Digital Communications I: Modulation and Coding Course

Spring – 2015

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Lecture 2: Formatting and Baseband Modulation

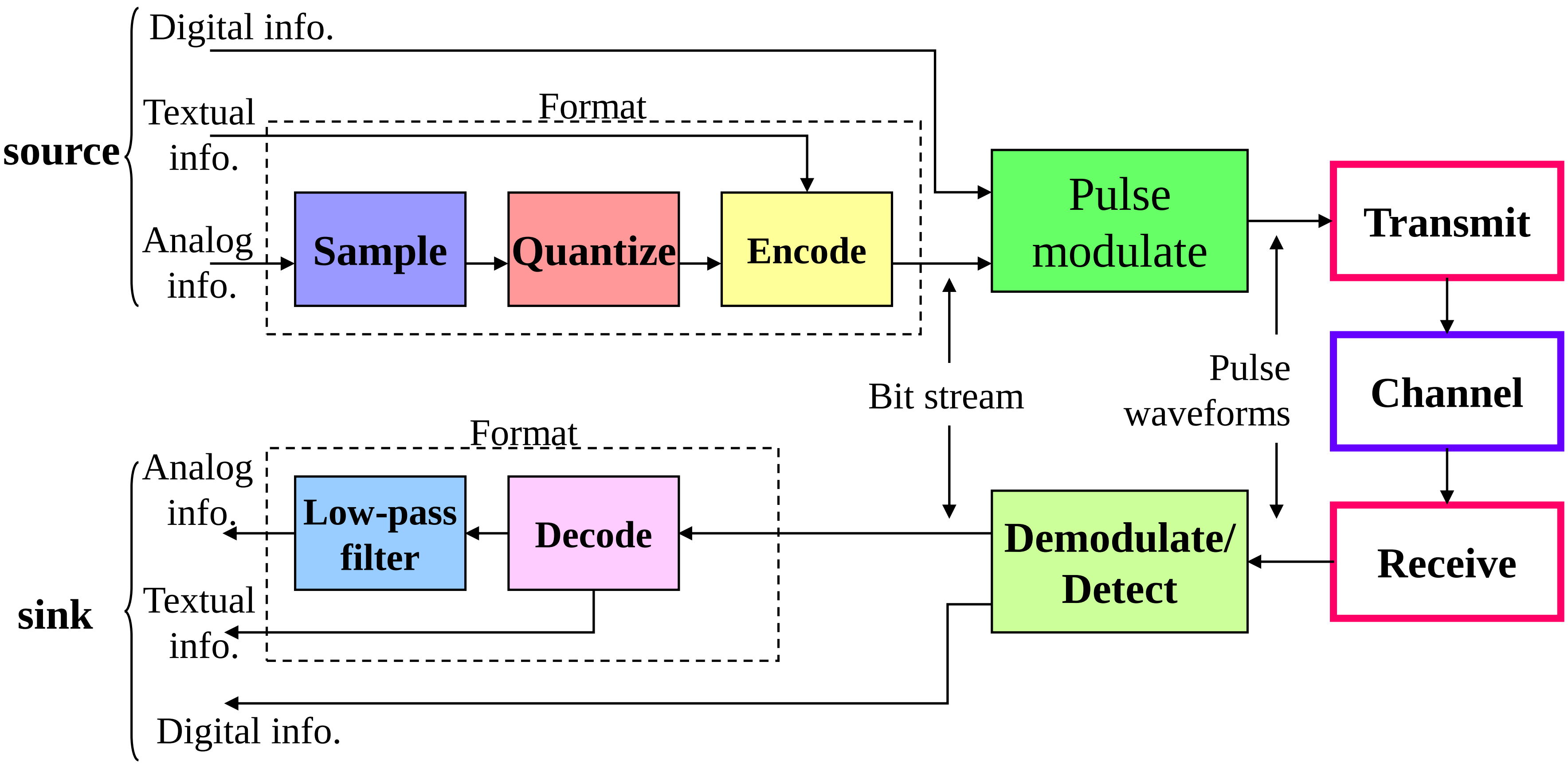
In our first two Lectures, we talked about:

- Important features of digital communication systems
- Some basic concepts/definitions:
  - Signal classification,
  - Fourier Series/Transform,
  - Spectral density,
  - Random processes,
  - Linear systems and
  - Signal bandwidth.

Today, we are going to talk about:

- The first important step in any DCS:
  - Transforming the information source to a form compatible with a digital system

