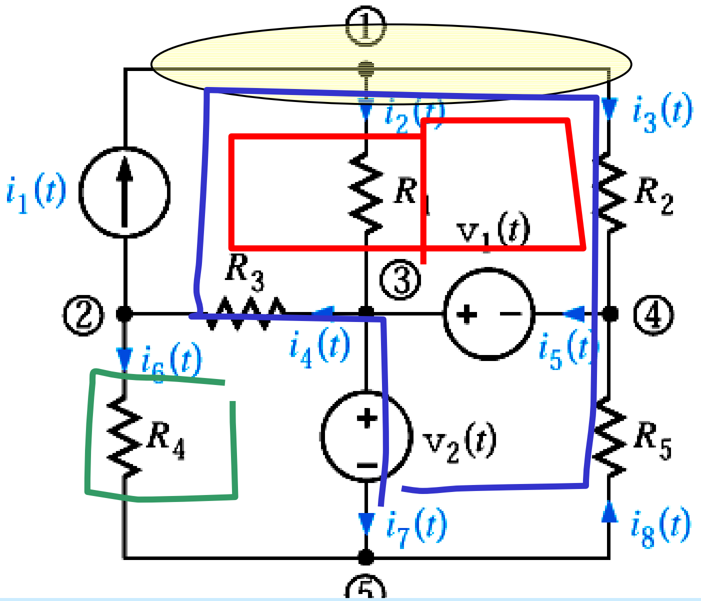
KIRCHHOFF CURRENT LAW

ONE OF THE FUNDAMENTAL CONSERVATION PRINCIPLES IN ELECTRICAL ENGINEERING

"CHARGE CANNOT BE CREATED NOR DESTROYED"

NODES, BRANCHES, LOOPS



NODE: point where two, or more, elements are joined (e.g., big node 1)

LOOP: A closed path that never goes twice over a node (e.g., the blue line)

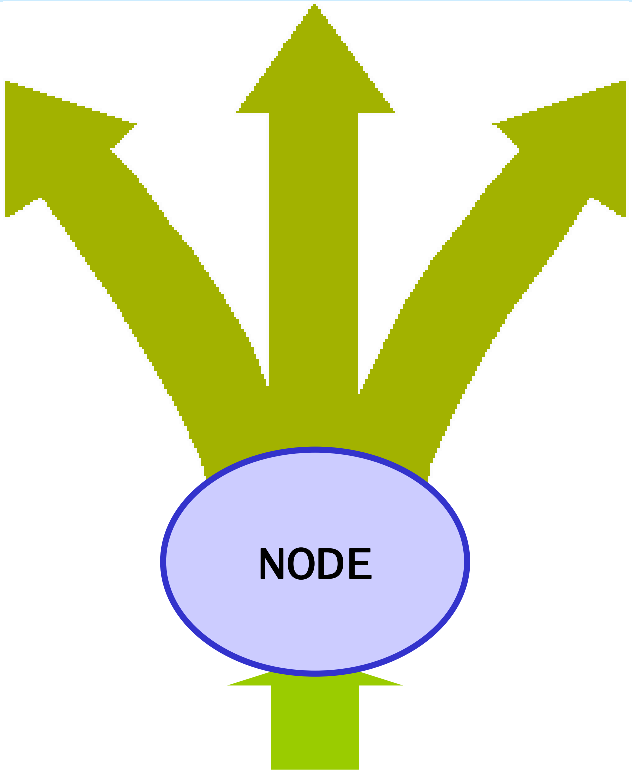
The red path is NOT a loop

BRANCH: Component connected between two nodes (e.g., component R4)

A NODE CONNECTS SEVERAL COMPONENTS. BUT IT DOES NOT HOLD ANY CHARGE.

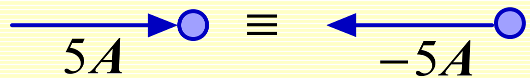
TOTAL CURRENT FLOWING INTO THE NODE MUST BE EQUAL TO TOTAL CURRENT OUT OF THE NODE

(A CONSERVATION OF CHARGE PRINCIPLE)



KIRCHHOFF CURRENT LAW (KCL)

SUM OF CURRENTS FLOWING INTO A NODE IS EQUAL TO SUM OF CURRENTS FLOWING OUT OF THE NODE

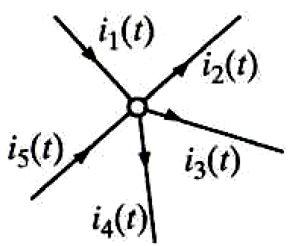


A current flowing into a node is equivalent to the negative flowing out of the node

ALGEBRAIC SUM OF CURRENT (FLOWING) OUT OF A NODE IS ZERO

ALGEBRAIC SUM OF CURRENTS FLOWING INTO A NODE IS ZERO

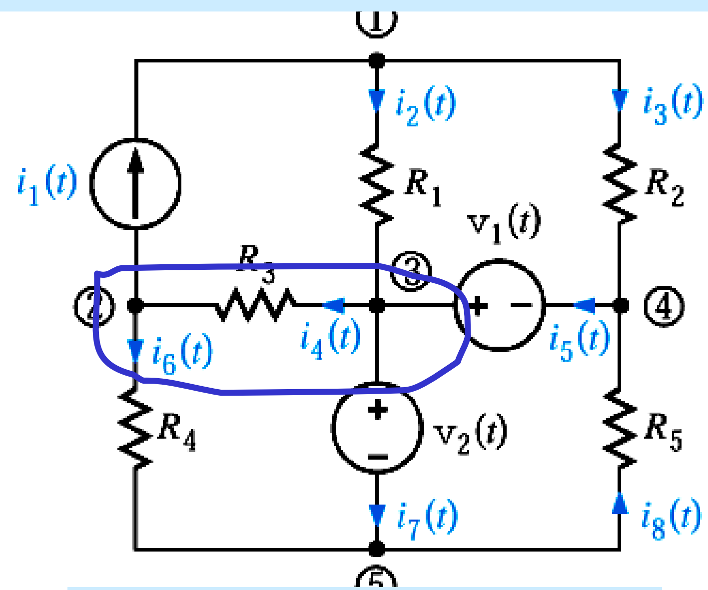
D2.3 Write the KCL equation for the following node:



$$i_1(t) - i_2(t) - i_3(t) - i_4(t) + i_5(t) = 0$$

A GENERALIZED NODE IS ANY PART OF A CIRCUIT WHERE THERE IS NO ACCUMULATION OF CHARGE

... OR WE CAN MAKE SUPERNODES BY AGGREGATING NODES



Leaving 2: $i_1 + i_6 - i_4 = 0$

Leaving 3: $-i_2 + i_4 - i_5 + i_7 = 0$

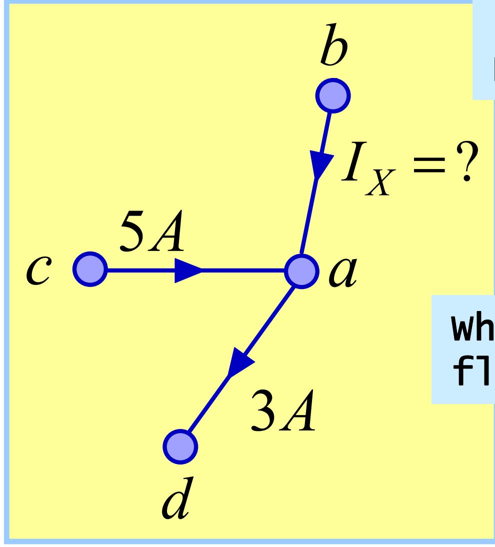
Adding 2 & 3: $i_1 - i_2 - i_5 + i_6 + i_7 = 0$

INTERPRETATION: SUM OF CURRENTS LEAVING NODES 2&3 IS ZERO

VISUALIZATION: WE CAN ENCLOSE NODES 2&3 INSIDE A SURFACE THAT IS VIEWED AS A GENERALIZED NODE (OR SUPERNODE)

