CAPACITANCE AND INDUCTANCE

Introduces two passive, energy storing devices: Capacitors and Inductors

LEARNING GOALS

CAPACITORS
Store energy in their electric field (electrostatic energy)
Model as circuit element

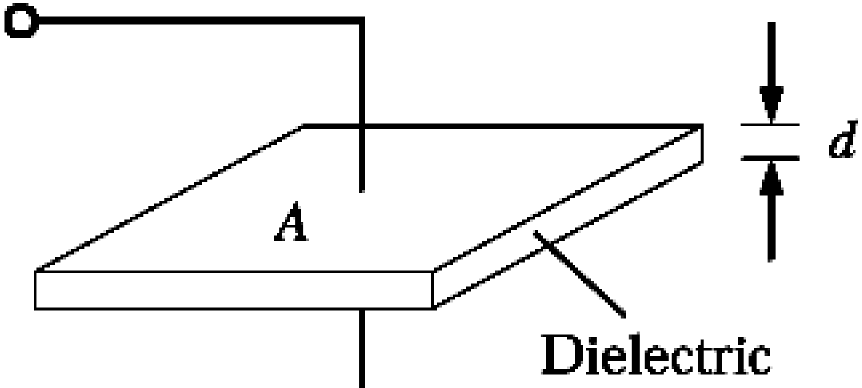
INDUCTORS
Store energy in their magnetic field
Model as circuit element

CAPACITOR AND INDUCTOR COMBINATIONS
Series/parallel combinations of elements

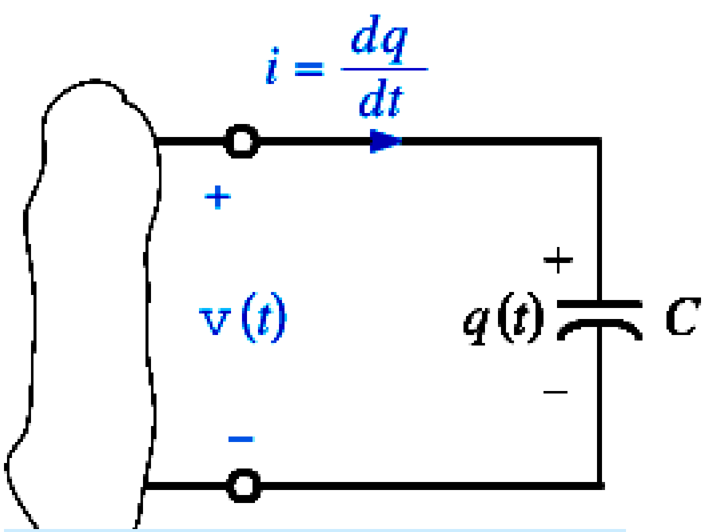
RC OP-AMP CIRCUITS
Integration and differentiation circuits

CAPACITORS

First of the energy storage devices to be discussed



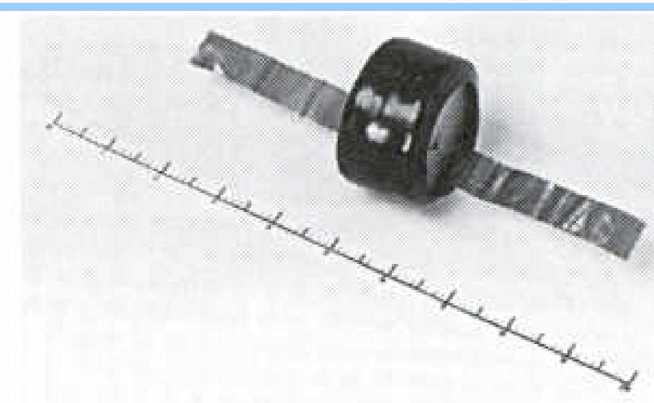
Basic parallel-plates capacitor



CIRCUIT REPRESENTATION

NOTICE USE OF PASSIVE SIGN CONVENTION

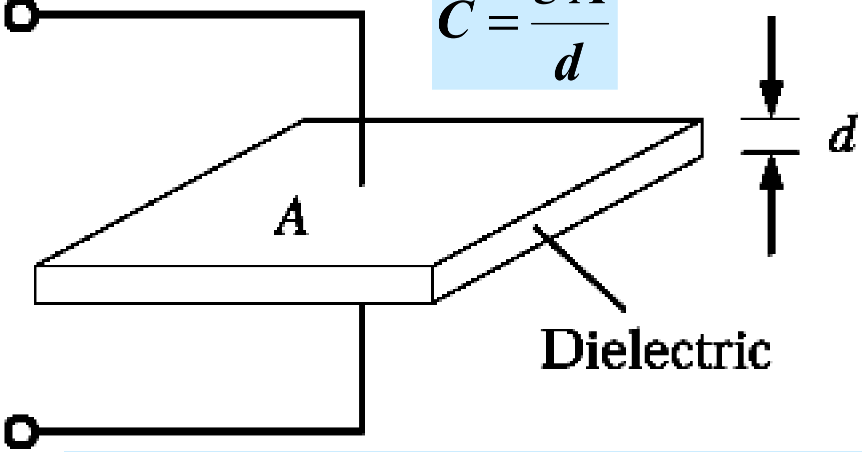
Typical Capacitors



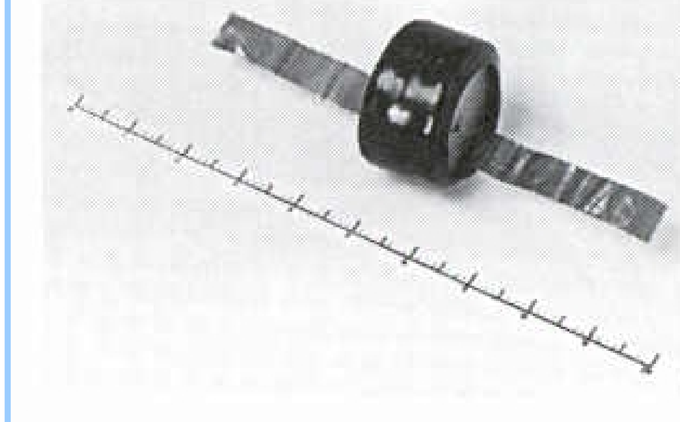
A 55-F, 500-J capacitor and a 10-in ruler for comparison.

GEAUX

$$C = \frac{\varepsilon A}{d}$$



ε Dielectric constant of material in gap



A 55-F, 500-J capacitor and a 10-in ruler for comparison.

PLATE SIZE FOR EQUIVALENT AIR-GAP CAPACITOR

$$55F = \frac{8.85\times10^{-12} A}{1.016\times10^{-4}} \Rightarrow A = 6.3141\times10^{8} m^{2}$$

Normal values of capacitance are small.
Microfarads is common.
For integrated circuits nano or pico farads are not unusual

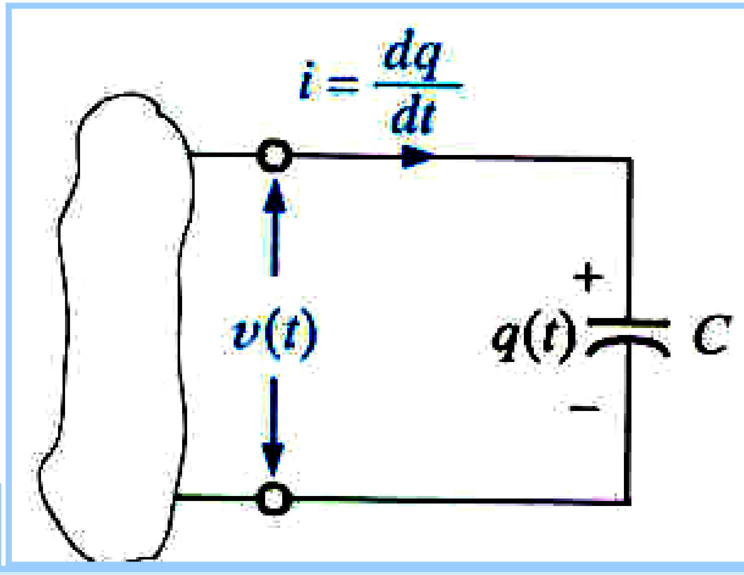
Basic capacitance law $Q = f(V_C)$

Linear capacitors obey Coulomb's law $Q = CV_C$

C is called the CAPACITANCE of the device and has units of $\frac{\text{charge}}{\text{voltage}}$

One Farad(F)is the capacitance of a device that can store one Coulomb of charge at one Volt.

$$\text{Farad} = \frac{\text{Coulomb}}{\text{Volt}}$$



Linear capacitor circuit representation

EXAMPLE Voltage across a capacitor of 2 micro Farads holding 10mC of charge

$$V_C = \frac{1}{C}Q = \frac{1}{2*10^{-6}}10*10^{-3} = 5000 \text{ V}$$

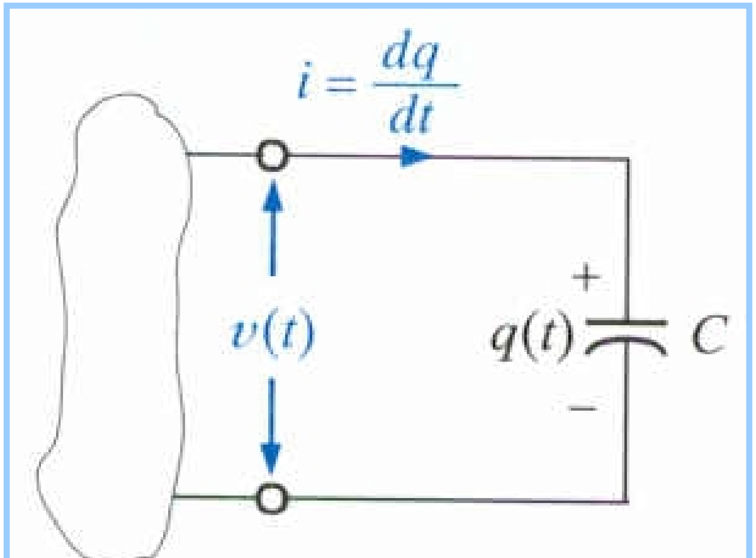
Capacitance in Farads, charge in Coulombs result in voltage in Volts

Capacitors can be dangerous!!!

GEAUX

Capacitors only store and release ELECTROSTATIC energy. They do not "create"

The capacitor is a passive element and follows the passive sign convention

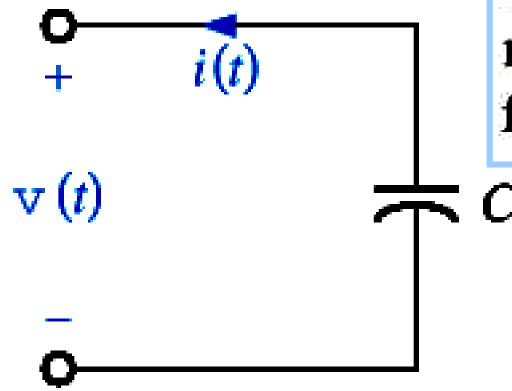


Linear capacitor circuit representation

$$i(t) = C\frac{dv}{dt}(t)$$

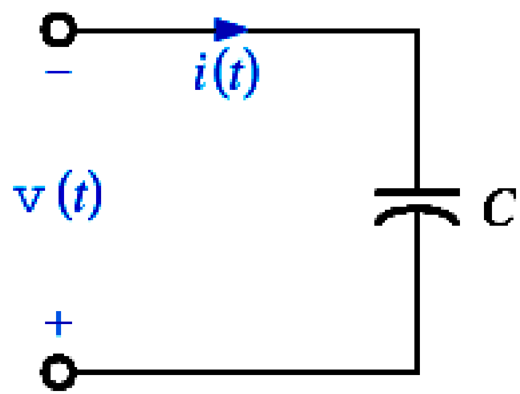
LEARNING BY DOING

Write the i-v relationship for the following capacitors.



(a)

$$i(t) = -C\frac{dv(t)}{dt}$$



(b)

$$i(t) = -C\frac{dv(t)}{dt}$$

GEAUX

$$Q_C = CV_C$$

Capacitance Law

If the voltage varies the charge varies and there is a displacement current

One can also express the voltage across in terms of the current

$$V_C(t) = \frac{1}{C}Q = \frac{1}{C}\int_{-\infty}^{t} i_C(x)dx$$

Integral form of Capacitance law

The mathematical implication of the integral form is ...

$$V_C(t-) = V_C(t+);\ \forall t$$

Voltage across a capacitor MUST be continuous

... Or one can express the current through in terms of the voltage across

$$i_C = \frac{dQ}{dt} = C\frac{dV_C}{dt}$$

Differential form of Capacitance law

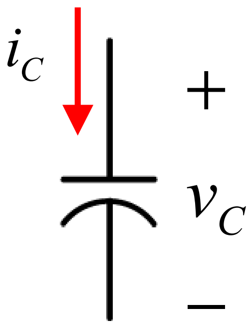
Implications of differential form??

$$V_C = Const \Rightarrow i_C = 0$$

DC or steady state behavior

A capacitor in steady state acts as an OPEN CIRCUIT

CAPACITOR AS CIRCUIT ELEMENT



$$i_C(t) = C\frac{dv_c}{dt}(t)$$

$$i_R = \frac{1}{R}v_R$$

$$v_R = Ri_R$$

Ohm's Law

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(x)dx$$

$$\int_{-\infty}^{t} = \int_{-\infty}^{t_0} + \int_{t_0}^{t}$$

$v_c(t_O)$

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t_0} i_C(x)dx + \frac{1}{C}\int_{t_0}^{t} i_C(x)dx$$

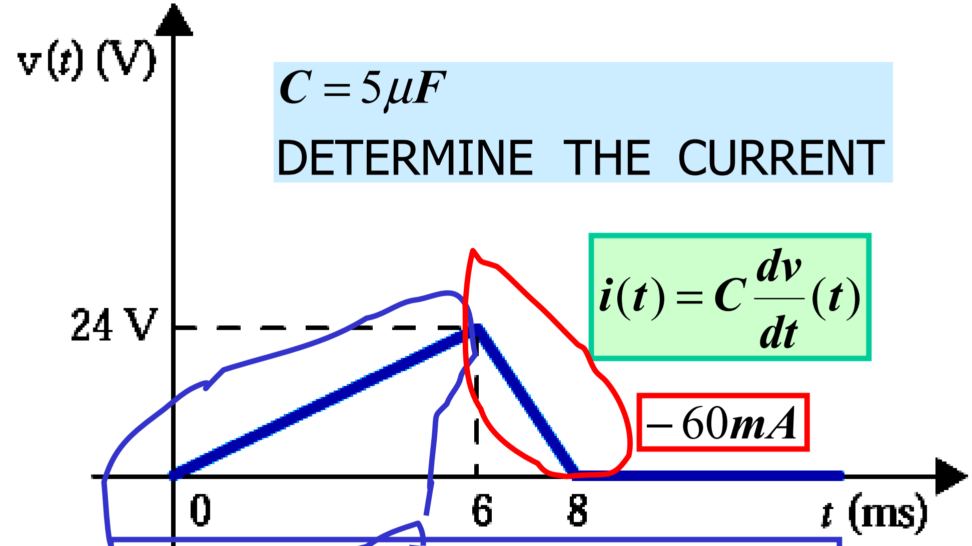
$$v_C(t) = v_C(t_0) + \frac{1}{C}\int_{t_0}^{t} i_C(x)dx$$

The fact that the voltage is defined through an integral has important implications...

LEARNING EXAMPLE

$C = 5\mu F$

DETERMINE THE CURRENT

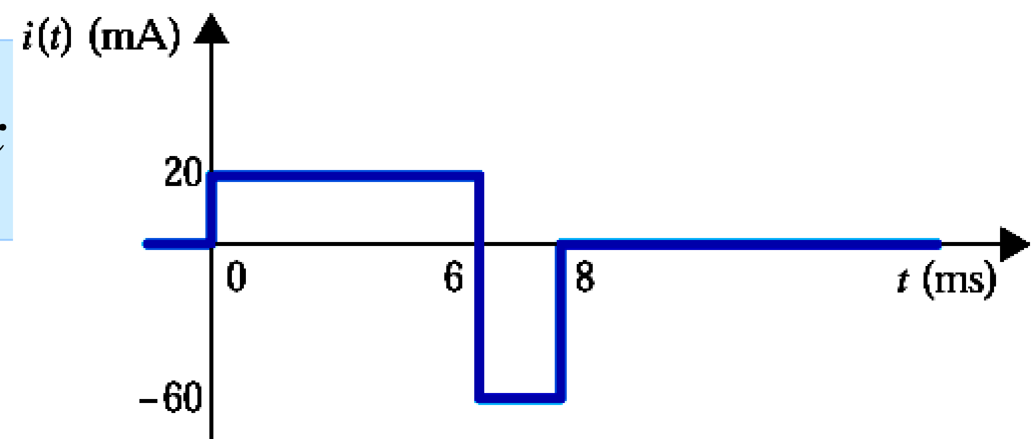


$$i(t) = C\frac{dv}{dt}(t)$$

$-60mA$

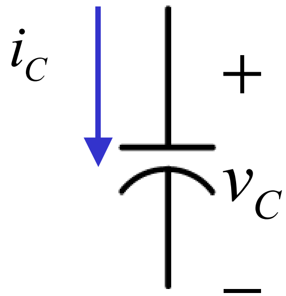
$$i = 5\times10^{-6}[F]\times\frac{24}{6\times10^{-3}}\left[\frac{V}{s}\right] = 20mA$$

$i(t) = 0$ *elsewhere*



GEAUX

CAPACITOR AS ENERGY STORAGE DEVICE



Instantaneous power

$$p_C(t) = v_C(t) i_C(t) \quad \textbf{W}$$

$$i_C(t) = C\frac{dv_c}{dt}(t)$$

$$p_C(t) = Cv_C(t)\frac{dv_c}{dt}$$

$$p_C(t) = C\frac{d}{dt}\left(\frac{1}{2}v_C^2(t)\right)$$

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(x)dx = \frac{q_C(t)}{C}$$

$$p_C(t) = \frac{1}{C}q_C(t)\frac{dq_C}{dt}(t)$$

$$p_C(t) = \frac{1}{C}\frac{d}{dt}\left(\frac{1}{2}q_c^2(t)\right)$$

Energy is the integral of power

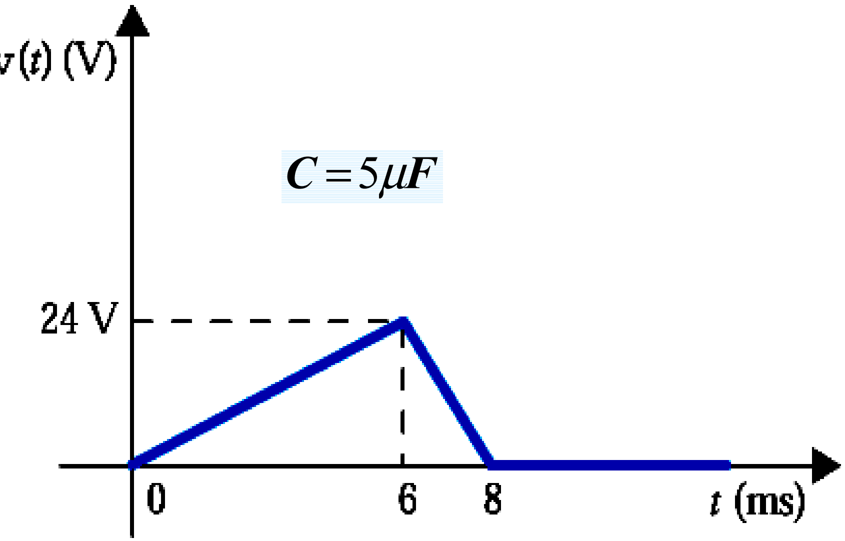
$$w_C(t_2, t_1) = \int_{t_1}^{t_2} p_C(x)dx$$

If t1 is minus infinity we talk about "energy stored at time t2."

$$w_C(t_2, t_1) = \frac{1}{2}Cv_C^2(t_2) - \frac{1}{2}Cv_C^2(t_1)$$

If both limits are infinity then we talk about the "total energy stored."

$$w_C(t_2, t_1) = \frac{1}{C}q_C^2(t_2) - \frac{1}{C}q_C^2(t_1)$$



LEARNING EXAMPLE

Energy stored in 0 - 6 msec

$$w_C(0,6) = \frac{1}{2} C v_C^2(6) - \frac{1}{2} C v_C^2(0)$$

$$w_C(0,6) = \frac{1}{2} 5*10^{-6}[F]*(24)^2[V^2]$$

Charge stored at 3msec

$$q_C(3) = C v_C(3)$$

$$q_C(3) = 5*10^{-6}[F]*12[V] = 60\mu C$$

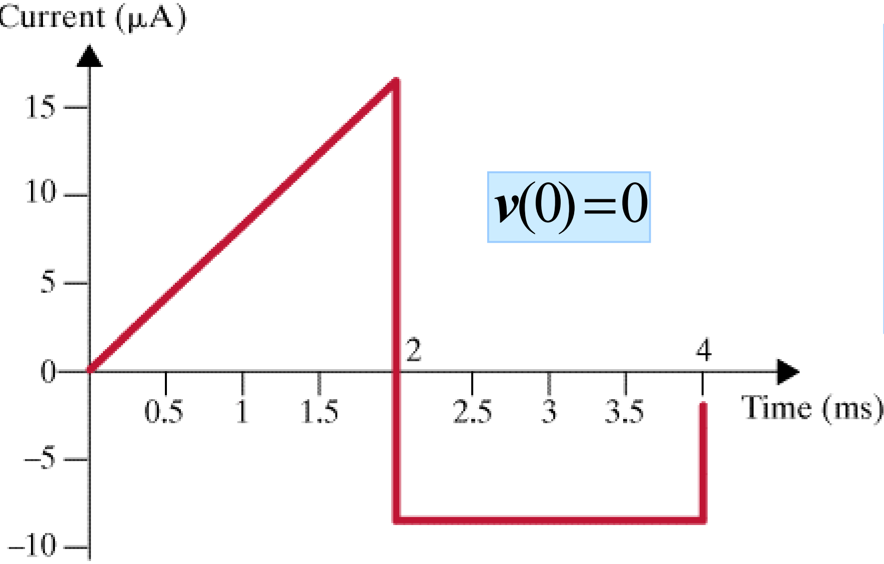
"total energy stored?" ....

"total charge stored?" ...

If charge is in Coulombs and capacitance in Farads then the energy is in ….