A Digital Communication System

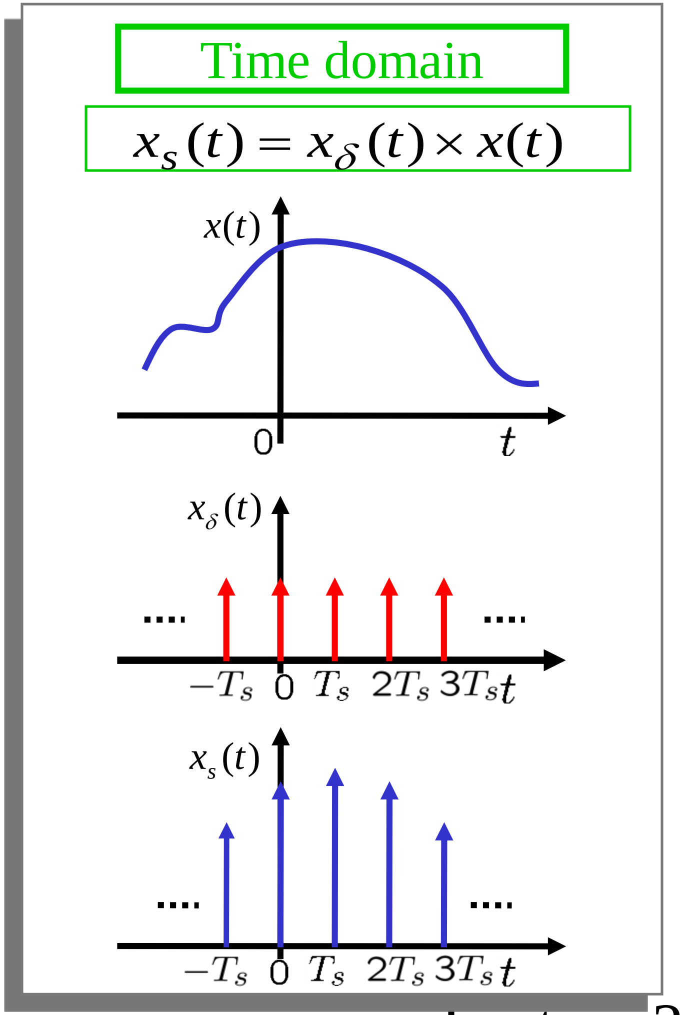


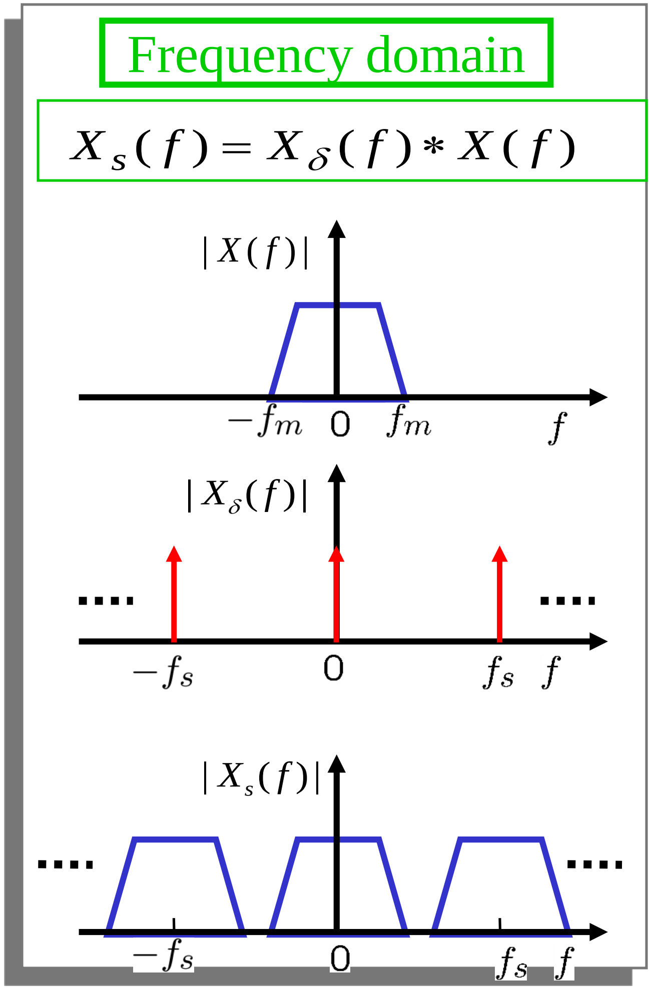
Format analog signals

- To transform an analog waveform into a form that is compatible with a digital communication system, the following steps are taken:
  1. Sampling – See my notes on **Sampling**
  2. Quantization and encoding
  3. Baseband transmission

