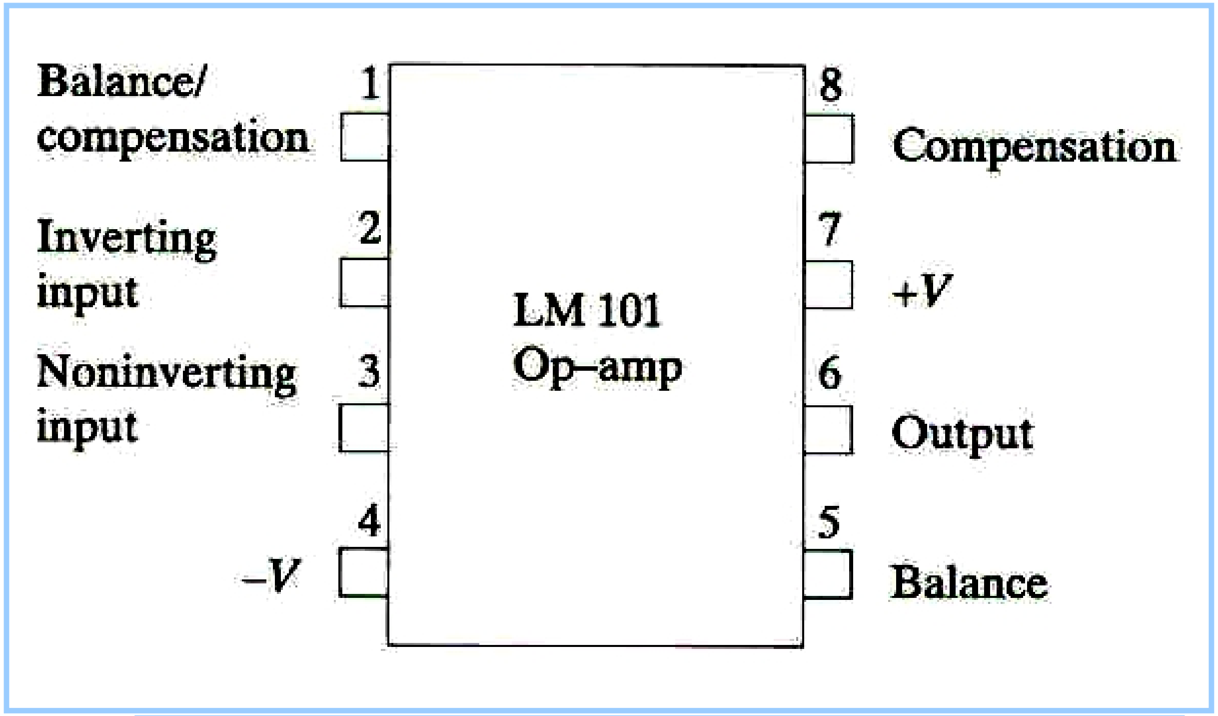
CIRCUITS WITH OPERATIONAL AMPLIFIERS

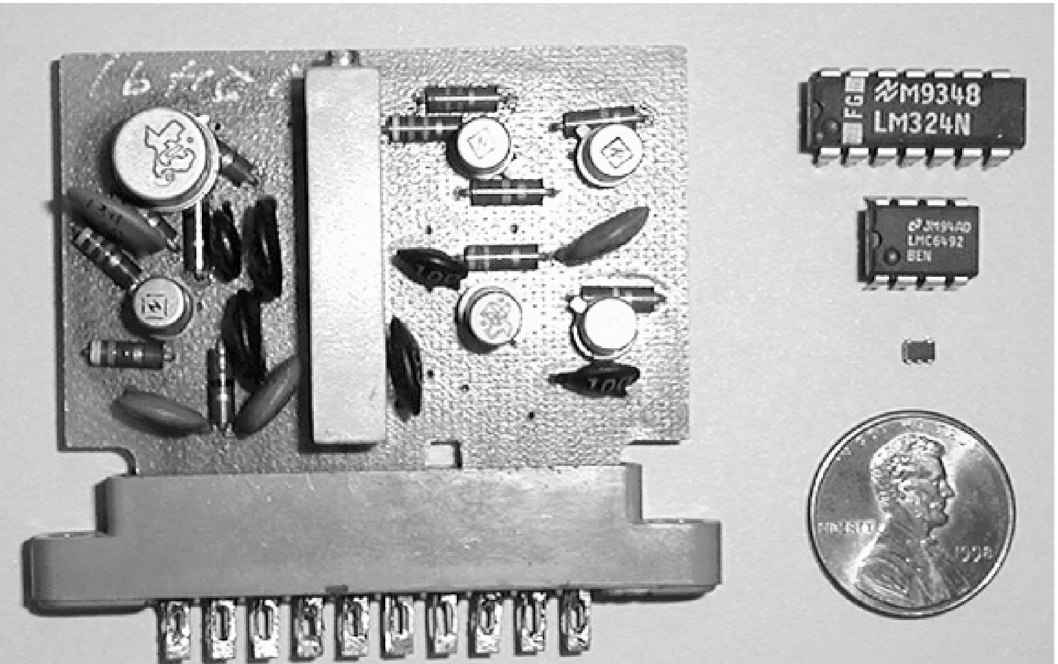
Why do we study them at this point???

1. OpAmps are very useful electronic components
2. We have already the tools to analyze practical circuits using OpAmps
3. The linear models for OpAmps include dependent sources



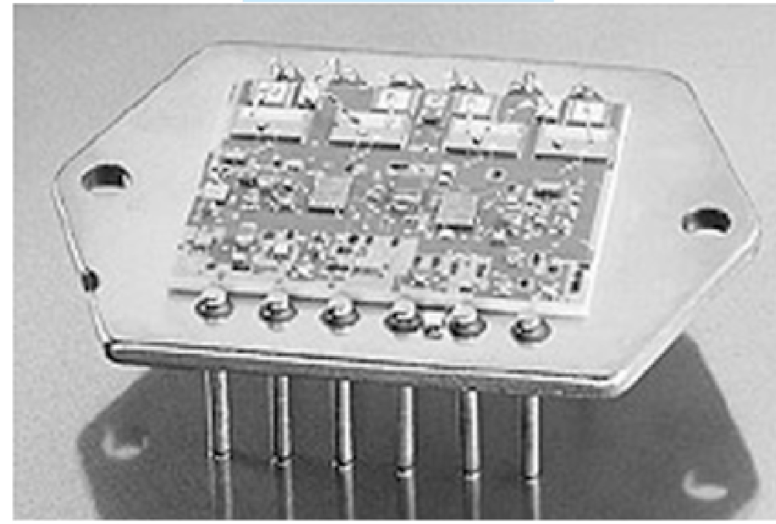
COMMERCIAL PACKAGING OF TYPICAL OP-AMP

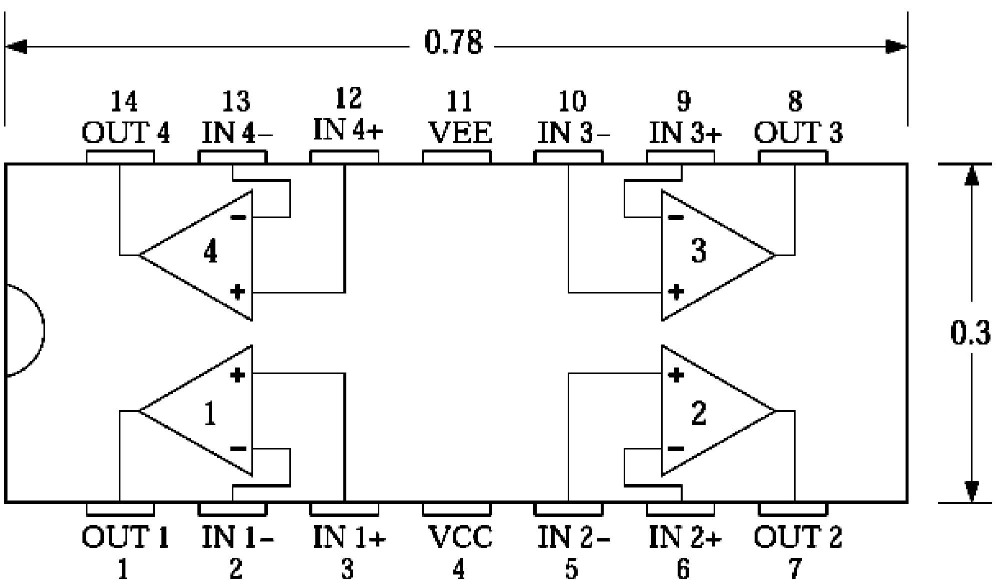
GEAUX



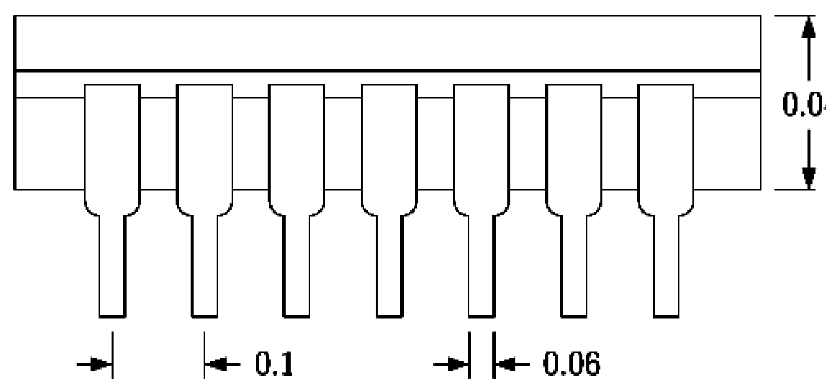
OP-AMP ASSEMBLED ON PRINTED CIRCUIT BOARD

