ECE321 Electronics I: Lecture 4

Chapter 3
Solid-State Diodes and Diode Circuits
Sections 3.1-3.5

Microelectronic Circuit Design

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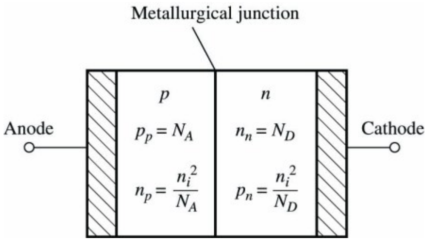
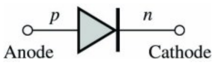
Lecture Goals

- Understand diode structure and basic layout
- Develop electrostatics of the *pn* junction
- Define regions of operation of the diode (forward bias, reverse bias, and reverse breakdown)

Diode Introduction



+ v_D --



i_D →

Diode symbol

- A diode is formed by joining an *n*-type semiconductor with a *p*-type semiconductor.
- A ***pn* junction** is the interface between *n* and *p* regions.
- e.g. $N_D=10^{16}/\text{cm}^3$, $N_A=10^{17}/\text{cm}^3$
  N side: $n_n=N_D=10^{16}/\text{cm}^3$
  $p_n=n_i^2/n_n=10^{20}/10^{16}=10^4/\text{cm}^3$
  P side: $p_p=N_A=10^{17}/\text{cm}^3$
  $n_p=n_i^2/p_p=10^{20}/10^{17}=10^3/\text{cm}^3$

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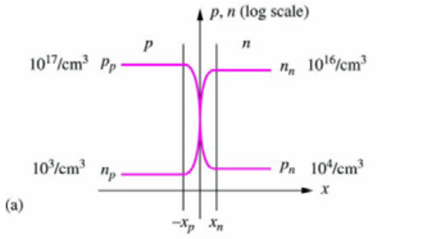
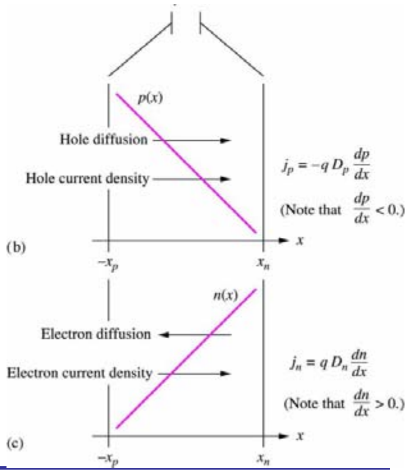
pn Junction Electrostatics





Donor and acceptor concentration on either side of the junction. Concentration gradients give rise to diffusion currents.

Drift Currents

- Diffusion currents lead to localized charge density variations near the *pn* junction.
- Gauss' law predicts an electric field due to the charge distribution:

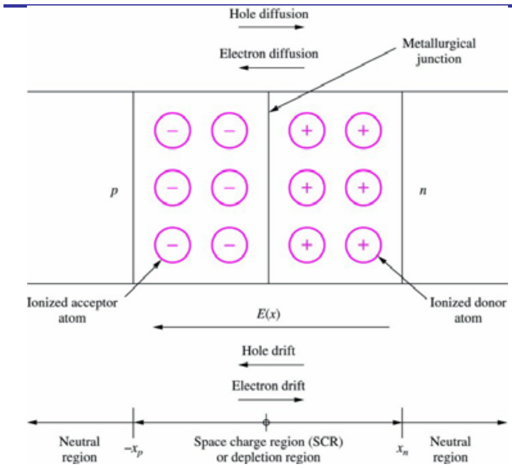
$$\nabla \cdot E = \frac{\rho_c}{\varepsilon_s}$$

- Assuming constant permittivity,

$$E(x) = \frac{1}{\varepsilon_s} \int \rho(x)dx$$

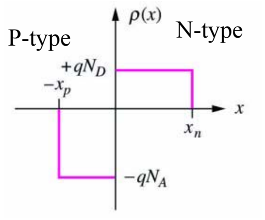
- Resulting electric field gives rise to a drift current. <u>With no external circuit connections, drift and diffusion currents cancel.</u> There is no actual current, since this would imply power dissipation, rather the electric field cancels the diffusion current 'tendency.'