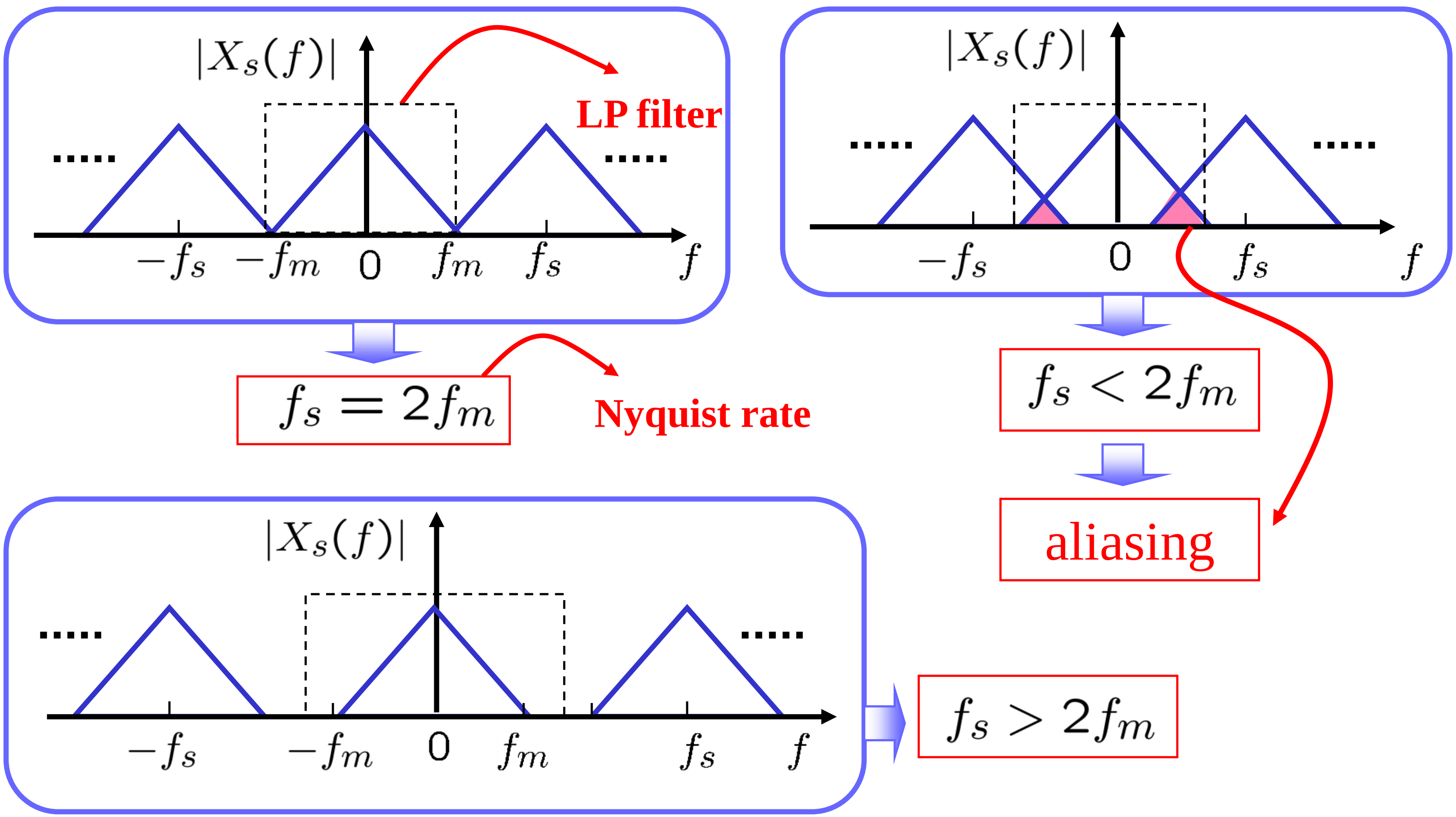
Sampling

See my notes on Fourier Series, Fourier Transform and Sampling

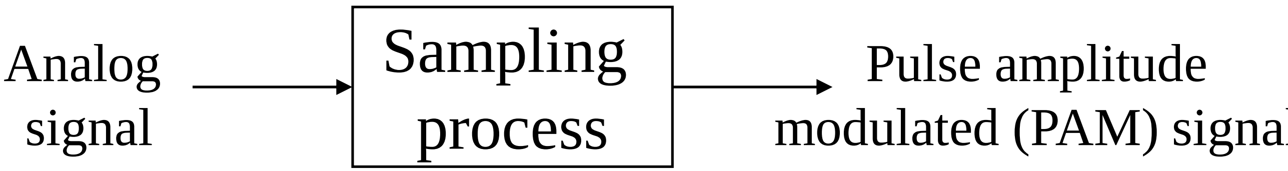




Aliasing effect



Sampling theorem



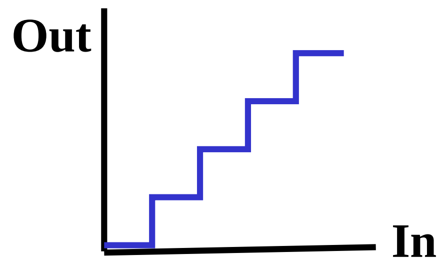
- **Sampling theorem:** A band-limited signal with no spectral components beyond f_m, can be uniquely determined by values sampled at uniform intervals of

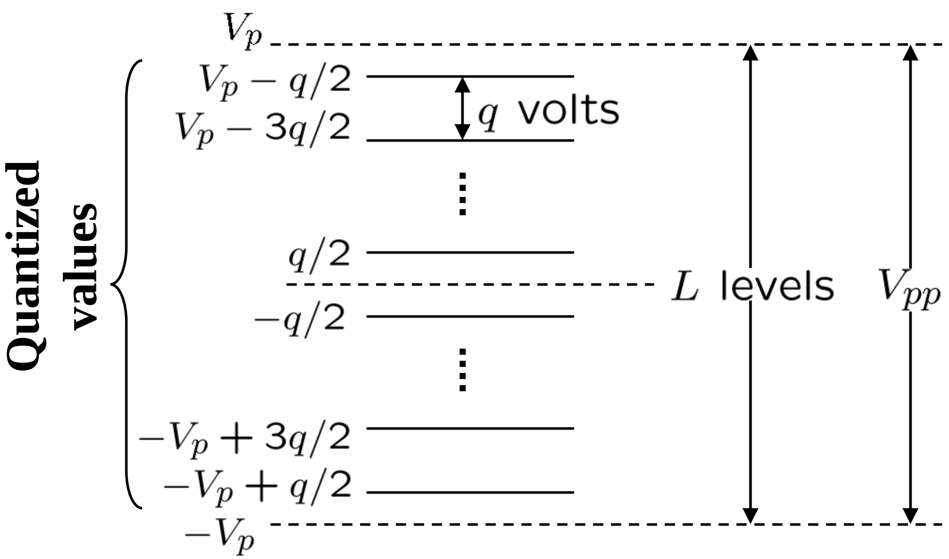
$$T_s \leq \frac{1}{2f_m}$$

- The sampling rate, $f_s = \frac{1}{T_s} = 2f_m$ is called the Nyquist rate.