LMC 6294 DIP





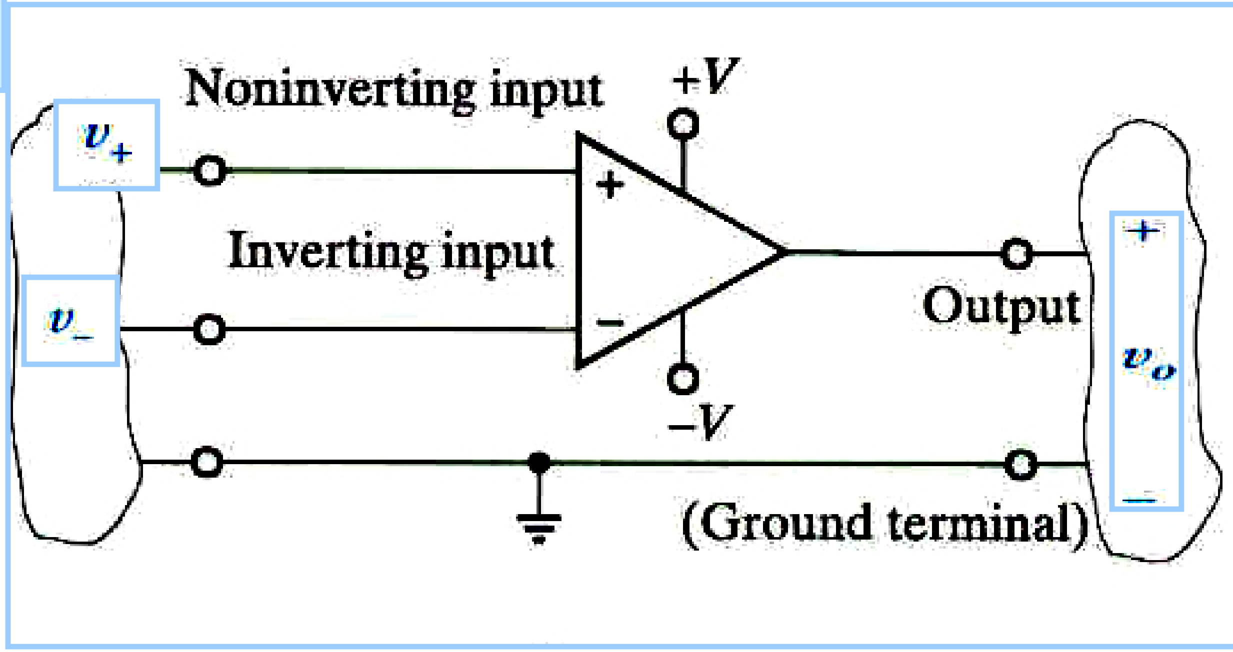
PIN OUT FOR LM324



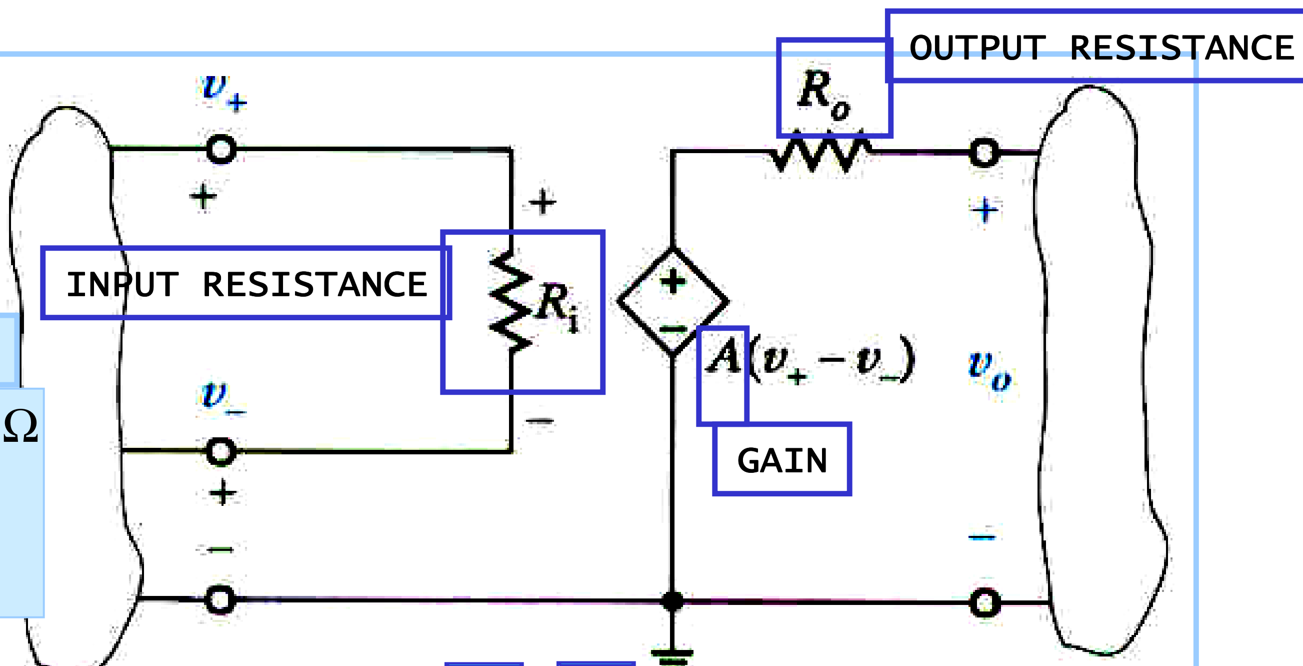
DIMENSIONAL DIAGRAM LM 324

GEAUX

CIRCUIT SYMBOL FOR AN OP-AMP



LINEAR MODEL



TYPICAL VALUES

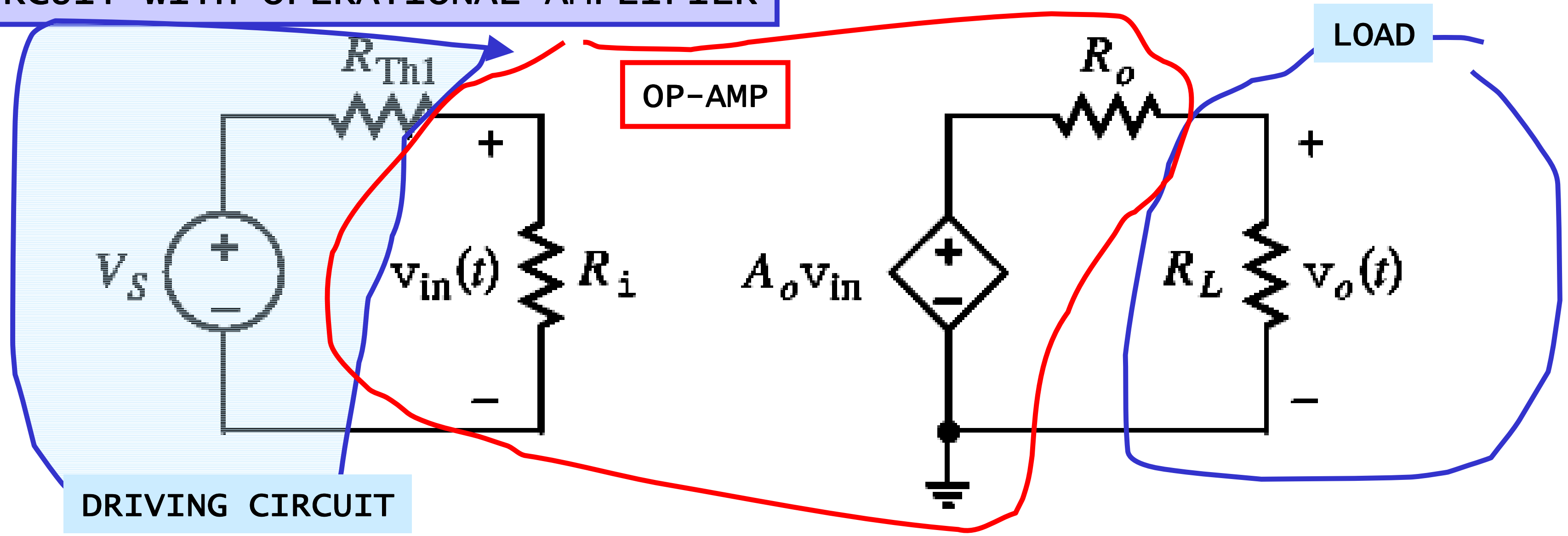
R_i : $10^5\Omega - 10^{12}\Omega$

R_O : $1\Omega - 50\Omega$

$A : 10^5 - 10^7$

GEAUX

CIRCUIT WITH OPERATIONAL AMPLIFIER

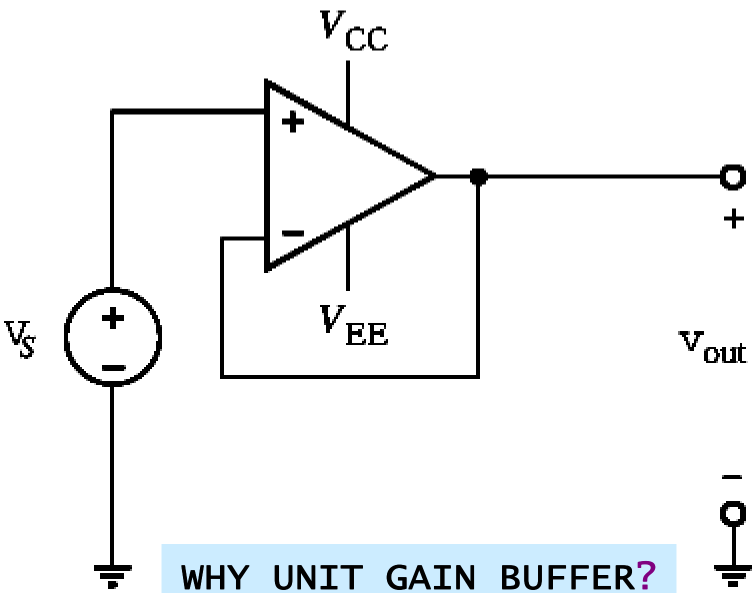


COMMERCIAL OP-AMPS AND THEIR MODEL VALUES

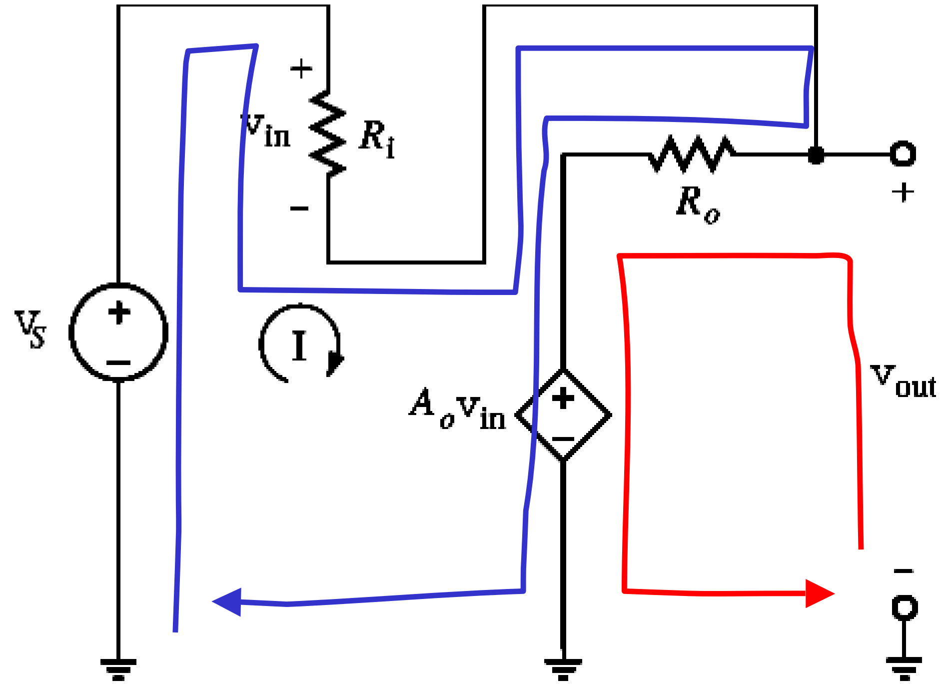
MANUFACTURER	PART No	A	Ri[MOhm]	Ro[Ohm]
National	LM324	100,000	1	20
National	LMC6492	50,000	10	150
Maxim	MAX4240	20,000	45	160

GEAUX

CIRCUIT AND MODEL FOR UNITY GAIN BUFFER



WHY UNIT GAIN BUFFER?



PERFORMANCE OF REAL OP-AMPS

Op-Amp	BUFFER GAIN
LM324	0.99999
LMC6492	0.9998
MAX4240	0.99995

KVL: $-V_s + R_i I + R_O I + A_O V_{in} = 0$

KVL: $-V_{out} + R_O I + A_O V_{in} = 0$

CONTROLLING VARIABLE: $V_{in} = R_i I$

SOLVING

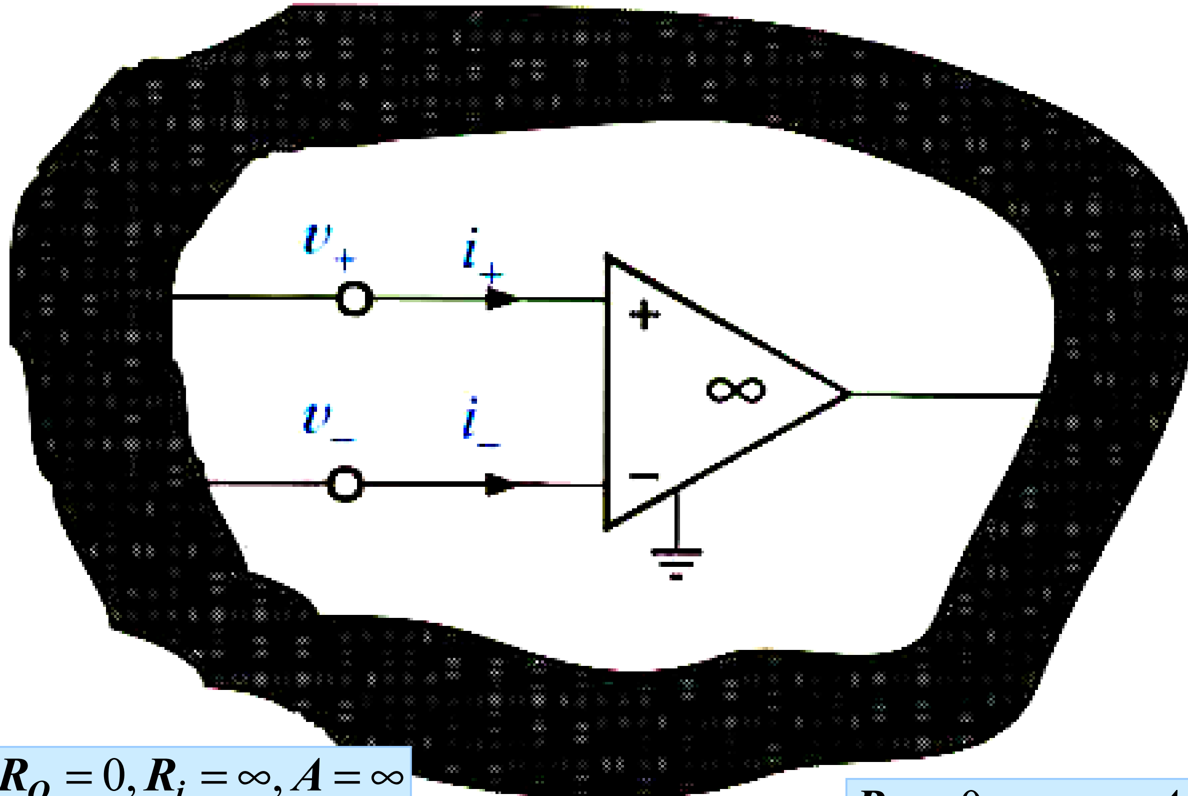
BUFFER GAIN

$$\frac{V_{out}}{V_s} = \frac{1}{1 + \dfrac{R_i}{R_O + A_O R_i}}$$

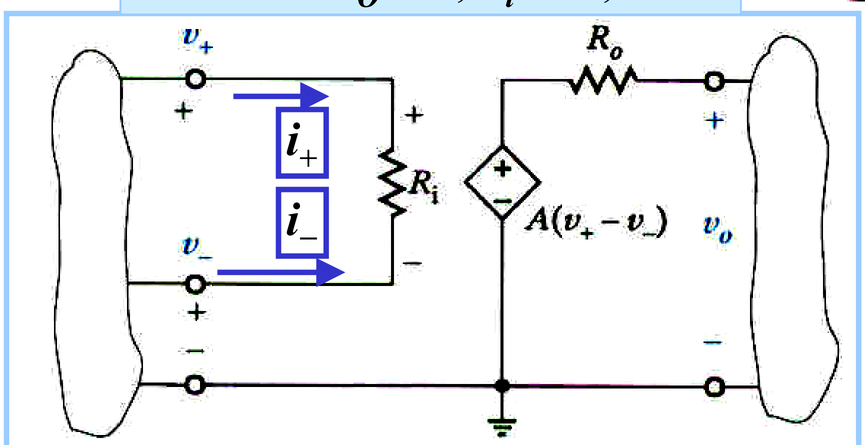
$$A_O \to \infty \Rightarrow \frac{V_{out}}{V_S} \to 1$$

GEAUX

THE IDEAL OP-AMP



IDEAL $\Rightarrow R_O = 0, R_i = \infty, A = \infty$



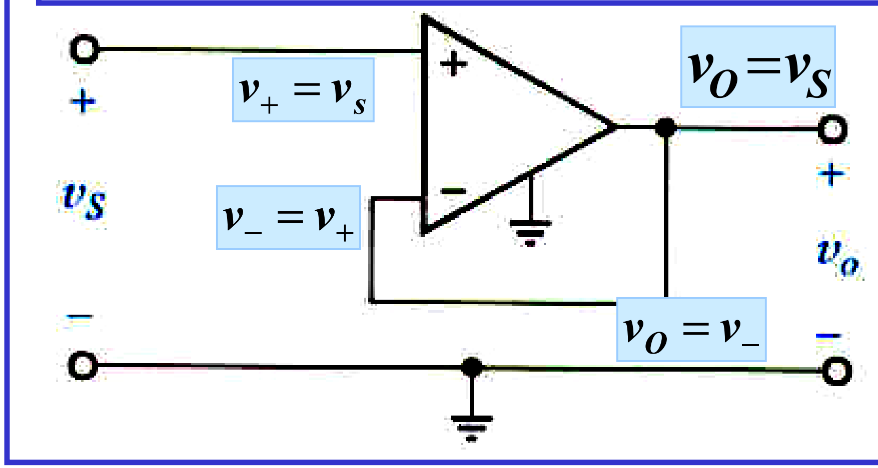
$R_O = 0 \Rightarrow v_O = A(v_+ - v_-)$

$R_i = \infty \Rightarrow i_+ = i_- = 0$

$A = \infty \Rightarrow v_+ = v_-$

GEAUX

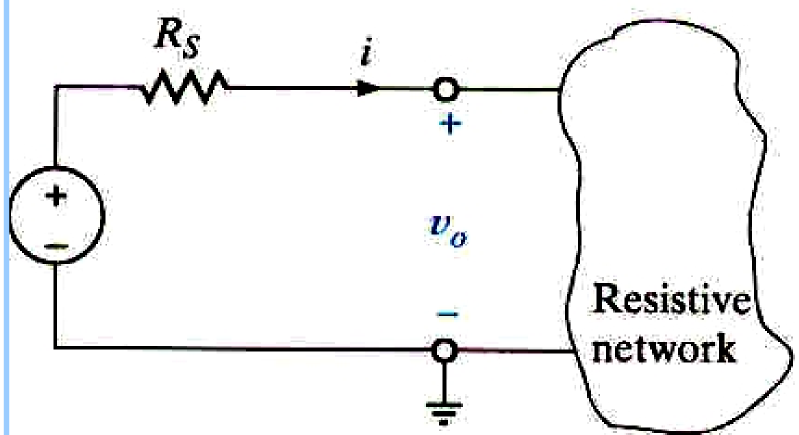
THE VOLTAGE FOLLOWER OR UNITY GAIN BUFFER