Space-Charge Region (SCR) Formation at the *pn* Junction



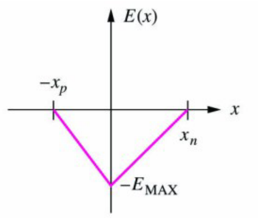
Electrons from N-type and holes from P-type migrate across the boundary by diffusion, and recombine, leaving immobile doping impurity ions (charges) to form the SCR (no mobile carriers.) The electric field in this "depletion region" inhibits further diffusion.

Potential Across the Junction



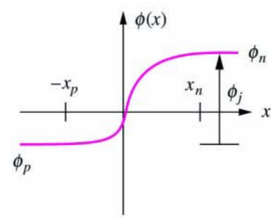
Charge Density

$qN_Ax_p = qN_Dx_n$



Electric Field

$$E(x) = \frac{1}{\varepsilon_s}\int \rho(x)dx$$



Potential

$$\phi_j = -\int E(x)dx = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right), \quad V_T = \frac{kT}{q}$$

Width of Depletion Region

Combining the previous expressions, we can form an expression for the width of the space-charge region, or depletion region. It is called the depletion region since the excess holes and electrons are depleted from the dopant atoms on either side of the junction.

$$w_{d0} = (x_n + x_p) = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\phi_j}$$

Junction Width and Potential

N-type side: $\rho_C = +qN_D$ (P-type side: $\rho_C = -qN_A$)

$d^2V/dx^2 = -\rho_C/\varepsilon_s$ so $E(x) = -dV/dx = \int(\rho_C/\varepsilon_s)dx$

$= (q/\varepsilon_s)N_D x + \text{const}$

$= 0$ at $x = x_n$ so const $= -(q/\varepsilon_s)N_D x_n$

and $E(x) = -(q/\varepsilon_s)N_D(x_n - x)$

and $E_{max} = -(q/\varepsilon_s)N_D x_n$ at $x = 0$

So $V(x) = -\int E(x)dx = +(q/\varepsilon_s)N_D(x_n x - x^2/2) + \text{const}$

$= 0$ at $x = 0$, so const $= 0$

Hence $V(x_n) = \varphi_n = (qN_D/2\varepsilon_s)x_n^2$ so $\varphi_j = \varphi_n + \varphi_p = (q/2\varepsilon_s)(x_n^2N_D + x_p^2N_A)$

Junction Width (cont'd)

From last slide: $\varphi_j = \varphi_n + \varphi_p = (q/2\varepsilon_s)(x_n^2N_D + x_p^2N_A)$

Substitute for x_n from: $N_A x_p = N_D x_n$

to get: $\varphi_j = (q/2\varepsilon_s)([N_A^2x_p^2/N_D^2]N_D + x_p^2N_A)$

Rearrange, and hence: $x_p^2 = (2\varepsilon_s\varphi_j/q)(N_A + N_A^2/N_D)$

$x_p = (2\varepsilon_s\varphi_j/q)^{½}(N_A+N_D)^{-½}(N_D/N_A)^{½}$

So $W = x_p + x_n = (2\varepsilon_s\varphi_j/q)^{½}(N_A+N_D)^{-½}\,[(N_D/N_A)^{½} + (N_A/N_D)^{½}]$

$= (2\varepsilon_s\varphi_j/q)^{½}(N_A+N_D)^{½}\,(N_DN_A)^{-½}$

$= (2\varepsilon_s\varphi_j/q)^{½}(1/N_A + 1/N_D)^{½}$

Also $E_{max} = -(q/\varepsilon_s)N_A x_p$

$= -(2q\varphi_j/\varepsilon_s)^{½}(N_A+N_D)^{-½}(N_DN_A)^{½} = -2\varphi_j / W$

Internal Diode Currents

Mathematically, for a diode with no external connections, the total current expressions developed in Chapter 2 are equal to zero. The equations only dictate that the total currents are zero. However, as mentioned earlier, since there is no power dissipation, we must assume that the field and diffusion current tendencies cancel and the actual currents are zero.

$$j_n^T = q\mu_n nE + qD_n \frac{\partial n}{\partial x} = 0$$

$$j_p^T = q\mu_p pE - qD_p \frac{\partial p}{\partial x} = 0$$

When external bias voltage is applied to the diode, the above equations are no longer equal to zero.