Quantization

- Amplitude quantizing: Mapping samples of a continuous amplitude waveform to a finite set of amplitudes.





- Average quantization noise power

$$\sigma^2 = \frac{q^2}{12}$$

- Signal peak power

$$V_p^2 = \frac{L^2 q^2}{4}$$

- Signal power to average quantization noise power

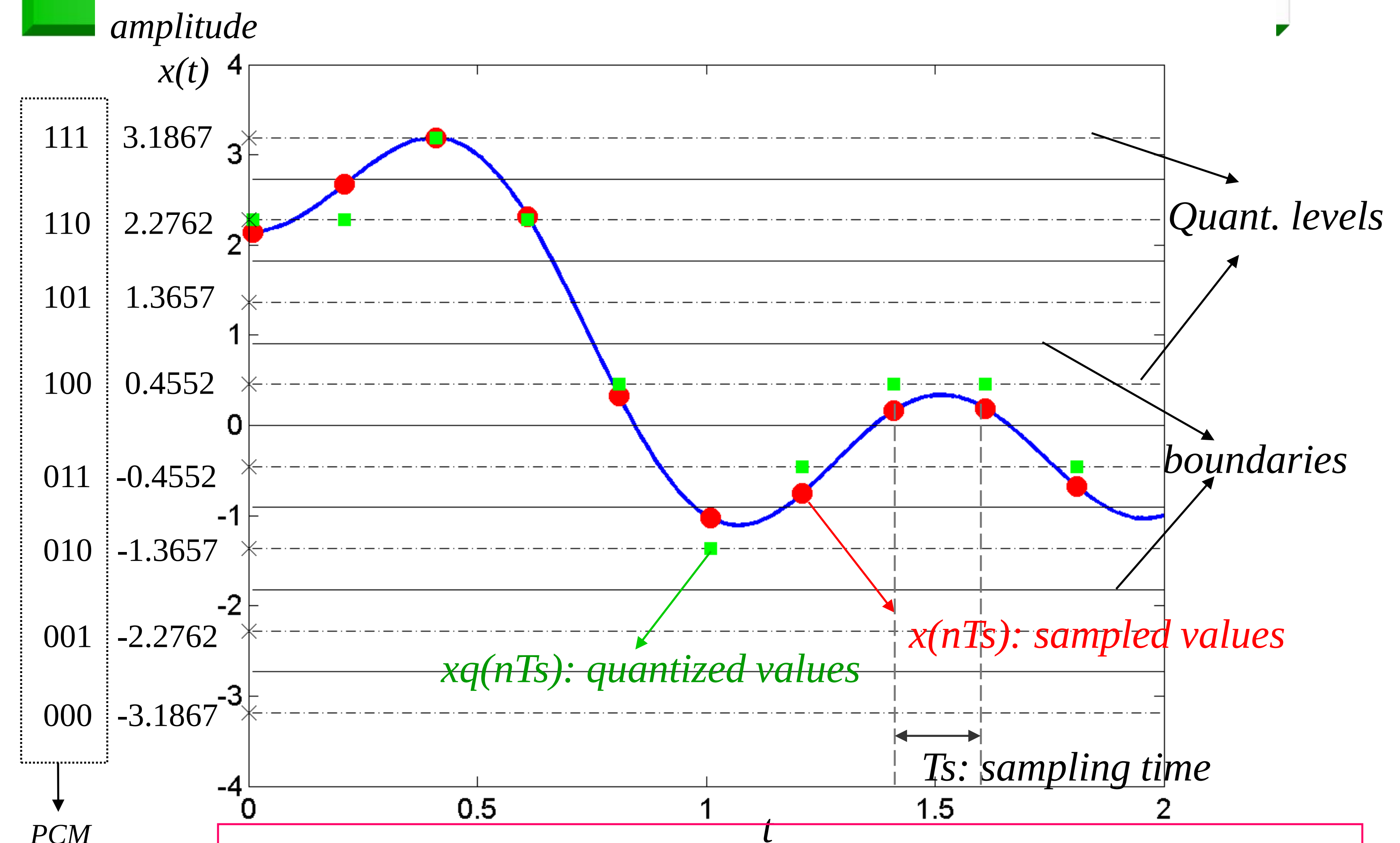
$$\left(\frac{S}{N}\right)_q = \frac{V_p^2}{\sigma^2} = 3L^2$$

Encoding (PCM)

- A uniform linear quantizer is called Pulse Code Modulation (PCM).
- Pulse code modulation (PCM): Encoding the quantized signals into a digital word (PCM word or codeword).
  - Each quantized sample is digitally encoded into an l bits codeword where L in the number of quantization levels and