GEAUX

PROBLEM SOLVING HINT: KCL CAN BE USED TO FIND A MISSING CURRENT



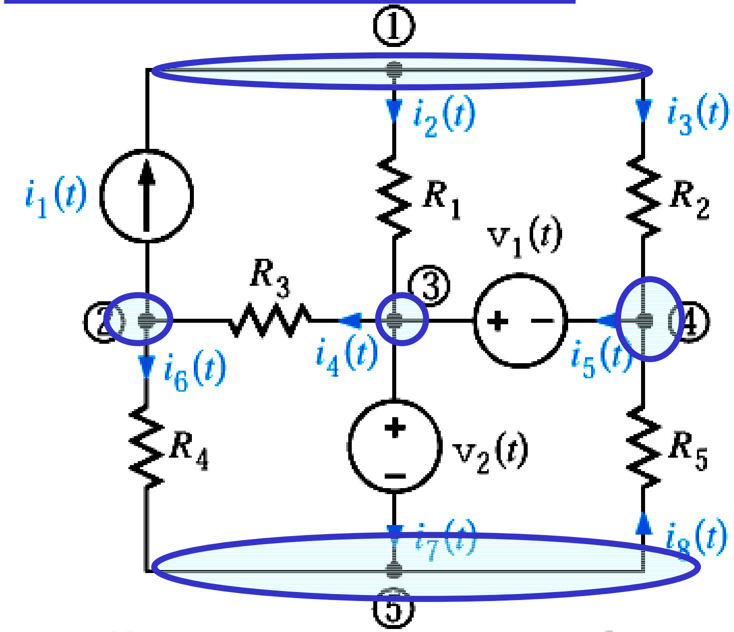
SUM OF CURRENTS INTO NODE IS ZERO

$$5A + I_X + (-3A) = 0$$

$$I_X = -2A$$

Which way are charges flowing on branch a-b?

WRITE ALL KCL EQUATIONS



$$-i_1(t) + i_2(t) + i_3(t) = 0$$

$$i_1(t) - i_4(t) + i_6(t) = 0$$

$$-i_2(t) + i_4(t) - i_5(t) + i_7(t) = 0$$

$$-i_3(t) + i_5(t) - i_8(t) = 0$$

$$-i_6(t) - i_7(t) + i_8(t) = 0$$

THE FIFTH EQUATION IS THE SUM OF THE FIRST FOUR... IT IS REDUNDANT!!!

...AND PRACTICE NOTATION CONVENTION AT THE SAME TIME...

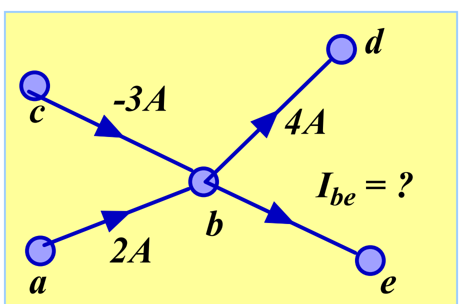
$I_{ab} = 2A,$

$I_{cb} = -3A$

$I_{bd} = 4A$

$I_{be} = ?$

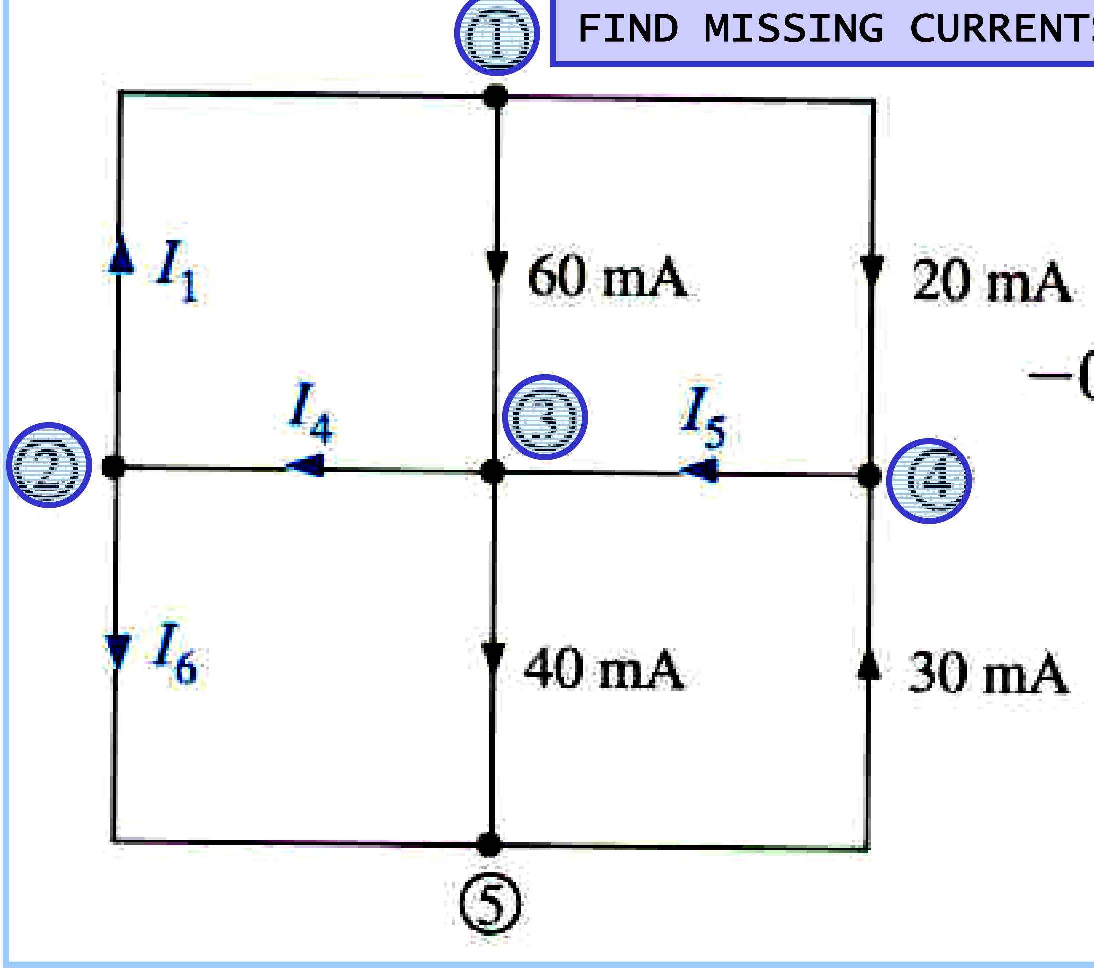
NODES: *a,b,c,d,e*
BRANCHES: *a-b,c-b,d-b,e-b*