Junction Potential (cont'd)

Considering total hole current:

$j_p = q\mu_p pE - qD_p(dp/dx) = 0$ at equilibrium

So $-\mu_p p\,(dV/dx) = D_p(dp/dx)$

$-\mu_p/D_p\ dV = dp/p$

and

$$-\frac{\mu_p}{D_p}\int_{-x_p}^{x_n} dV = \int_{p_p}^{p_n} \frac{dp}{p}$$

Einstein -> $-(kT/q)\varphi_j = \log_e p_n - \log_e p_p$

$\varphi_j = (q/kT)\log_e(p_p/p_n)$

$= (q/kT)\log_e(N_A N_D/n_i^2)$

Width of Depletion Region (Example)

Problem: Find built-in potential and depletion-region width for given diode

Given data:On *p*-type side: $N_A = 10^{17}/\text{cm}^3$ on *n*-type side: $N_D = 10^{20}/\text{cm}^3$

Assumptions: Room-temperature operation with V_T = 0.025 V

Analysis:

$$\phi_j = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = (0.025\ \text{V}) \ln\left[\frac{\left(10^{17}/\text{cm}^3\right)\left(10^{20}/\text{cm}^3\right)}{\left(10^{20}/\text{cm}^6\right)}\right] = 0.979\ \text{V}$$

$$w_{d0} = \sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\phi_j} = 0.113\,\mu\text{m}$$

Diode Electric Field (Example)

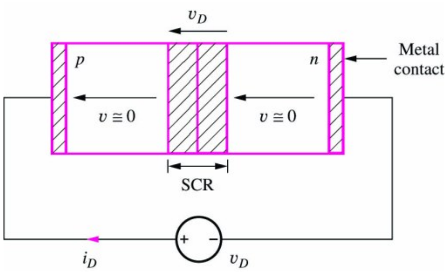
- **Problem:** Find the electric field and size of the individual depletion layers on either side of a *pn* junction for a given diode
- **Given data**: On the *p*-type side: $N_A = 10^{17}/\text{cm}^3$ on the *n*-type side: $N_D = 10^{20}/\text{cm}^3$ from earlier example, $\phi_j = 0.979\text{V}$ $w_{d0} = 0.113\ \mu\text{m}$
- **Assumptions**: Room-temperature operation
- **Analysis:**

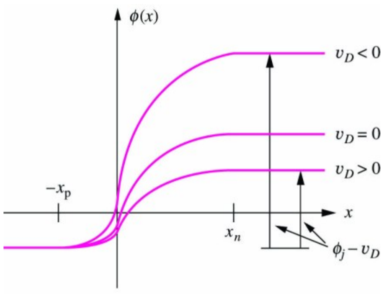
$$w_{d0} = x_n + x_p = x_n\left(1 + \frac{N_D}{N_A}\right) = x_p\left(1 + \frac{N_A}{N_D}\right)$$

$$x_n = \frac{w_{d0}}{\left(1 + \frac{N_D}{N_A}\right)} = 1.13 \times 10^{-4}\ \mu m \qquad x_p = \frac{w_{d0}}{\left(1 + \frac{N_A}{N_D}\right)} 0.113\ \mu m$$

$$E_{MAX} = \frac{2\phi_j}{w_{d0}} = \frac{2(0.979V)}{0.113\mu m} = 173\ kV/cm$$

Diode Junction Potential for Different Applied Voltages



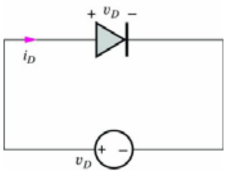


Changes height of potential barrier to diffusion current

$\varphi_j \rightarrow \varphi_j - V$ in all previous formulae

Diode Equation

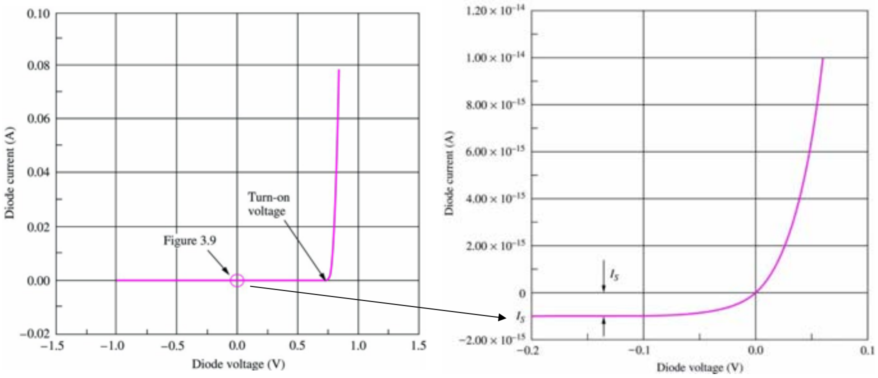
$$i_D = I_S\left[\exp\left(\frac{qv_D}{nkT}\right) - 1\right] = I_S\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right]$$



where I_S = reverse saturation current (A)
v_D = voltage applied to diode (V)
q = electronic charge (1.60 x 10-19 C)
k = Boltzmann’s constant (1.38 x 10^{-23} J/K)
T = absolute temperature
n = nonideality factor (dimensionless)
V_T = kT/q = thermal voltage (V) (25 mV at room temp.)