$$l = \log_2 L$$

Quantization example

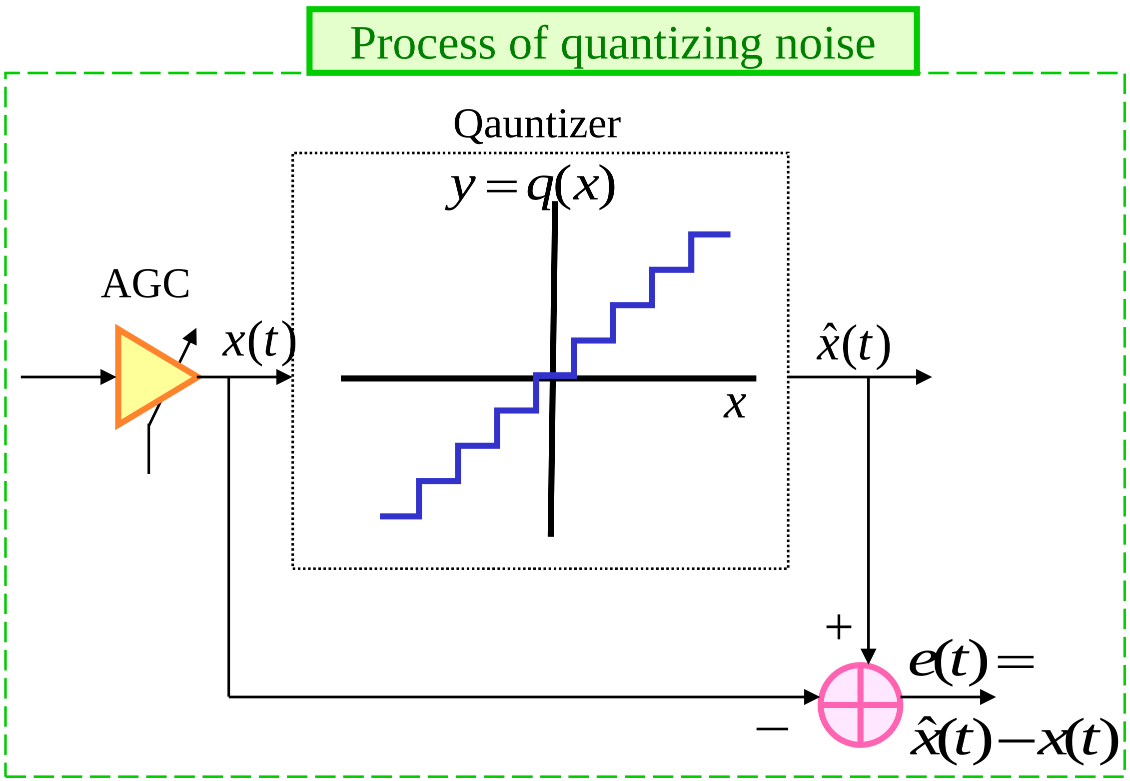


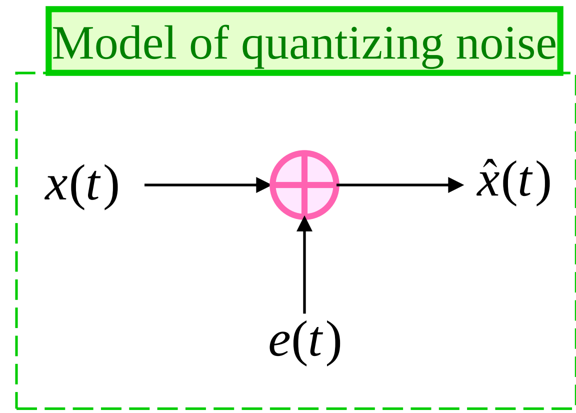
110	110	111	110	100	010	011	100	100	011	PCM sequence

Quantization error

- Quantizing error: The difference between the input and output of a quantizer

$$\Rightarrow \quad e(t) = \hat{x}(t) - x(t)$$





The Noise Model is an approximation!

Quantization error …

- Quantizing error:
  - **Granular or linear errors** happen for inputs within the dynamic range of quantizer
  - **Saturation errors** happen for inputs outside the dynamic range of quantizer
    - Saturation errors are larger than linear errors (AKA as "Overflow" or "Clipping")
    - Saturation errors can be avoided by proper tuning of AGC
    - Saturation errors need to be handled by Overflow Detection!
- Quantization noise variance:

$$\sigma_q^2 = \mathbf{E}\{[x - q(x)]^2\} = \int_{-\infty}^{\infty} e^2(x) p(x) dx = \sigma_{\text{Lin}}^2 + \sigma_{\text{Sat}}^2$$

$$\sigma_{\text{Lin}}^2 = 2 \sum_{l=0}^{L/2-1} \frac{q_l^2}{12} p(x_l) q_l \quad \xrightarrow{\text{Uniform q.}} \quad \sigma_{\text{Lin}}^2 = \frac{q^2}{12}$$

Uniform and non-uniform quant.