$$I_{be} + 4A + [-(-3A)] + (-2A) = 0$$

GEAUX

FIND MISSING CURRENTS



$$-I_1 + 0.06 + 0.02 = 0$$

$$I_1 - I_4 + I_6 = 0$$

$$-0.06 + I_4 - I_5 + 0.04 = 0$$

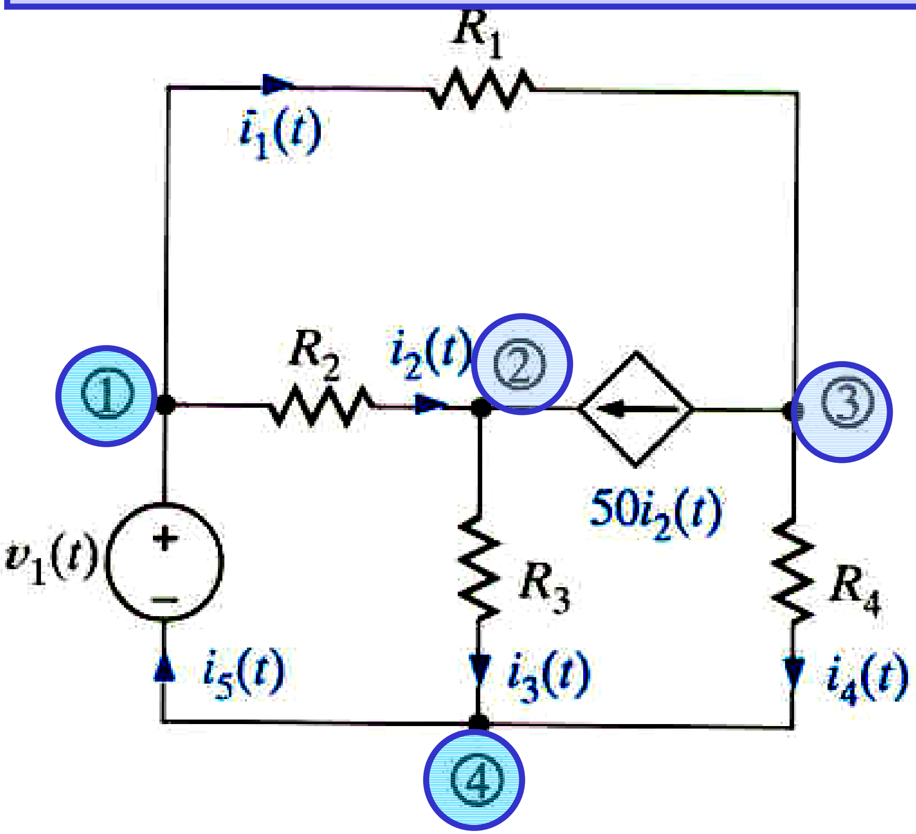
$$-0.02 + I_5 - 0.03 = 0$$

KCL DEPENDS ONLY ON THE INTERCONNECTION.
THE TYPE OF COMPONENT IS IRRELEVANT

KCL DEPENDS ONLY ON THE TOPOLOGY OF THE CIRCUIT

GEAUX

WRITE KCL EQUATIONS FOR THIS CIRCUIT



$$i_1(t) + i_2(t) - i_5(t) = 0$$

$$-i_2(t) + i_3(t) - 50i_2(t) = 0$$

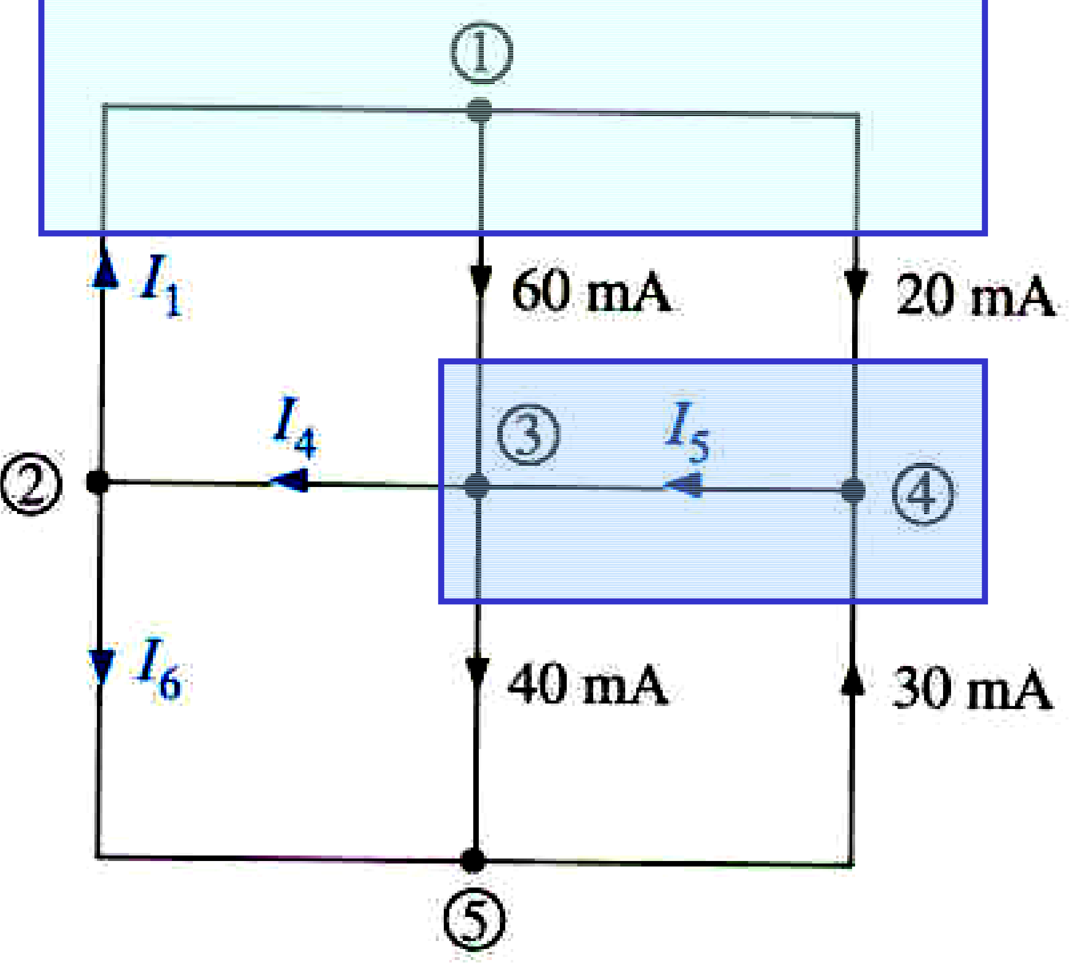
$$-i_1(t) + 50i_2(t) + i_4(t) = 0$$

$$i_5(t) - i_3(t) - i_4(t) = 0$$

- THE LAST EQUATION IS AGAIN LINEARLY DEPENDENT OF THE PREVIOUS THREE

- THE PRESENCE OF A DEPENDENT SOURCE DOES NOT AFFECT APPLICATION OF KCL KCL DEPENDS ONLY ON THE TOPOLOGY

GEAUX



Here we illustrate the use of a more general idea of node. The shaded surface encloses a section of the circuit and can be considered as a BIG node

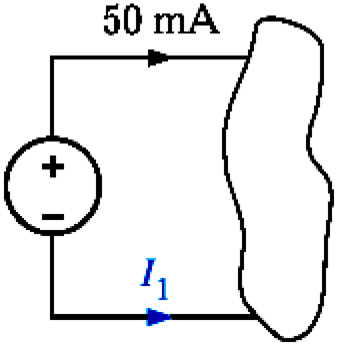
SUM OF CURRENTS LEAVING BIG NODE = 0

$$I_4 + 40mA - 30mA - 20mA - 60mA = 0$$

$$I_4 = 70mA$$

THE CURRENT I5 BECOMES INTERNAL TO THE NODE AND IT IS NOT NEEDED!!!

