

EE-387 Probability for Electrical and Computer Engineers
Assignment 4 (due on Thursday, August 4, 2005 before lecture)

Problem 1: (Problem 3.8.5 from Yates and Goodman) The time between telephone calls at a telephone switch is an exponential random variable T with expected value 0.01. Given $T > 0.02$,

- (a) What is $E[T|T > 0.02]$, the conditional expected value of T ?
- (b) What is $\text{Var}[T|T > 0.02]$, the conditional variance of T ?

Problem 2: (Problem 4.2.2 from Yates and Goodman) Random variables X and Y have the joint PMF

$$P_{X,Y}(x,y) = \begin{cases} c|x+y| & x = -2, 0, 2 \\ & y = -1, 0, 1 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) What is the value of the constant c ?
- (b) What is $P[Y < X]$?
- (c) What is $P[Y > X]$?
- (d) What is $P[Y = X]$?
- (e) What is $P[X < 1]$?

Problem 3: (Problem 4.3.2 from Yates and Goodman) Given the random variables X and Y in Problem 2, find (a) The marginal PMFs $P_X(x)$ and $P_Y(y)$, (b) The expected values $E[X]$ and $E[Y]$, (c) The standard deviations σ_X and σ_Y .

Problem 4: (Problem 4.4.1 from Yates and Goodman) Random variables X and Y have the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} c & x+y \leq 1, x \geq 0, y \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) What is the value of the constant c ?
- (b) What is $P[X \leq Y]$?
- (c) What is $P[X + Y \leq 1/2]$?

Problem 5: (Problem 4.4.3 from Yates and Goodman) Random variables X and Y have joint

PDF

$$f_{X,Y}(x,y) = \begin{cases} 6e^{-(2x+3y)} & x \geq 0, y \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $P[X > Y]$ and $P[X + Y \leq 1]$. (b) Find $P[\min(X, Y) \geq 1]$. (c) Find $P[\max(X, Y) \leq 1]$.

Problem 6: (Problem 4.5.5 from Yates and Goodman) Over the circle $X^2 + Y^2 \leq r^2$, random variable X and Y have the PDF

$$f_{X,Y}(x,y) = \begin{cases} 2|xy|/r^4 & x^2 + y^2 \leq r^2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) What is the marginal PDF $f_X(x)$? (b) What is the marginal PDF $f_Y(y)$?

Problem 7: (Problem 4.5.6 from Yates and Goodman) Random variable X and Y have the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} cy & 0 \leq y \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Draw the region of nonzero probability. (b) What is the value of the constant c ? (c) What is $F_X(x)$? (d) What is $F_Y(y)$? (e) What is $P[Y \leq X/2]$?

Problem 8: (Problem 4.6.2 from Yates and Goodman) Given random variables X and Y in Problem 2 and the function $W = X + 2Y$, find (a) The probability mass function $P_W(w)$, (b) The expected value $E[W]$, (c) $P[W > 0]$.

Problem 9: (Problem 4.6.6 from Yates and Goodman) Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} x+y & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Let $W = \max(X, Y)$. (a) What is S_W , the range of W ? (b) Find $F_W(w)$ and $f_W(w)$.

Problem 10: (Problem 4.6.8 from Yates and Goodman) Random variables X and Y have joint

PDF

$$f_{X,Y}(x,y) = \begin{cases} 2 & 0 \leq y \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Let $W = Y/X$. (a) What is S_W , the range of W ? (b) Find $F_W(w)$, $f_W(w)$, and $E[W]$.

Problem 11: (Problem 4.7.10 from Yates and Goodman) Random variables X and Y have joint

PDF

$$f_{X,Y}(x,y) = \begin{cases} 5x^2/2 & -1 \leq x \leq 1; \\ & 0 \leq y \leq x^2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) What are $E[X]$ and $\text{Var}[X]$? (b) What are $E[Y]$ and $\text{Var}[Y]$? (c) What is $\text{Cov}[X, Y]$? (d) What is $E[X + Y]$? (e) What is $\text{Var}[X + Y]$?