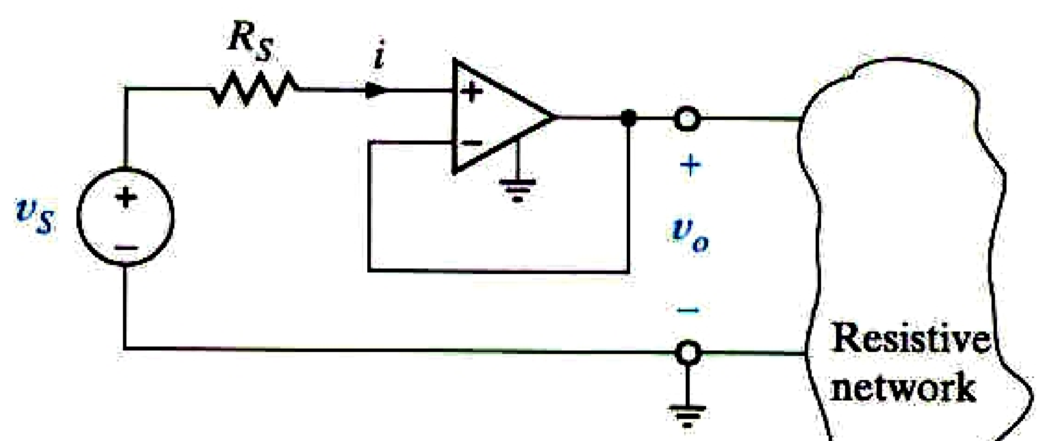
THE VOLTAGE FOLLOWER ACTS AS BUFFER AMPLIFIER

THE VOLTAGE FOLLOWER ISOLATES ONE CIRCUIT FROM ANOTHER
ESPECIALLY USEFUL IF THE SOURCE HAS VERY LITTLE POWER

CONNECTION WITHOUT BUFFER



CONNECTION WITH BUFFER



$$v_o = v_S - iR_S$$

THE SOURCE SUPPLIES POWER

$$v_O = v_S$$

THE SOURCE SUPPLIES NO POWER

GEAUX

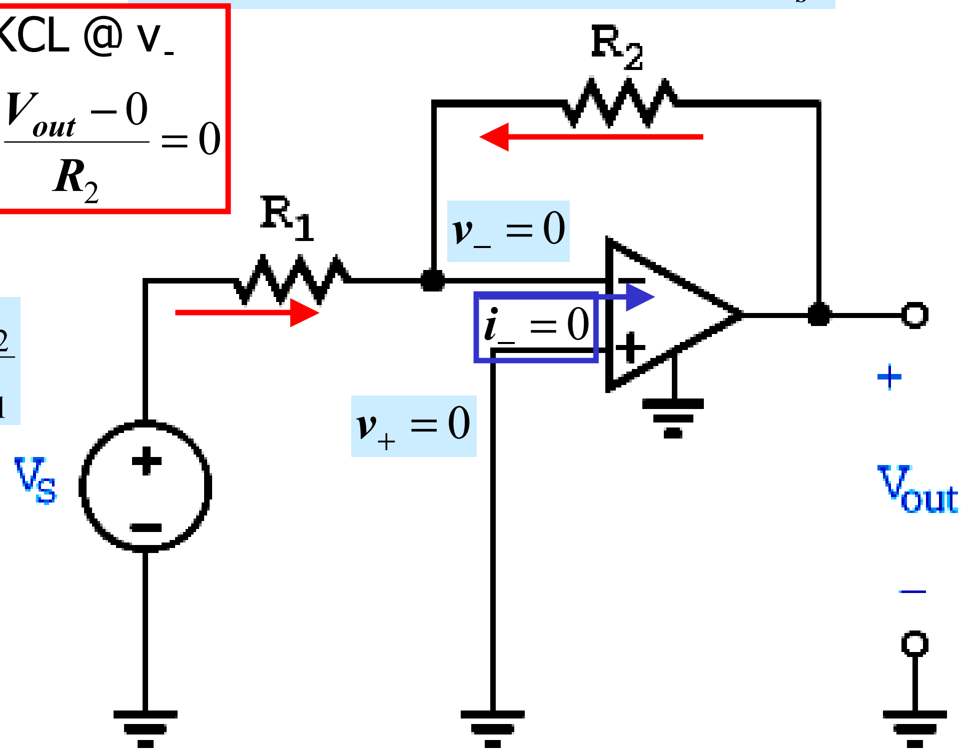
LEARNING EXAMPLE

DETERMINE THE GAIN $G = \frac{V_{out}}{V_s}$

APPLY KCL @ v_-

$$\frac{V_s - 0}{R_1} + \frac{V_{out} - 0}{R_2} = 0$$

$$G = \frac{V_{out}}{V_s} = -\frac{R_2}{R_1}$$



$$A_o = \infty \Rightarrow v_+ = v_- \quad \therefore v_- = 0$$

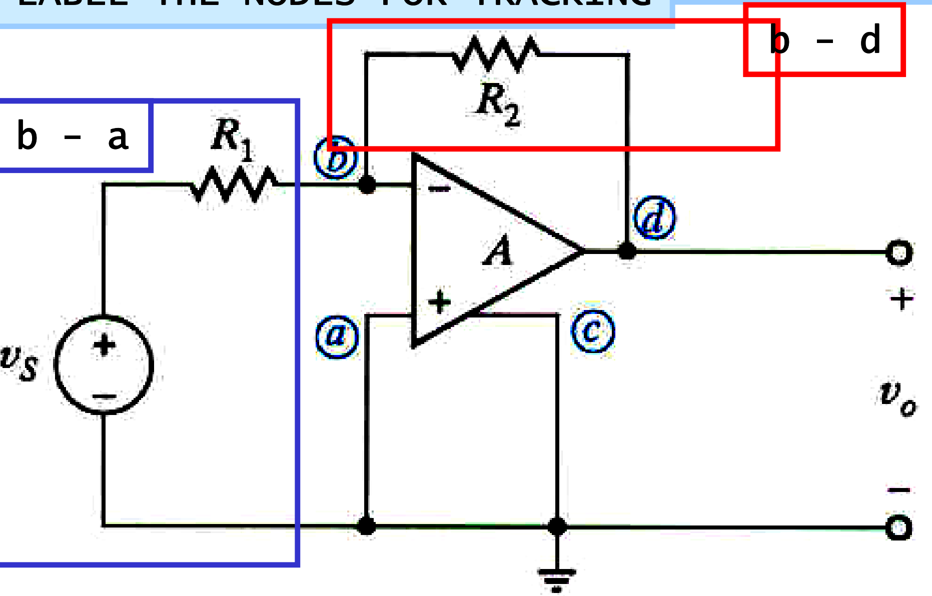
$$R_i = \infty \Rightarrow i_- = i_+ = 0$$

NEXT WE EXAMINE THE SAME CIRCUIT WITHOUT THE ASSUMPTION OF IDEAL OP-AMP

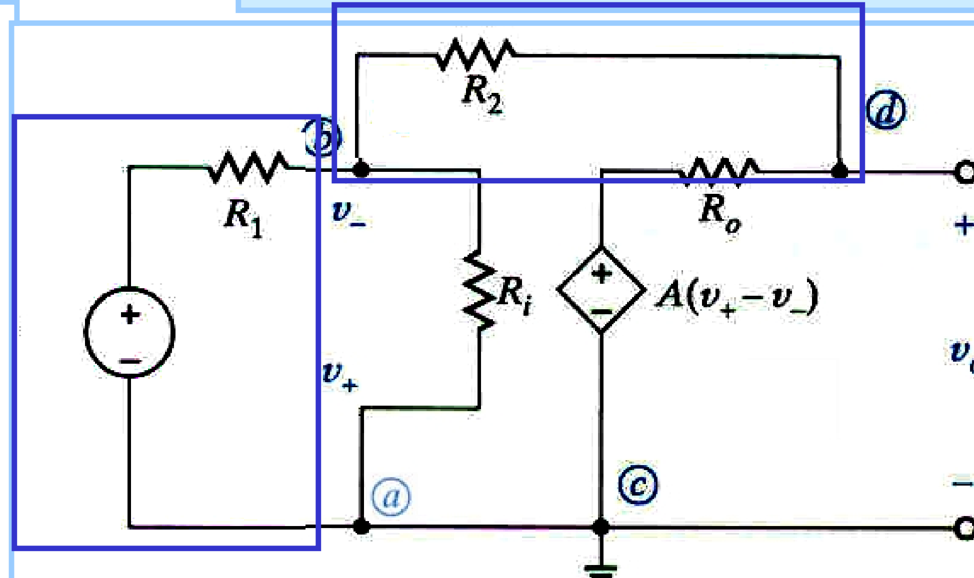
GEAUX

REPLACING OP-AMPS BY THEIR LINEAR MODEL

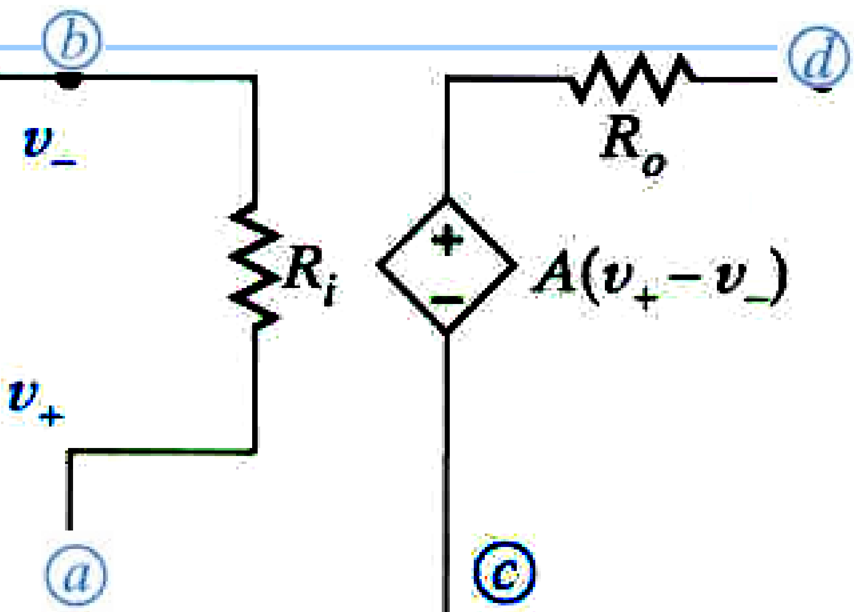
LABEL THE NODES FOR TRACKING



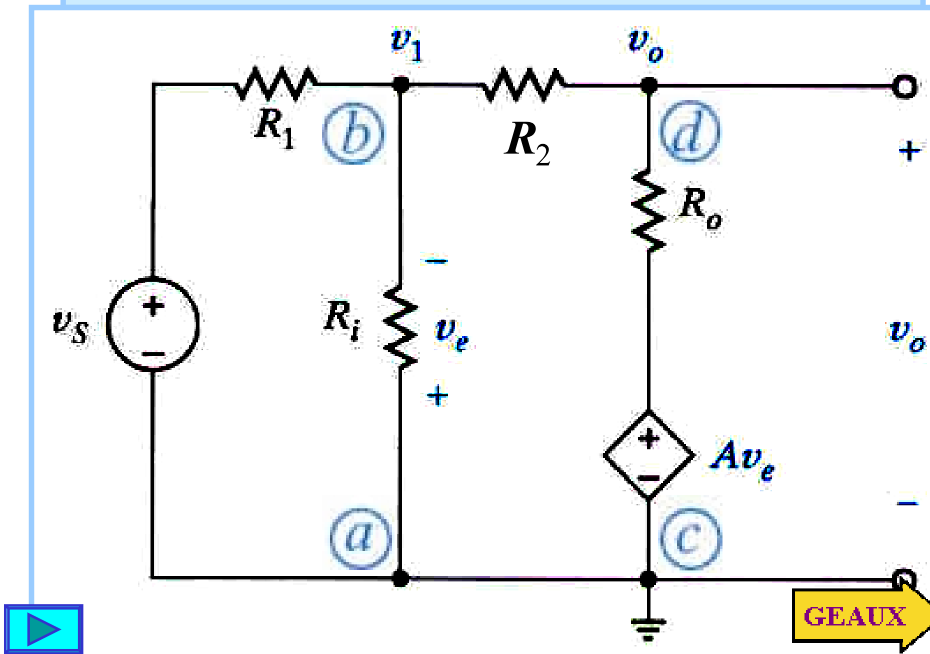
CONNECT THE EXTERNAL COMPONENTS



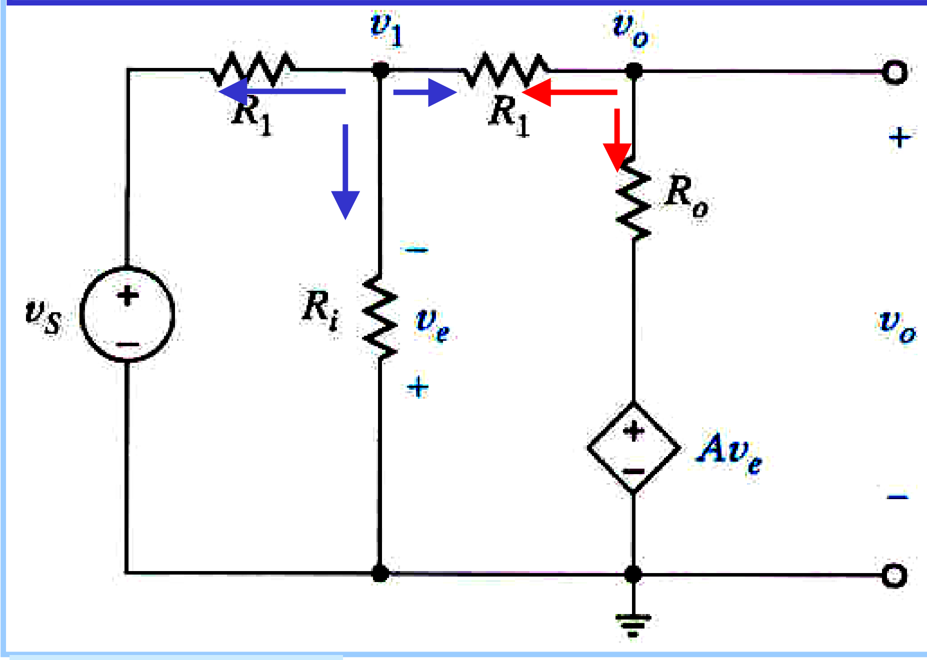
DRAW THE LINEAR EQUIVALENT FOR OP-AMP



REDRAW CIRCUIT FOR INCREASED CLARITY



INVERTING AMPLIFIER: ANALYSIS OF NON IDEAL CASE



NODE ANALYSIS

$$\frac{v_1 - v_S}{R_1} + \frac{v_1}{R_i} + \frac{v_1 - v_o}{R_2} = 0$$

$$\frac{v_o - v_1}{R_2} + \frac{v_o - Av_e}{R_o} = 0$$

CONTROLLING VARIABLE IN TERMS OF NODE VOLTAGES

$$v_e = -v_1$$

TYPICAL OP-AMP: $A = 10^5$, $R_i = 10^8\Omega, R_O = 10\Omega$

USE LINEAR ALGEBRA

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2} & -\left(\frac{1}{R_2}\right) \\ -\left(\frac{1}{R_2} - \frac{A}{R_o}\right) & \frac{1}{R_2} + \frac{1}{R_o} \end{bmatrix} \begin{bmatrix} v_1 \\ v_0 \end{bmatrix} = \begin{bmatrix} \frac{v_S}{R_1} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_0 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \frac{1}{R_2} + \frac{1}{R_o} & \frac{1}{R_2} \\ \frac{1}{R_2} - \frac{A}{R_o} & \frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_o} \end{bmatrix} \begin{bmatrix} \frac{v_S}{R_1} \\ 0 \end{bmatrix}$$

$$\Delta = \left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}\right)\left(\frac{1}{R_2} + \frac{1}{R_o}\right) - \left(\frac{1}{R_2}\right)\left(\frac{1}{R_2} - \frac{A}{R_o}\right)$$

$$v_o = \frac{\left(\frac{1}{R_2} - \frac{A}{R_o}\right)\left(\frac{v_S}{R_1}\right)}{\left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}\right)\left(\frac{1}{R_2} + \frac{1}{R_o}\right) - \left(\frac{1}{R_2}\right)\left(\frac{1}{R_2} - \frac{A}{R_o}\right)}$$