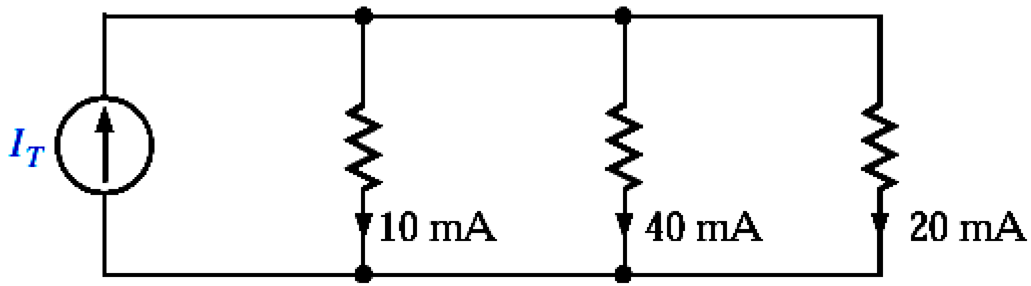
Find I_1



(a)

$I_1 = -50mA$

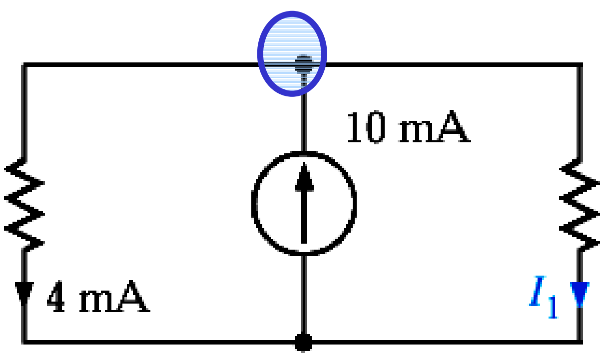
Find I_T



(b)

$I_T = 10mA + 40mA + 20mA$

Find I_1



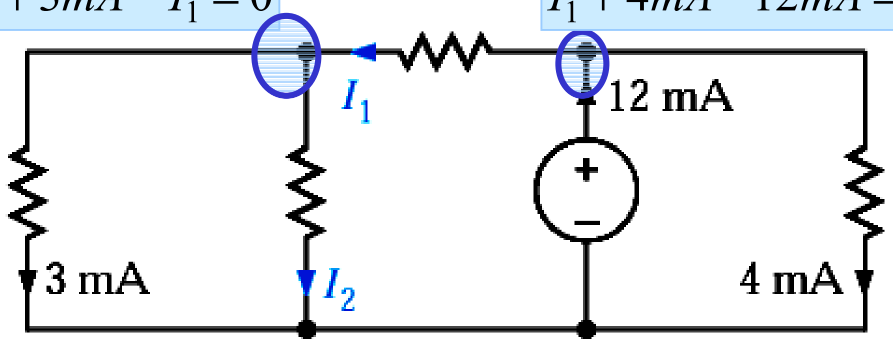
$10mA - 4mA - I_1 = 0$

(a)

Find I_1 and I_2

$I_2 + 3mA - I_1 = 0$

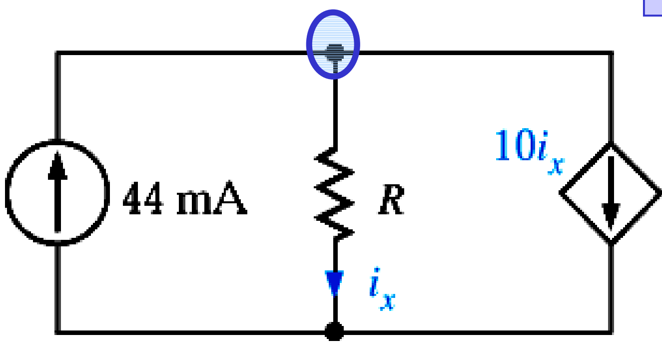
$I_1 + 4mA - 12mA = 0$



(b)

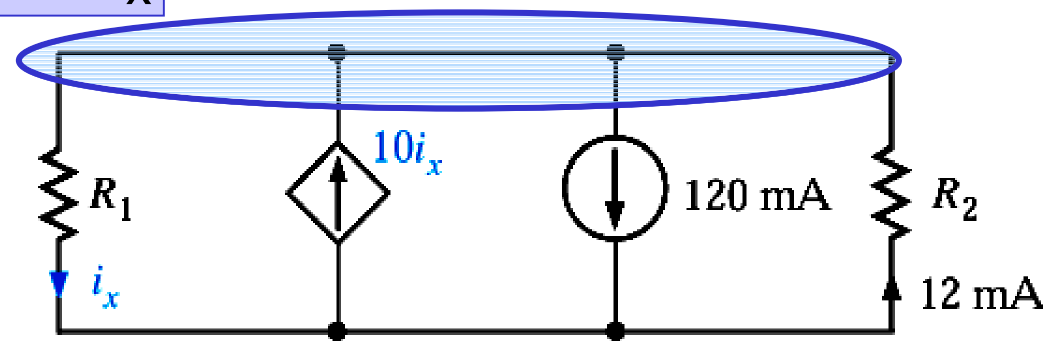
GEAUX

Find i_x



(a)

$$10i_x + i_x - 44mA = 0$$
$$i_x = 4mA$$

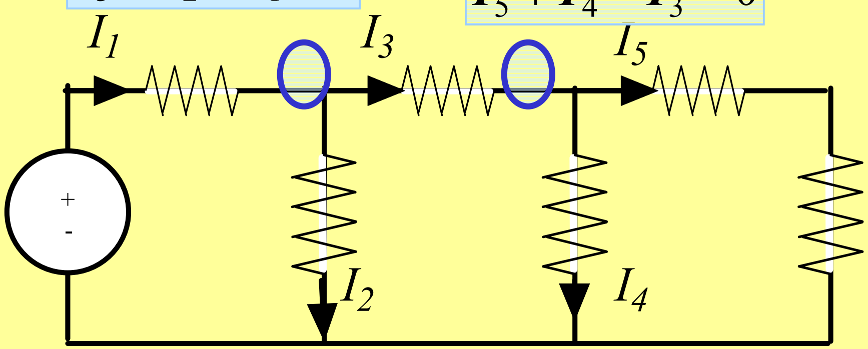


(b)