GEAUX

$C = 4\mu F$. FIND THE VOLTAGE



$v(0) = 0$

$$i(t) = \frac{16 \times 10^{-6} t}{2 \times 10^{-3}} \quad 0 \le t \le 2 \text{ ms}$$
$$= -8 \times 10^{-6} \quad 2 \text{ ms} \le t \le 4 \text{ ms}$$
$$= 0 \quad 4 \text{ ms} < t$$

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(x)dx; \;\; t > 0$$

$$v(t) = \frac{1}{(4)(10^{-6})}\int_0^t 8(10^{-3})x\,dx = 10^3 t^2$$

$0 \le t \le 2$

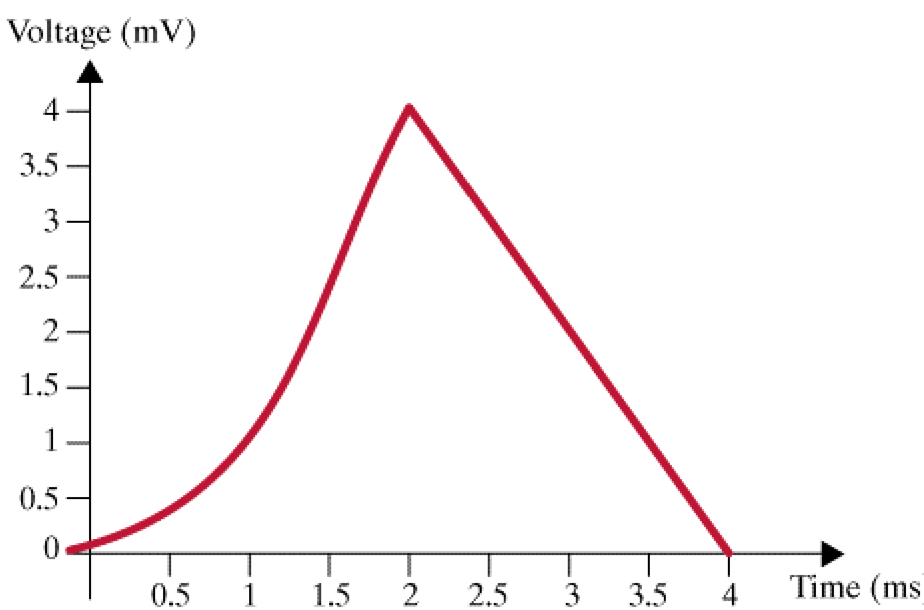
$$v(t) = v(2) + \frac{1}{C}\int_2^t i(x)dx; \;\; t > 2$$

$$v(2 \text{ ms}) = 10^3(2 \times 10^{-3})^2 = 4 \text{ mV}$$

$$v(t) = \frac{1}{(4)(10^{-6})}\int_{2(10^{-3})}^t -(8)(10^{-6})\,dx + (4)(10^{-3})$$

$2 < t \le 4ms$

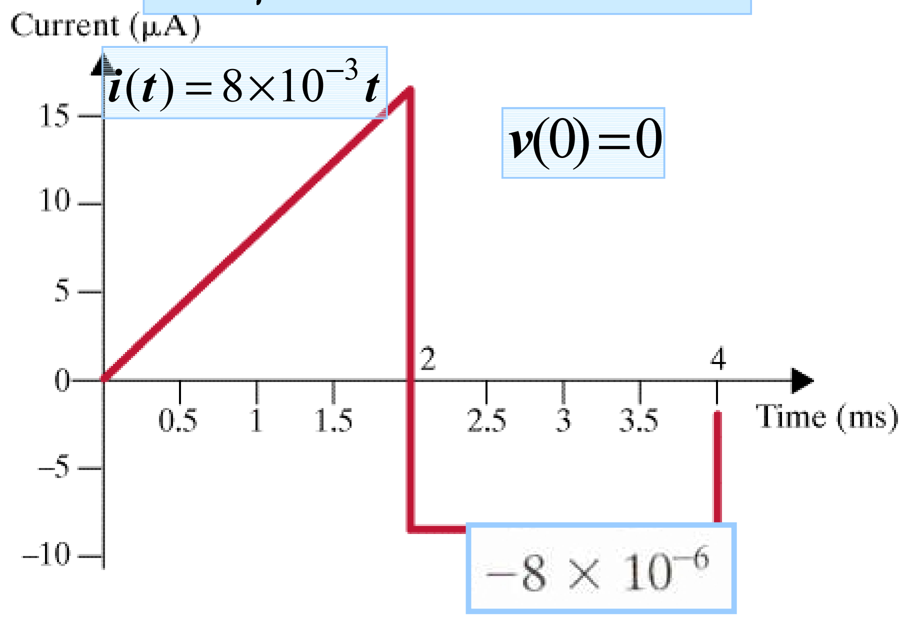
$$v(t) = -2t + 8\times 10^{-3}[V]$$



(b)

GEAUX

$C = 4\mu F$. FIND THE POWER



(a)

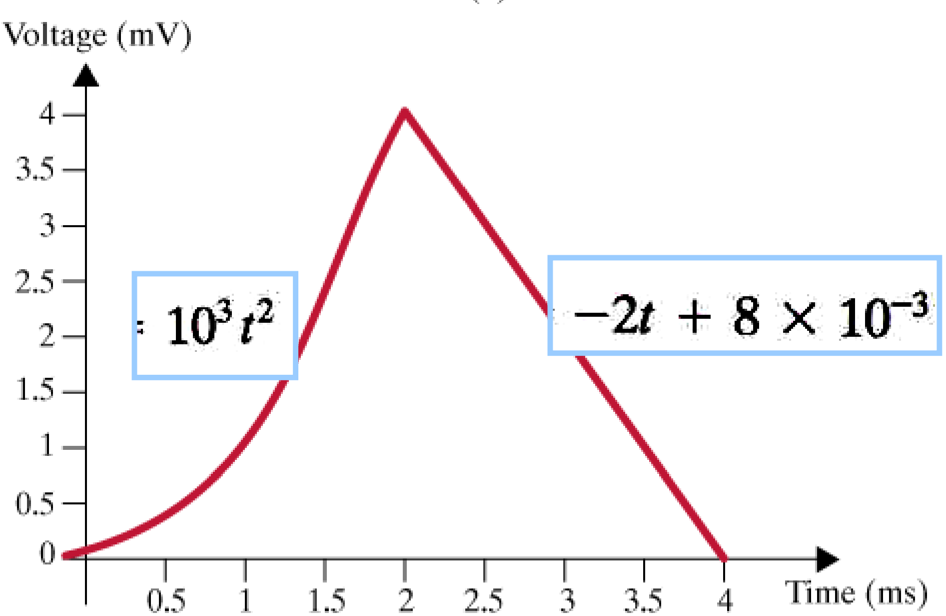
$$p(t) = v(t)i(t)$$

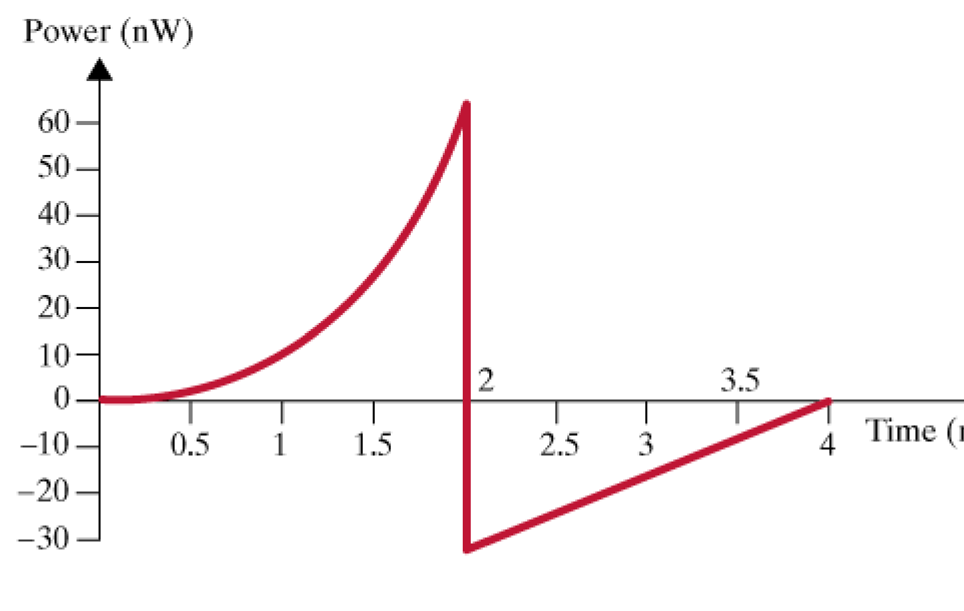
$$p(t) = 8t^3,\ 0 \le t \le 2ms$$

$$p(t) = -(8)(10^{-6})(-2t + 8 \times 10^{-3}) = 16(10^{-6})t - 64(10^{-9})$$

$$2 < t \le 4ms$$

$$p(t) = 0,\ elsewhere$$

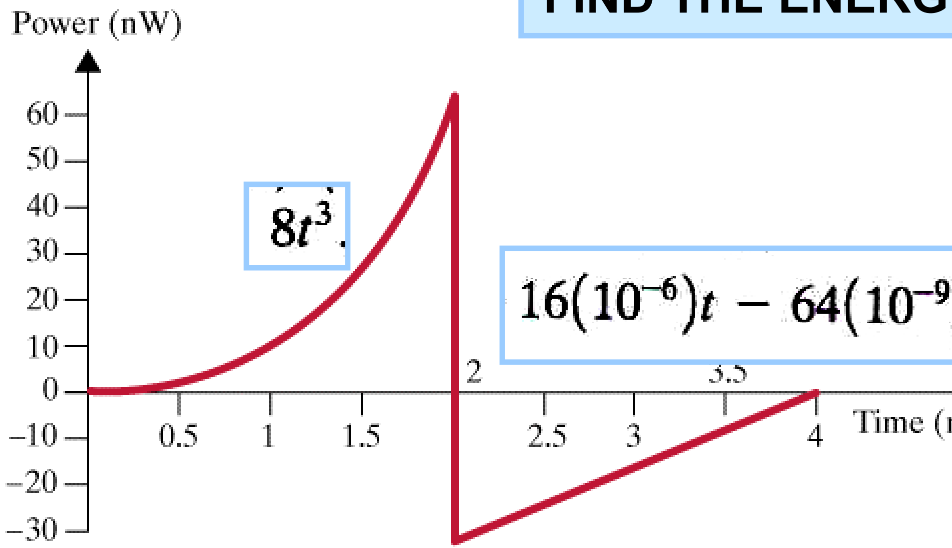


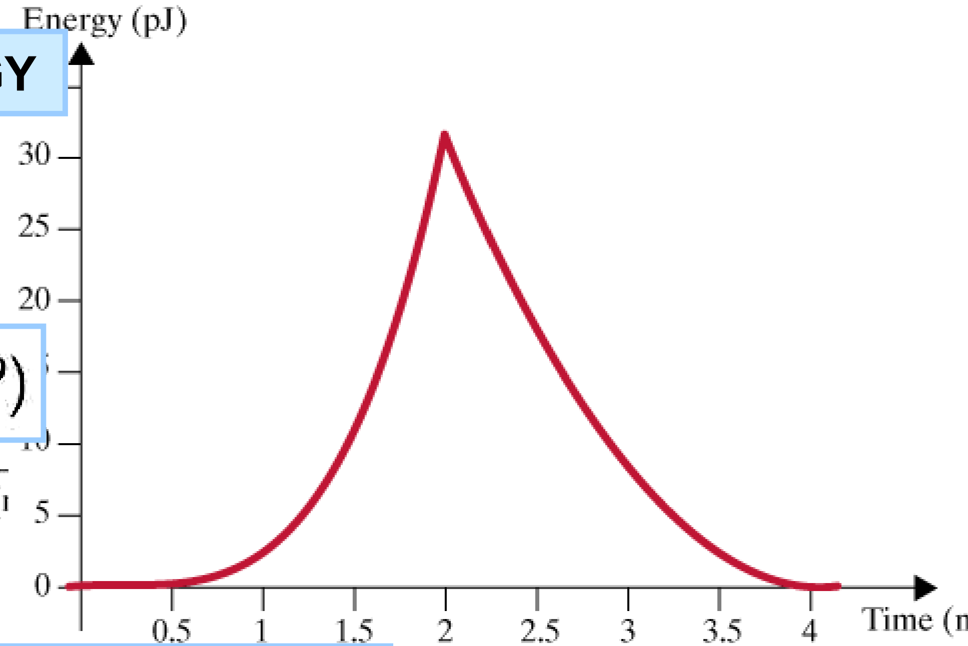


(c)

GEAUX

FIND THE ENERGY





$$p(t) = 8t^3, \quad 0 \le t \le 2ms$$

$$w(t) = \int_0^t 8x^3\,dx = 2t^4$$

$$w(2\text{ ms}) = 32\text{ pJ}$$

$$p(t) = -(8)(10^{-6})(-2t + 8 \times 10^{-3}) = 16(10^{-6})t - 64(10^{-9})$$

$$2 < t \le 4ms$$

$$p(t) = 0, \quad elsewhere$$

$$w(t) = \int_{t_0}^{t} p(x)\,dx + w(t_0)$$

$$w(t) = \int_{2\times10^{-3}}^{t} [(16 \times 10^{-6})x - (64 \times 10^{-9})]\,dx + 32 \times 10^{-12}$$

$$= (8 \times 10^{-6})t^2 - (64 \times 10^{-9})t + 128 \times 10^{-12}$$

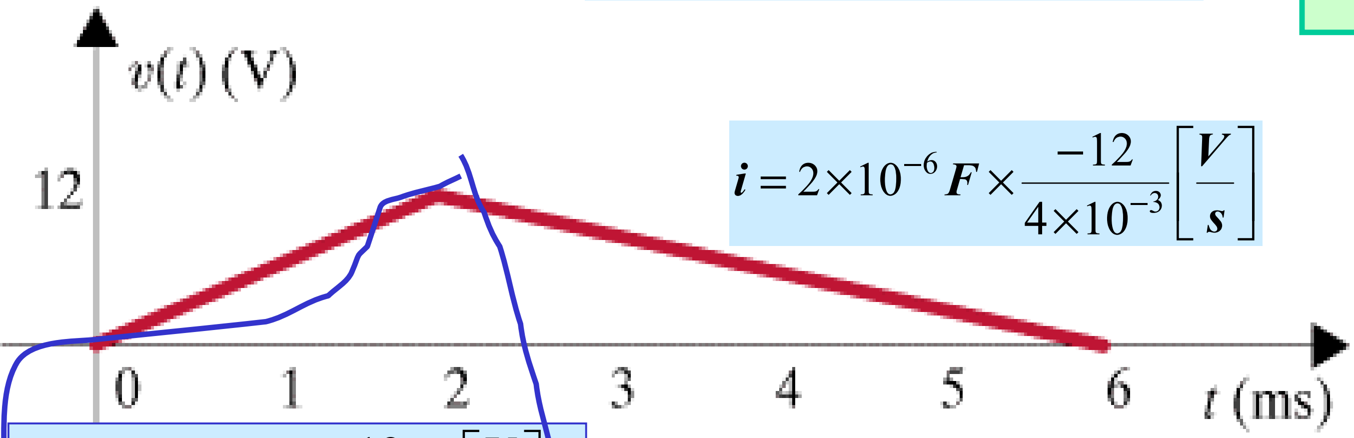
GEAUX

LEARNING EXTENSION

$C = 2\mu F$

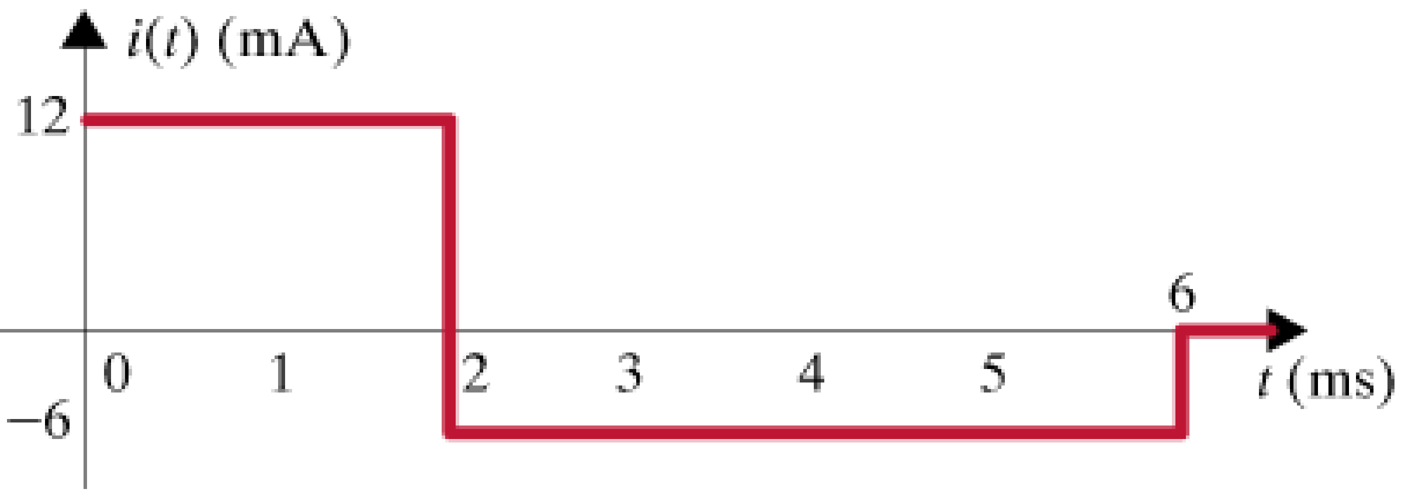
DETERMINE THE CURRENT

$$i(t) = C\frac{dv}{dt}(t)$$



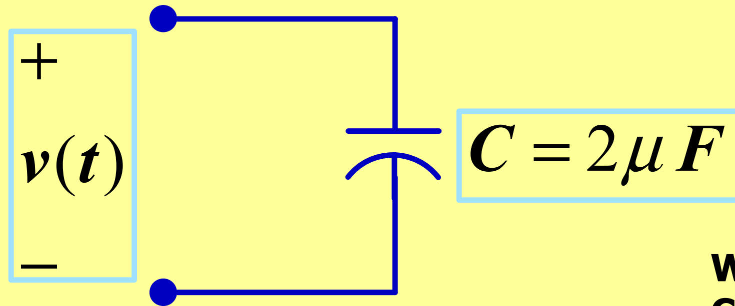
$$i = 2\times10^{-6} F \times \frac{-12}{4\times10^{-3}}\left[\frac{V}{s}\right]$$

$$i = 2\times10^{-6}\ F \times \frac{12}{2\times10^{-3}}\left[\frac{V}{s}\right] =$$



GEAUX

SAMPLE PROBLEM



$$v(t) = 130\sin(120\pi t)$$

WHAT VARIABLES CAN BE COMPUTED?

Energy stored at a given time t

$$E(t) = \frac{1}{2}Cv_C^2(t)$$

$$E(1/240) = \frac{1}{2}2*10^{-6}[F]*130^2\sin^2\left(\frac{\pi}{2}\right) \quad \text{J}$$

Charge stored at a given time

$$q_C(t) = Cv_C(t)$$

$$q_C(1/120) = 2*10^{-6}[C]*\sin(\pi)[V] = 0 \quad \text{C}$$

Current through the capacitor

$$i_C = C\frac{dv_C}{dt}(t)$$

$$i_C(1/120) = 2*10^{-6}*130*120\pi\cos(\pi) \quad \text{A}$$

Electric power supplied to capacitor at a given time

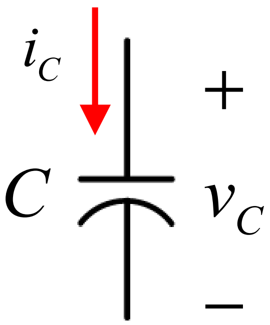
$$p_C(t) = v_C(t)i_C(t) \quad \text{W}$$

Energy stored over a given time interval

$$w(t_2, t_1) = \frac{1}{2}Cv_C^2(t_2) - \frac{1}{2}Cv_C^2(t_1) \quad \text{J}$$

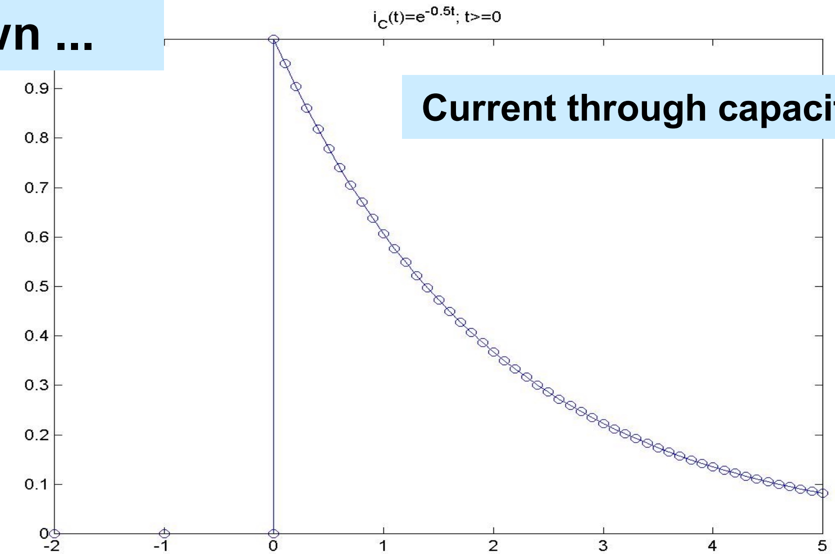
SAMPLE PROBLEM

If the current is known ...



$C = 2\mu F$

$$i_C(t) = \begin{cases} e^{-0.5t};\ t \ge 0 \\ 0;\ t < 0 \end{cases} [mA]$$



Voltage at a given time t

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(x)dx$$

$$v_C(0) = 0[V]$$

Voltage at a given time *t* when voltage at time *to<t* is also known

$$v_C(t) = v_C(t_0) + \frac{1}{C}\int_{t_0}^{t} i_C(x)dx$$

$$v_C(2) = v_C(0) + \frac{1}{C}\int_{0}^{2} e^{-0.5x}dx = \frac{1}{2*10^{-6}}\left[-\frac{1}{0.5}e^{-0.5x}\right]_0^2 = \frac{1}{2*10^{-6}}\frac{1}{0.5}\left(1-e^{-1}\right) = 0.6321*10^6 \text{ V}$$

Charge at a given time

$$q_C(t) = Cv_C(t)$$

$$q_C(2) = 2*0.6321 \text{ C}$$

Voltage as a function of time

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(x)dx$$

$$v_C(t) = 0;\ t \le 0$$

$$v_C(t) = v_C(0) + \frac{1}{C}\int_{0}^{t} e^{-0.5x}dx$$

$$v_C(t) = \begin{cases} 10^6(1-e^{-0.5t});\ t \ge 0 \\ 0;\ t < 0 \end{cases} \text{ V}$$

Electric power supplied to capacitor

$$p_C(t) = v_C(t)i_C(t) \text{ W}$$

Energy stored in capacitor at a given time

$$w(t) = \frac{1}{2}Cv_C^2(t) \text{ J}$$

"Total" energy stored in the capacitor

$$w_T = \frac{1}{2}Cv_C^2(\infty)$$

$$w_T = \frac{1}{2}2*10^{-6}*(10^6)^2 = 10^6 \text{ J}$$

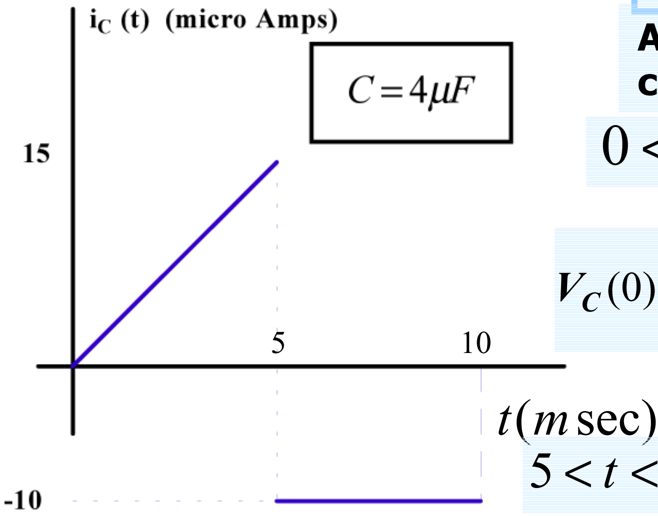
GEAUX

SAMPLE PROBLEM

Given current and capacitance

Compute voltage as a function of time

$$V_c(t) = \frac{1}{C}\int_{t_o}^{t} i_C(t)dt + V_C(t_o)$$



At minus infinity everything is zero. Since current is zero for t<0 we have

$$V_C(t) = 0; t \le 0$$

$$0 < t < 5m\sec \Rightarrow i_C(t) = \frac{15}{5}\frac{\mu A}{ms}t = 3\frac{10^{-6}}{10^{-3}}\frac{A}{s}t = 3*10^{-3}[A/s]t$$

$$V_C(0) = 0 \Rightarrow V_C(t) = \frac{3*10^{-3}}{4*10^{-6}}\int_0^t xdx\ [V] = \frac{3*10^{3}}{8}t^2\ [V]; 0 < t < 5*10^{-3}[s]$$

In particular

$$V_C(5ms) = \frac{3*10^3*(5*10^{-3})^2}{8}[V] = \frac{75}{8}[mV]$$

$$5 < t < 10\ ms \Rightarrow i_C(t) = -10\ [\mu A]$$

Charge stored at 5ms

$$q_C(t) = CV_C(t)$$

$$q(5ms) = 4*10^{-6}[F]*\frac{75*10^{-3}}{8}[V]$$

$$q(5ms) = (75/2)\ [nC]$$

$$V_C(5ms) = \frac{75}{8}[mV] \Rightarrow V_C(t) = \frac{75*10^{-3}}{8} + \frac{1}{4*10^{-6}}\int_{5*10^{-3}}^{t}(-10*10^{-6})[A/s]dx$$

$$V_C(t) = \frac{75*10^{-3}}{8} - \frac{10}{4}\left(t - 5*10^{-3}\right)[V];\ 5*10^{-3} < t < 10*10^{-3}[s]$$

Total energy stored

$$E = \frac{1}{2}CV_C^2$$

Total means at infinity. Hence

$$E_T = 0.5*4*10^{-6}\left(\frac{25*10^{-3}}{8}\right)^2\ [J]$$

Before looking into a formal way to describe the current we will look at additional questions that can be answered.