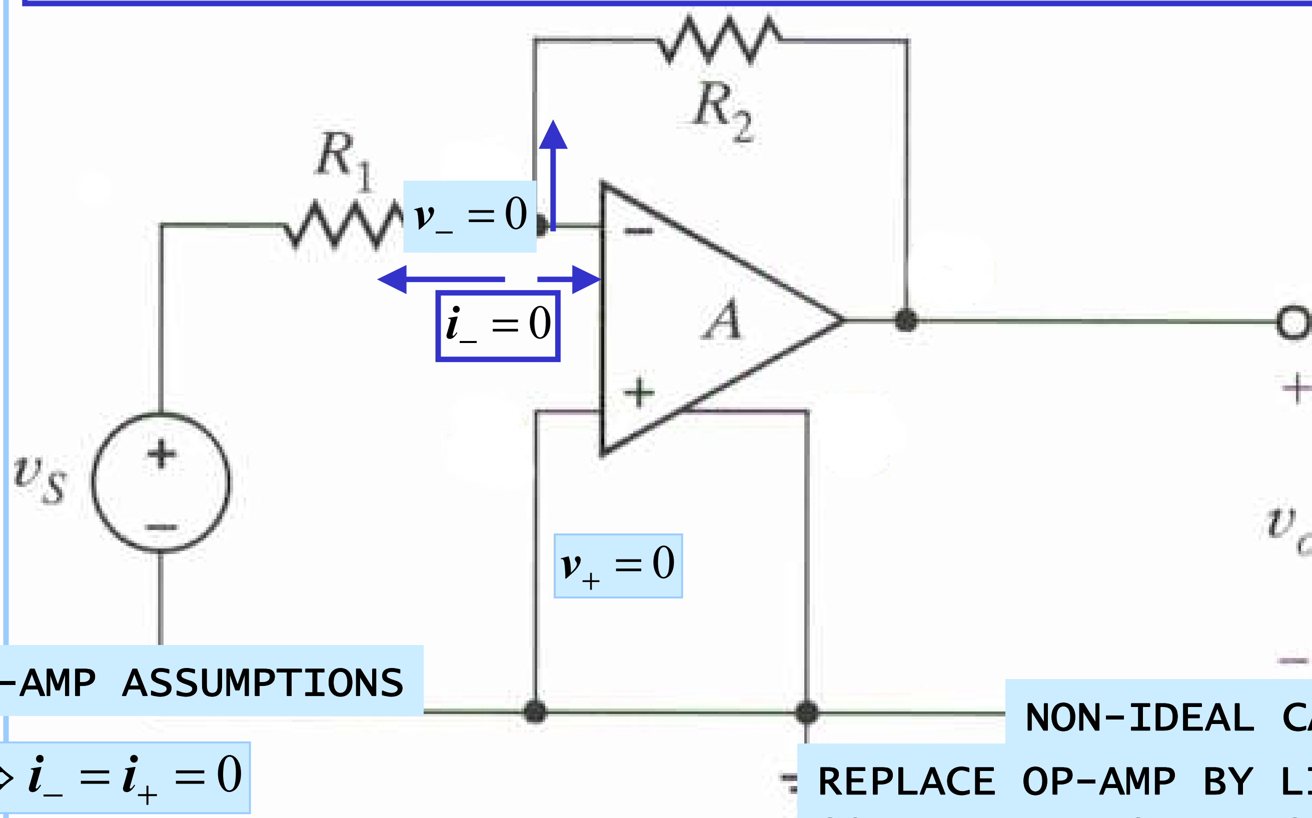
$$\frac{v_o}{v_S} = \frac{-(R_2/R_1)}{1 - \left[\left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}\right)\left(\frac{1}{R_2} + \frac{1}{R_o}\right) \Big/ \left(\frac{1}{R_2}\right)\left(\frac{1}{R_2} - \frac{A}{R_o}\right)\right]}$$

$$R_1 = 1k\Omega, R_2 = 5k\Omega \Rightarrow \frac{v_O}{v_S} = -4.9996994$$

$$A = \infty \Rightarrow \frac{v_O}{v_S} = -5.000$$

GEAUX

SUMMARY COMPARISON: IDEAL OP-AMP AND NON-IDEAL CASE



IDEAL OP-AMP ASSUMPTIONS

$R_i = \infty \Rightarrow i_- = i_+ = 0$

$A = \infty \Rightarrow v_+ = v_-$

KCL @ INVERTING TERMINAL

$$\frac{0 - v_S}{R_1} + \frac{0 - v_O}{R_2} = 0 \Rightarrow \frac{v_O}{v_s} = -\frac{R_2}{R_1}$$

NON-IDEAL CASE

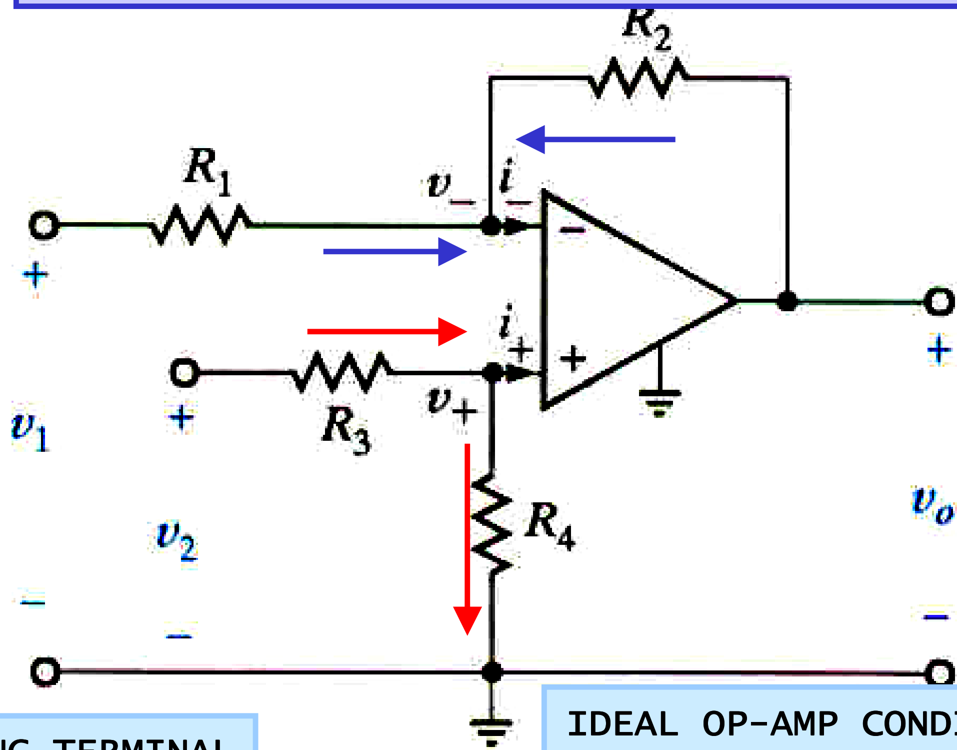
REPLACE OP-AMP BY LINEAR MODEL
SOLVE THE RESULTING CIRCUIT WITH DEPENDENT SOURCES

GAIN FOR NON-IDEAL CASE

$$\frac{v_o}{v_s} = \frac{-(R_2/R_1)}{1 - \left[\left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}\right)\left(\frac{1}{R_2} + \frac{1}{R_o}\right) \Big/ \left(\frac{1}{R_2}\right)\left(\frac{1}{R_2} - \frac{A}{R_o}\right)\right]}$$

THE IDEAL OP-AMP ASSUMPTION PROVIDES EXCELLENT APPROXIMATION.
(UNLESS FORCED OTHERWISE WE WILL ALWAYS USE IT!)

LEARNING EXAMPLE: DIFFERENTIAL AMPLIFIER



KCL @ INVERTING TERMINAL

$$\frac{v_1 - v_-}{R_1} + \frac{v_o - v_-}{R_2} = i_-$$

KCL @ NON INVERTING TERMINAL

$$\frac{v_2 - v_+}{R_3} = \frac{v_+}{R_4} + i_+$$

IDEAL OP-AMP CONDITIONS

$$i_+ = i_- = 0 \qquad v_+ = v_-.$$

$$i_+ = 0 \Rightarrow v_+ = \frac{R_4}{R_3 + R_4} v_2 \Rightarrow v_- = \frac{R_4}{R_3 + R_4} v_2$$

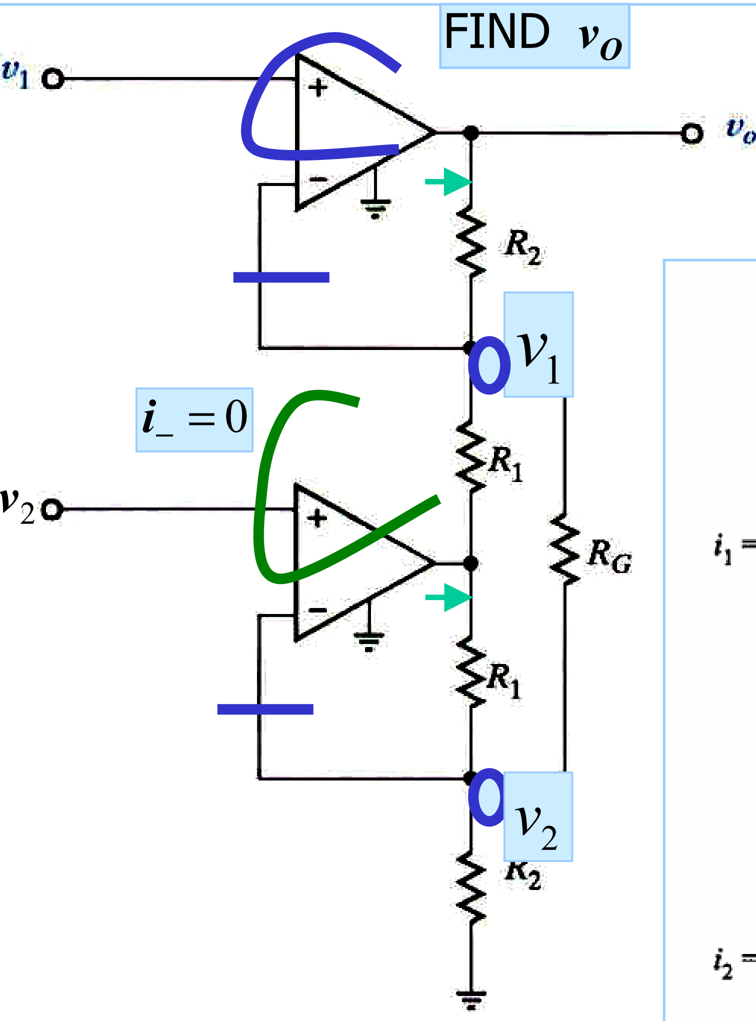
$$v_O = \left(1 + \frac{R_2}{R_1}\right) v_- - \frac{R_2}{R_1} v_1 = \frac{R_2}{R_1}\left(\left(1 + \frac{R_1}{R_2}\right) v_- - v_1\right)$$

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{R_1}{R_2}\right) \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1$$

$$R_4 = R_2, R_3 = R_1 \Rightarrow v_O = \frac{R_2}{R_1}(v_2 - v_1)$$

GEAUX

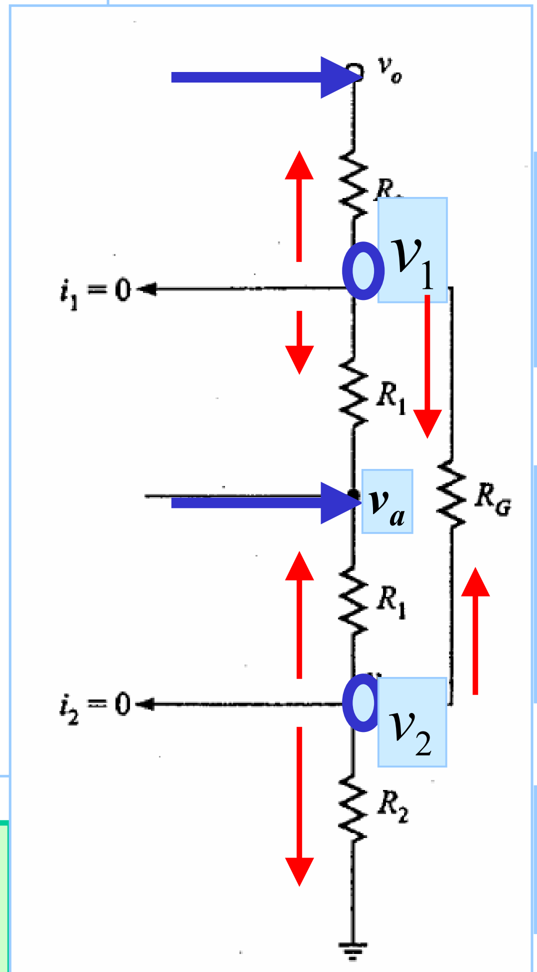
FIND v_o



Which voltages are set? $v_{+1} = v_1, v_{+2} = v_2$

What voltages are also known due to infinite gain assumption?