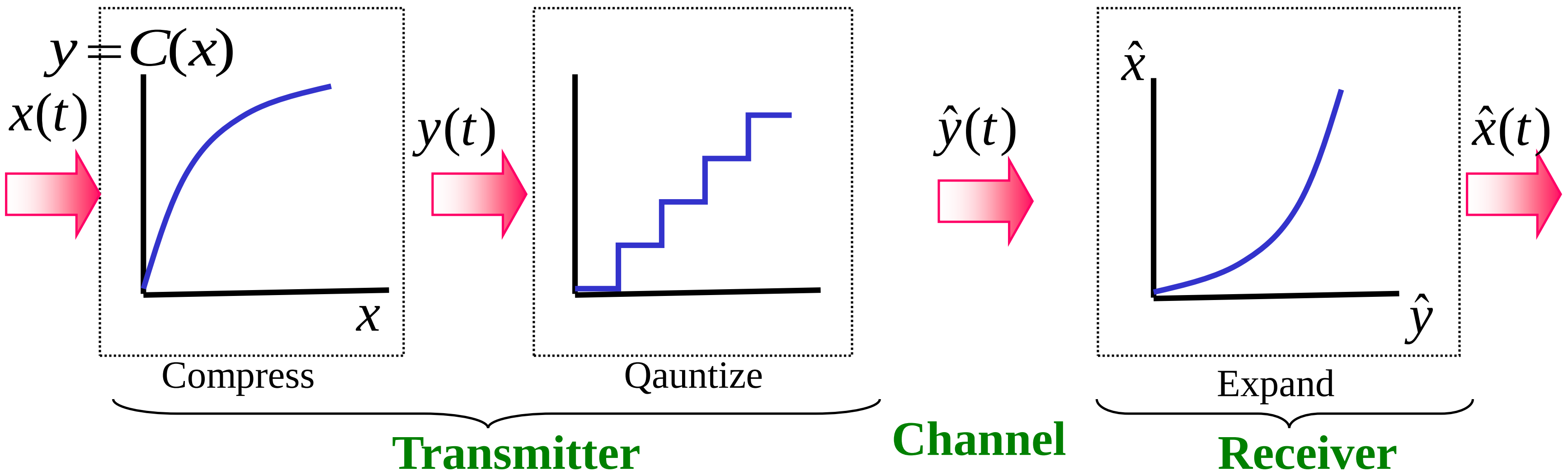
- Uniform (linear) quantizing:
  - No assumption about amplitude statistics and correlation properties of the input.
  - Not using the user-related specifications
  - Robust to small changes in input statistic by not finely tuned to a specific set of input parameters
  - Simple implementation
- Application of linear quantizer:
  - Signal processing, graphic and display applications, process control applications
- Non-uniform quantizing:
  - Using the input statistics to tune quantizer parameters
  - Larger SNR than uniform quantizing with same number of levels
  - Non-uniform intervals in the dynamic range with same quantization noise variance
- Application of non-uniform quantizer:
  - Commonly used for speech

    Examples are μ-law (US) and A-law (international)

Non-uniform quantization

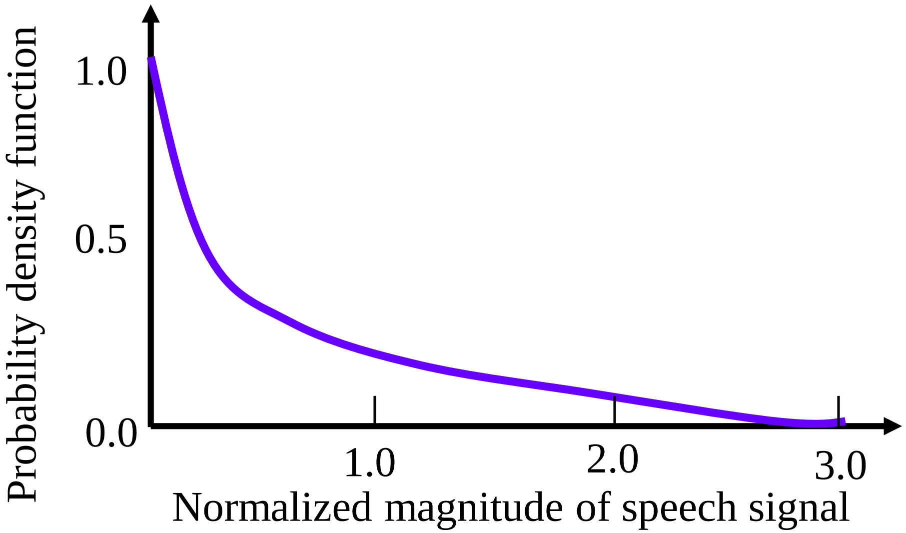
- It is achieved by uniformly quantizing the "compressed" signal. (actually, modern A/D converters use Uniform quantizing at 12-13 bits and compand digitally)
- At the receiver, an inverse compression characteristic, called "expansion" is employed to avoid signal distortion.

compression+expansion ⇨ companding



Statistics of speech amplitudes

- In speech, weak signals are more frequent than strong ones.