$$i_x - 10i_x + 120mA - 12mA = 0$$

$$I_3 + I_2 - I_1 = 0$$

$$I_5 + I_4 - I_3 = 0$$



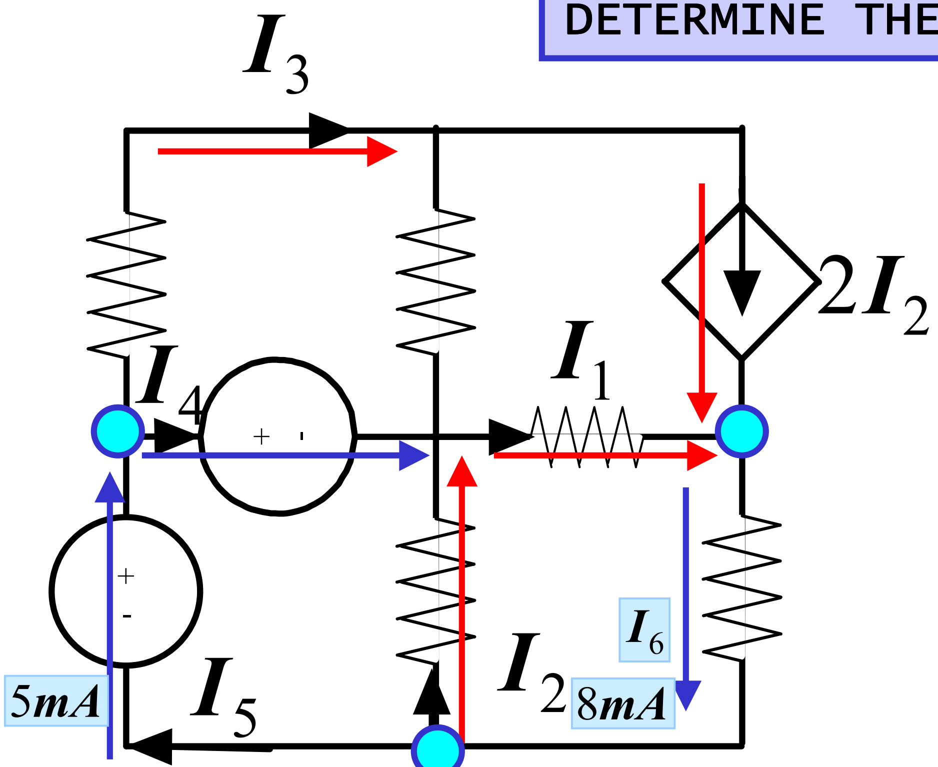
I_1 = $14mA$

I_5 = $5mA$

$I_2 = 6mA$, $I_3 = 8mA$, $I_4 = 4mA$

GEAUX

DETERMINE THE CURRENTS INDICATED



$I_4 = 2mA$

$I_5 = 5mA$

$$I_1 = 2mA,\ I_2 = 3mA,\ I_3 = 5mA$$

$$I_6 - I_1 - 2I_2 = 0 \Rightarrow I_6 = 8mA$$

$$I_5 + I_2 - I_6 = 0$$

$$I_4 + I_3 - I_5 = 0$$

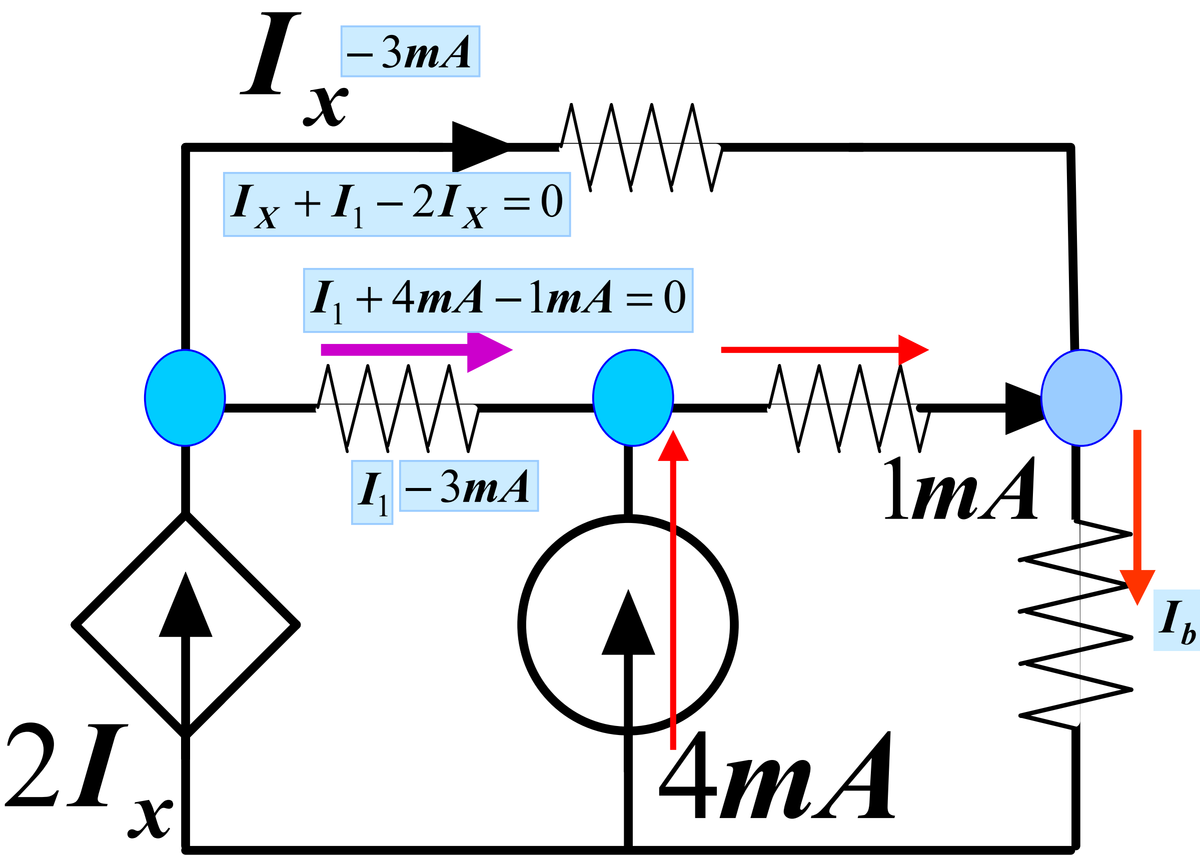
THE PLAN

MARK ALL THE KNOWN CURRENTS

FIND NODES WHERE ALL BUT ONE CURRENT ARE KNOWN

GEAUX

FIND I_x



VERIFICATION

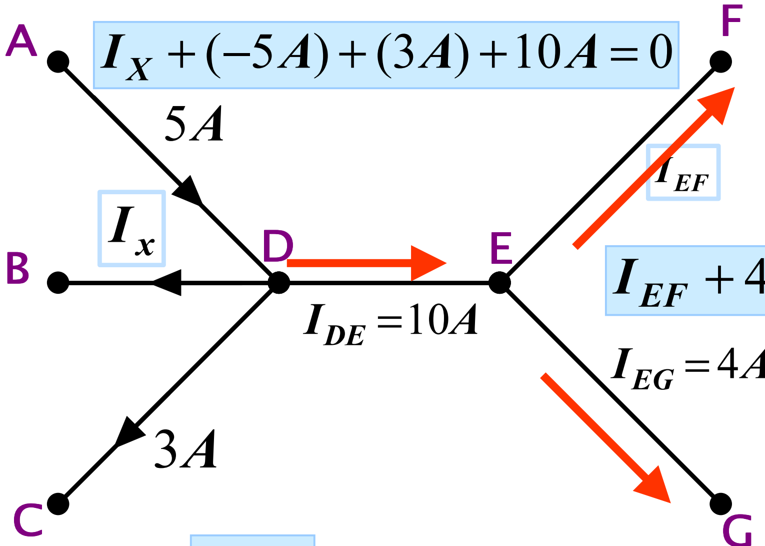
$I_b = 1mA + I_X = -2mA$

$2I_X + 4mA = I_b$

GEAUX

This question tests KCL and convention to denote currents

Use sum of currents leaving node = 0



$$I_X + (-5A) + (3A) + 10A = 0$$

$$I_{EF} + 4A - 10A = 0$$

$I_x =$ **-8A**

On BD current flows from **B** to **D**

$I_{EF} =$ **6A**

On EF current flows from **E** to **F**

KIRCHHOFF VOLTAGE LAW

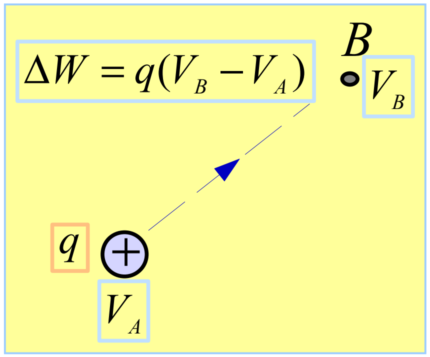
ONE OF THE FUNDAMENTAL CONSERVATION LAWS IN ELECTRICAL ENGINERING

THIS IS A CONSERVATION OF ENERGY PRINCIPLE
"ENERGY CANNOT BE CREATE NOR DESTROYED"

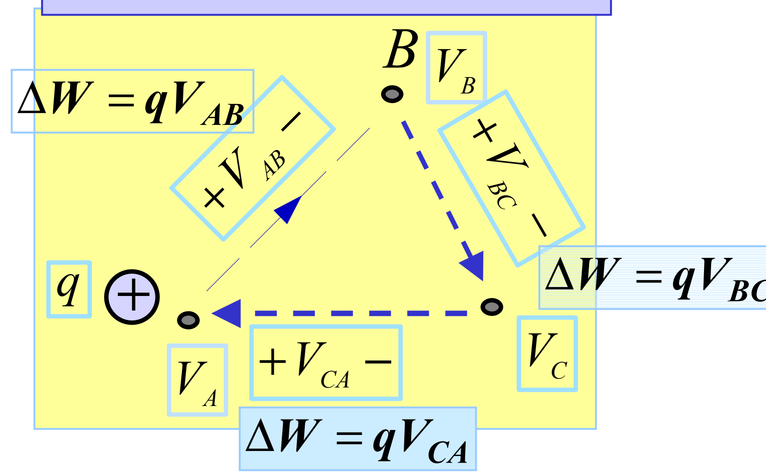
KIRCHHOFF VOLTAGE LAW (KVL)