Use now the infinite resistance assumption



KCL@v1

$$\frac{v_1 - v_o}{R_2} + \frac{v_1 - v_a}{R_1} + \frac{v_1 - v_2}{R_G} = 0$$

KCL@v2

$$\frac{v_2 - v_a}{R_1} + \frac{v_2 - v_1}{R_G} + \frac{v_2}{R_2} = 0$$

Solve for vo

$$v_o = (v_1 - v_2)\left(1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G}\right)$$

CAUTION: There could be currents flowing OUT of the OpAmps

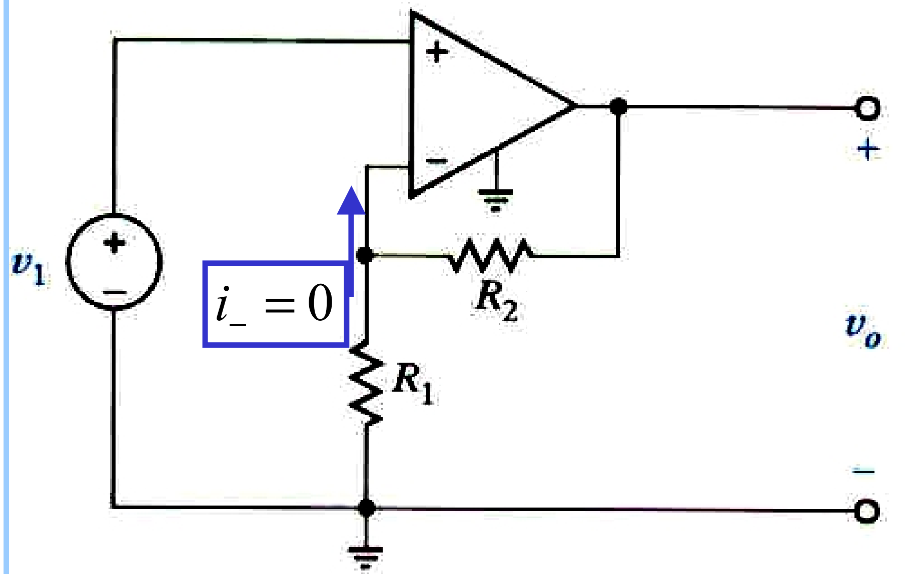
THE CIRCUIT REDUCES TO THIS

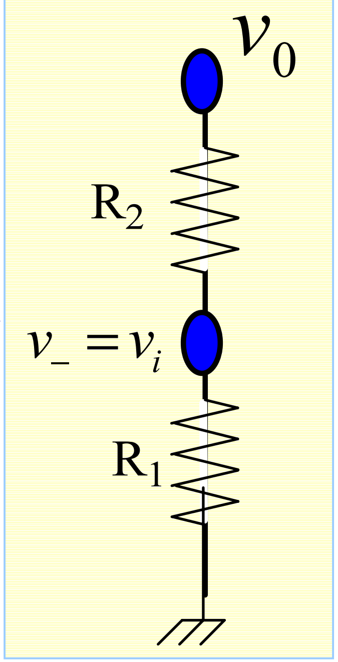
v_O, v_a to be determined

GEAUX

LEARNING EXTENSION

NONINVERTING AMPLIFIER - IDEAL OP-AMP





SET VOLTAGE $v_+ = v_1$

$$v_+ = v_1 \Rightarrow v_- = v_1$$

INFINITE GAIN ASSUMPTION

INFINITE INPUT RESISTANCE

"inverse voltage divider"

$$v_i = \frac{R_1}{R_1 + R_2} v_0 \Rightarrow v_0 = \frac{R_1 + R_2}{R_1} v_i$$

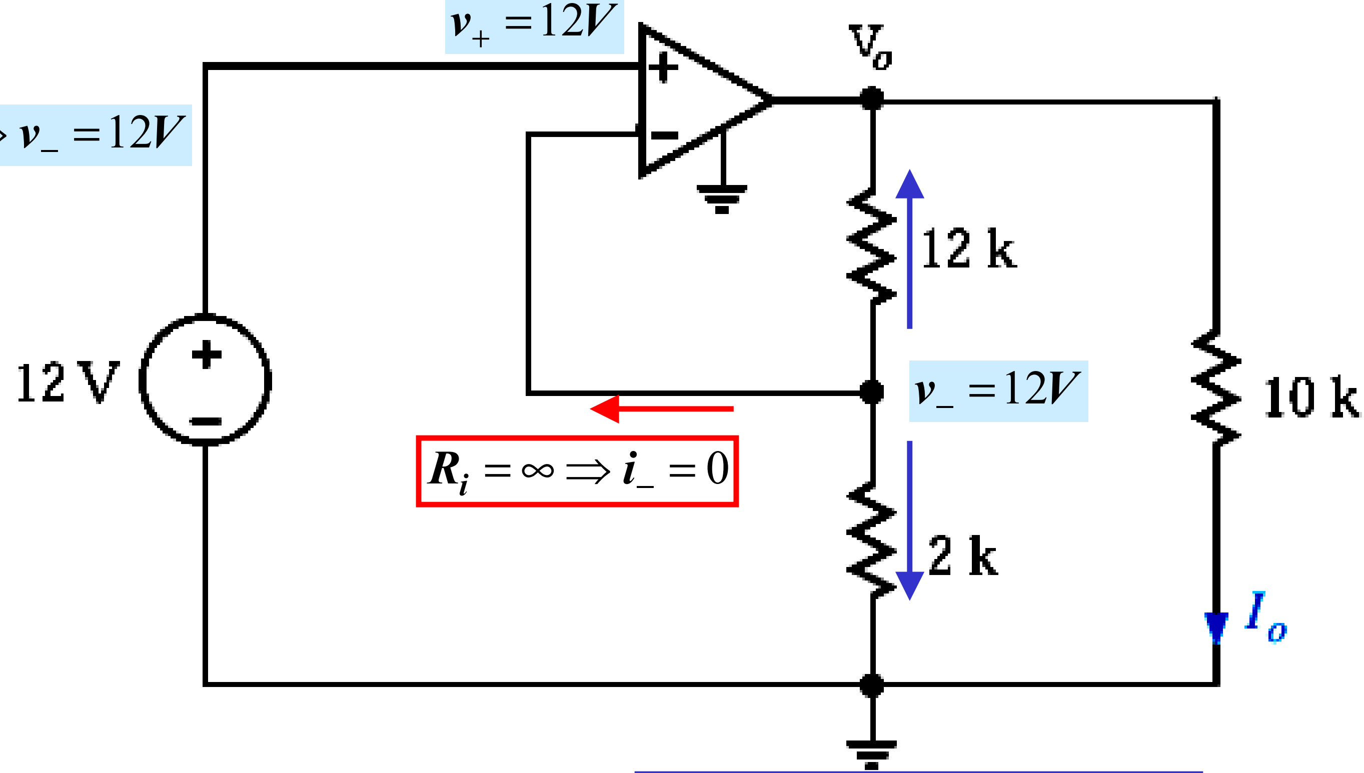
GEAUX

LEARNING EXTENSION

FIND I_O. ASSUME IDEAL OP-AMP

$v_+ = 12V$

$A_O = \infty \Rightarrow v_- = 12V$



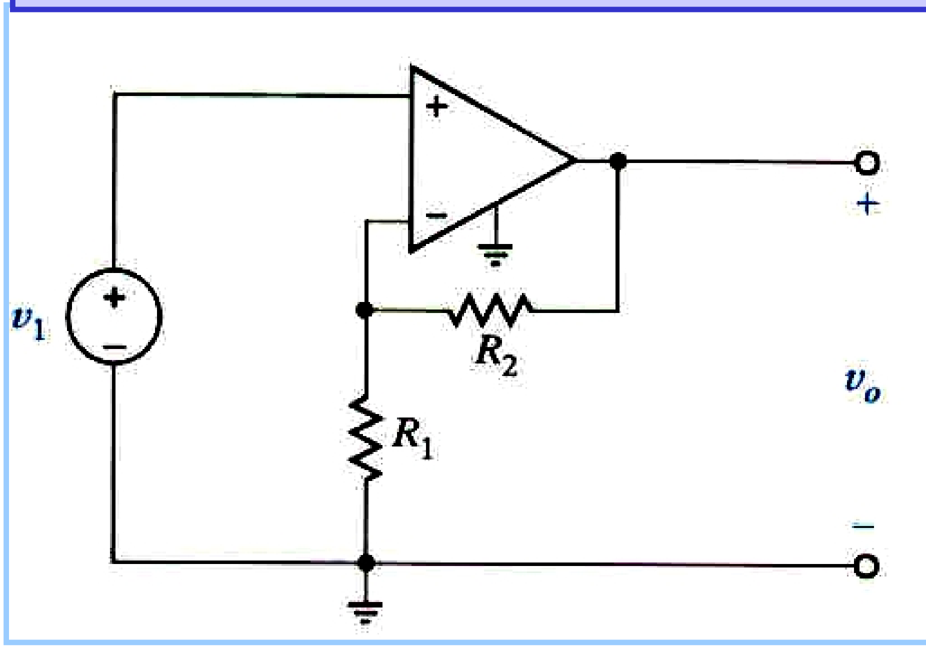
$v_- = 12V$

$R_i = \infty \Rightarrow i_- = 0$

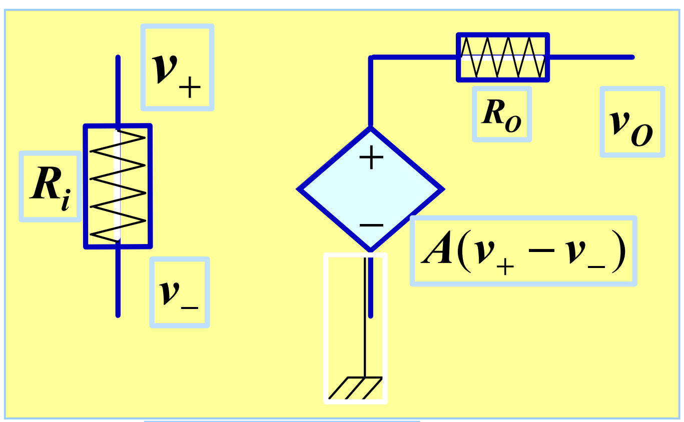
$$\text{KCL@ } v_- : \frac{12 - V_o}{12k} + \frac{12}{2k} = 0 \Rightarrow V_o = 84V$$

$$\therefore I_O = \frac{V_o}{10k} = 8.4mA$$

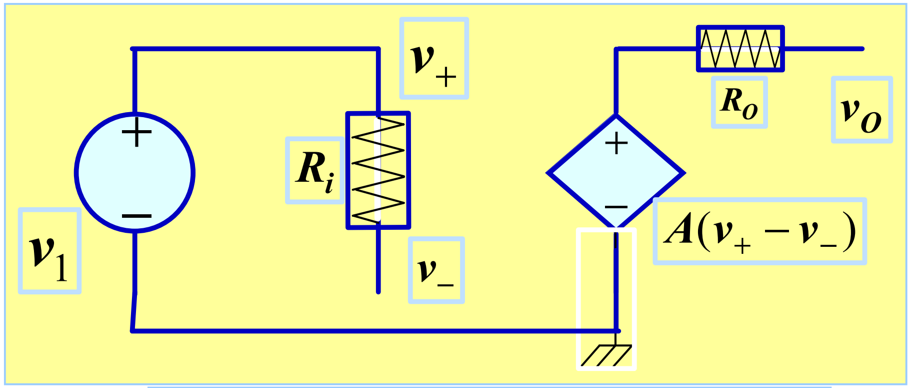
FIND GAIN AND INPUT RESISTANCE - NON IDEAL OP-AMP