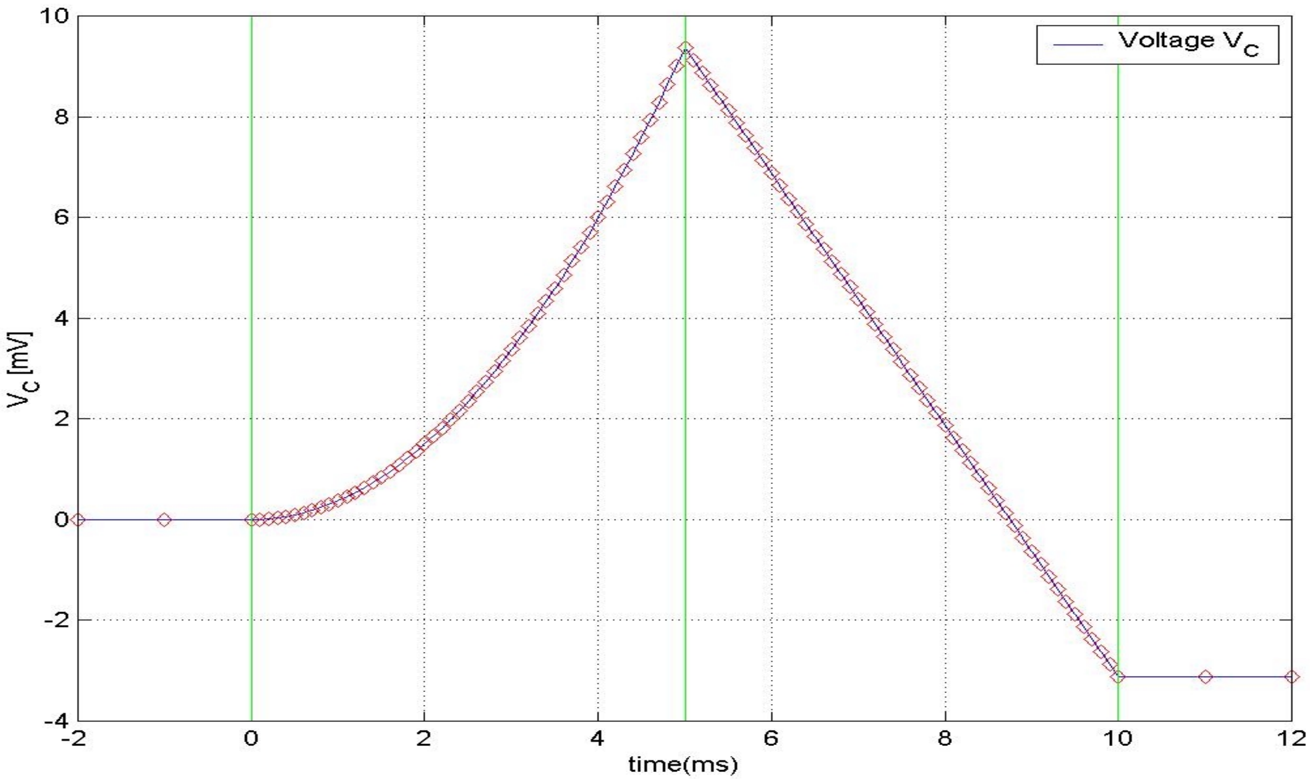
Now, for a formal way to represent piecewise functions....

GEAUX

Formal description of a piecewise analytical signal

$$V_c(t) = \begin{cases} 0; & t \le 0 \\ \frac{3}{8}t^2; & 0 < t < 5ms \\ \frac{75}{8} - \frac{10}{4}(t-5); & 5 < t \le 10\ [ms] \\ -\frac{25}{8}; & t > 10[ms] \end{cases}$$

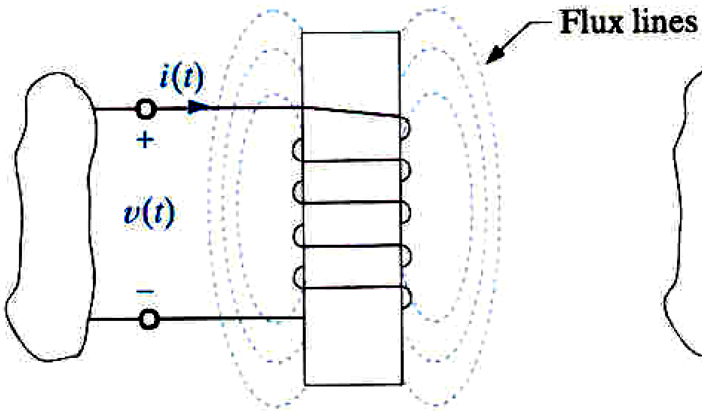
$[mV]$

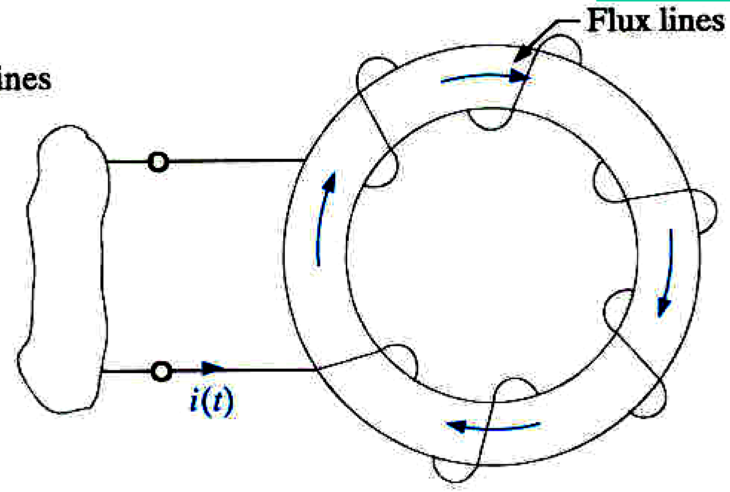


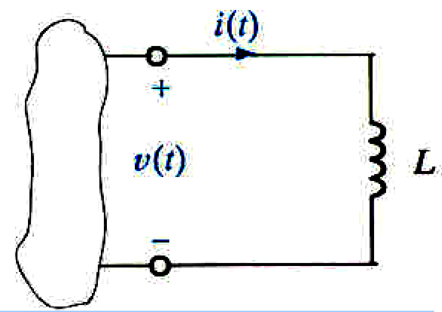
GEAUX

INDUCTORS

NOTICE USE OF PASSIVE SIGN CONVENTION

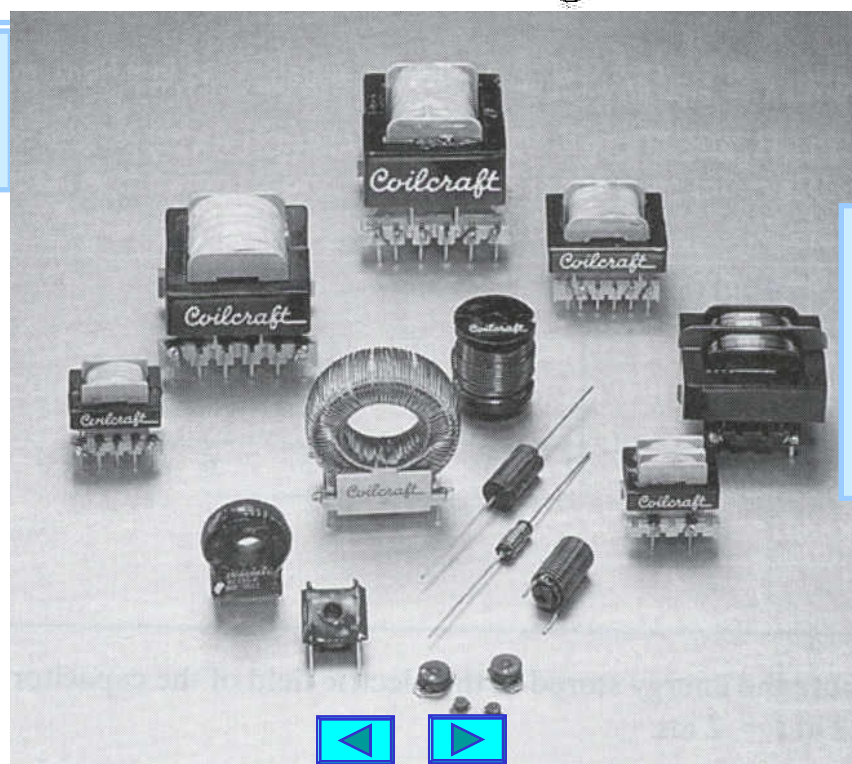






Circuit representation for an inductor

Flux lines may extend beyond inductor creating stray inductance effects



A TIME VARYING FLUX CREATES A COUNTER EMF AND CAUSES A VOLTAGE TO APPEAR AT THE TERMINALS OF THE DEVICE

A TIME VARYING MAGNETIC FLUX INDUCES A VOLTAGE

$$v_L = \frac{d\phi}{dt}$$

Induction law

FOR A LINEAR INDUCTOR THE FLUX IS PROPORTIONAL TO THE CURRENT

$$\phi = Li_L \Rightarrow \quad v_L = L\frac{di_L}{dt}$$

DIFFERENTIAL FORM OF INDUCTION LAW

THE PROPORTIONALITY CONSTANT, L, IS CALLED THE INDUCTANCE OF THE COMPONENT

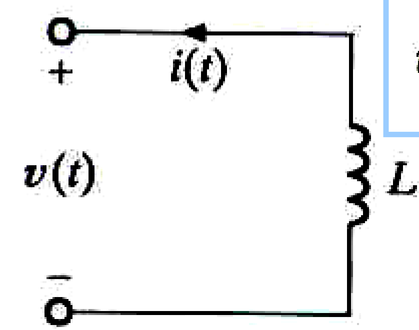
INDUCTANCE IS MEASURED IN UNITS OF henry (H). DIMENSIONALLY

$$\text{HENRY} = \frac{\text{Volt}}{\text{Amp}/\text{sec}}$$

INDUCTORS STORE ELECTROMAGNETIC ENERGY. THEY MAY SUPPLY STORED ENERGY BACK TO THE CIRCUIT BUT THEY CANNOT CREATE ENERGY. THEY MUST ABIDE BY THE PASSIVE SIGN CONVENTION

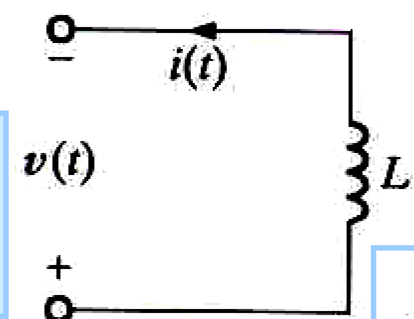
LEARNING by Doing

Write the i-v relationship for the following inductors.



(a)

$$v(t) = -L\frac{di(t)}{dt}$$

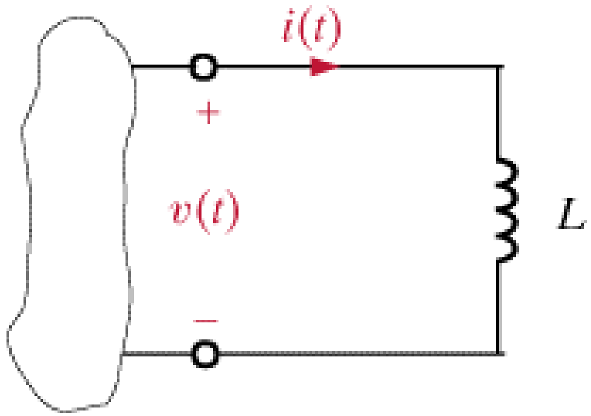


(b)

$$v(t) = L\frac{di(t)}{dt}$$

Follow passive sign convention

GEAUX



$$v_L = L\frac{di_L}{dt}$$

Differential form of induction law

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(x)dx$$

Integral form of induction law

$$i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(x)dx; \; t \ge t_0$$

A direct consequence of integral form $i_L(t-) = i_L(t+); \;\; \forall t$ Current MUST be continuous

A direct consequence of differential form $i_L = Const. \Rightarrow v_L = 0$ DC (steady state) behavior

Power and Energy stored

$$p_L(t) = v_L(t)i_L(t)$$ W

$$p_L(t) = L\frac{di_L}{dt}(t)i_L(t) = \frac{d}{dt}\left(\frac{1}{2}Li_L^2(t)\right)$$

$$w_L(t_2,t_1) = \int_{t_1}^{t_2}\frac{d}{dt}\left(\frac{1}{2}Li_L^2(x)\right)dx$$ J

Current in Amps, Inductance in Henrys
yield energy in Joules

$$w(t_2,t_1) = \frac{1}{2}Li_L^2(t_2) - \frac{1}{2}Li_L^2(t_1)$$

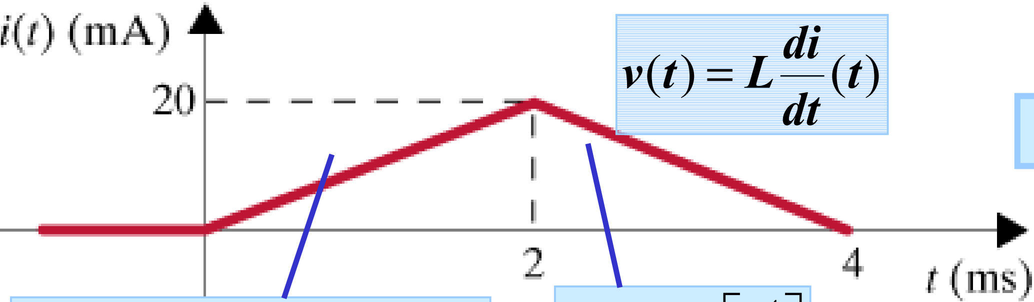
Energy stored on the interval
Can be positive or negative

$$w_L(t) = \frac{1}{2}Li_L^2(t)$$

"Energy stored at time t"
Must be non-negative.
Passive element!!!

LEARNING EXAMPLE

L=10mH. FIND THE VOLTAGE



$$v(t) = L\frac{di}{dt}(t)$$

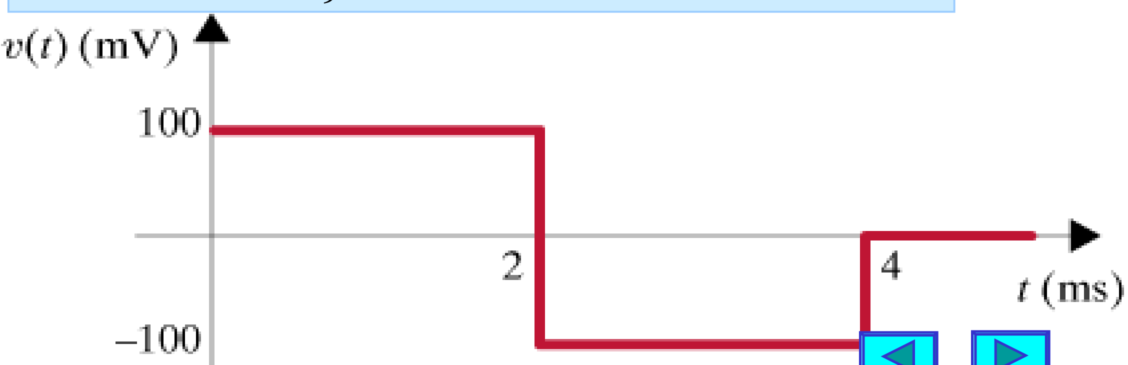
$$m = \frac{20\times10^{-3}A}{2\times10^{-3}s} = 10\left[\frac{A}{s}\right]$$

$$m = -10\left[\frac{A}{s}\right]$$

THE DERIVATIVE OF A STRAIGHT LINE IS ITS SLOPE

$$\frac{di}{dt} = \begin{cases} 10(A/s) & 0 \le t \le 2ms \\ -10(A/s) & 2 < t \le 4ms \\ 0 & elsewhere \end{cases}$$

$$\left.\begin{array}{l} \frac{di}{dt}(t) = 10(A/s) \\ L = 10\times10^{-3}H \end{array}\right\} \Rightarrow v(t) = 100\times10^{-3}V = 100mV$$



ENERGY STORED BETWEEN 2 AND 4 ms

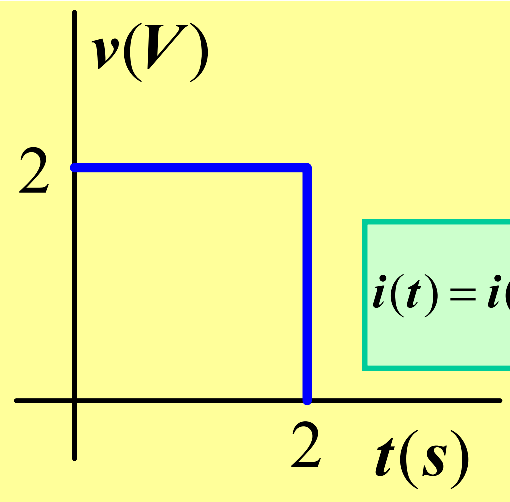
$$w(4,2) = \frac{1}{2}Li_L^2(4) - \frac{1}{2}Li_L^2(2)$$

$$w(4,2) = 0 - 0.5*10*10^{-3}(20*10^{-3})^2 \text{ J}$$

THE VALUE IS NEGATIVE BECAUSE THE INDUCTOR IS SUPPLYING ENERGY PREVIOUSLY STORED

GEAUX

SAMPLE PROBLEM *L=0.1H, i(0)=2A*. Find *i(t), t>0*

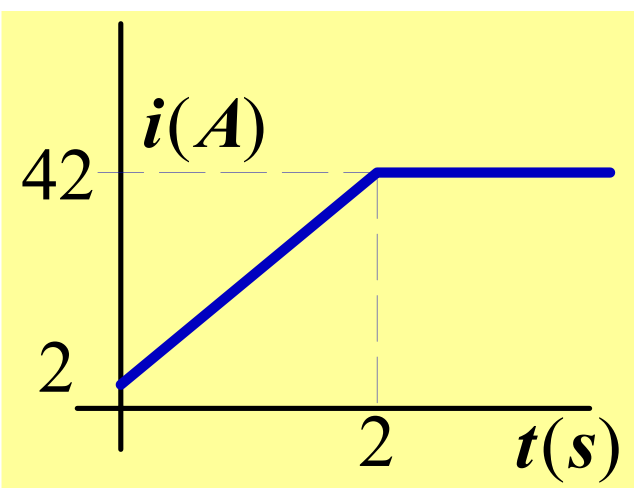


$$i(t) = i(0) + \frac{1}{L}\int_0^t v(x)dx$$

$$v(x) = 2 \Rightarrow \int_0^t v(x)dx = 2t; \quad 0 < t \le 2$$

$$L = 0.1H \Rightarrow i(t) = 2 + 20t;\ 0 \le t \le 2s$$

$$v(x) = 0;\ t > 2 \Rightarrow i(t) = i(2);\ t > 2s$$



ENERGY COMPUTATIONS

$$w(t_2, t_1) = \frac{1}{2}Li_L^2(t_2) - \frac{1}{2}Li_L^2(t_1)$$

Energy stored on the interval
Can be positive or negative

Initial energy stored in inductor

$$w(0) = 0.5 * 0.1[H](2A)^2 = 0.2[J]$$

"Total energy stored in the inductor"

$$w(\infty) = 0.5 * 0.1[H] * (42A)^2 = 88.2J$$

Energy stored between 0 and 2 sec

$$w(2,0) = \frac{1}{2}Li_L^2(2) - \frac{1}{2}Li_L^2(0)$$

$$w(2,0) = 0.5 * 0.1 * (42)^2 - 0.5 * 0.1 * (2)^2$$

$$w(2,0) = 88[J]$$

GEAUX

LEARNING EXAMPLE

The current in a 2-mH inductor is
$$i(t) = 2 \sin 377t \text{ A}$$

FIND THE VOLTAGE ACROSS AND THE ENERGY STORED (AS FUNCTION OF TIME)