KVL IS A CONSERVATION OF ENERGY PRINCIPLE

A POSITIVE CHARGE GAINS ENERGY AS IT MOVES TO A POINT WITH HIGHER VOLTAGE AND RELEASES ENERGY IF IT MOVES TO A POINT WITH LOWER VOLTAGE



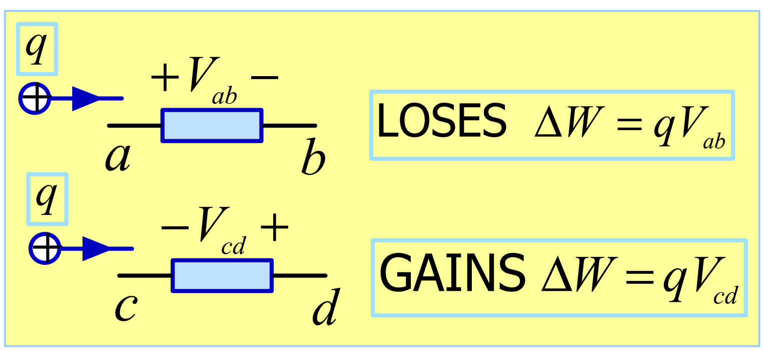
A "THOUGHT EXPERIMENT"



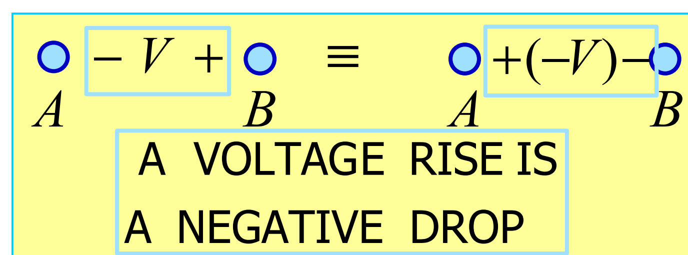
IF THE CHARGE COMES BACK TO THE SAME INITIAL POINT THE NET ENERGY GAIN MUST BE ZERO (Conservative network)

OTHERWISE THE CHARGE COULD END UP WITH INFINITE ENERGY, OR SUPPLY AN INFINITE AMOUNT OF ENERGY

$$q(V_{AB} + V_{BC} + V_{CD}) = 0$$

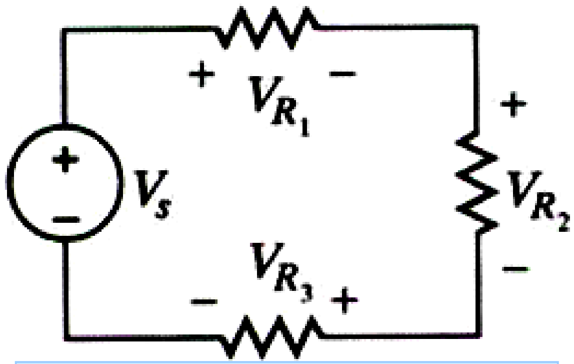


KVL: THE ALGEBRAIC SUM OF VOLTAGE DROPS AROUND ANY LOOP MUST BE ZERO

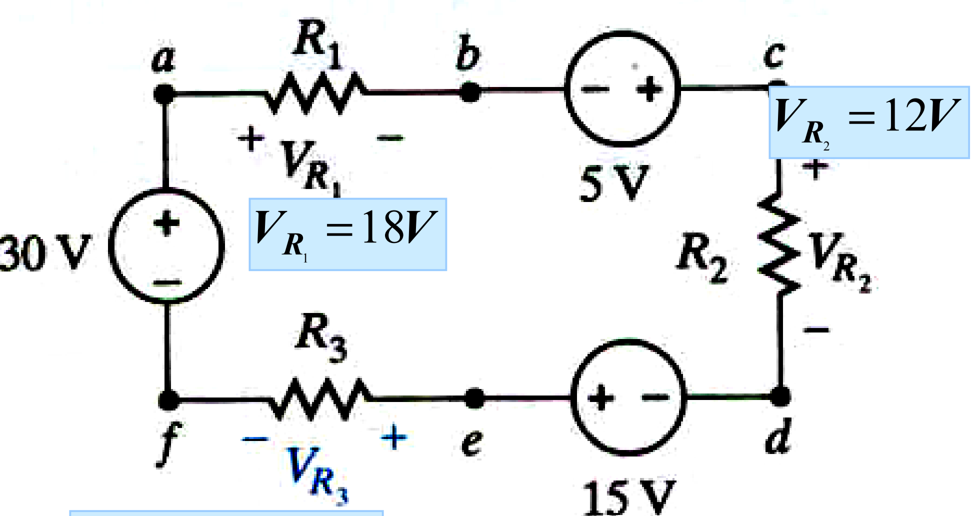


GEAUX

D2.4 Write the KVL equation for the following loop, traveling clockwise:



$$-V_S + V_{R_1} + V_{R_2} + V_{R_3} = 0$$

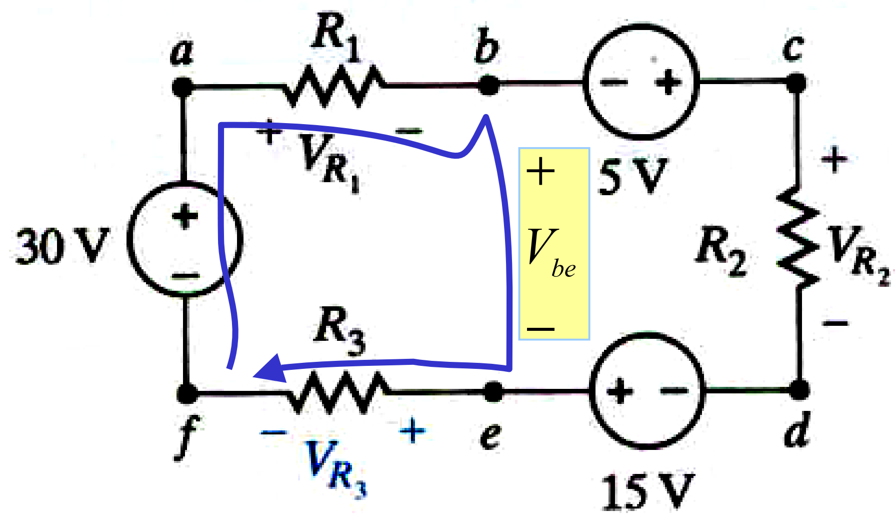


LOOP *abcdefa*

$$+V_{R_1} - 5 + V_{R_2} - 15 + V_{R_3} - 30 = 0$$

PROBLEM SOLVING TIP: KVL IS USEFUL TO DETERMINE A VOLTAGE - FIND A LOOP INCLUDING THE UNKNOWN VOLTAGE

THE LOOP DOES NOT HAVE TO BE PHYSICAL



EXAMPLE: V_{R1}, V_{R3} ARE KNOWN
DETERMINE THE VOLTAGE V_{be}

$$V_{R_1} + V_{be} + V_{R_3} - 30[V] = 0$$

GEAUX

BACKGROUND: WHEN DISCUSSING KCL WE SAW THAT NOT ALL POSSIBLE KCL EQUATIONS ARE INDEPENDENT. WE SHALL SEE THAT THE SAME SITUATION ARISES WHEN USING KVL