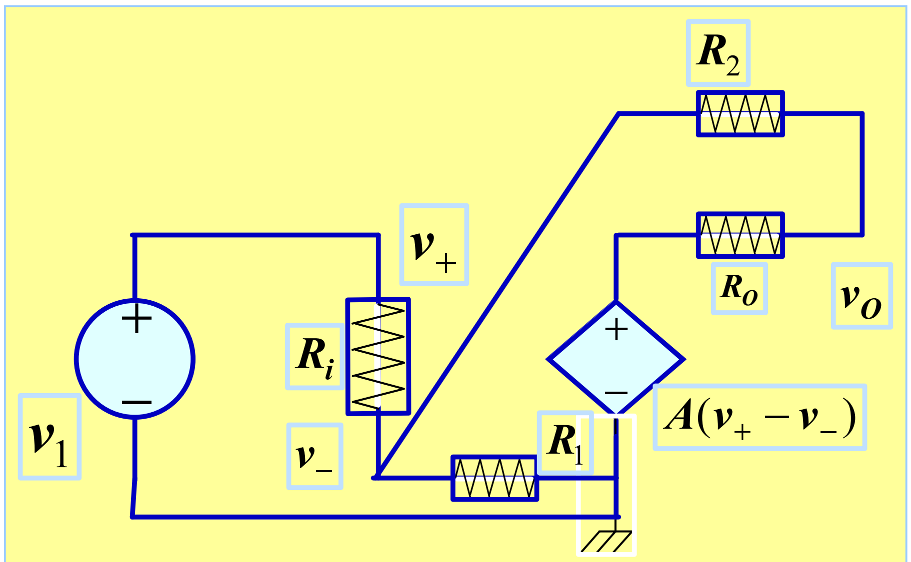
DETERMINE EQUIVALENT CIRCUIT
USING LINEAR MODEL FOR OP-AMP



THE OP-AMP



ADD THE INPUT VOLTAGE SOURCE

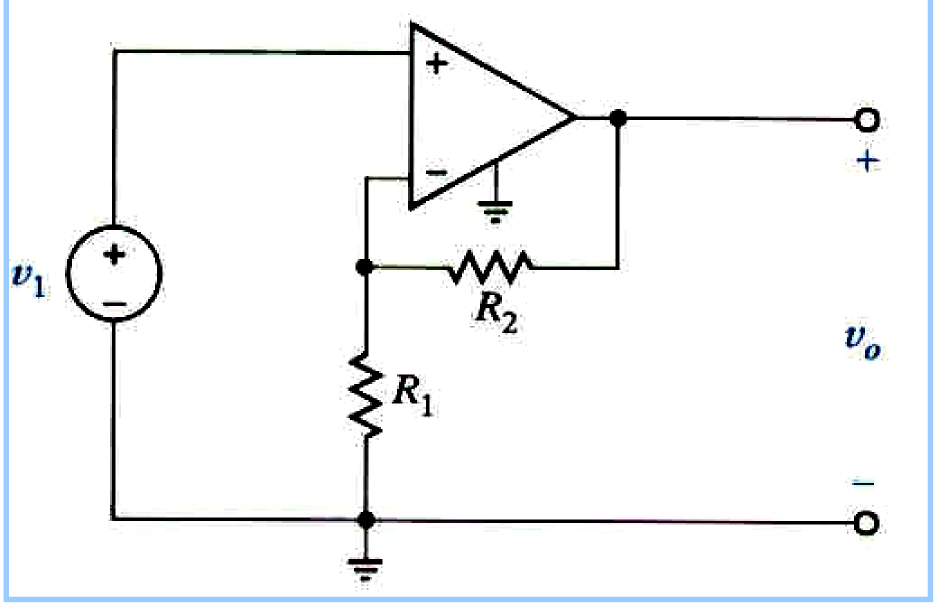


AND THE EXTERNAL RESISTORS

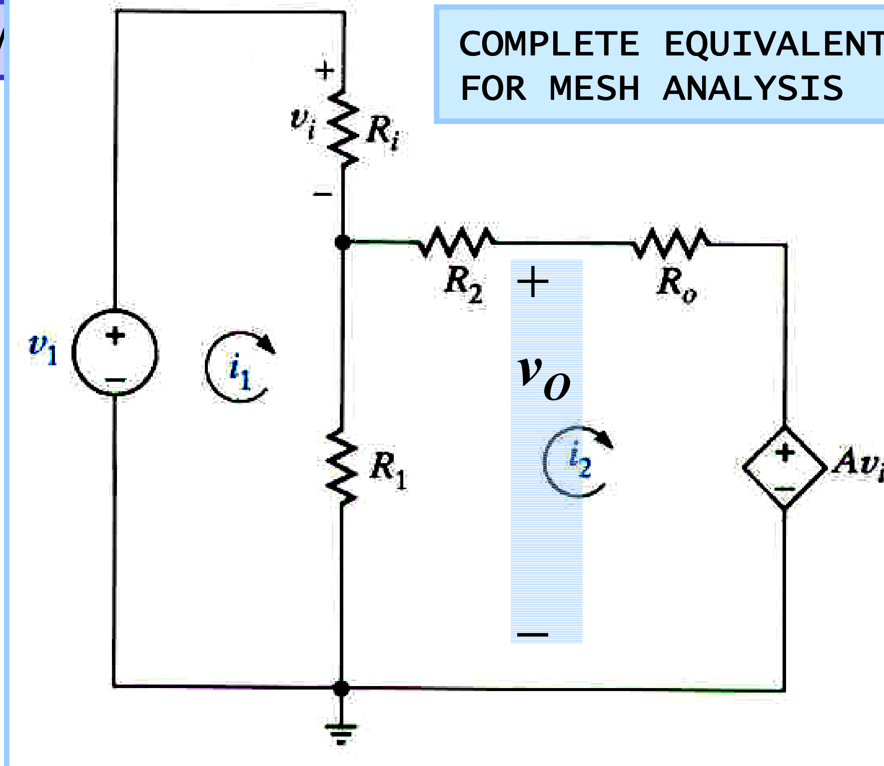
NOW RE-DRAW CIRCUIT TO ENHANCE
CLARITY. THERE ARE ONLY TWO LOOPS

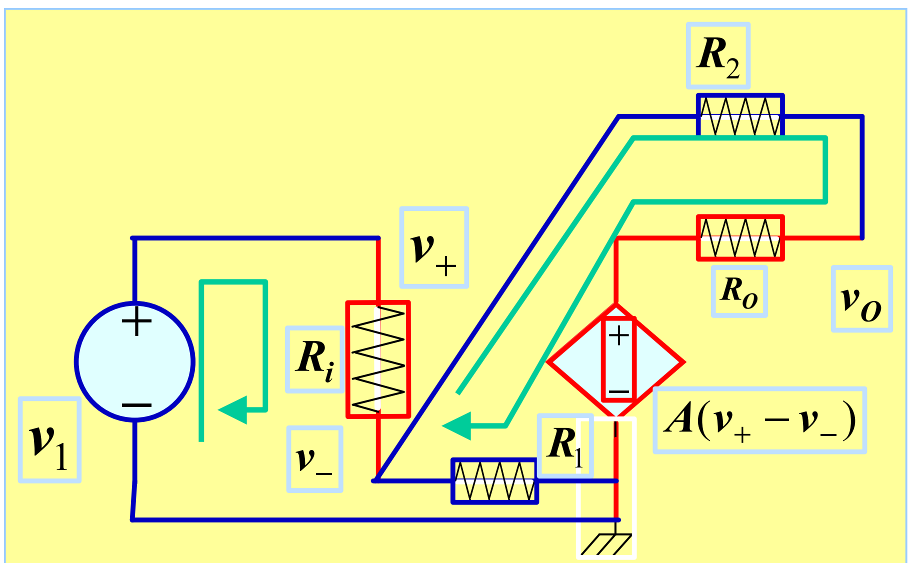
GEAUX

NON-INVERTING OP-AMP. NON IDEAL OP-AMP



COMPLETE EQUIVALENT FOR MESH ANALYSIS





COMPLETE EQUIVALENT CIRCUIT

MESH 1

$$v_1 = i_1(R_i + R_1) - i_2 R_1$$

MESH 2

$$-Av_i = -i_1 R_1 + i_2(R_1 + R_2 + R_o)$$

CONTROLLNG VARIABLE IN TERMS OF LOOP CURRENTS

$$v_i = R_i i_1$$

$$v_O = -R_2 i_2 + R_1(i_1 - i_2)$$

GEAUX

MATHEMATICAL MODEL

MESH 1

$$v_1 = i_1(R_i + R_1) - i_2 R_1$$

MESH 2

$$-Av_i = -i_1 R_1 + i_2(R_1 + R_2 + R_o)$$

CONTROLLNG VARIABLE IN TERMS OF LOOP CURRENTS

$$v_i = R_i i_1$$

$$v_O = -R_2 i_2 + R_1(i_1 - i_2)$$

INPUT RESISTANCE $R_{in} = \frac{v_1}{i_1}$

GAIN $G = \frac{v_O}{v_i}$

REPLACE AND PUT IN MATRIX FORM

$$\begin{bmatrix} (R_1+R_2) & -R_1 \\ AR_i - R_1 & (R_1+R_2+R_O) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ 0 \end{bmatrix}$$

THE FORMAL SOLUTION

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} (R_1+R_2) & -R_1 \\ AR_i - R_1 & (R_1+R_2+R_O) \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ 0 \end{bmatrix}$$

$$\Delta = (R_1+R_2+R_O)(R_1+R_2) + R_1(AR_i - R_1)$$

$$Adj = \begin{bmatrix} (R_1+R_2+R_O) & R_1 \\ -(AR_i - R_1) & (R_1+R_2) \end{bmatrix}$$

THE SOLUTIONS

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} (R_1+R_2+R_O) & R_1 \\ -(AR_i - R_1) & (R_1+R_2) \end{bmatrix} \begin{bmatrix} v_1 \\ 0 \end{bmatrix}$$

$$i_1 = \frac{R_1+R_2+R_O}{\Delta} v_1 \qquad i_2 = \frac{-(AR_i - R_1)}{\Delta}$$

$$R_{in} = \frac{v_1}{i_1} = \frac{(R_i+R_1)(R_1+R_2+R_o) + R_1(AR_i - R_1)}{R_1+R_2+R_o} = R_i + \frac{R_1(R_2+R_o+AR_i)}{R_1+R_2+R_o}$$

$$v_O = R_1 i_1 - (R_1+R_2) i_2 = \frac{R_1(R_1+R_2+R_O)}{\Delta} v_1 + \frac{(R_1+R_2)(AR_i - R_1)}{\Delta} v_1$$

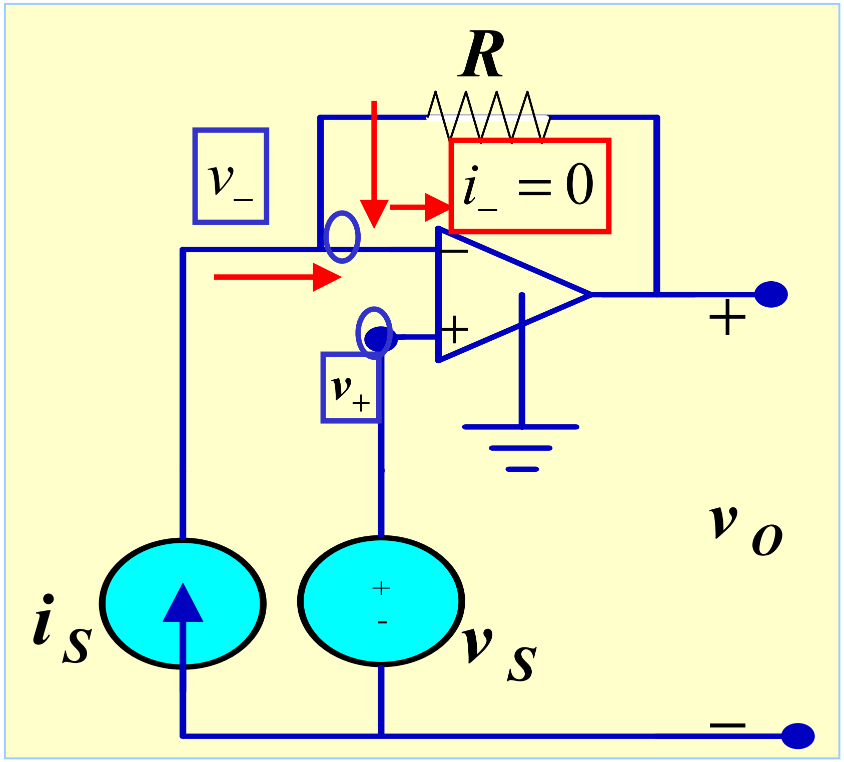
$$A \to \infty ???$$

$$A \to \infty \Rightarrow \Delta \to AR_1R_i \qquad R_{in} \to \infty$$

$$G = \frac{v_O}{v_1} \to \frac{R_1+R_2}{R_1}$$

GEAUX

Sample Problem



Set voltages? $v_+ = v_S$

Use infinite gain assumption $v_- = v_S$

Use infinite input resistance assumption and apply KCL to inverting input

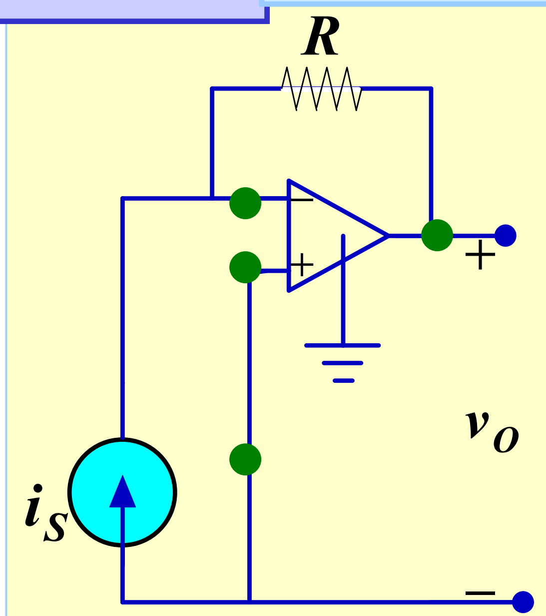
$$i_S + \frac{v_o - v_-}{R} = 0$$

$$v_o = v_S - Ri_S$$

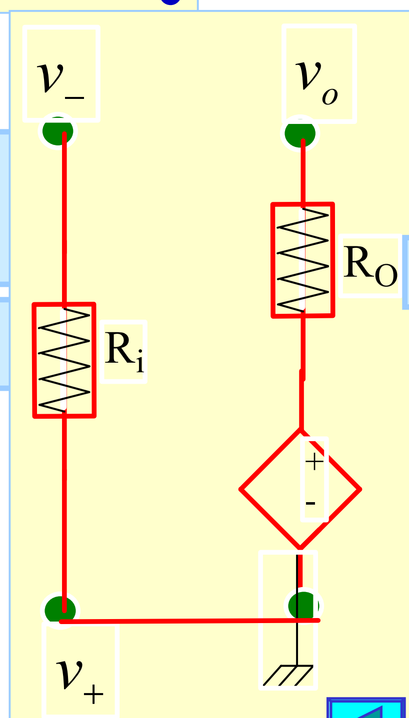
Find the expression for Vo. Indicate where and how you are using the Ideal OpAmp assumptions

Sample Problem

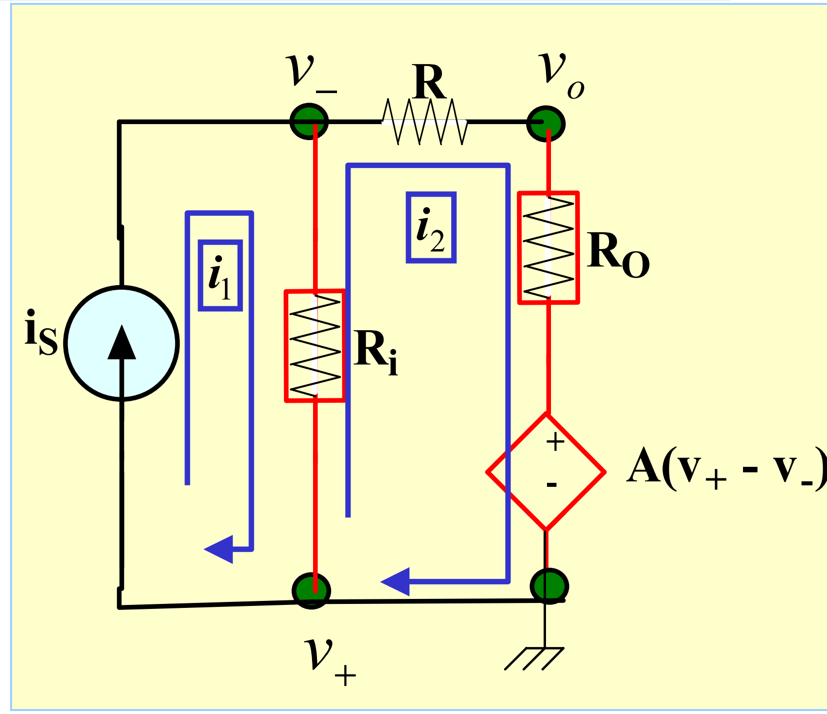
DRAW THE LINEAR EQUIVALENT CIRCUIT AND WRITE THE LOOP EQUATIONS



1. Locate nodes
2. Place the nodes in linear circuit model
3. Place linear model



4. Place remaining components



TWO LOOPS. ONE CURRENT SOURCE. USE MESHES

MESH 1 $i_1 = i_s$

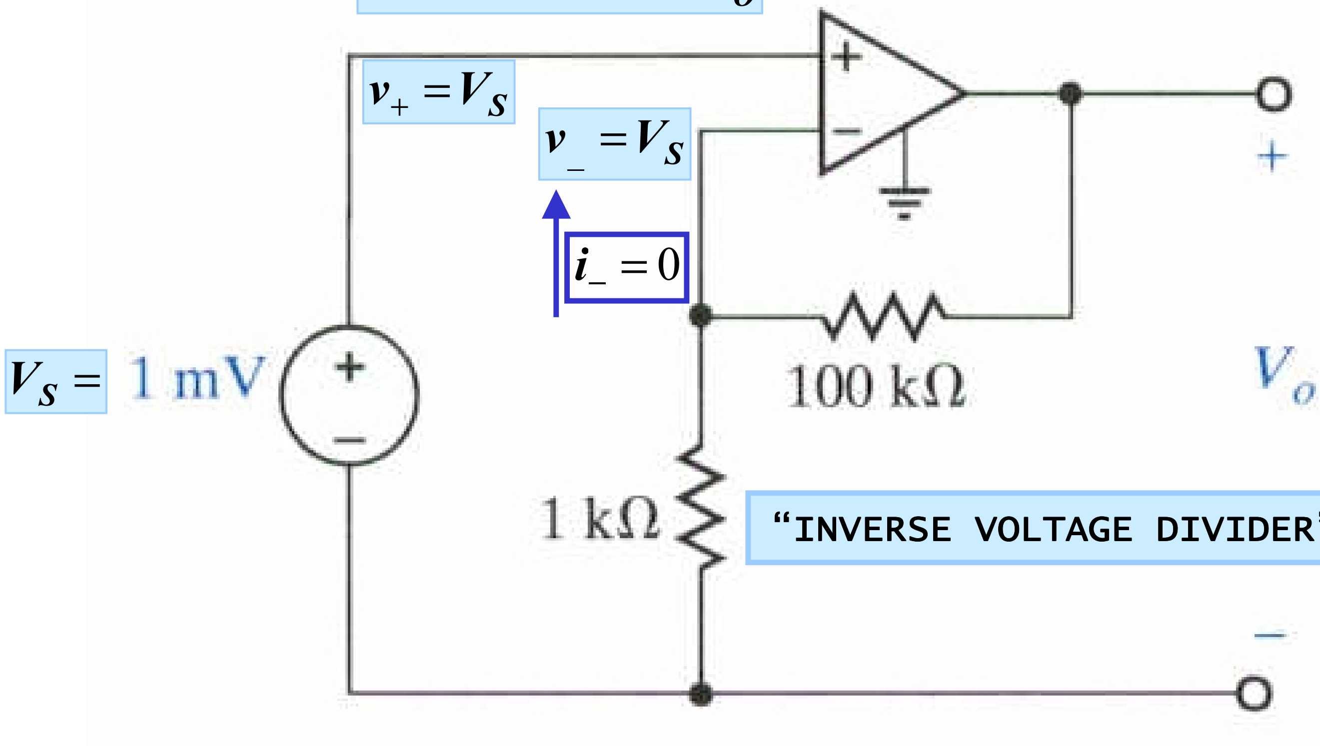
MESH 2 $R_i(i_2 - i_S) + (R + R_O)i_2 + A(v_+ - v_-)$