$$v(t) = L\frac{di(t)}{dt} = (2 \times 10^{-3})\frac{d}{dt}(2 \sin 377t)$$

$$v(t) = 1.508 \cos 377t \text{ V}$$

FOR ENERGY STORED IN THE INDUCTOR

$$w_L(t) = \frac{1}{2}Li^2(t)$$

$$w_L(t) = \frac{1}{2}(2 \times 10^{-3})(2 \sin 377t)^2$$

$$= 0.004 \sin^2 377t$$

NOTICE THAT ENERGY STORED AT ANY GIVEN TIME IS NON NEGATIVE
-THIS IS A PASSIVE ELEMENT-

GEAUX

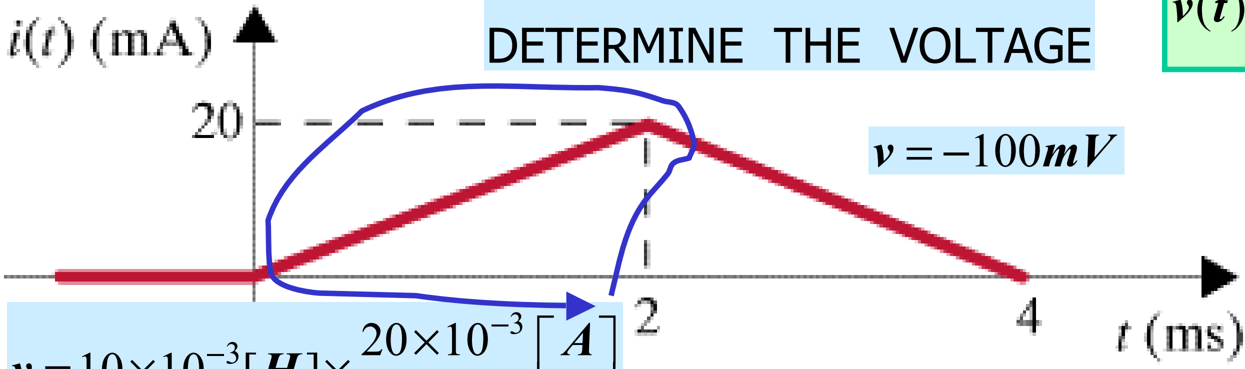
LEARNING EXAMPLE

$L = 10mH$

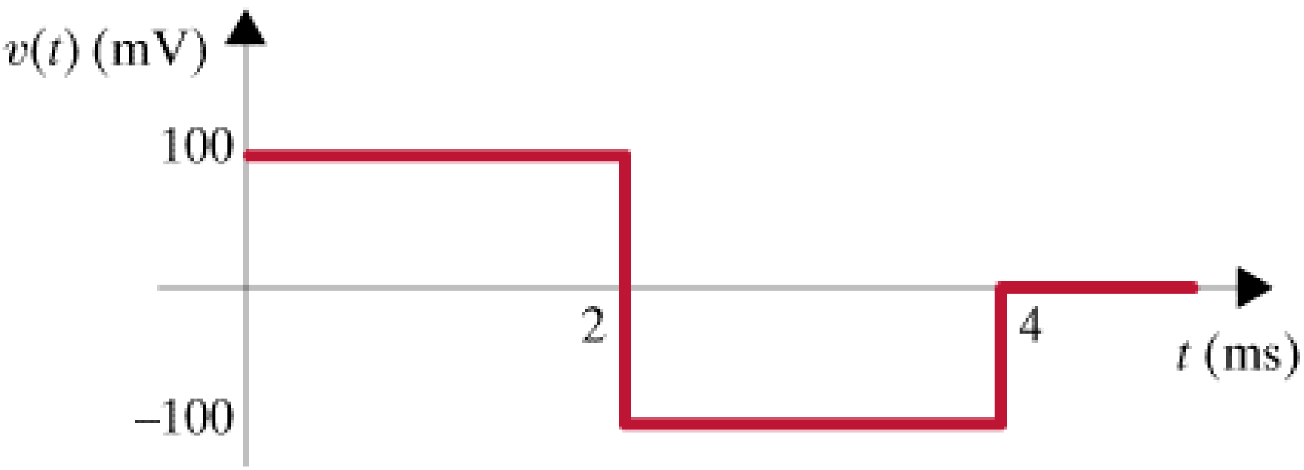
DETERMINE THE VOLTAGE

$$v(t) = L\frac{di}{dt}(t)$$

$v = -100mV$

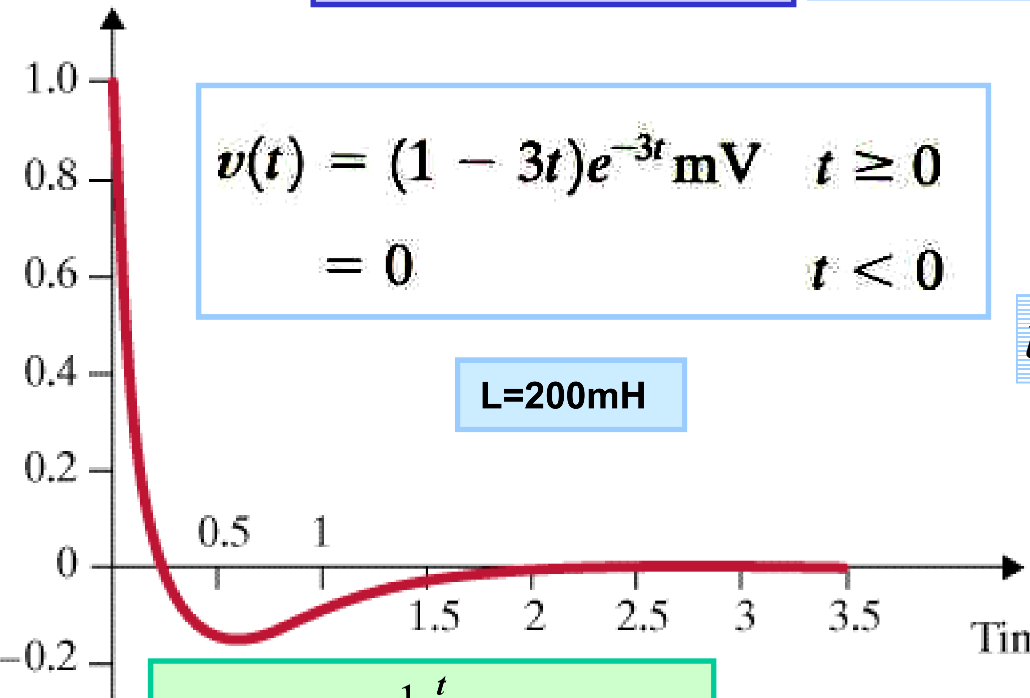


$$v = 10\times10^{-3}[H]\times\frac{20\times10^{-3}}{2\times10^{-3}}\left[\frac{A}{s}\right]$$



GEAUX

LEARNING EXAMPLE **FIND THE CURRENT**



$$v(t) = (1 - 3t)e^{-3t}\ \text{mV} \quad t \geq 0$$
$$= 0 \quad t < 0$$

L=200mH

$$i(t) = i(0) + \frac{1}{L}\int_0^t v(x)dx;\ t > 0$$

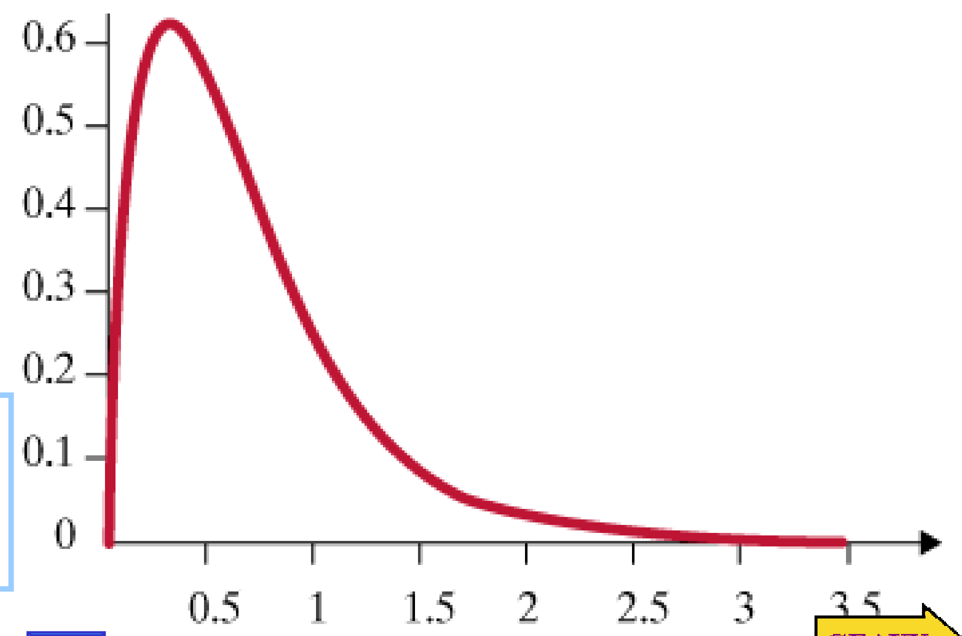
$$v(t) = 0;\ t < 0 \Rightarrow i(0) = 0$$

$$i(t) = \frac{10^3}{200}\int_0^t (1 - 3x)e^{-3x}\,dx$$

$$= 5\left\{\int_0^t e^{-3x}\,dx - 3\int_0^t xe^{-3x}\,dx\right\}$$

$$i(t) = 5\left\{\frac{e^{-3x}}{-3}\Big|_0^t - 3\left[-\frac{e^{-3x}}{9}(3x + 1)\right]_0^t\right\}$$

$$i(t) \begin{cases} = 5te^{-3t}\ \text{mA} & t \geq 0 \\ = 0 & t < 0 \end{cases}$$



GEAUX

L=200mH

$$v(t) = (1 - 3t)e^{-3t}\,\text{mV} \quad t \geq 0$$
$$= 0 \quad t < 0$$

$$i(t) = 5te^{-3t}\,\text{mA} \quad t \geq 0$$
$$= 0 \quad t < 0$$

POWER $p(t) = v(t)i(t)$

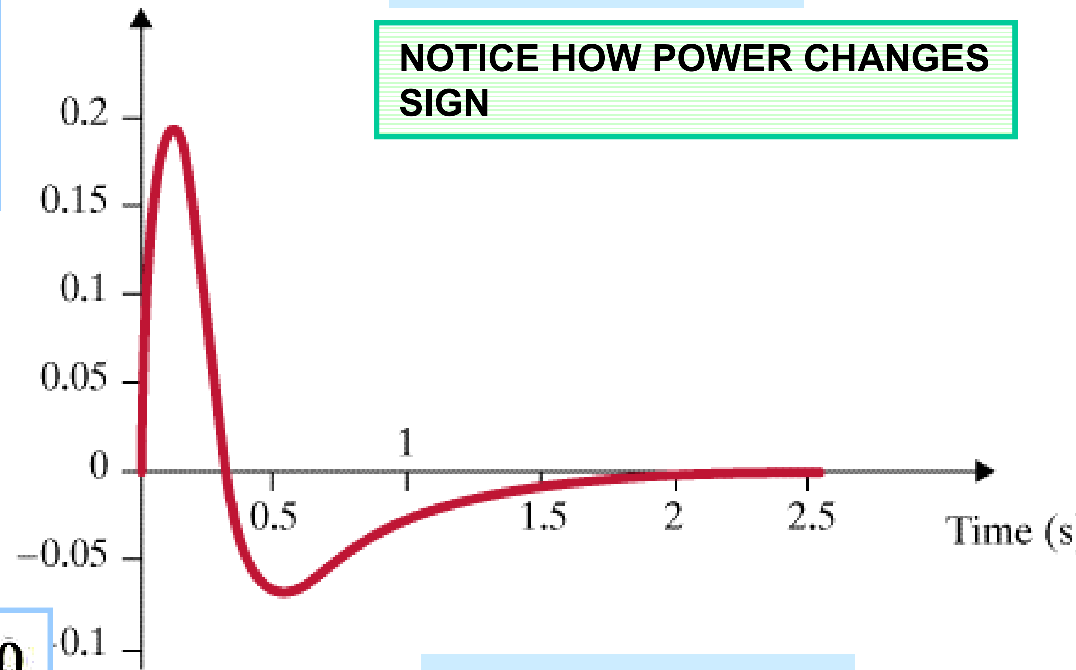
$$p(t) = 5t(1 - 3t)e^{-6t}\,\mu\text{W} \quad t \geq 0$$
$$= 0 \quad t < 0$$

ENERGY $w(t) = \frac{1}{2}Li^2(t)$

$$w(t) = 2.5t^2 e^{-6t}\,\mu\text{J} \quad t \geq 0$$
$$= 0 \quad t < 0$$

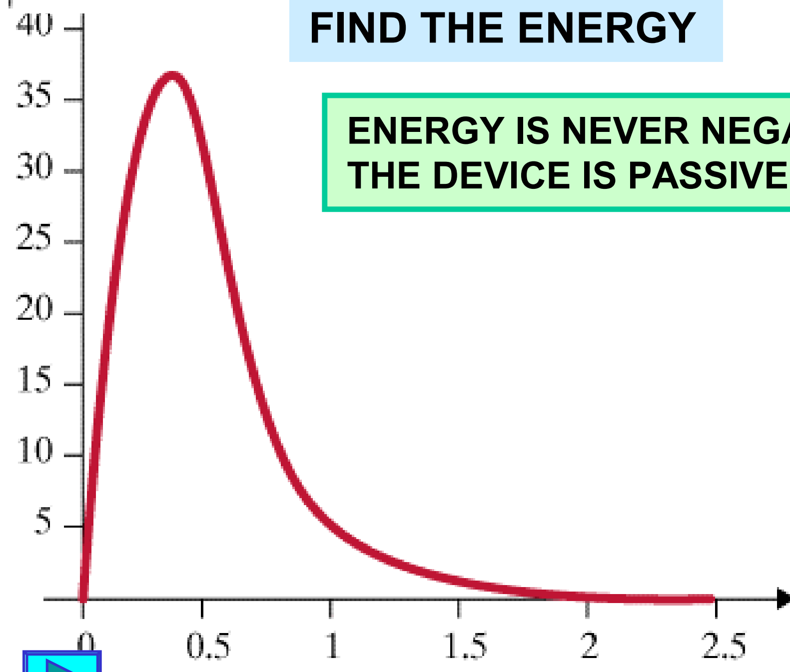
FIND THE POWER

NOTICE HOW POWER CHANGES SIGN



FIND THE ENERGY

ENERGY IS NEVER NEGATIVE. THE DEVICE IS PASSIVE

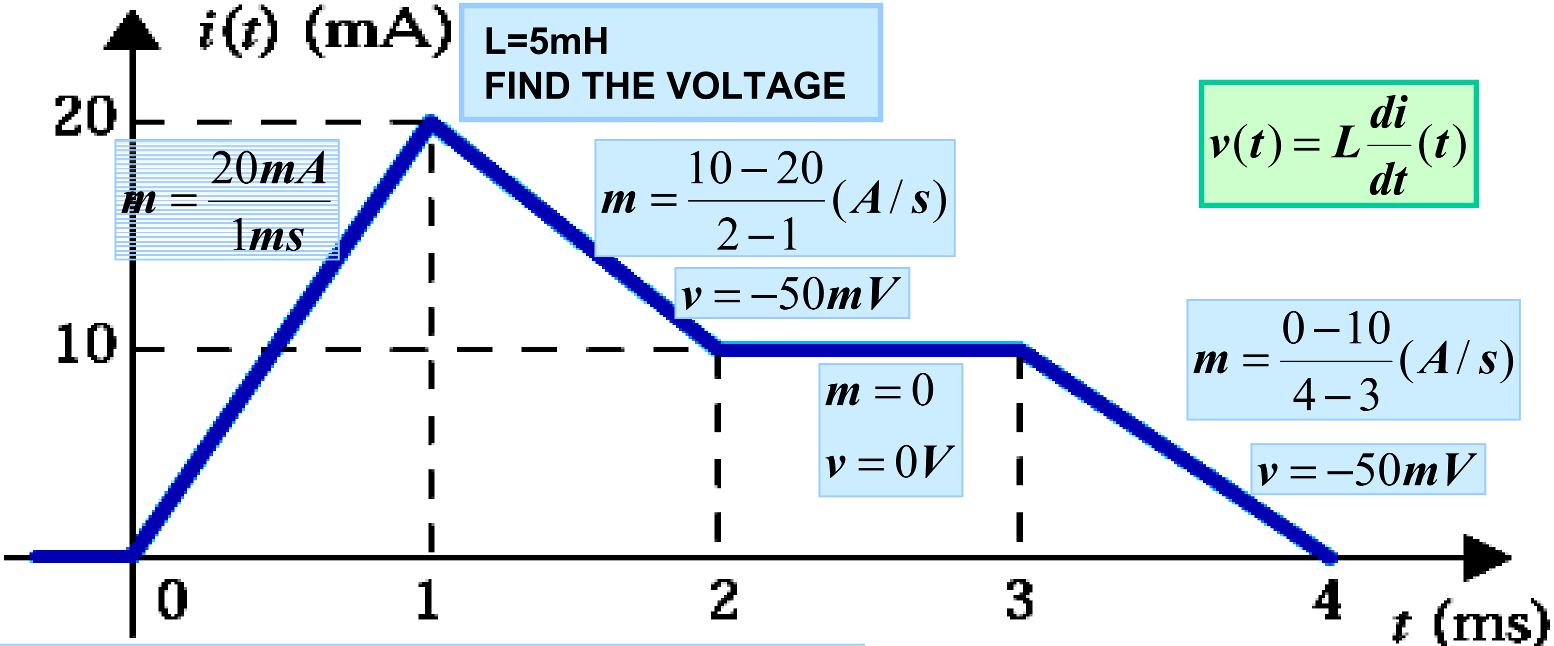


GEAUX

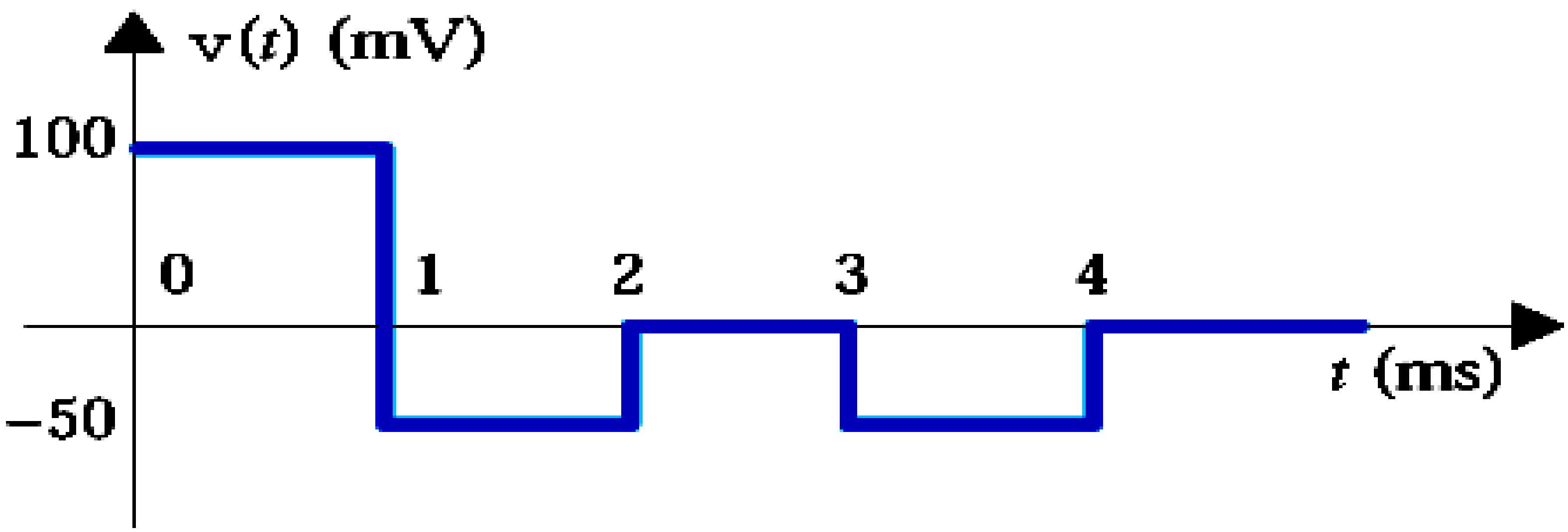
LEARNING EXTENSION

L=5mH
FIND THE VOLTAGE

$$v(t) = L\frac{di}{dt}(t)$$



$$v = 5\times10^{-3}(H)\times 20(A/s);\ 0 \le t < 1ms = 100mV$$



GEAUX

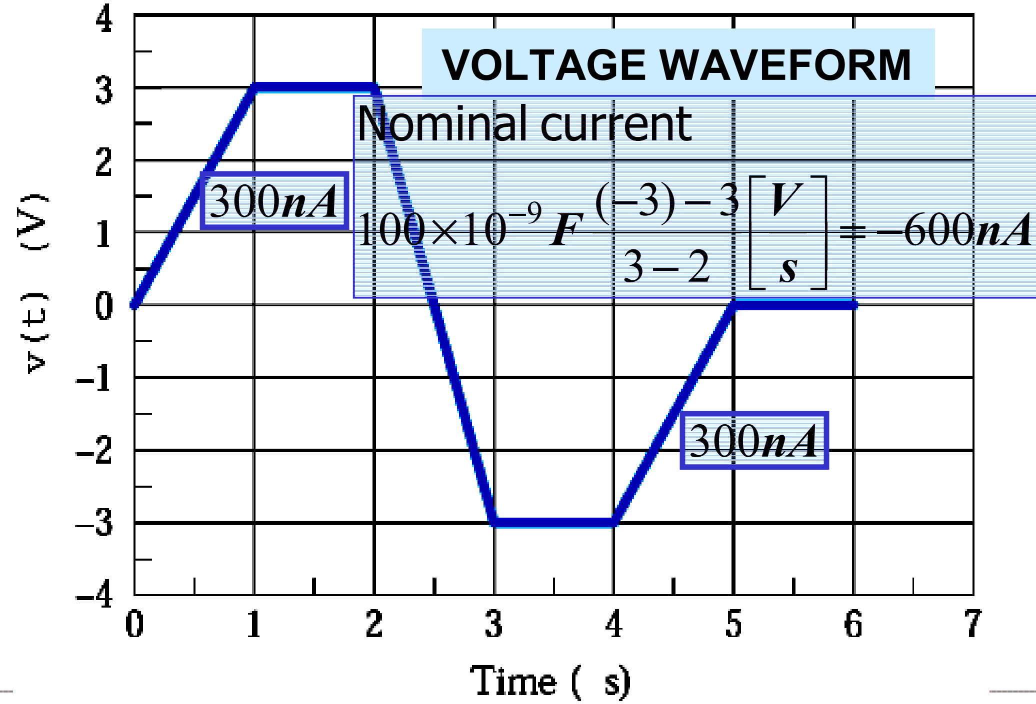
CAPACITOR SPECIFICATIONS

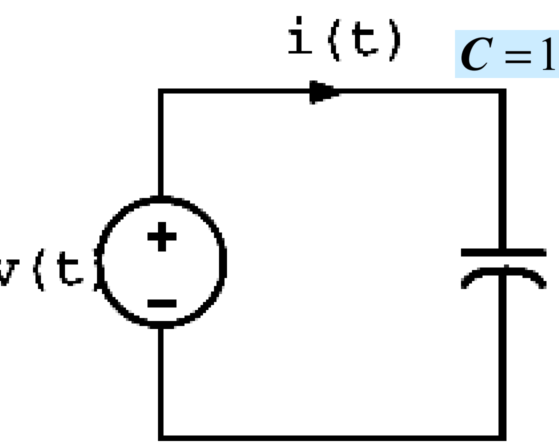
CAPACITANCE RANGE $pF \approx C \approx 50mF$
IN STANDARD VALUES