A SNEAK PREVIEW ON THE NUMBER OF LINEARLY INDEPENDENT EQUATIONS

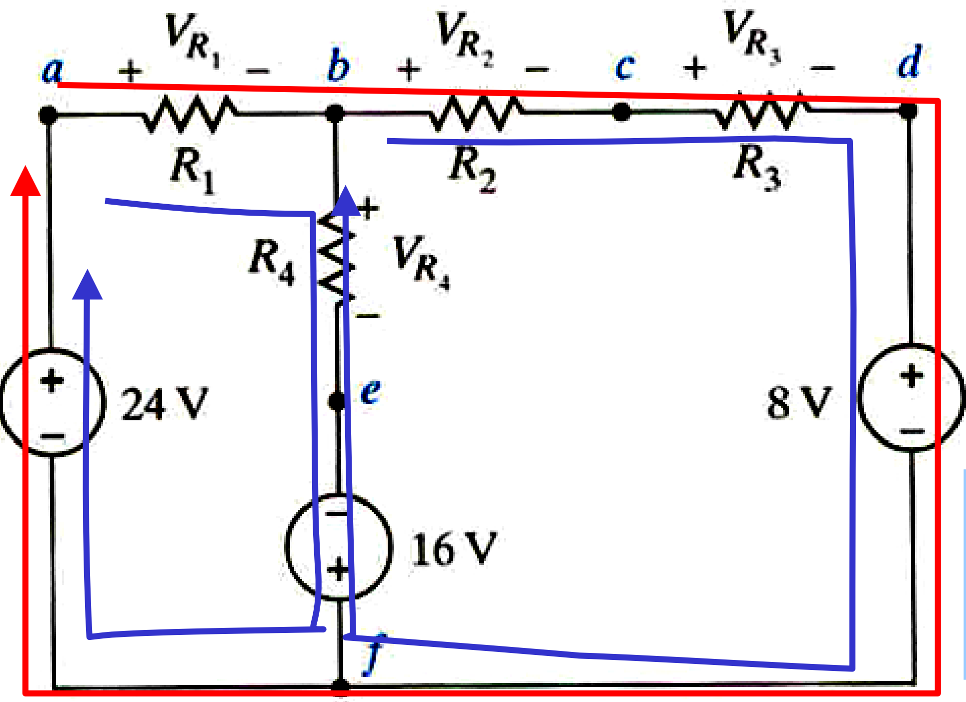
IN THE CIRCUIT DEFINE

N NUMBER OF NODES

B NUMBER OF BRANCHES

$N-1$ LINEARLY INDEPENDENT KCL EQUATIONS

$B-(N-1)$ LINEARLY INDEPENDENT KVL EQUATIONS



EXAMPLE: FOR THE CIRCUIT SHOWN WE HAVE N = 6, B = 7.
HENCE THERE ARE ONLY TWO INDEPENDENT KVL EQUATIONS

$$V_{R_1} + V_{R_4} - 16 - 24 = 0$$

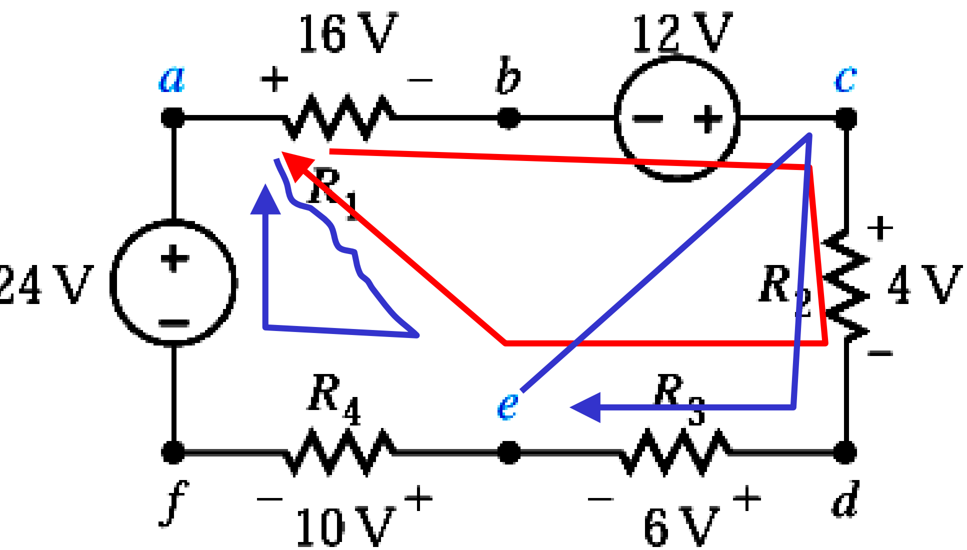
$$V_{R_2} + V_{R_3} + 8 + 16 - V_{R_4} = 0$$

$$V_{R_1} + V_{R_2} + V_{R_3} + 8 - 24 = 0$$

THE THIRD EQUATION IS THE SUM OF THE OTHER TWO!!