CONTROLLING VARIABLE $v_+ - v_- = R_i(i_2 - i_1)$

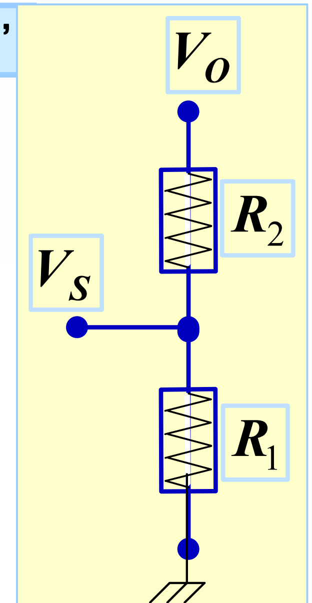
GEAUX

LEARNING EXTENSION

FIND GAIN AND V_O



“INVERSE VOLTAGE DIVIDER”



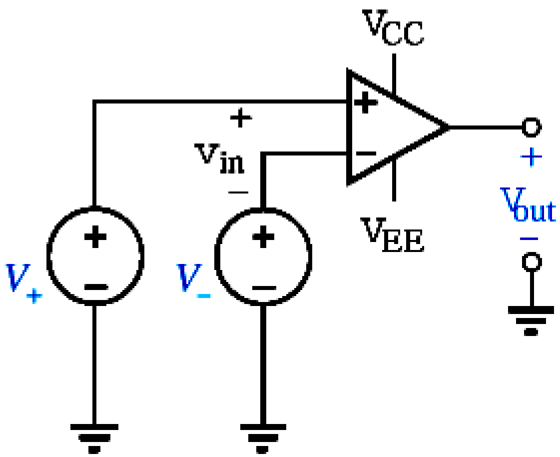
$$V_O = \frac{100k + 1k}{1k} V_S$$

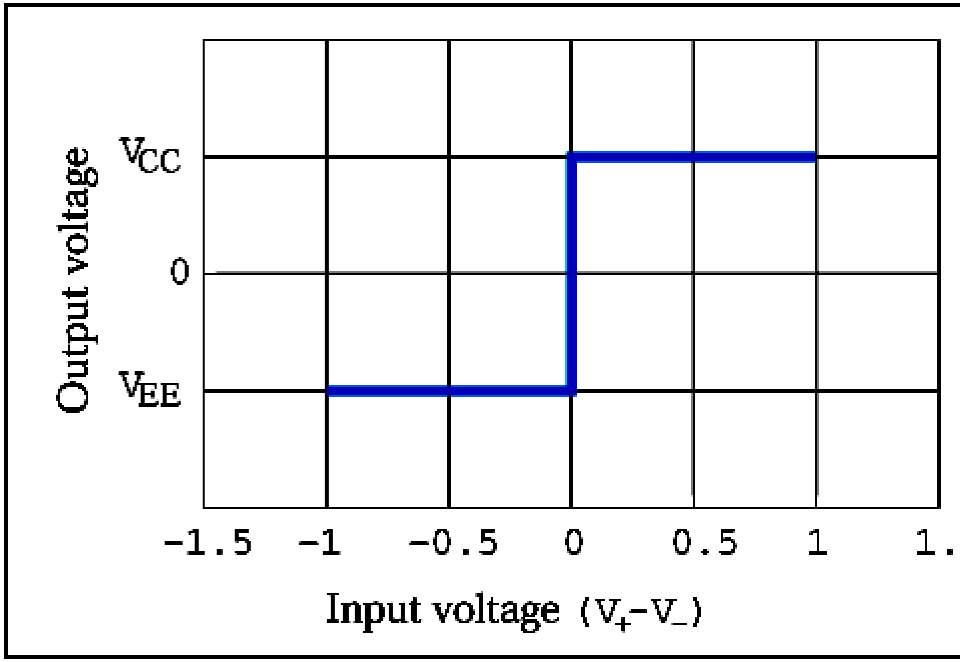
$$G = \frac{V_O}{V_S} = 101$$

$$V_S = 1mV \Rightarrow V_O = 0.101V$$

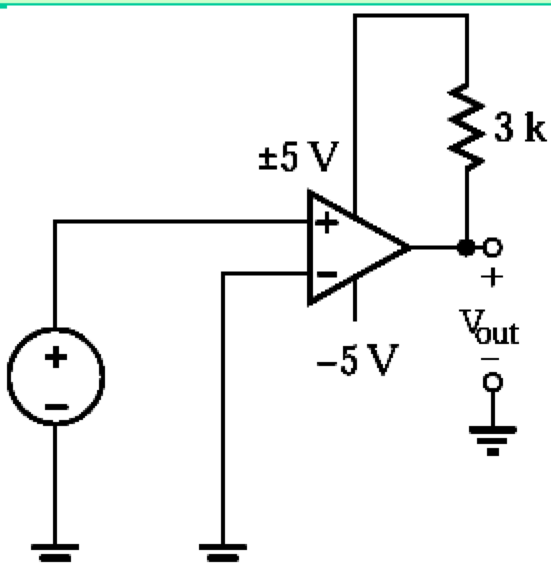
GEAUX

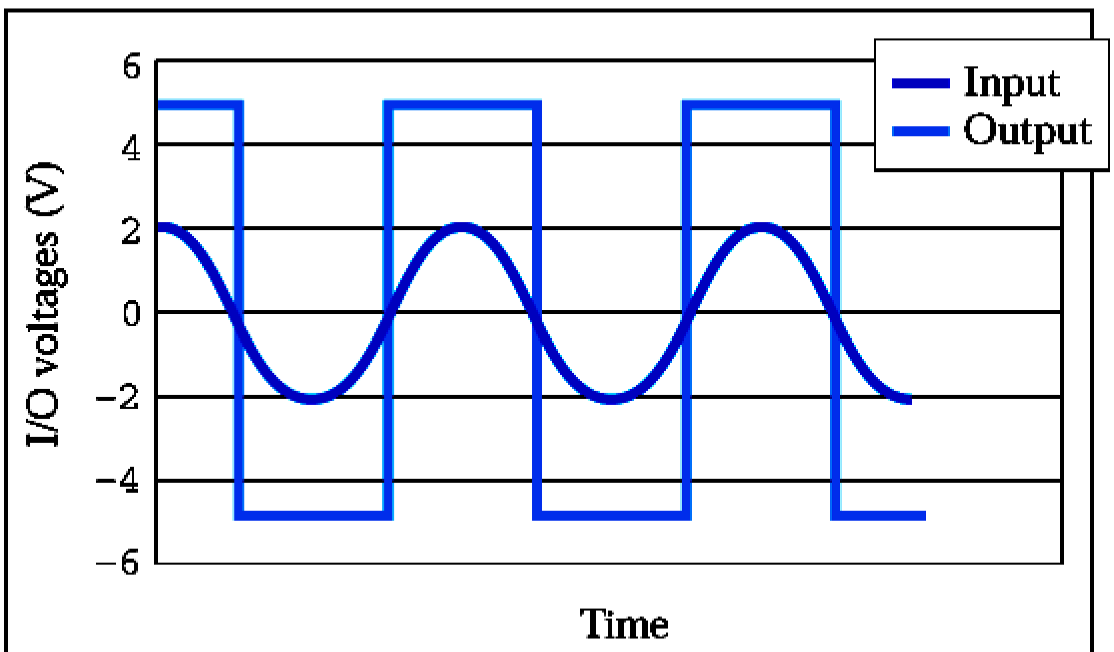
COMPARATOR CIRCUITS





Some REAL OpAmps require a “pull up resistor.”