I_S is typically between 10^{-18} and 10^{-9} A, and is strongly temperature dependent due to its dependence on n_i^2. The nonideality factor is typically close to 1, but approaches 2 for devices with high current densities. It is assumed to be 1 in this text.

Diode *i*-*v* Characteristics



The turn-on voltage marks the point of significant current flow.
$\boldsymbol{I_s}$ is called the reverse saturation current.

Diode Voltage and Current Calculations (Example)

Problem: Find diode voltage for diode with given specifications

Given data: I_S = 0.1 fA, I_D = 300 μA

Assumptions: Room-temperature dc operation with V_T = 0.025 V

Analysis:

With I_S = 0.1 fA $\quad V_D = nV_T \ln\left(1+\frac{I_D}{I_S}\right) = 1(0.0025V)\ln(1+\frac{3\times10^{-4}A}{10^{-16}A}) = 0.718\ V$

With I_S = 10 fA $\quad V_D = 0.603\text{V}$

With I_D = 1 mA, I_S = 0.1 fA $\quad V_D = 0.748\text{V}$

Diode Current for Reverse, Zero, and Forward Bias

- Reverse bias:

$$i_D = I_S\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right] \cong I_S\left[0 - 1\right] \cong -I_S$$

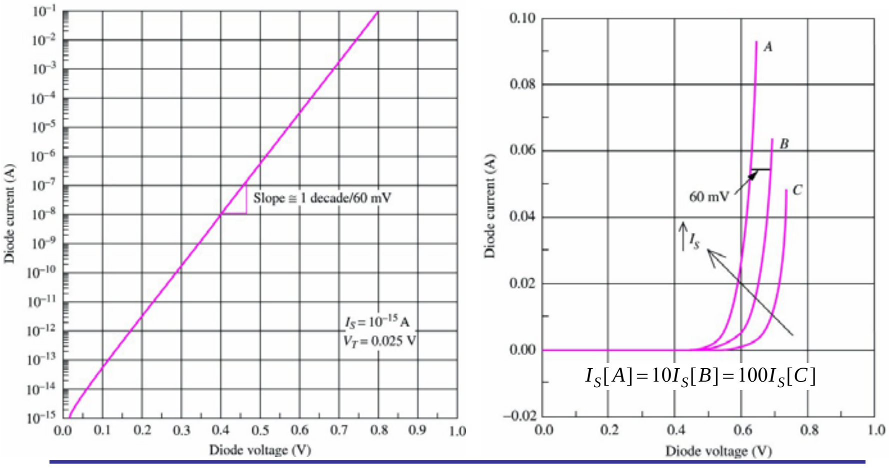
- Zero bias:

$$i_D = I_S\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right] \cong I_S\left[1 - 1\right] \cong 0$$

- Forward bias:

$$i_D = I_S\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right] \cong I_S \exp\left(\frac{v_D}{nV_T}\right)$$

Semi-log Plot of Forward Diode Current and Current for Three Different Values of I_S



Diode Temperature Coefficient

Diode voltage under forward bias:

$$v_D = V_T \ln\left(\frac{i_D}{I_S}+1\right) = \frac{kT}{q}\ln\left(\frac{i_D}{I_S}+1\right) \cong \frac{kT}{q}\ln\left(\frac{i_D}{I_S}\right)$$

Taking the derivative with respect to temperature yields

$$\frac{dv_D}{dT} = \frac{k}{q}\ln\left(\frac{i_D}{I_S}\right) - \frac{kT}{q}\frac{1}{I_S}\frac{dI_S}{dT} = \frac{v_D}{T} - V_T\frac{1}{I_S}\frac{dI_S}{dT} = \frac{v_D - V_{GO} - 3V_T}{T} \quad \text{V/K}$$

Assuming $i_D >> I_S$, $I_S \propto n_i^2$, and V_{GO} is the silicon bandgap energy at 0K. For a typical silicon diode

$$\frac{dv_D}{dT} = \frac{(0.65-1.12-0.075)V}{300K} = \text{-1.82 mV/K} \approx \textbf{-1.8 mV/°C}$$

End of Lecture 4