GEAUX

FIND THE VOLTAGES V_{ae}, V_{ec}



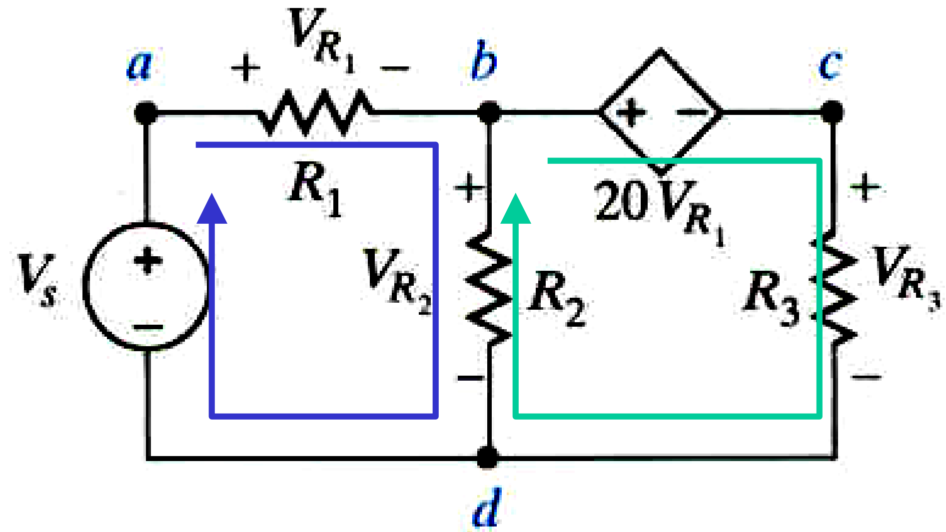
$$V_{ae} + 10 - 24 = 0$$

$$16 - 12 + 4 + 6 - V_{ae} = 0$$

GIVEN THE CHOICE USE THE SIMPLEST LOOP

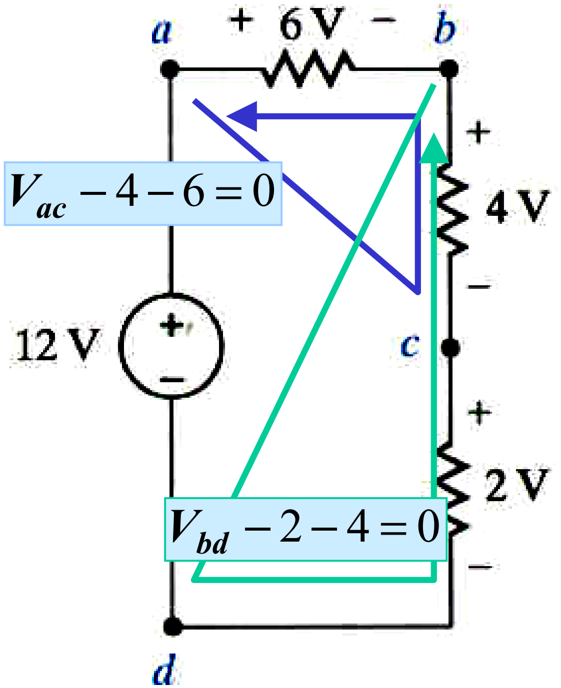
$$4 + 6 + V_{ec} = 0$$

DEPENDENT SOURCES ARE HANDLED WITH THE SAME EASE



$$V_{R_1} + V_{R_2} - V_S = 0$$

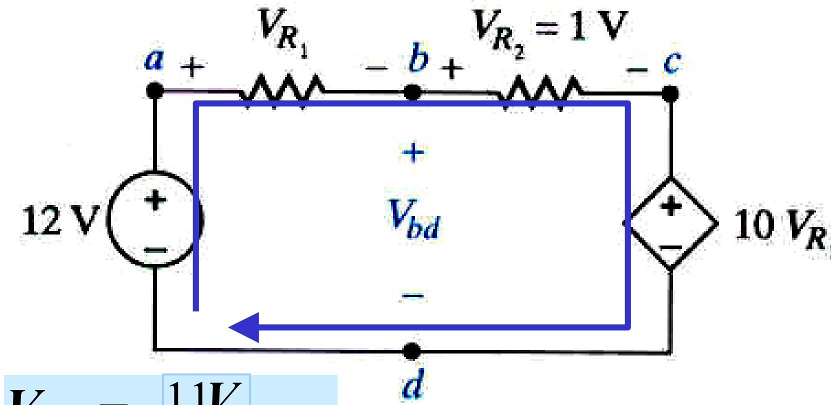
$$20V_{R_1} + V_{R_3} - V_{R_2} = 0$$



V_{ad} = ______

V_{ac} = $10V$

V_{bd} = $6V$



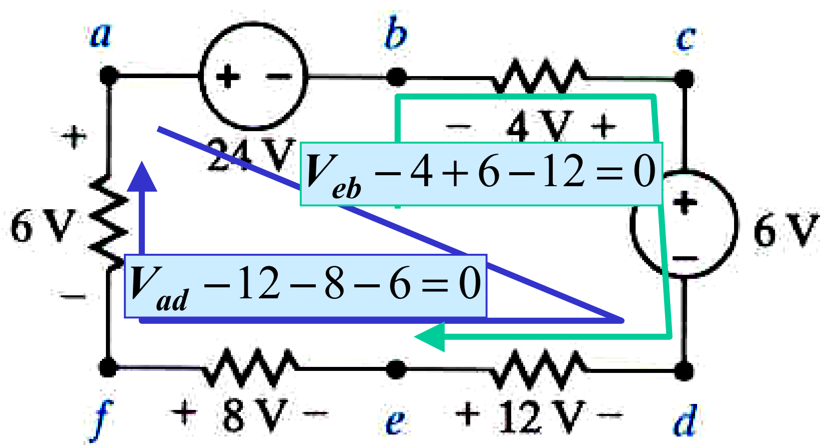
V_{bd} = $11V$

MUST FIND V_{R_1} FIRST

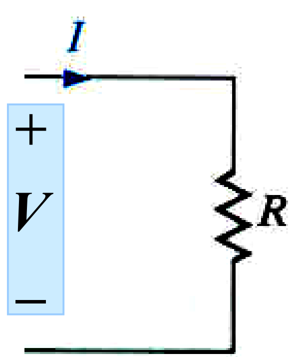
$$-12 + V_{R_1} + 1 + 10V_{R_1} = 0 \Rightarrow V_{R_1} = 1V$$

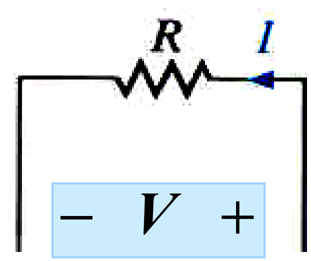
DEPENDENT SOURCES ARE NOT REALLY DIFFICULT TO ANALYZE

REMINDER: IN A RESISTOR THE VOLTAGE AND CURRENT DIRECTIONS MUST SATISFY THE PASSIVE SIGN CONVENTION



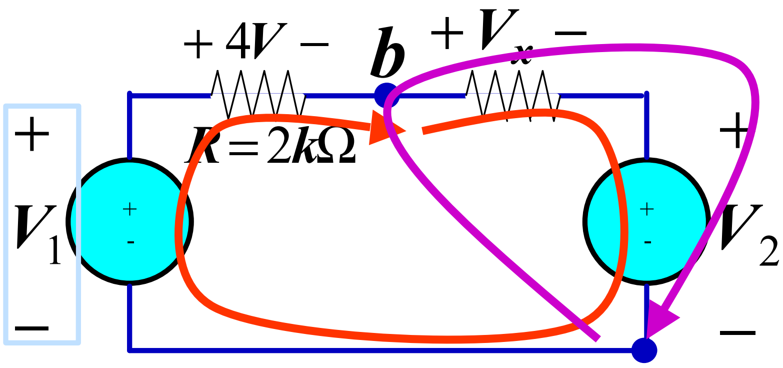
V_{ad} = ______, V_{eb} = ______





GEAUX

SAMPLE PROBLEM



$V_1 = 12V, V_2 = 4V$

DETERMINE

V_x = **4V**

V_{ab} = **-8V**

Power disipated on the 2k resistor

P_{2k} =

Remember past topics

We need to find a closed path where only one voltage is unknown

FOR V_X

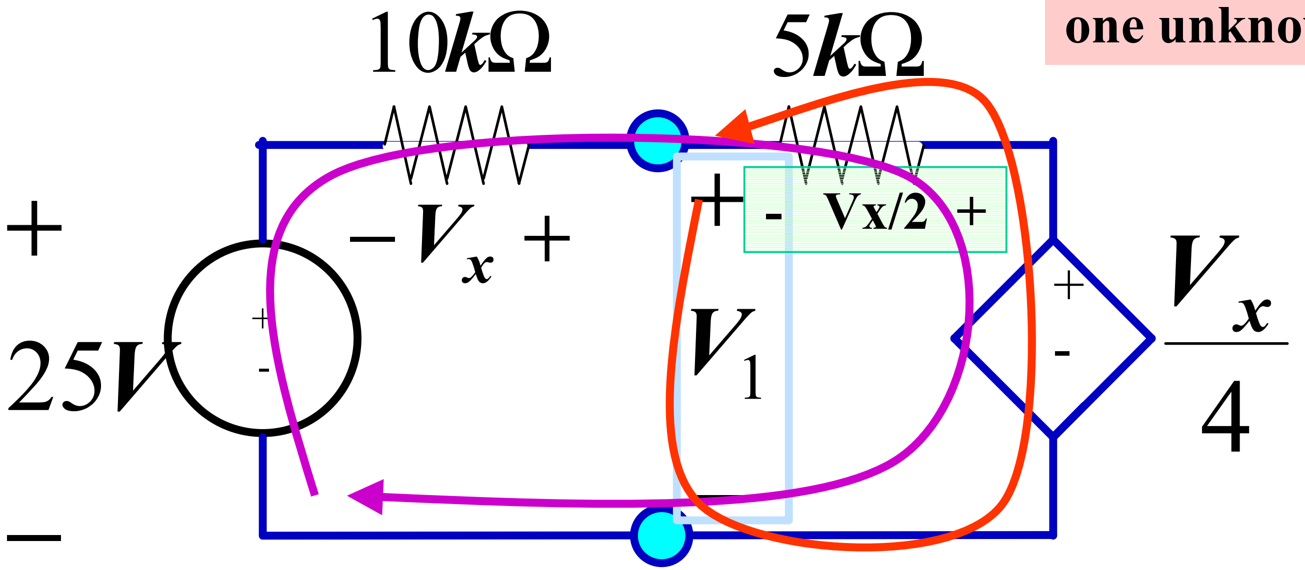
$$V_X + V_2 - V_1 + 4 = 0$$

$$V_X + 4 - 12 + 4 = 0$$

$$V_X + V_2 + V_{ab} = 0$$

$$V_{ab} = -V_X - V_2$$

GEAUX



There are no loops with only one unknown!!!

The current through the 5k and 10k resistors is the same. Hence the voltage drop across the 5k is one half of the drop across the 10k!!!

$$-25[V] - V_X - \frac{V_X}{2} + \frac{V_X}{4} = 0$$

$$V_X = -20[V]$$

$$V_1 - \frac{V_X}{4} + \frac{V_X}{2} = 0$$

$$V_1 = -\frac{V_X}{4} = 5[V]$$