ZERO-CROSSING DETECTOR

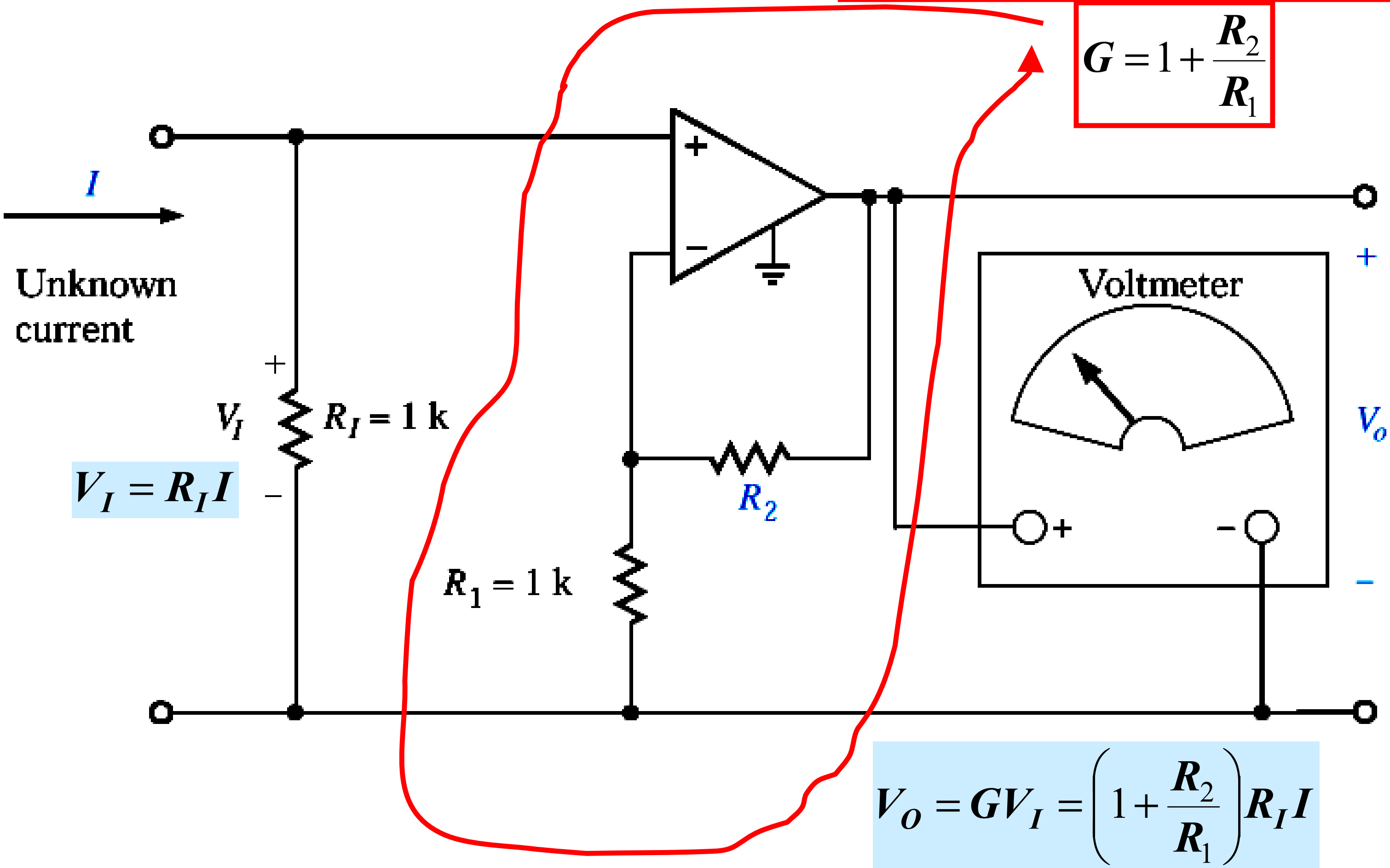
GEAUX

LEARNING BY APPLICATION

OP-AMP BASED AMMETER

NON-INVERTING AMPLIFIER

$$G = 1 + \frac{R_2}{R_1}$$

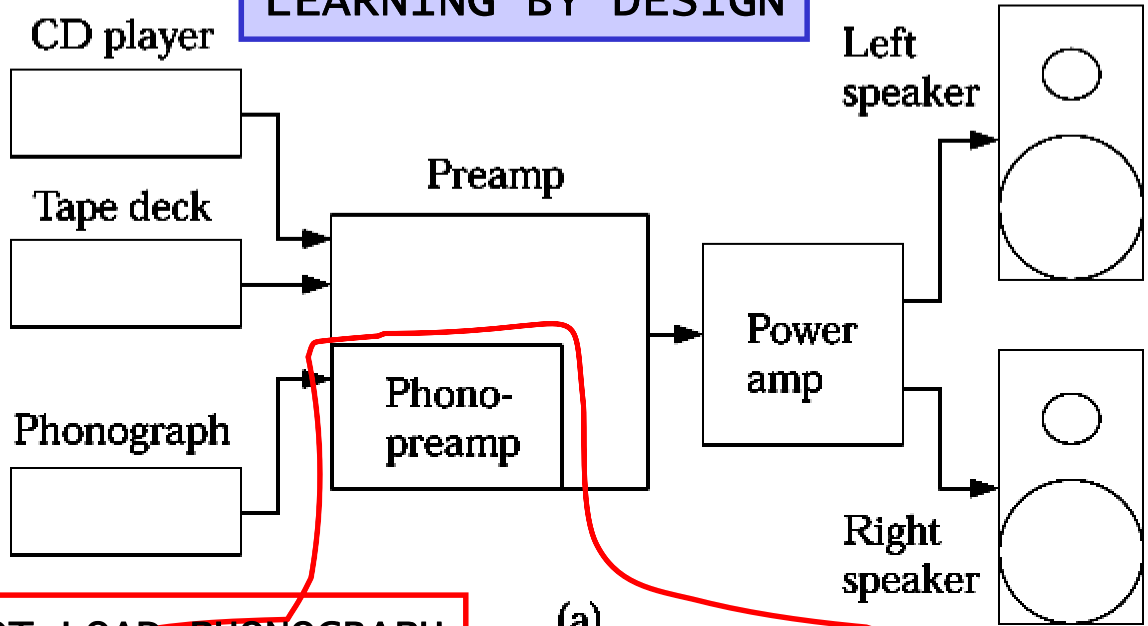


$V_I = R_I I$

$$V_O = GV_I = \left(1 + \frac{R_2}{R_1}\right) R_I I$$

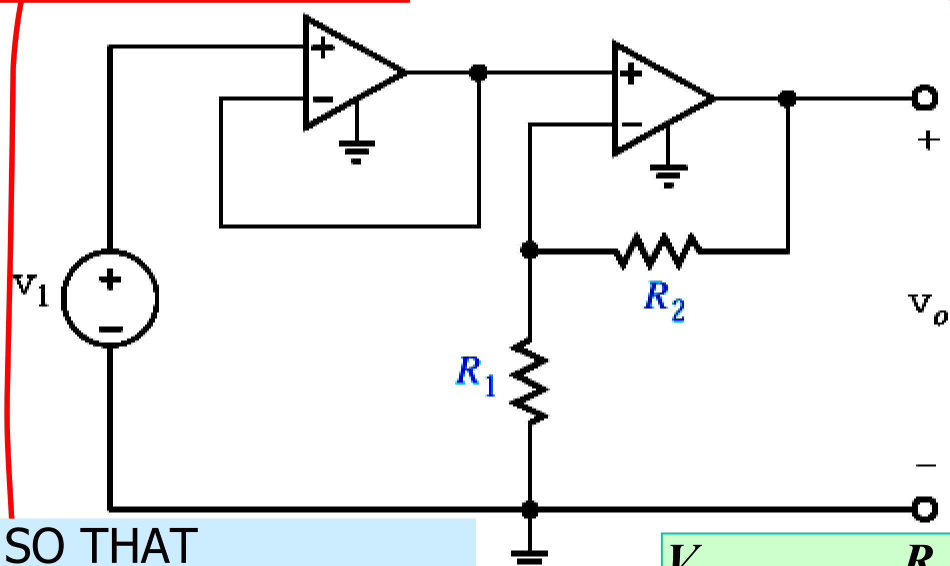
GEAUX

LEARNING BY DESIGN



(a)

DOES NOT LOAD PHONOGRAPH



(b)

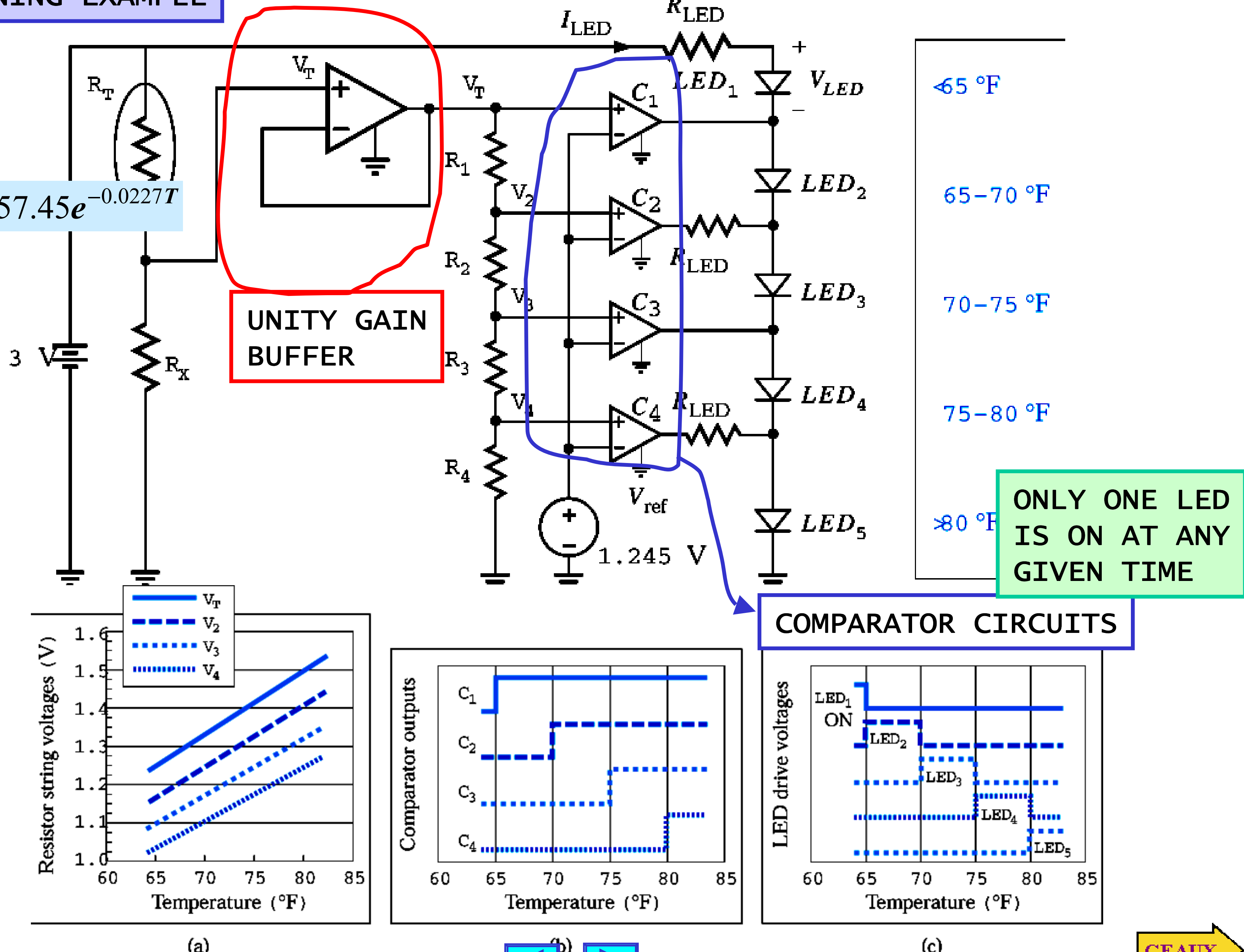
DETERMINE R_2, R_1 SO THAT

IT PROVIDES AN AMPLIFICATION OF 1000

$$\frac{V_o}{V_1} = (1)(1 + \frac{R_2}{R_1})$$

GEAUX

LEARNING EXAMPLE



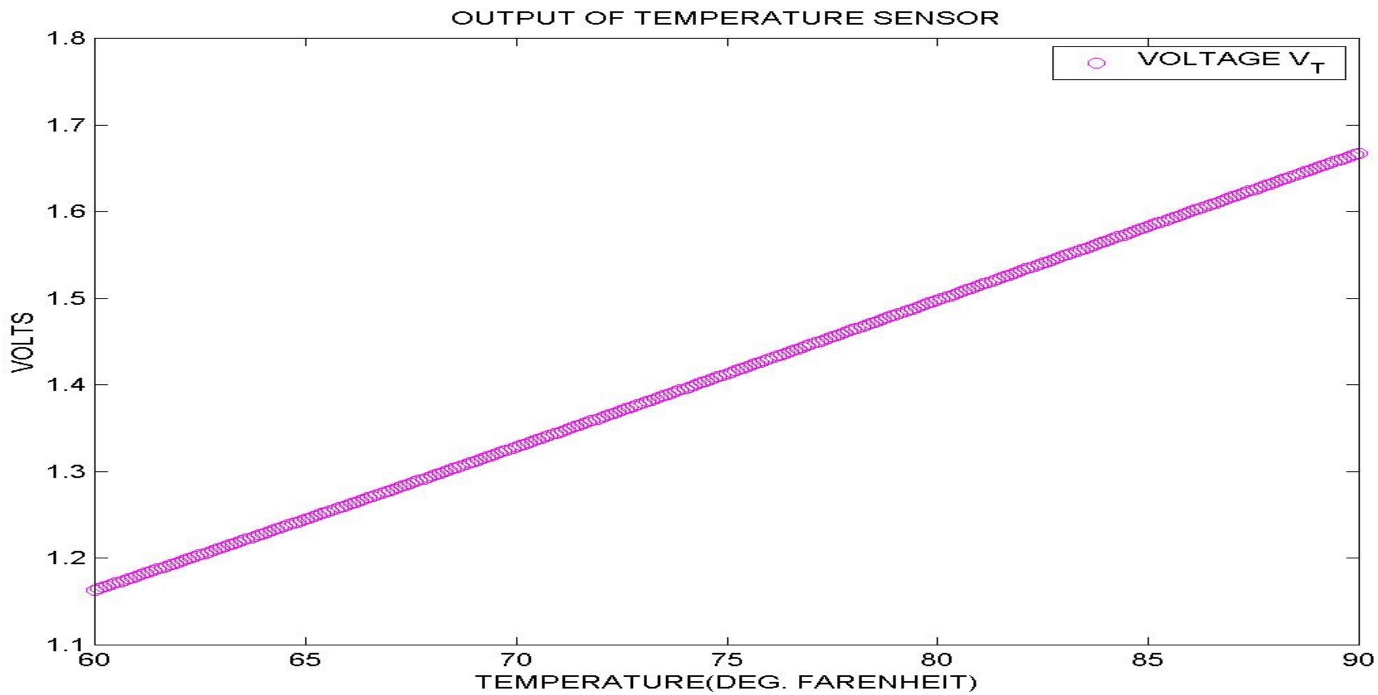
$$R_T = 57.45e^{-0.0227T}$$

GEAUX

MATLAB SIMULATION OF TEMPERATURE SENSOR

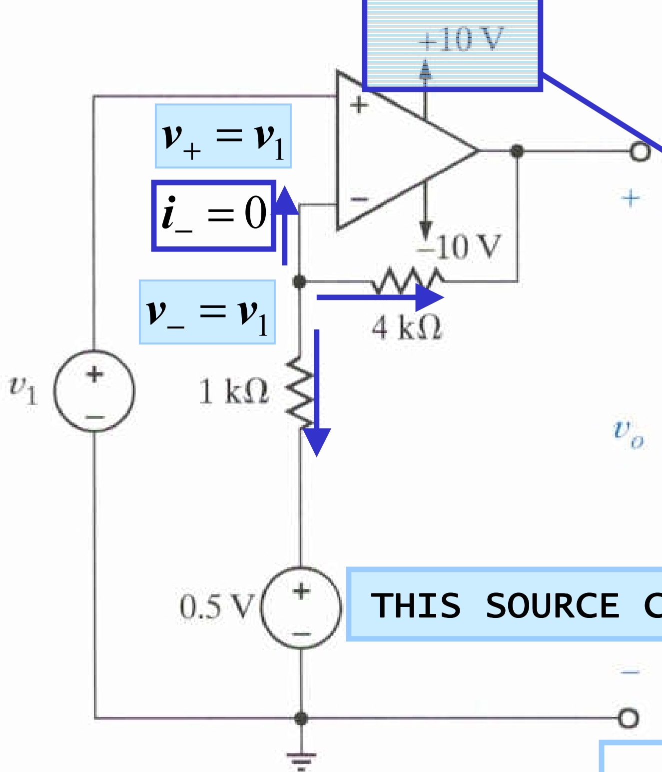
WE SHOW THE SEQUENCE OF MATLAB INSTRUCTIONS USED TO OBTAIN THE PLOT OF THE VOLTAGE AS FUNCTION OF THE TEMPERATURE

```
»T=[60:0.1:90]'; %define a column array of temperature values
» RT=57.45*exp(-0.0227*T); %model of thermistor
» RX=9.32; %computed resistance  needed for voltage divider
» VT=3*RX./(RX+RT); %voltage divider equation. Notice “./” to create output array
» plot(T,VT, ‘mo’); %basic plotting instruction
» title('OUTPUT OF TEMPERATURE SENSOR'); %proper graph labeling tools
» xlabel('TEMPERATURE(DEG. FARENHEIT)')
» ylabel('VOLTS')
» legend('VOLTAGE V_T')
```



GEAUX

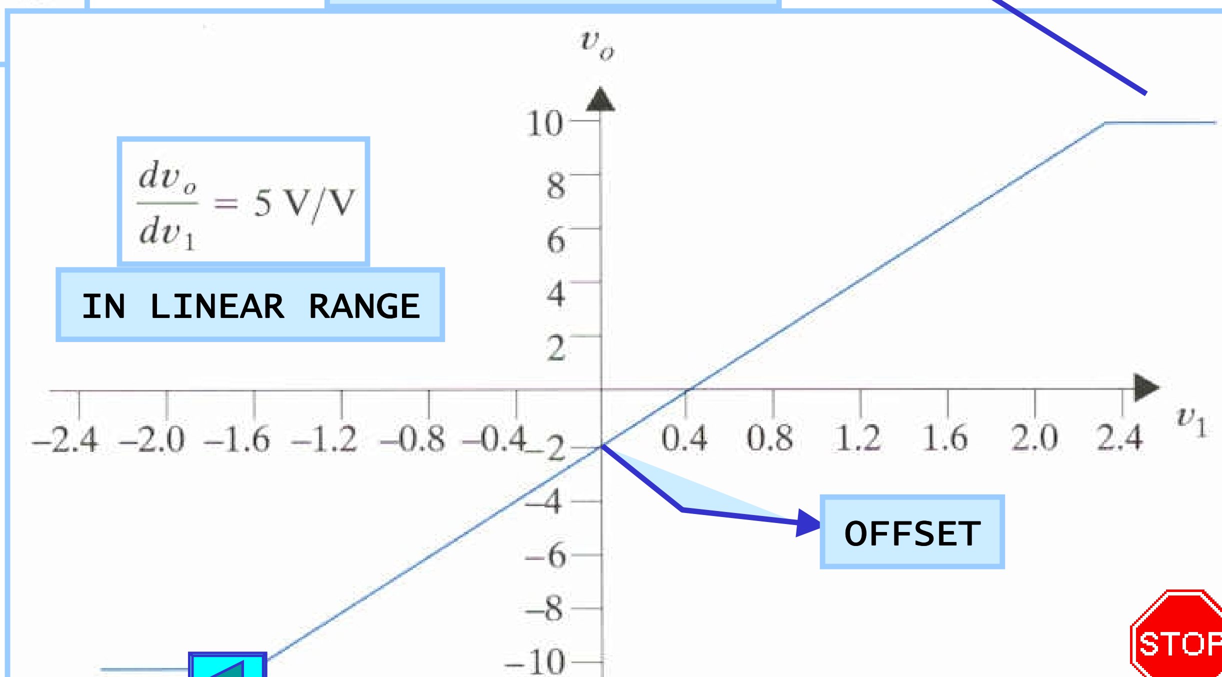
EXAMPLE OF TRANSFER CURVE SHOWING SATURATION



THIS SOURCE CREATES THE OFFSET

OUTPUT CANNOT EXCEED SUPPLY (10v)

THE TRANSFER CURVE



KCL @ v_

$$\frac{v_1 - 0.5}{1k} + \frac{v_1 - v_o}{4k} = 0$$

$$v_o = 5v_1 - 2$$