STANDARD CAPACITOR RATINGS
$6.3V \quad - \quad 500V$

STANDARD TOLERANCE
$\pm 5\%, \pm 10\%, \pm 20\%$

LEARNING EXAMPLE

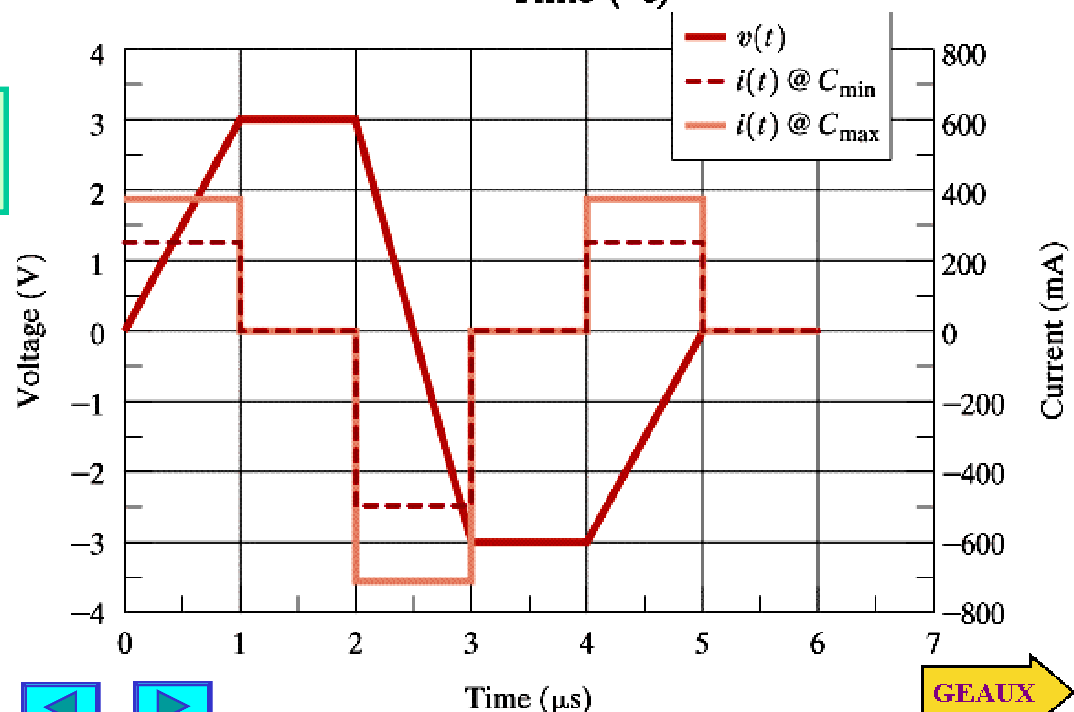




$C = 100nF \pm 20\%$

$$i(t) = C\frac{dv}{dt}(t)$$

GIVEN THE VOLTAGE WAVEFORM DETERMINE THE VARIATIONS IN CURRENT



INDUCTOR SPECIFICATIONS

INDUCTANCE RANGES $\approx 1nH \leq L \leq \approx 100$

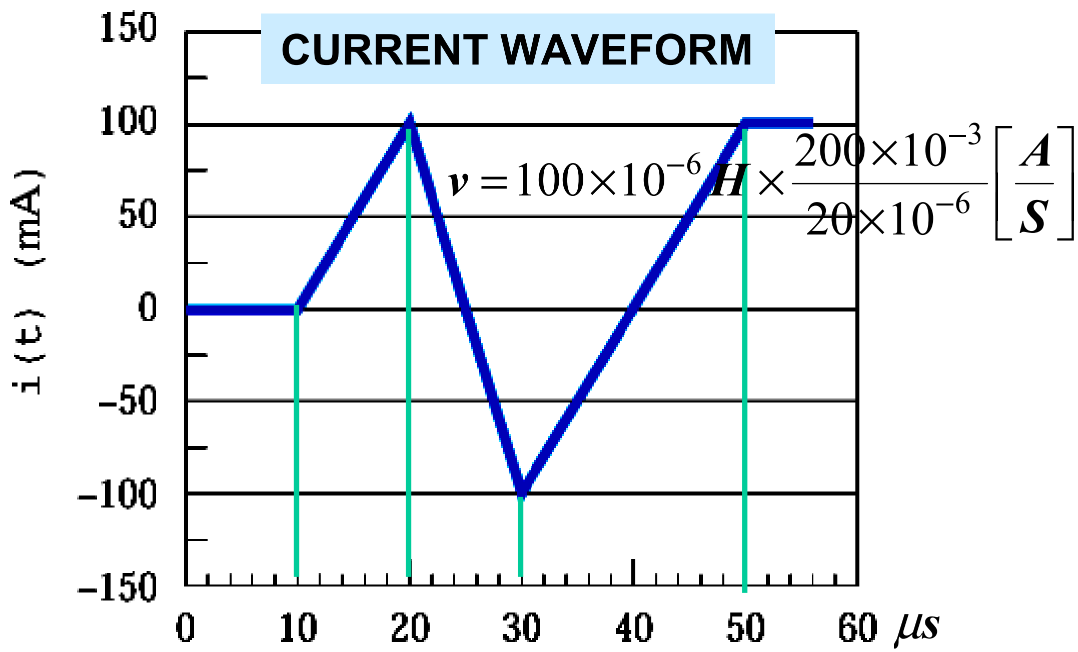
IN STANDARD VALUES

STANDARD INDUCTOR RATINGS

$\approx mA \quad - \quad \approx 1A$

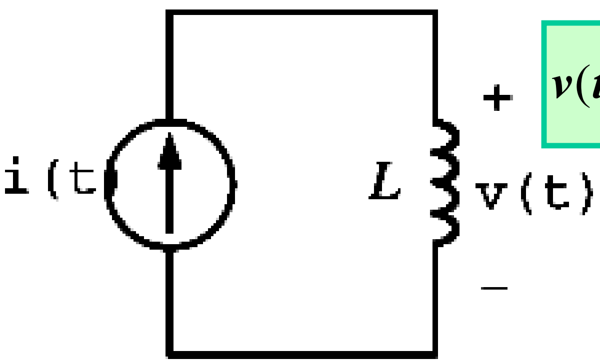
STANDARD TOLERANCE

$\pm 5\%, \pm 10\%$



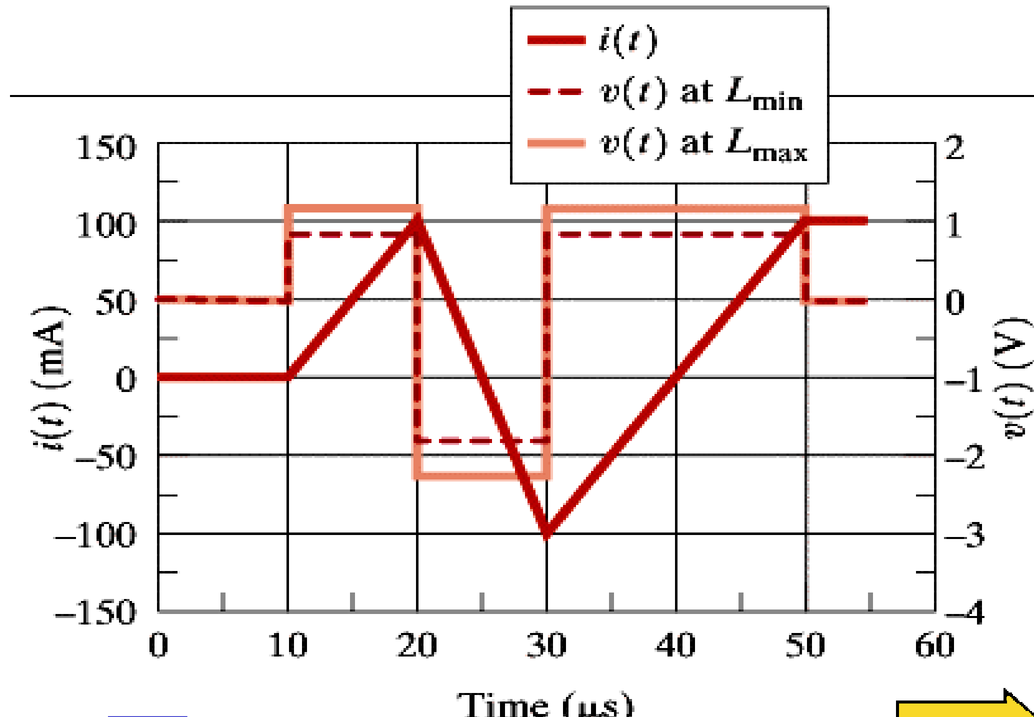
LEARNING EXAMPLE

$L = 100\mu H \pm 10\%$



$$v(t) = L\frac{di}{dt}(t)$$

GIVEN THE CURRENT WAVEFORM DETERMINE THE VARIATIONS IN VOLTAGE



GEAUX

The Dual Relationship for Capacitors and Inductors

Capacitor	Inductor
$i(t) = C\dfrac{dv(t)}{dt}$	$v(t) = L\dfrac{di(t)}{dt}$
$v(t) = \dfrac{1}{C}\int_{t_0}^{t} i(x)\,dx + v(t_0)$	$i(t) = \dfrac{1}{L}\int_{t_0}^{t} v(x)\,dx + i(t_0)$
$p(t) = Cv(t)\dfrac{dv(t)}{dt}$	$p(t) = Li(t)\dfrac{di(t)}{dt}$
$w(t) = \frac{1}{2}Cv(t)^2$	$w(t) = \frac{1}{2}Li^2(t)$

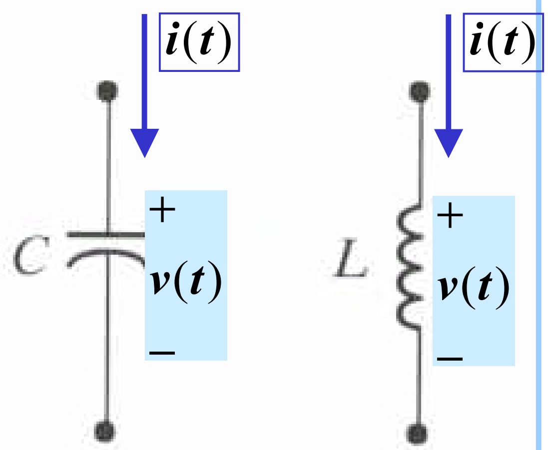
$C \rightarrow L$

$v \rightarrow i$

$i \rightarrow v$

GEAUX

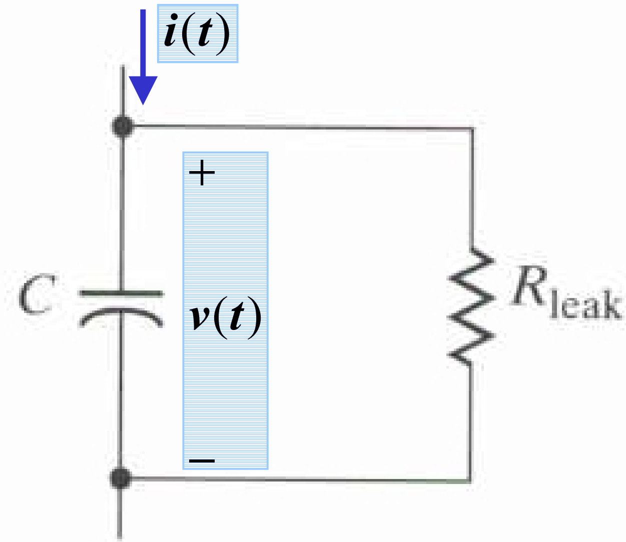
IDEAL AND PRACTICAL ELEMENTS

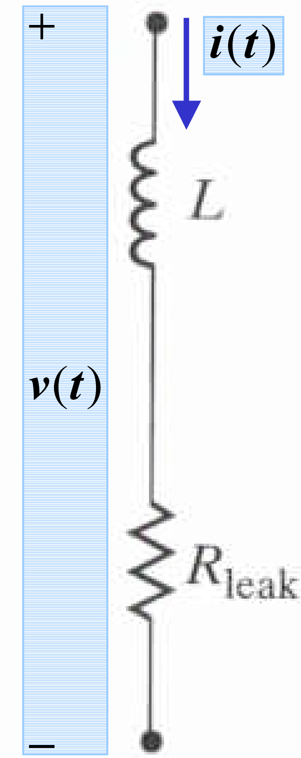


IDEAL ELEMENTS

$$i(t) = C\frac{dv}{dt}(t)$$

$$v(t) = L\frac{di}{dt}(t)$$





CAPACITOR/INDUCTOR MODELS INCLUDING LEAKAGE RESISTANCE

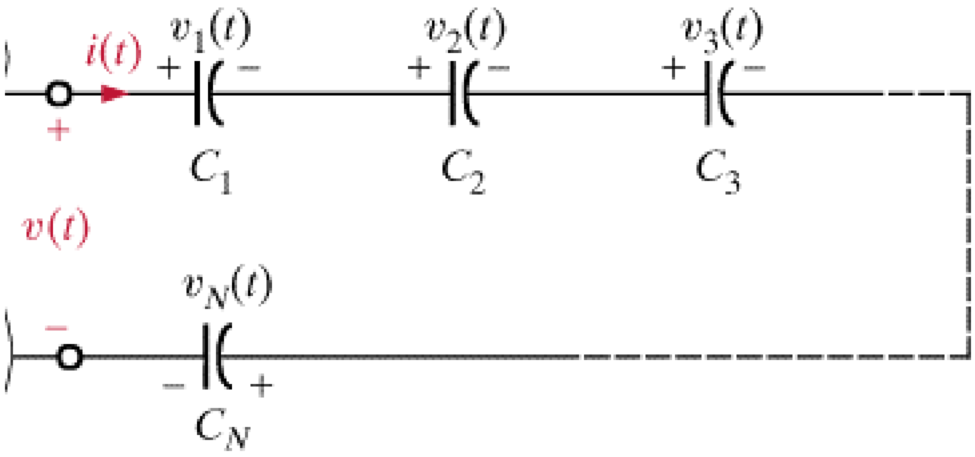
$$i(t) = \frac{v(t)}{R_{leak}} + C\frac{dv}{dt}(t)$$

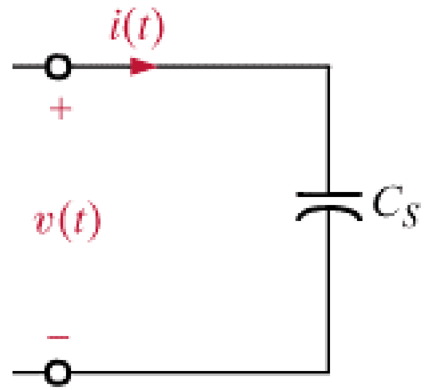
MODEL FOR “LEAKY” CAPACITOR

$$v(t) = R_{leak}\,i(t) + L\frac{di}{dt}(t)$$

MODEL FOR “LEAKY” INDUCTORS

SERIES CAPACITORS





$$v(t) = v_1(t) + v_2(t) + v_3(t) + \cdots + v_N(t)$$

$$v_i(t) = \frac{1}{C_i}\int_{t_0}^{t} i(t)\,dt + v_i(t_0)$$

$$v(t) = \left(\sum_{i=1}^{N}\frac{1}{C_i}\right)\int_{t_0}^{t} i(t)\,dt + \sum_{i=1}^{N} v_i(t_0)$$

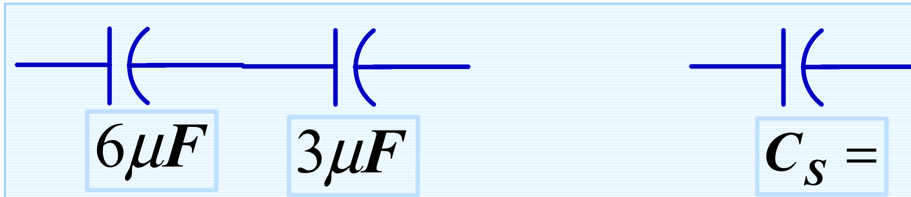
$$\frac{1}{C_S} = \sum_{i=1}^{N}\frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}$$

$$v(t_0) = \sum_{i=1}^{N} v_i(t_0)$$

NOTICE SIMILARITY WITH RESITORS IN PARALLEL

$$C_s = \frac{C_1 C_2}{C_1 + C_2}$$

Series Combination of two capacitors

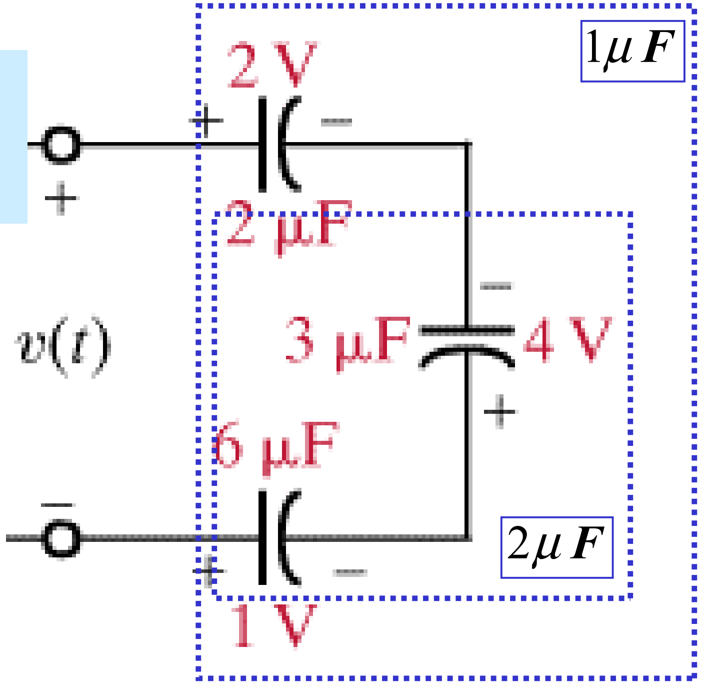


$2\mu F$

GEAUX

LEARNING EXAMPLE

DETERMINE EQUIVALENT CAPACITANCE AND THE INITIAL VOLTAGE



$$\frac{1}{C_s} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6}$$

OR WE CAN REDUCE TWO AT A TIME

$$v(t_0) = -3\text{ V} = +2V - 4V - 1V$$

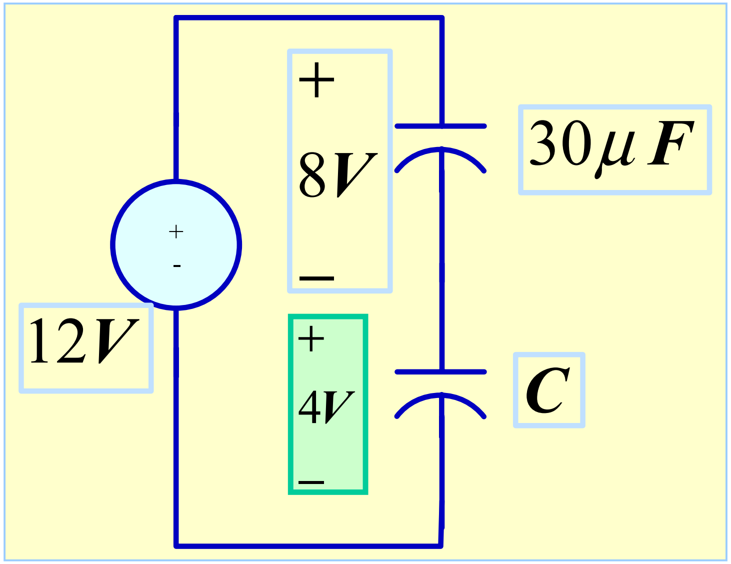
ALGEBRAIC SUM OF INITIAL VOLTAGES

POLARITY IS DICTATED BY THE REFERENCE DIRECTION FOR THE VOLTAGE

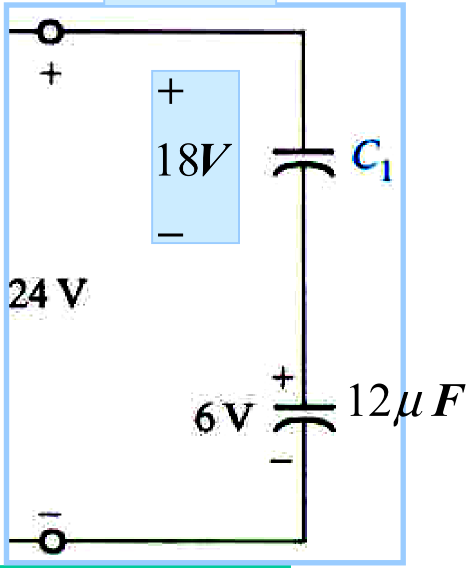
GEAUX

LEARNING EXAMPLE

Two uncharged capacitors are connected as shown.
Find the unknown capacitance



FIND C_1



SAME CURRENT. CONNECTED FOR THE SAME TIME PERIOD

SAME CHARGE ON BOTH CAPACITORS

$$Q = (30\mu F)(8V) = 240\mu C$$

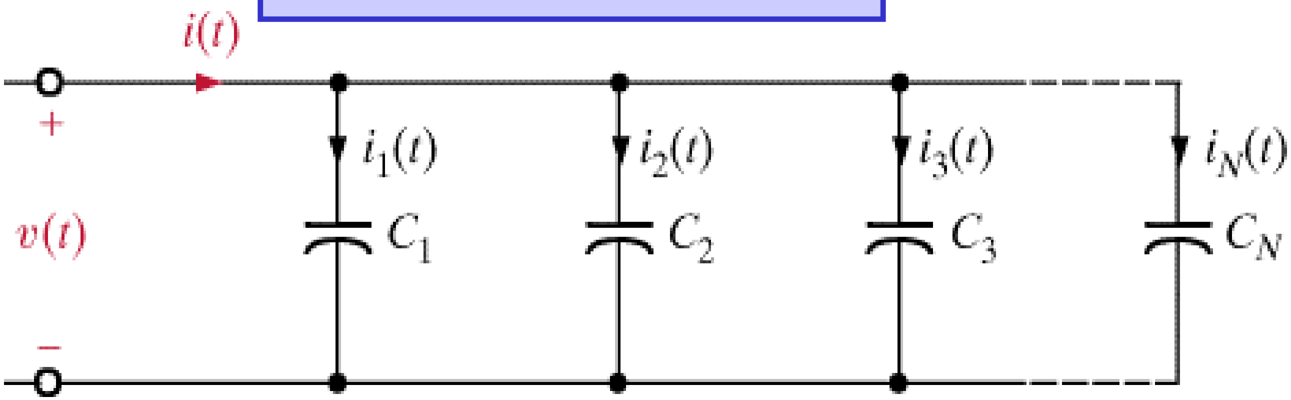
$$C = \frac{Q}{V} = \frac{240\mu C}{4V} = 60\mu F$$

$$Q = CV \Rightarrow Q = (12\mu F)(6V) = 72\mu C$$

$$C_1 = \frac{72\mu C}{18V} = 4\mu F$$

GEAUX

PARALLEL CAPACITORS



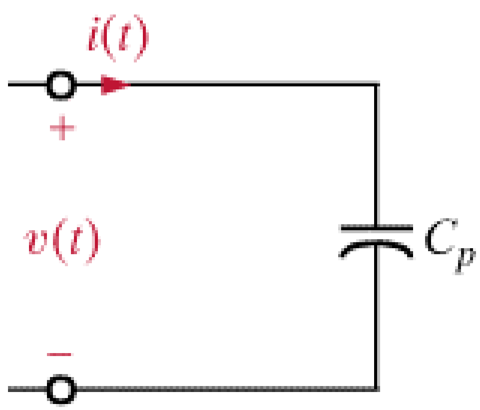
$$i(t) = i_1(t) + i_2(t) + i_3(t) + \cdots + i_N(t)$$

$$i_k(t) = C_k \frac{dv}{dt}(t)$$

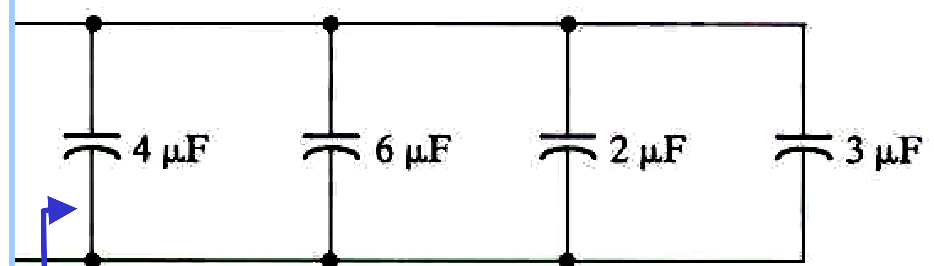
$$i(t) = C_1 \frac{dv(t)}{dt} + C_2 \frac{dv(t)}{dt} + C_3 \frac{dv(t)}{dt} + \cdots + C_N \frac{dv(t)}{dt}$$