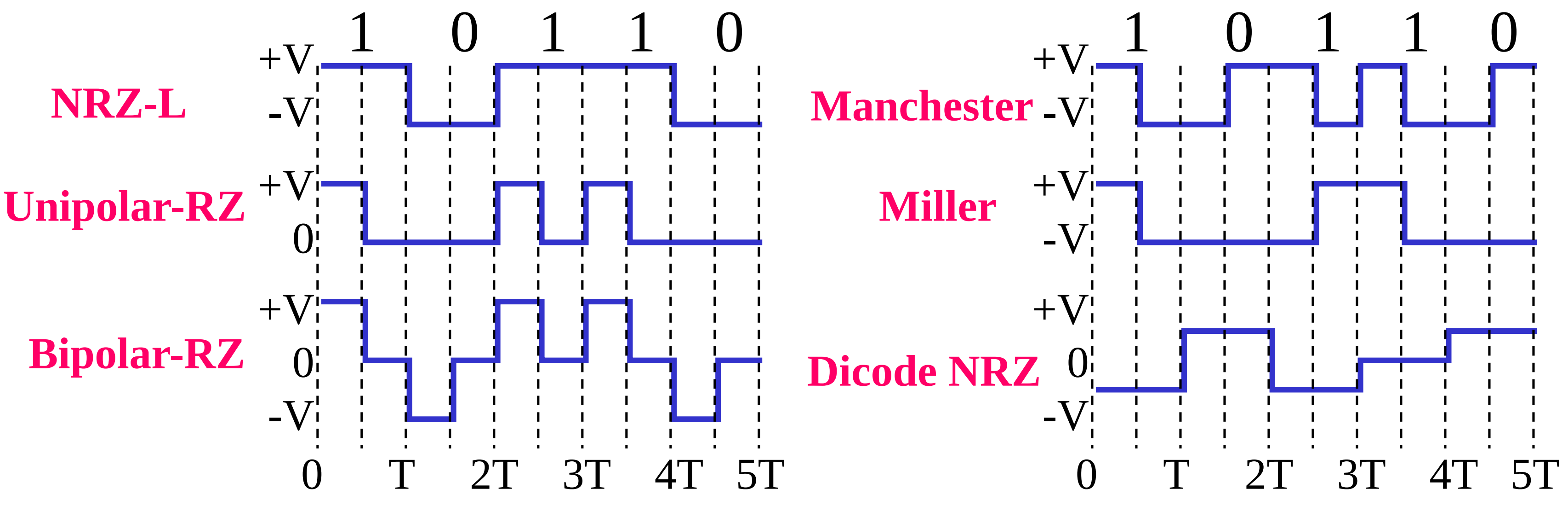
- Using equal step sizes (uniform quantizer) gives low $\left(\frac{S}{N}\right)_q$ for weak signals and high $\left(\frac{S}{N}\right)_q$ for strong signals.
  - Adjusting the step size of the quantizer by taking into account the speech statistics improves the average SNR for the input range.

Baseband transmission

- To transmit information through physical channels, PCM sequences (codewords) are transformed to pulses (waveforms).
  - Each waveform carries a symbol from a set of size M.
  - Each transmit symbol represents $k = \log_2 M$ bits of the PCM words.
  - PCM waveforms (line codes) are used for binary symbols (M=2).
  - M-ary pulse modulation are used for non-binary symbols (M>2).

PCM waveforms

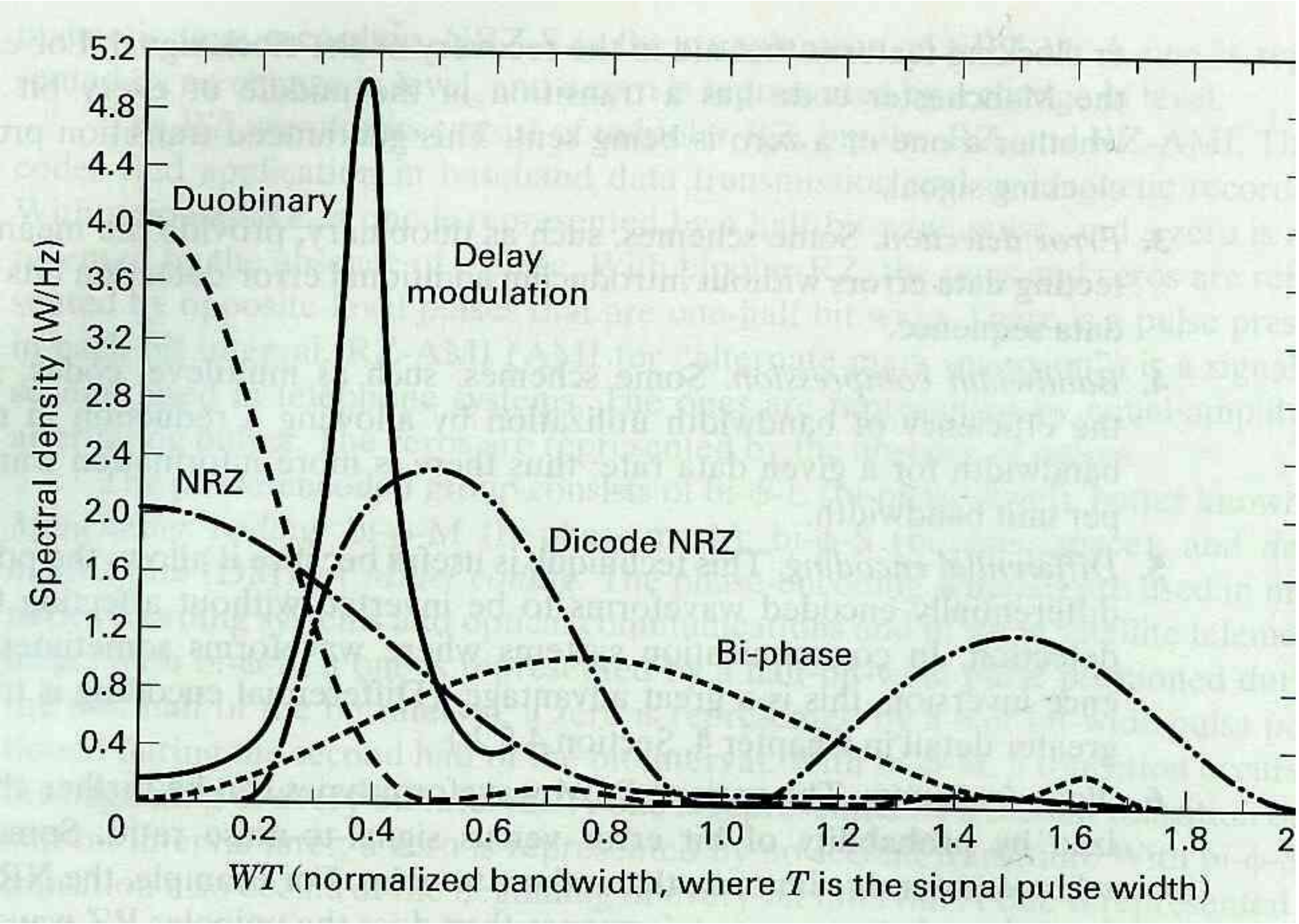
- PCM waveforms category:
  - Nonreturn-to-zero (NRZ)
  - Return-to-zero (RZ)
  - Phase encoded
  - Multilevel binary