$$= \left(\sum_{i=1}^{N} C_i \right) \frac{dv(t)}{dt}$$

$$C_p = C_1 + C_2 + C_3 + \cdots + C_N$$



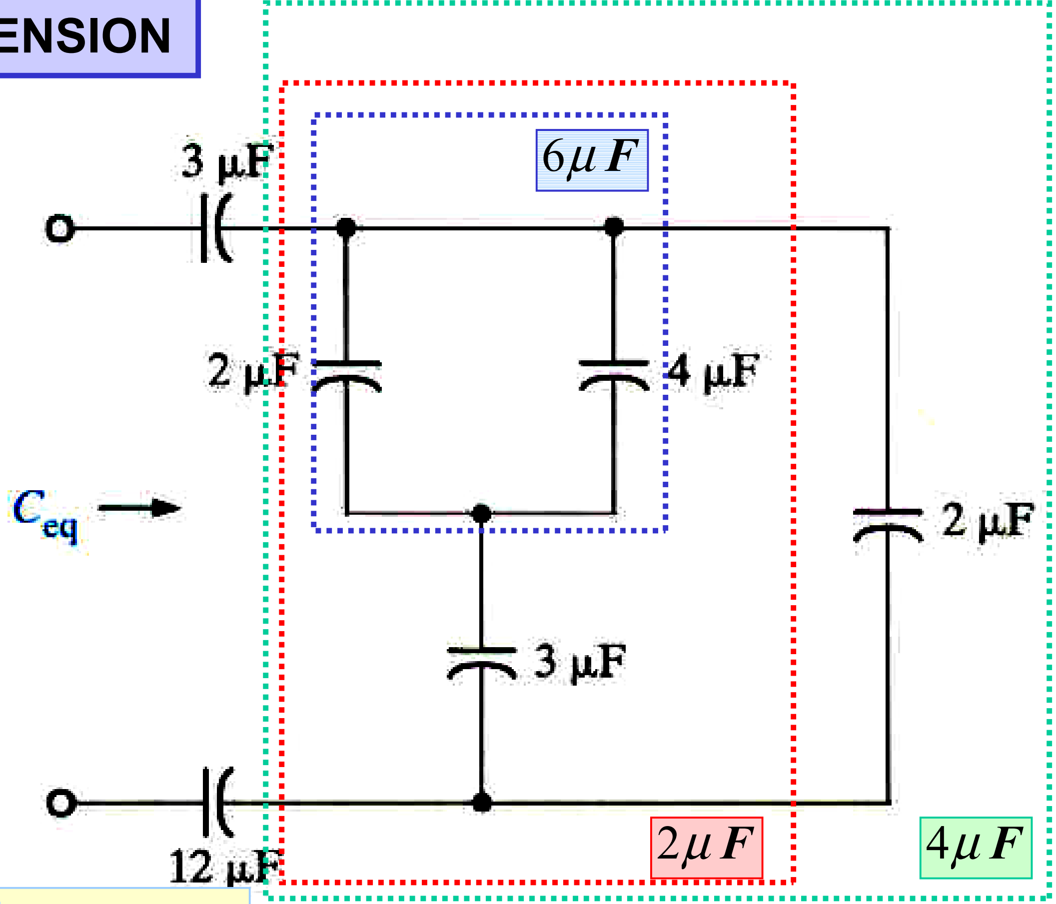
LEARNING EXAMPLE

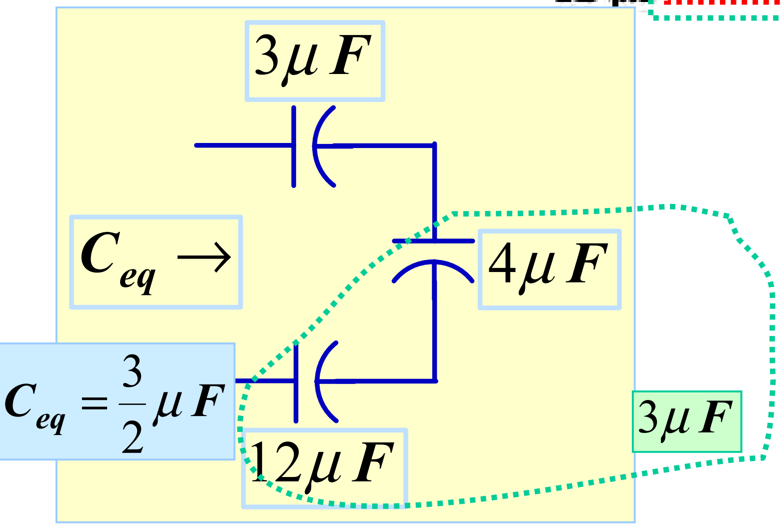


$$C_P = 4 + 6 + 2 + 3 = 15 \mu F$$

GEAUX

LEARNING EXTENSION



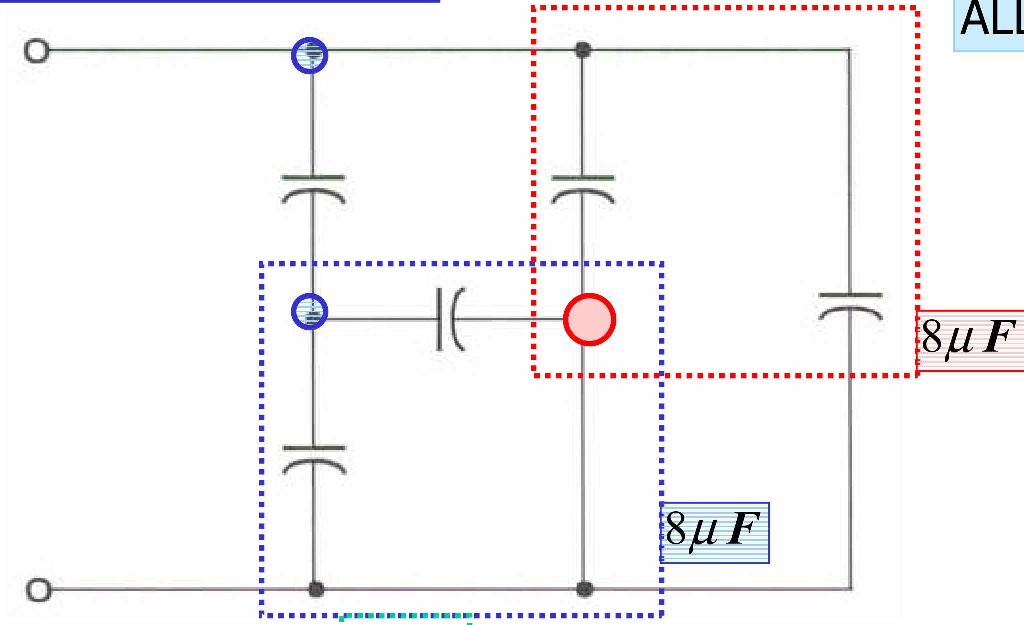


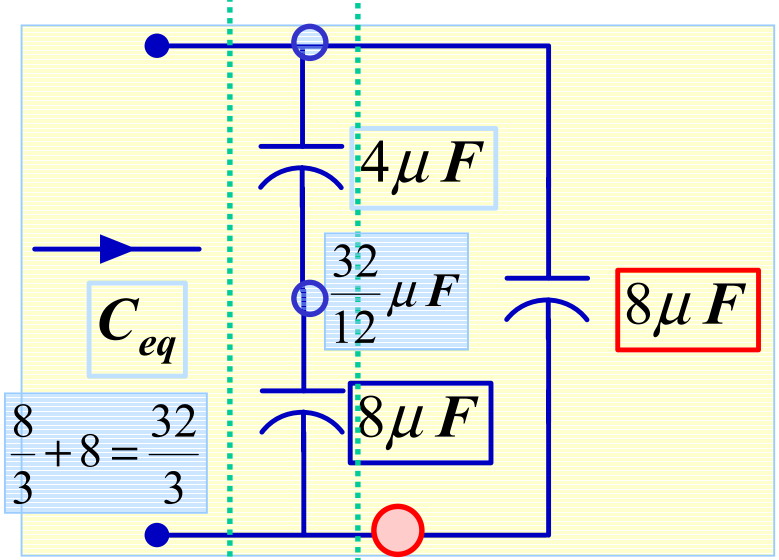
GEAUX

SAMPLE PROBLEM

FIND EQUIVALENT CAPACITANCE

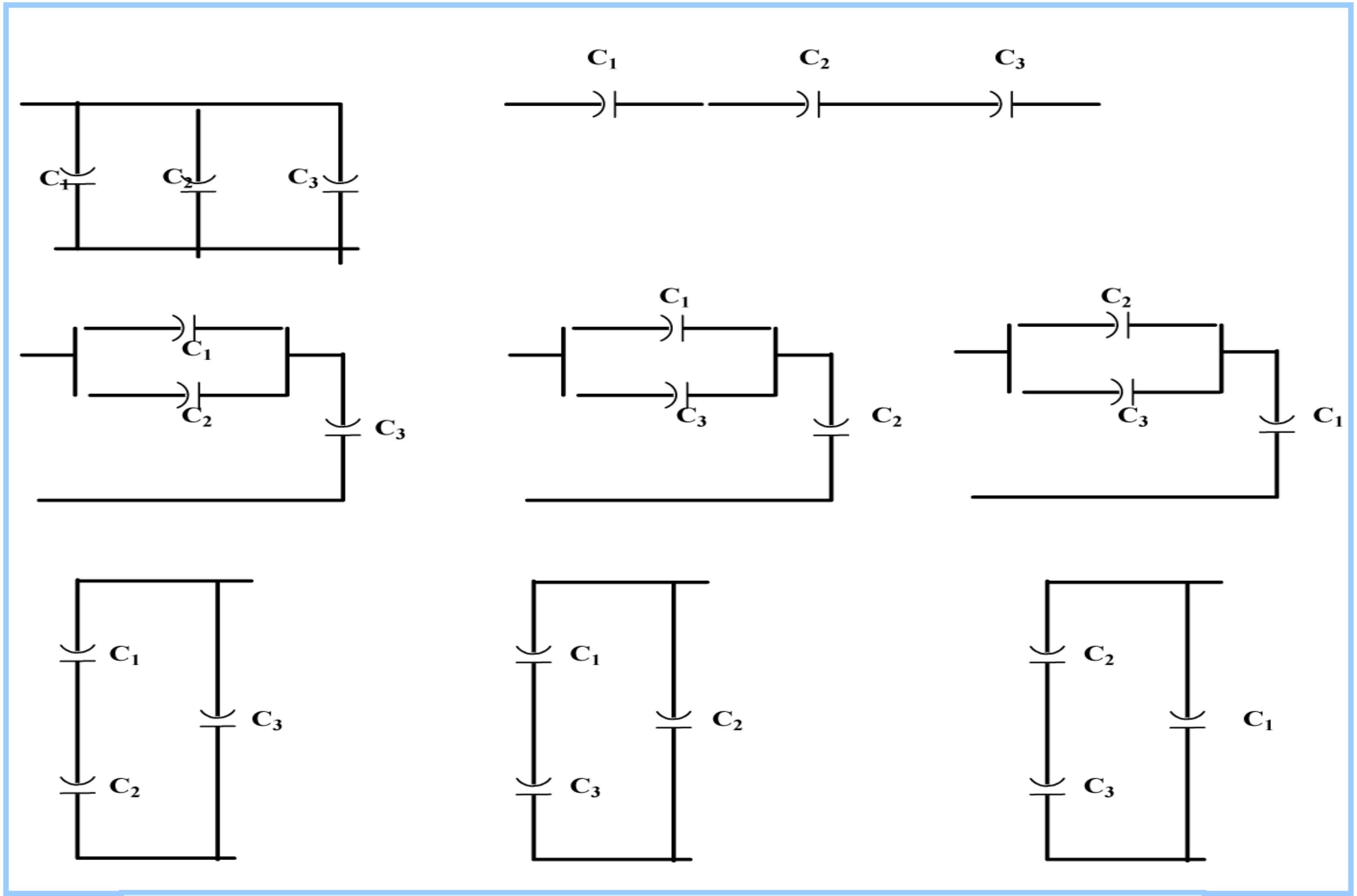
ALL CAPACITORS ARE 4μF





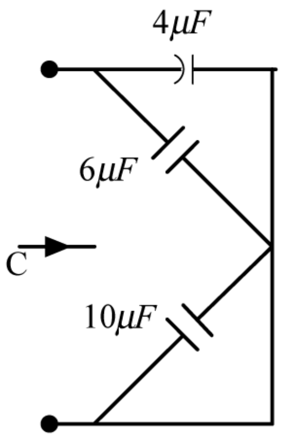
GEAUX

SAMPLE PROBLEM

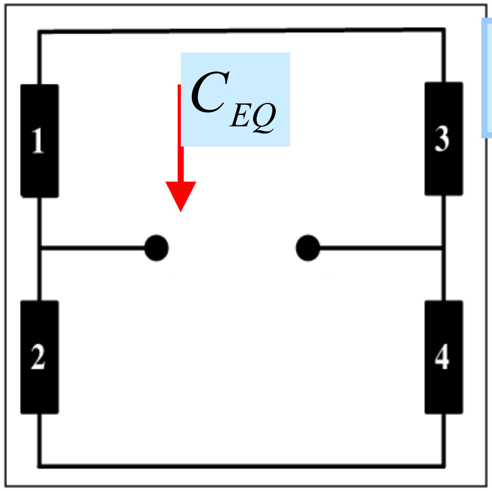


IF ALL CAPACITORS HAVE THE SAME CAPACITANCE VALUE *C* DETERMINE THE VARIOUS EQUIVALENT CAPACITANCES

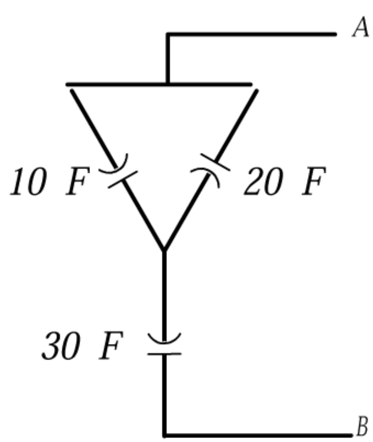
Examples of interconnections



C = ________



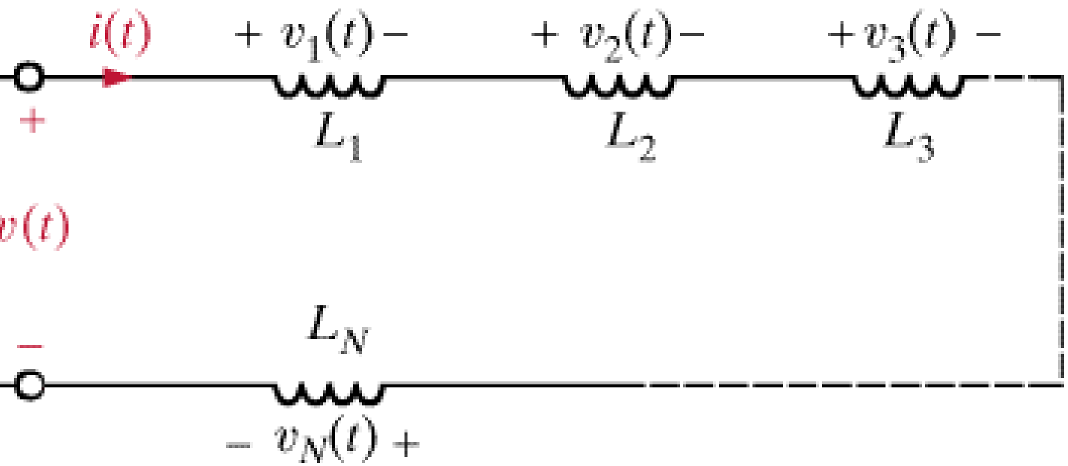
All capacitors are equal with C=8 microFarads

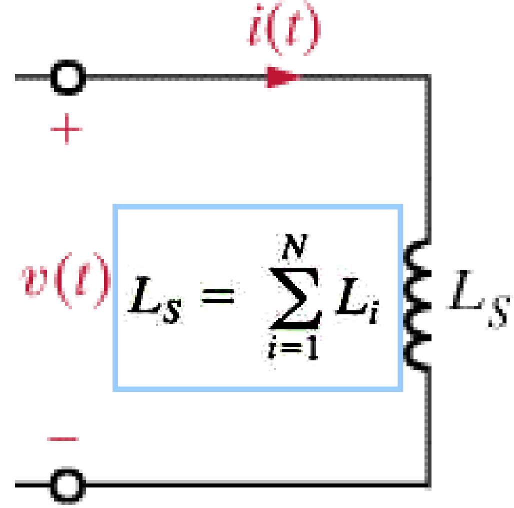


C_{AB} = ________

GEAUX

SERIES INDUCTORS





$$v(t) = v_1(t) + v_2(t) + v_3(t) + \cdots + v_N(t)$$

$$v_k(t) = L_k \frac{di}{dt}(t)$$

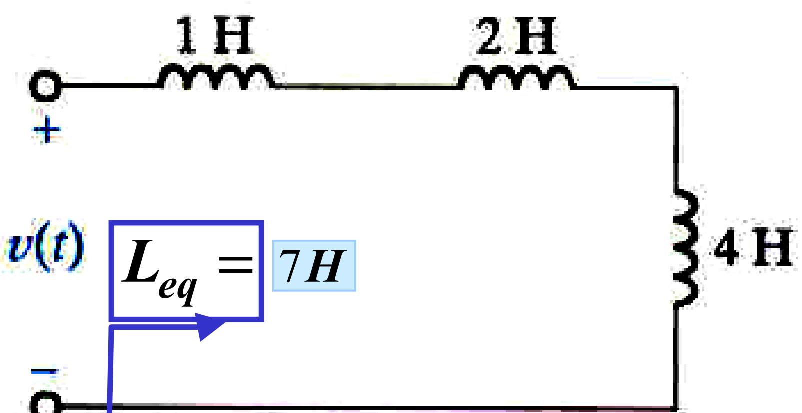
$$v(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + L_3 \frac{di(t)}{dt} + \cdots + L_N \frac{di(t)}{dt}$$

$$= \left(\sum_{i=1}^{N} L_i \right) \frac{di(t)}{dt}$$

$$L_S = \sum_{i=1}^{N} L_i$$

$$v(t) = L_S \frac{di}{dt}(t)$$

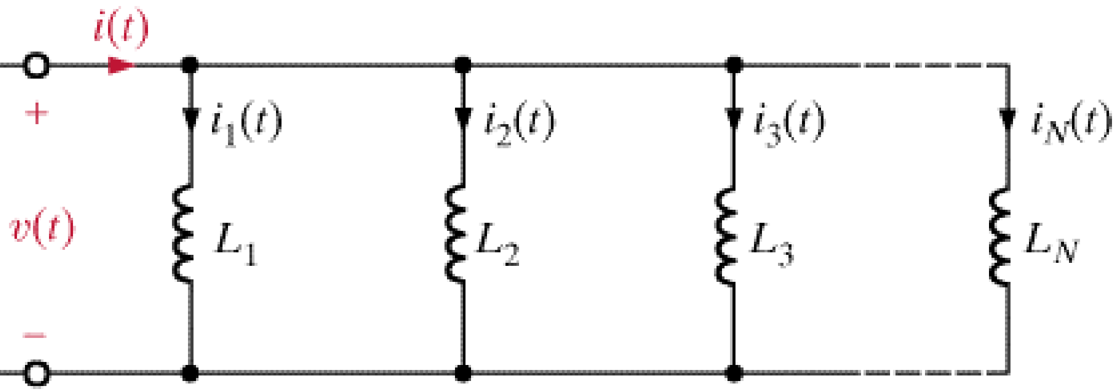
LEARNING EXAMPLE

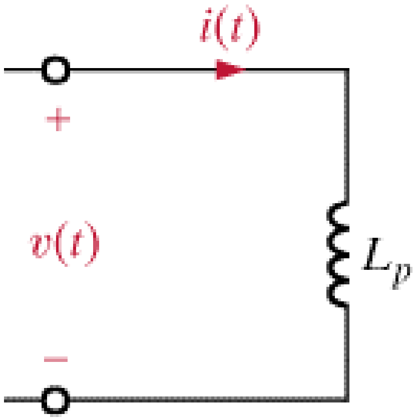


$L_{eq} = 7H$

GEAUX

PARALLEL INDUCTORS





$$i(t) = i_1(t) + i_2(t) + i_3(t) + \cdots + i_N(t)$$

$$i_j(t) = \frac{1}{L_j}\int_{t_0}^{t} v(x)\,dx + i_j(t_0)$$

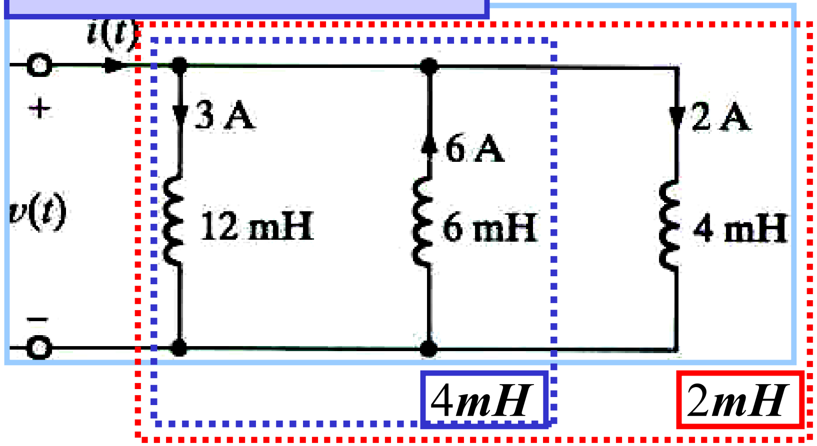
$$i(t) = \left(\sum_{j=1}^{N}\frac{1}{L_j}\right)\int_{t_0}^{t} v(x)\,dx + \sum_{j=1}^{N} i_j(t_0)$$

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

$$i(t_0) = \sum_{j=1}^{N} i_j(t_0)$$

$$i(t) = \frac{1}{L_P}\int_{t_0}^{t} v(x)\,dx + i(t_0)$$

LEARNING EXAMPLE



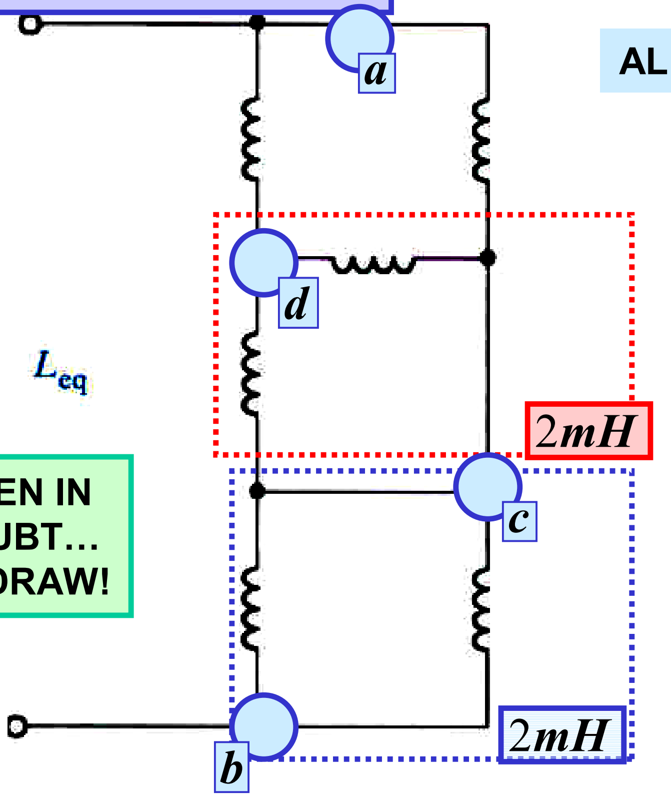
$$i(t_0) = 3A - 6A + 2A = -1A$$

INDUCTORS COMBINE LIKE RESISTORS
CAPACITORS COMBINE LIKE CONDUCTANCES

GEAUX

LEARNING EXTENSION

ALL INDUCTORS ARE 4mH

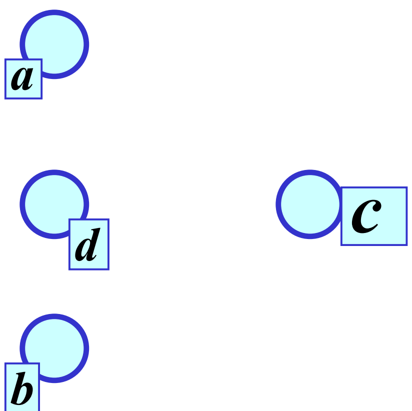


L_{eq}

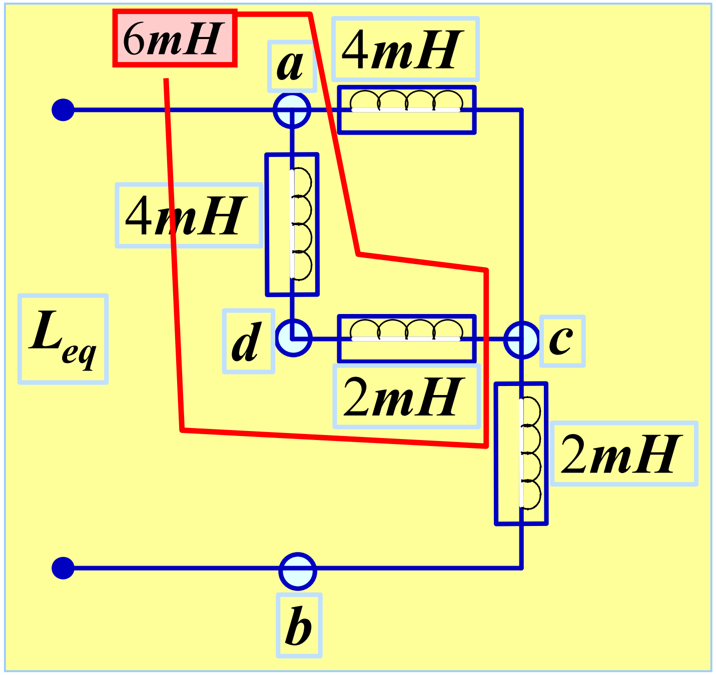
WHEN IN DOUBT… REDRAW!

IDENTIFY ALL NODES

PLACE NODES IN CHOSEN LOCATIONS



CONNECT COMPONENTS BETWEEN NODES

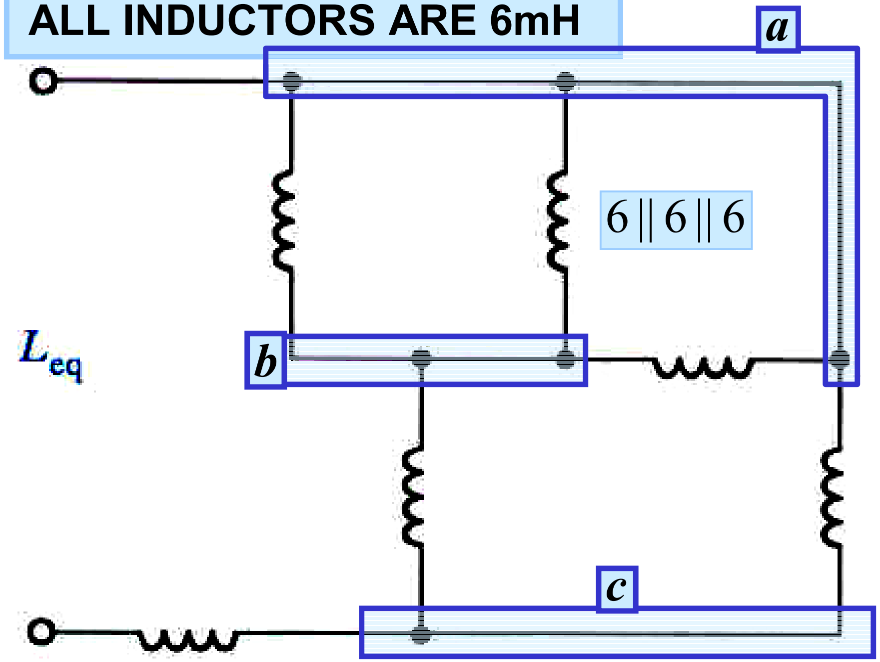


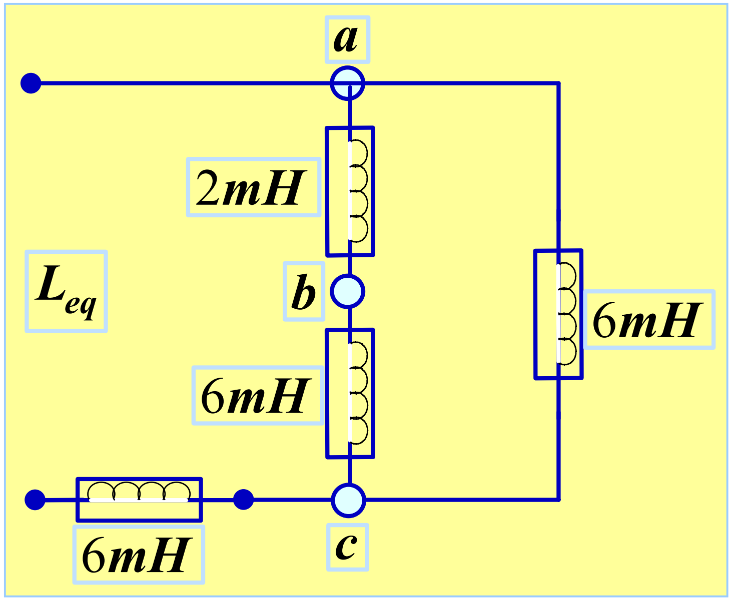
$$L_{eq} = (6mH \,||\, 4mH) + 2mH = 4.4mH$$

GEAUX

LEARNING EXTENSION

ALL INDUCTORS ARE 6mH





NODES CAN HAVE COMPLICATED SHAPES. KEEP IN MIND DIFFERENCE BETWEEN PHYSICAL LAYOUT AND ELECTRICAL CONNECTIONS

$$L_{eq} = 6 + [(6+2) \| 6] = 6 + \frac{48}{14} = 6\frac{24}{7}\, mH$$

$$L_{eq} = \frac{66}{7}\, mH$$

a

b

SELECTED LAYOUT

c

GEAUX

SUMMARY

- The important (dual) relationships for capacitors and inductors are as follows:

$$q = Cv$$

$$i(t) = C\frac{dv(t)}{dt} \qquad v(t) = L\frac{di(t)}{dt}$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(x)\,dx \qquad i(t) = \frac{1}{L}\int_{-\infty}^{t} v(x)\,dx$$

$$p(t) = Cv(t)\frac{dv(t)}{dt} \qquad p(t) = Li(t)\frac{di(t)}{dt}$$

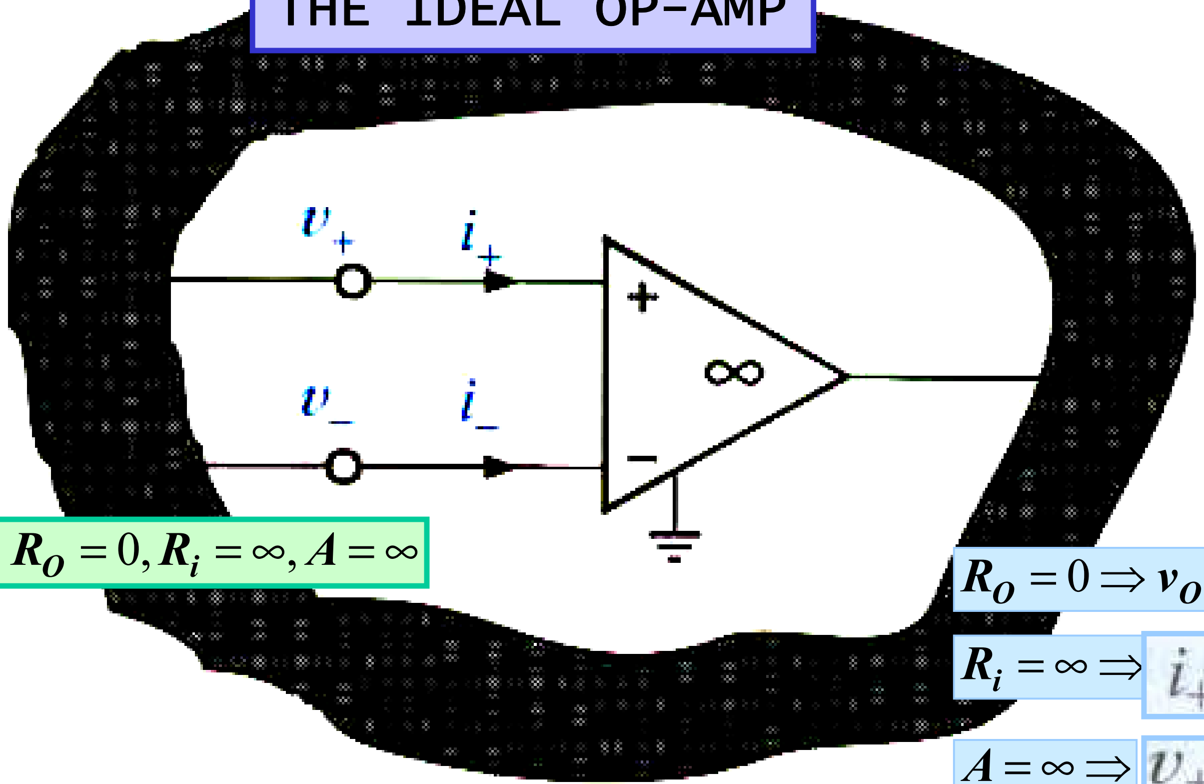
$$W_C(t) = 1/2Cv^2(t) \qquad W_L(t) = 1/2Li^2(t)$$

- The passive sign convention is used with capacitors and inductors.
- In dc steady state a capacitor looks like an open circuit and an inductor looks like a short circuit.
- Leakage resistance is present in practical capacitors and inductors.
- When capacitors are interconnected, their equivalent capacitance is determined as follows: Capacitors in series combine like resistors in parallel and capacitors in parallel combine like resistors in series.
- When inductors are interconnected, their equivalent inductance is determined as follows: Inductors in series combine like resistors in series and inductors in parallel combine like resistors in parallel.
- *RC* operational amplifier circuits can be used to differentiate or integrate an electrical signal.

RC OPERATIONAL AMPLIFIER CIRCUITS

INTRODUCES TWO VERY IMPORTANT PRACTICAL CIRCUITS BASED ON OPERATIONAL AMPLIFIERS

THE IDEAL OP-AMP



IDEAL $\Rightarrow R_O = 0, R_i = \infty, A = \infty$

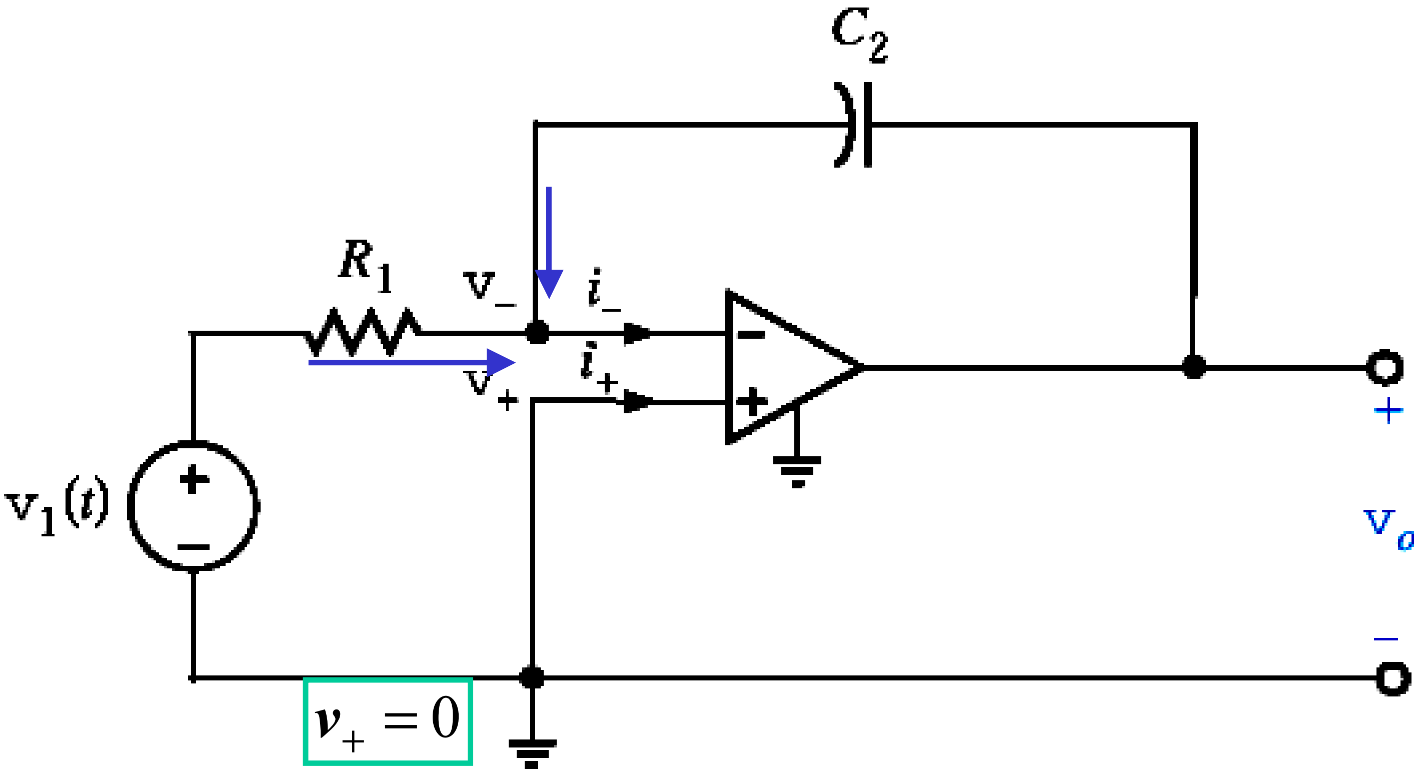
$R_O = 0 \Rightarrow v_O = A(v_+ - v_-)$

$R_i = \infty \Rightarrow i_+ = i_- = 0$

$A = \infty \Rightarrow v_+ = v_-$

GEAUX

RC OPERATIONAL AMPLIFIER CIRCUITS -THE INTEGRATOR



$$v_+ = 0$$

$$\frac{v_1 - v_-}{R_1} + C_2 \frac{d}{dt}(v_o - v_-) = i_-$$

IDEAL OP-AMP ASSUMPTIONS

$v_- = v_+ \quad (A = \infty)$

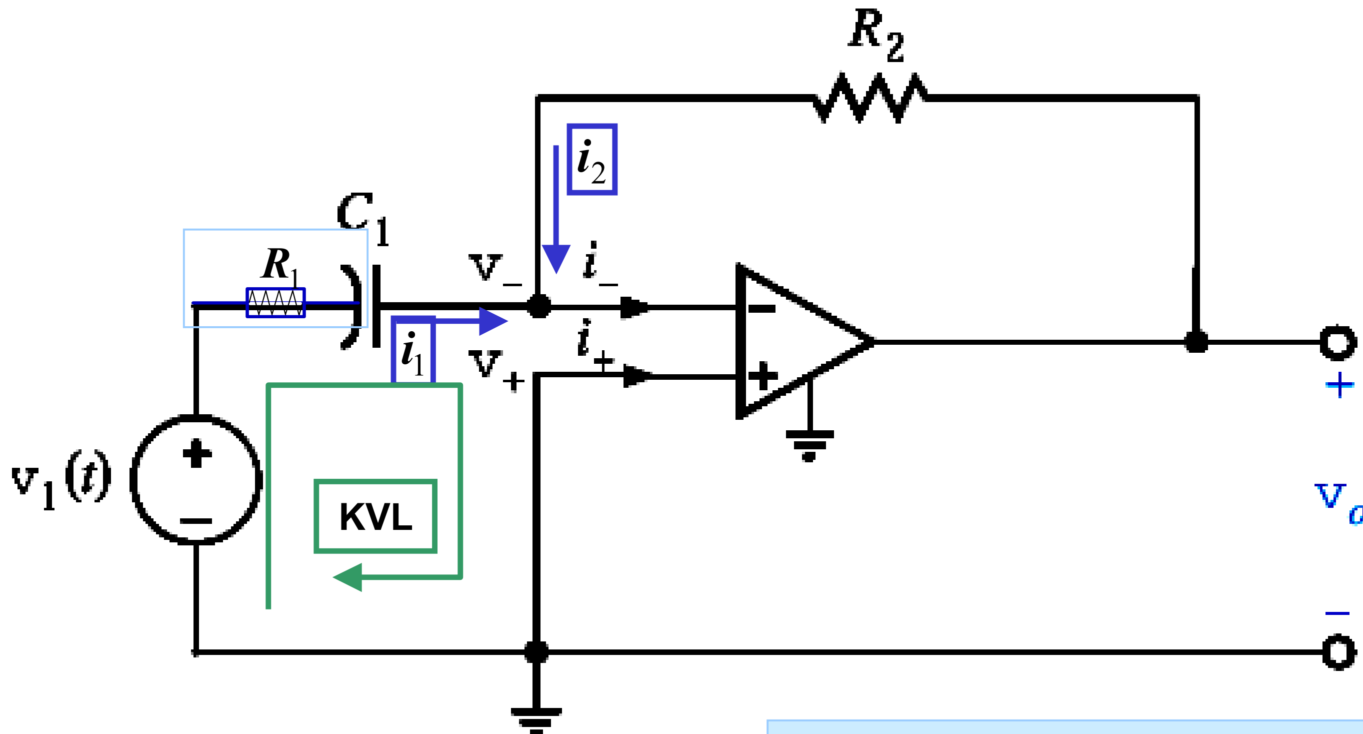
$i_- = 0 \quad (R_i = \infty)$

$$\frac{v_1}{R_1} = -C_2 \frac{dv_o}{dt}$$

$$v_o(t) = \frac{-1}{R_1 C_2} \int_{-\infty}^{t} v_1(x)\,dx$$

$$= \frac{-1}{R_1 C_2} \int_{0}^{t} v_1(x)\,dx + v_o(0)$$

RC OPERATIONAL AMPLIFIER CIRCUITS - THE DIFFERENTIATOR



$v_+ = 0$

KCL@v_- : $i_1 + i_2 = i_-$

IDEAL OP-AMP ASSUMPTIONS

$v_- = v_+ \quad (A = \infty)$

$i_- = 0 \quad (R_i = \infty)$

$$i_1 + \frac{v_O}{R_2} = 0$$

$$v_1(t) = R_1 i_1 + \frac{1}{C_1}\int_{-\infty}^{t} i_1(x)dx$$

DIFFERENTIATE

$$R_1 C_1 \frac{di_1}{dt} + i_1 = C_1 \frac{dv_1}{dt}(t)$$

replace i_1 in terms of v_o $(i_1 = -\frac{v_o}{R_2})$

$$R_1 C_1 \frac{dv_o}{dt} + v_o = -R_2 C_1 \frac{dv_1}{dt}(t)$$

IF R1 COULD BE SET TO ZERO WE WOULD HAVE AN <u>IDEAL</u> DIFFERENTIATOR.
IN PRACTICE AN IDEAL DIFFERENTIATOR AMPLIFIES ELECTRIC NOISE AND DOES NOT OPERATE.
THE RESISTOR INTRODUCES A FILTERING ACTION. ITS VALUE IS KEPT AS SMALL AS POSSIBLE TO APPROXIMATE A DIFFERENTIATOR

ABOUT ELECTRIC NOISE

ALL ELECTRICAL SIGNALS ARE CORRUPTED BY
EXTERNAL, UNCONTROLLABLE AND OFTEN UNMEASURABLE, SIGNALS. THESE UNDESIRED SIGNALS ARE REFERRED TO AS NOISE

SIMPLE MODEL FOR A NOISY 60Hz SINUSOID CORRUPTED WITH ONE MICROVOLT OF 1GHz INTERFERENCE.

$$y(t) = \sin(120\pi t) + 10^{-6}\sin(2\times 10^{9}\pi t)$$

signal noise

$$\frac{\text{noise amplitude}}{\text{signal amplitude}} = 10^{-6}$$

THE DERIVATIVE

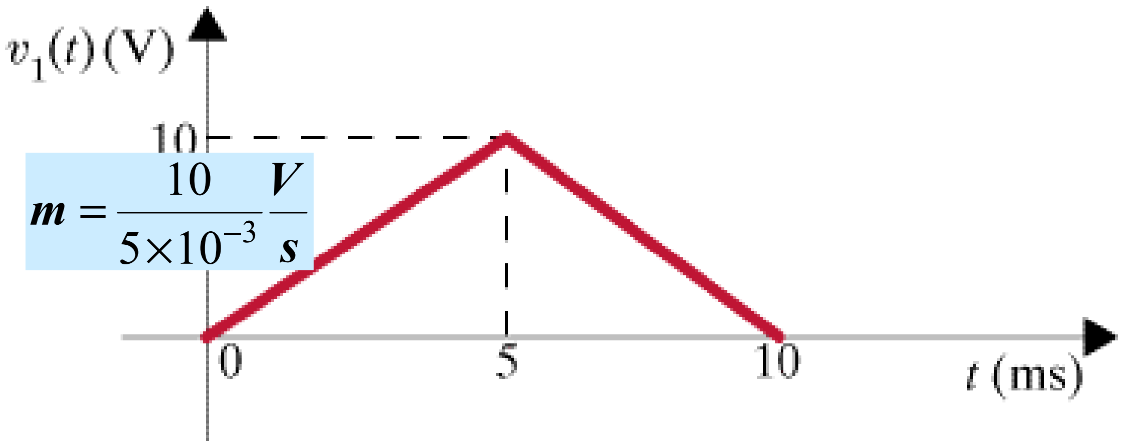
$$\frac{dy}{dt}(t) = 120\pi\cos(120\pi) + 2000\pi\cos(2\times 10^{9}\pi t)$$

signal noise

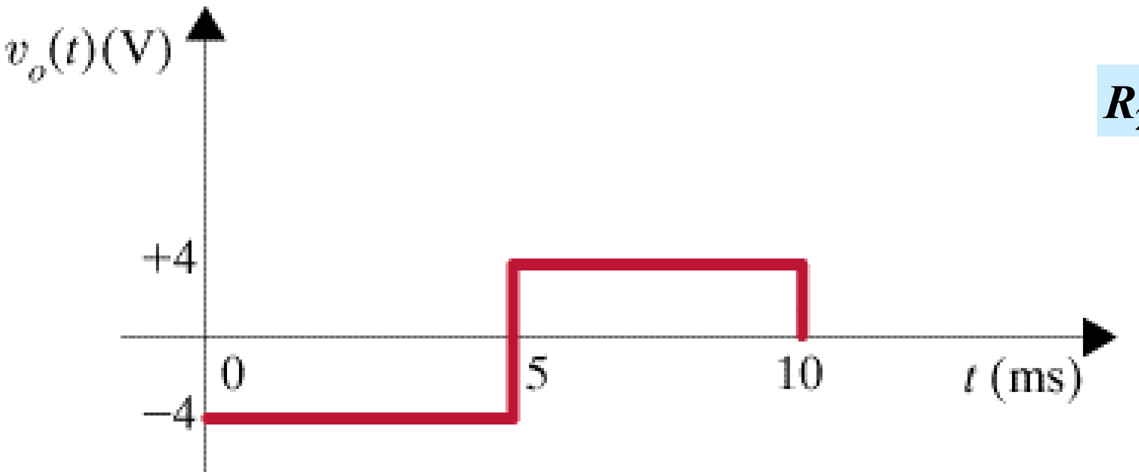
$$\frac{\text{noise amplitude}}{\text{signal amplitude}} = \frac{2000}{120} = 16.67$$