PCM waveforms …

- Criteria for comparing and selecting PCM waveforms:
  - Spectral characteristics (power spectral density and bandwidth efficiency)
  - Bit synchronization capability
  - Error detection capability
  - Interference and noise immunity
  - Implementation cost and complexity

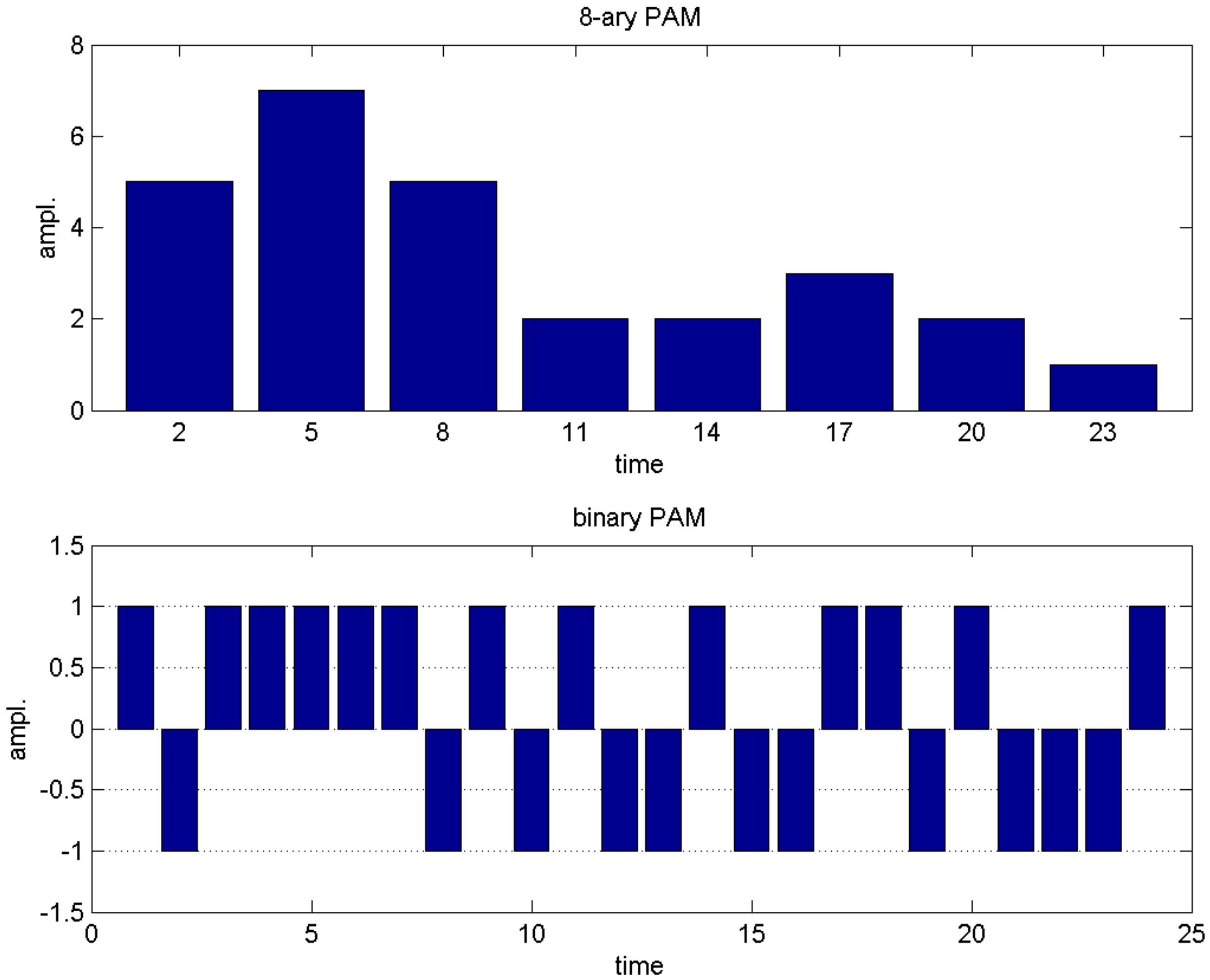
Spectra of PCM waveforms



M-ary pulse modulation

- M-ary pulse modulations category:
    - M-ary pulse-amplitude modulation (PAM)
    - M-ary pulse-position modulation (PPM)
    - M-ary pulse-duration modulation (PDM)
- M-ary PAM is a multi-level signaling where each symbol takes one of the M allowable amplitude levels, each representing $k = \log_2 M$ bits of PCM words.
- For a given data rate, M-ary PAM ($M>2$) requires less bandwidth than binary PCM.
- For a given average pulse power, binary PCM is easier to detect than M-ary PAM ($M>2$).

PAM example


8-ary PAM
ampl.
8
6
4
2
0
2
5
8
11
14
17
20
23
time
binary PAM
ampl.
1.5
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
time