LEARNING EXTENSION

INPUT TO IDEAL DIFFERENTIATOR WITH $R_2 = 1k\Omega, C_1 = 2\mu F$



$$m = \frac{10}{5\times10^{-3}}\frac{V}{s}$$



IDEAL DIFFERENTIATOR

$$v_o = -R_2C_1\frac{dv_1}{dt}(t)$$

DIMENSIONAL ANALYSIS

$$\Omega = \frac{V}{A} = \frac{V}{Q/s} = \frac{V\times s}{Q}$$

$$F = \frac{Q}{V} \Rightarrow \Omega\times F = s$$

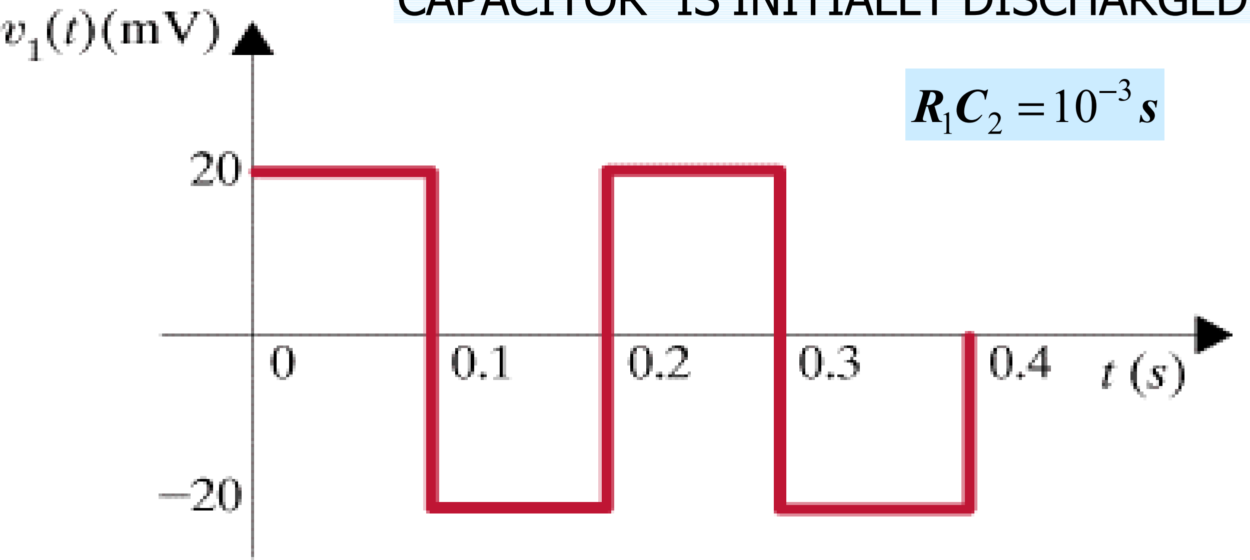
$$R_2C_1 = 1\times10^3\,\Omega\times2\times10^{-6}\,F = 2\times10^{-3}\,s$$

GEAUX

LEARNING EXTENSION

INPUT TO AN INTEGRATOR WITH $R_1 = 5k\Omega, C_2 = 0.2\mu F$

CAPACITOR IS INITIALLY DISCHARGED



$R_1 C_2 = 10^{-3}\, s$

INTEGRATOR

$$v_o(t) = v_o(0) - \frac{1}{R_1 C_2}\int_0^t v_i(x)dx$$

DIMENSIONAL ANALYSIS

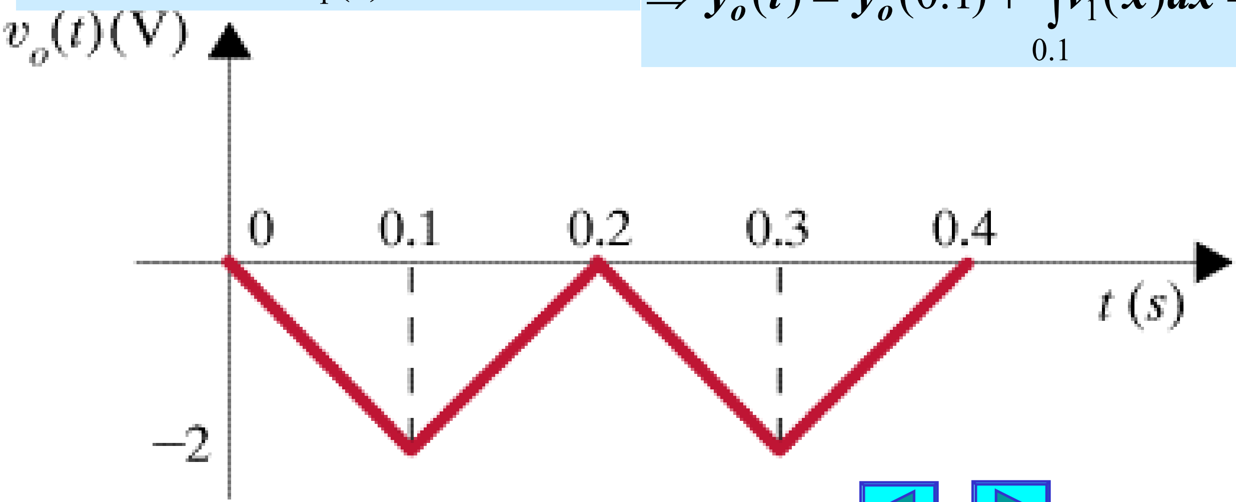
$$\Omega = \frac{V}{A} = \frac{V}{Q/s} = \frac{V \times s}{Q}$$

$$F = \frac{Q}{V} \Rightarrow \Omega \times F = s$$

$$0 < t < 0.1s : v_1(t) = 20\times10^{-3} \Rightarrow y_o(t) = \int_0^t v_1(x)dx = 20\times10^{-3}\, t(V\times s) \Rightarrow y(0.1) = 2\times10^{-3}(V\times s)$$

$$0.1 < t < 0.2s : v_1(t) = -20\times10^{-3} \Rightarrow y_o(t) = y_o(0.1) + \int_{0.1}^t v_1(x)dx = 2\times10^{-3} - 20\times10^{-3}(t-0.1)(V\times s)$$

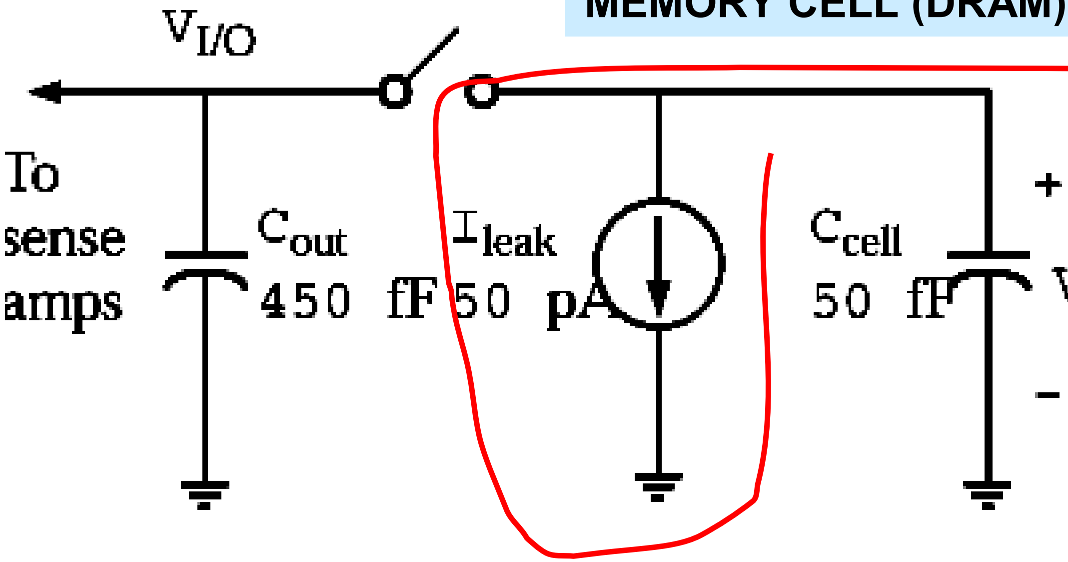
$$v_o(t) = \frac{1}{R_1 C_2} y_o(t)$$



GEAUX

LEARNING EXAMPLE

SIMPLE CIRCUIT MODEL FOR DYNAMIC RANDOM ACCESS MEMORY CELL (DRAM)



REPRESENTS CHARGE LEAKAGE FROM CELL CAPACITOR

NOTICE THE VALUES OF THE CAPACITANCES

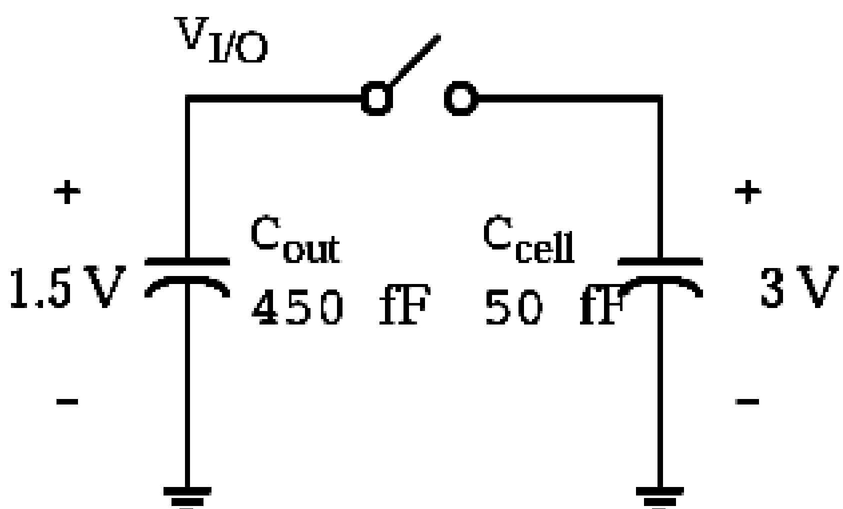
$V_{cell} > 1.5V$ FOR CORRECT STORAGE OF A LOGIC ONE

$$v_C = v_C(0) + \frac{1}{C}\int_0^t i_C(x)dx$$

$$V_{cell} = 3 - \frac{I_{leak}}{C_{cell}}t \geq 1.5 \Rightarrow \frac{I_{leak}}{C_{cell}}t \leq 1.5V$$

$$t_H = \frac{1.5(V) \times 50 \times 10^{-15}(F)}{50 \times 10^{-12} A} = 1.5 \times 10^{-3} s$$

THE CELL MUST BE "REFRESHED" AT A FREQUENCY HIGHER THAN $1/t_H$

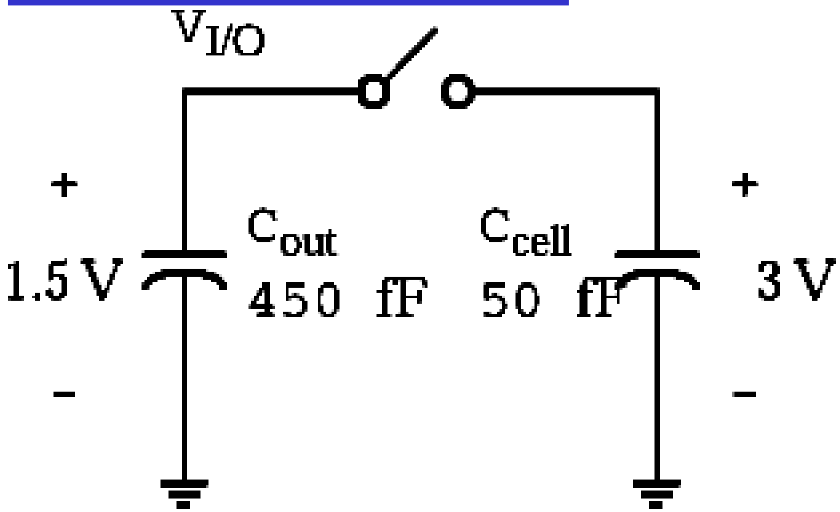


THE ANALYSIS OF THE READ OPERATION GIVES FURTHER INSIGHT ON THE REQUIREMENTS

CELL AT THE BEGINNING OF A MEMORY READ OPERATION

SWITCHED CAPACITOR CIRCUIT

CELL READ OPERATION



IF SWITCH IS CLOSED BOTH CAPACITORS MUST HAVE THE SAME VOLTAGE

ASSUMING NO LOSS OF CHARGE THEN THE CHARGE BEFORE CLOSING MUST BE EQUAL TO CHARGE AFTER CLOSING

$$Q_{before} = 1.5V \times 450 \times 10^{-15} F + 3V \times 50 \times 10^{-15} F$$

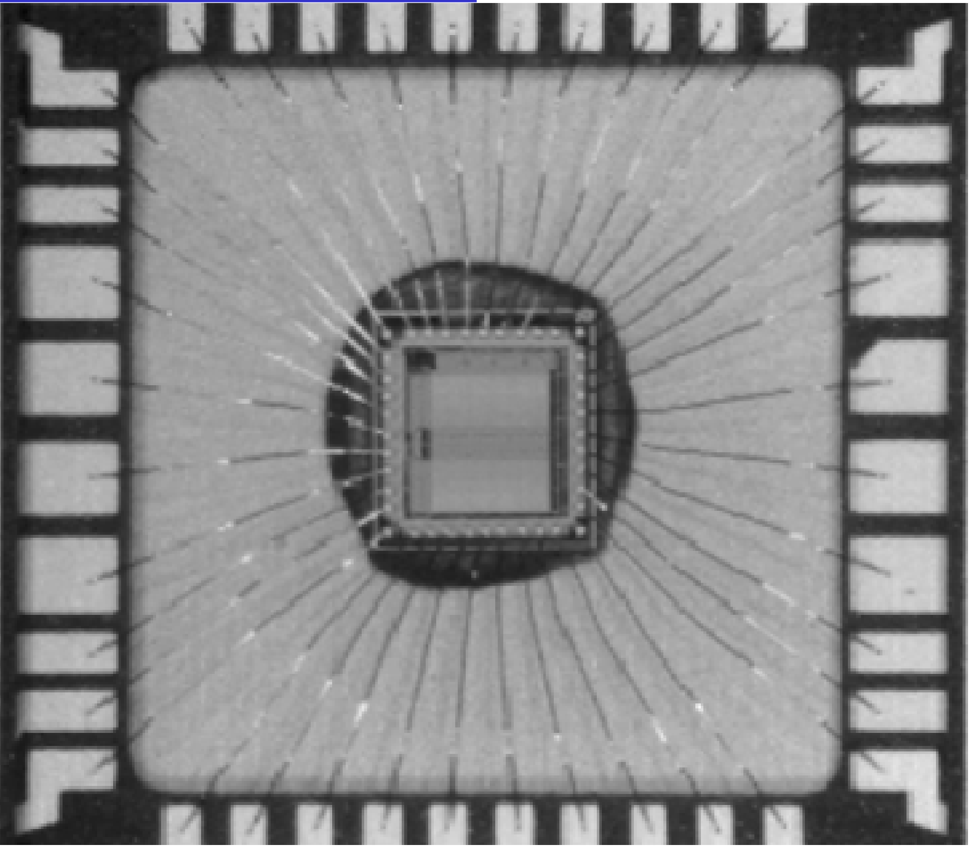
$$Q_{after} = V_{after} \times (500 \times 10^{-15} F)$$

$$V_{after} = 1.65V$$

Even at full charge the voltage variation is small. SENSOR amplifiers are required

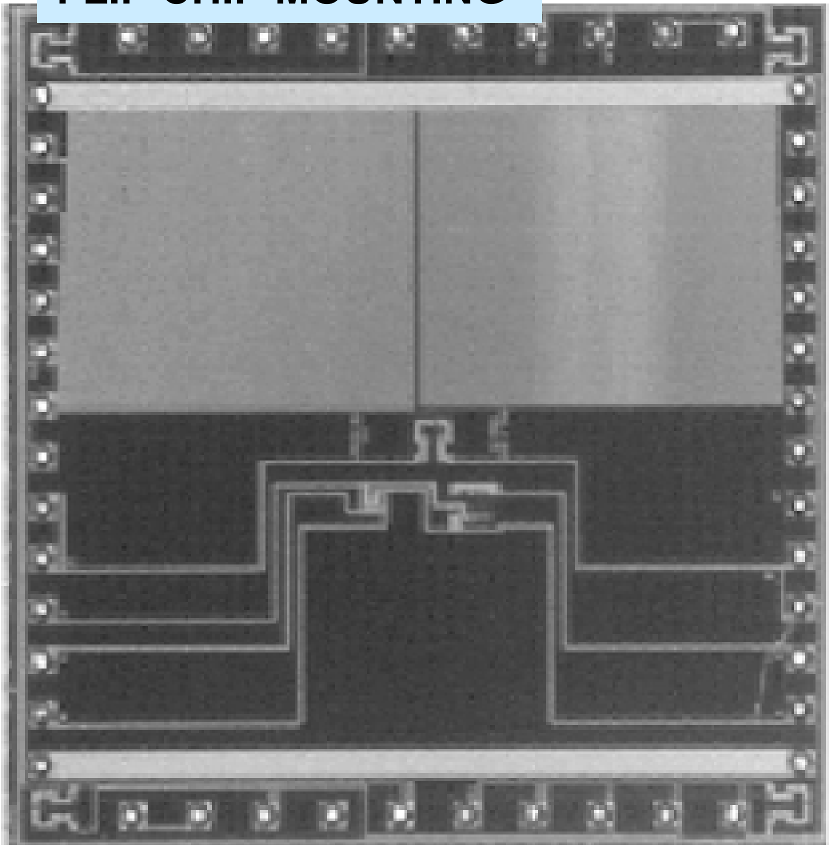
After a READ operation the cell must be refreshed

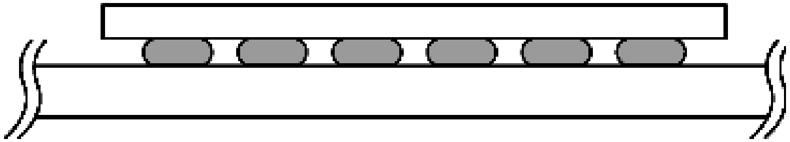
LEARNING EXAMPLE



IC WITH WIREBONDS TO THE OUTSIDE

FLIP CHIP MOUNTING



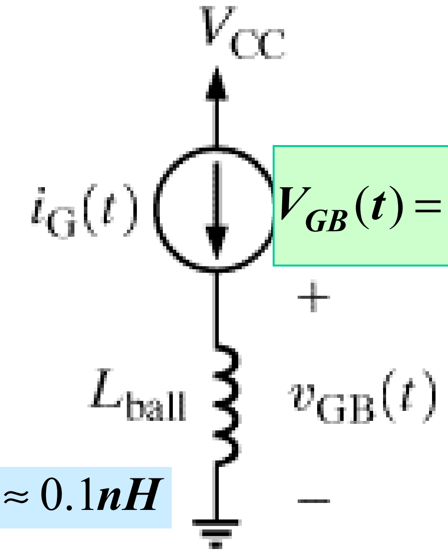


GOAL: REDUCE INDUCTANCE IN THE WIRING AND REDUCE THE "GROUND BOUNCE" EFFECT

A SIMPLE MODEL CAN BE USED TO DESCRIBE GROUND BOUNCE

GEAUX

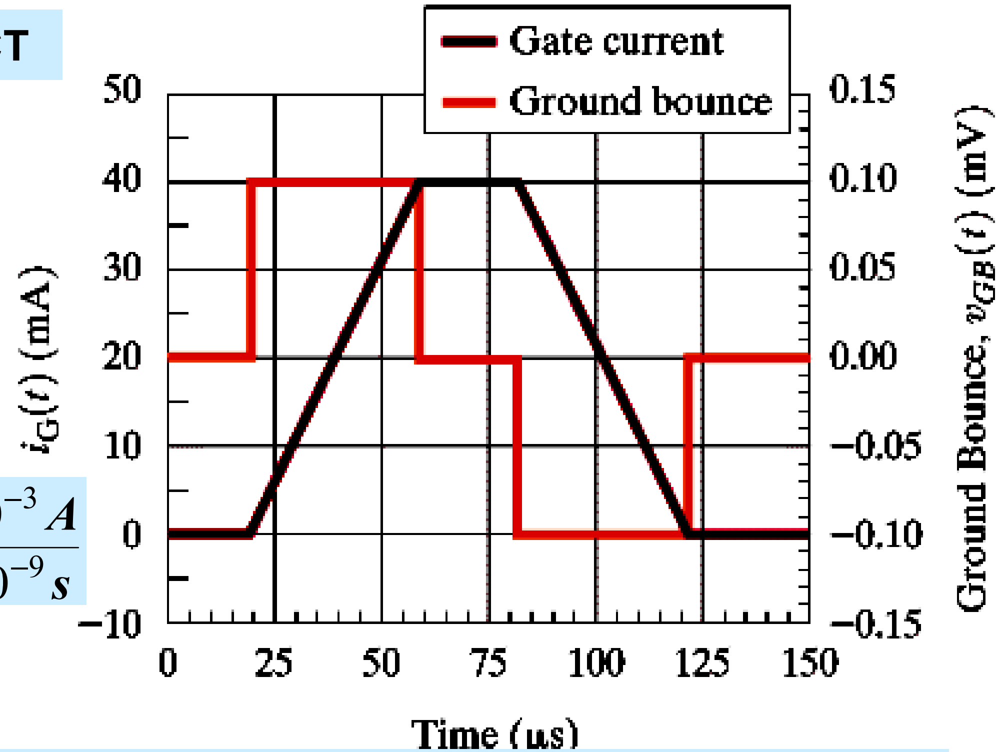
MODELING THE GROUND BOUNCE EFFECT



$$V_{GB}(t) = L_{ball}\frac{di_G}{dt}(t)$$

$$L_{ball} \approx 0.1nH$$

$$m = \frac{40\times10^{-3}\,A}{40\times10^{-9}\,s}$$



IF ALL GATES IN A CHIP ARE CONNECTED TO A SINGLE GROUND THE CURRENT CAN BE QUITE HIGH AND THE BOUNCE MAY BECOME UNACCEPTABLE

USE SEVERAL GROUND CONNECTIONS (BALLS) AND ALLOCATE A FRACTION OF THE GATES TO EACH BALL