LECTURE 2

- **Readings:** Sections 1.1, 1.2

Lecture outline

- Sample space of an experiment
  - Examples
- Axioms of probability
  - More examples

Probability and the "real world"

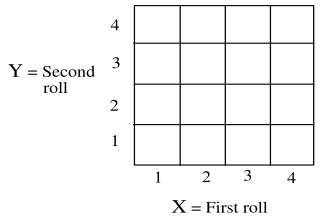
- A branch of math: Axioms $\Rightarrow$ theorems
  - **Thm:** Frequency of event A is $\mathbf{P}(A)$
- Are probabilities frequencies?
  - $\mathbf{P}$(coin toss yields heads) $= 1/2$
  - $\mathbf{P}$(a problem occurs with a nuclear plant) $= 10^{-8}$
- Probability models as a way of describing uncertainty
  - Use for consistent reasoning
  - Use for predictions, decisions

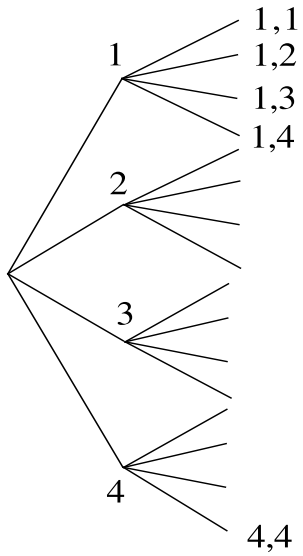
Sample space

- List of possible outcomes
- List must be:
  - Mutually exclusive
  - Collectively exhaustive
  - At the "right" granularity

Sample space examples

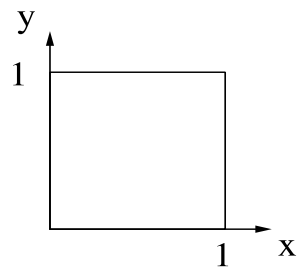
- Two rolls of a tetrahedral die
  - Sample space vs. sequential description





A continuous sample space

- (x, y) such that $0 \leq x, y \leq 1$



Axioms of probability

- **Event:** a subset of the sample space
- Probability is assigned to events

Axioms:

1. $\mathbf{P}(A) \geq 0$
2. $\mathbf{P}(\text{universe}) = 1$
3. If $A \cap B = \emptyset$,
   then $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$

- $\mathbf{P}(\{s_1, s_2, \ldots, s_k\}) = \mathbf{P}(s_1) + \cdots + \mathbf{P}(s_k)$

  – Axiom 3 needs strengthening
  – Do weird sets have probabilities?

Example 1

- Let every possible outcome have probability $1/16$
- $\mathbf{P}(X = 1) =$
- Let $Z = \min(X, Y)$
- $\mathbf{P}(Z = 1) =$
- $\mathbf{P}(Z = 2) =$
- $\mathbf{P}(Z = 3) =$
- $\mathbf{P}(Z = 4) =$

Discrete uniform law

- Let all sample points be equally likely
- Then,

$$\mathbf{P}(A) = \frac{\text{number of elements of } A}{\text{total number of sample points}}$$

- Just count...

Example 2

- Each of two people choose a number between zero and one. What is the probability that they are at most 1/4 apart?
- Draw sample space and event of interest:

- Need to choose a probability law:
  – Choose **uniform** law: probability = area

The probability is:

A word about infinite sample spaces

- Sample space: $\{1, 2, \ldots\}$
  – We are given $\mathbf{P}(n) = 2^{-n}$
  – Find $\mathbf{P}(\text{outcome is even})$
- Solution:

$$\begin{aligned} \mathbf{P}(\{2, 4, 6, \ldots\}) &= \mathbf{P}(2) + \mathbf{P}(4) + \cdots \\ &= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots = \frac{1}{3} \end{aligned}$$

- Axiom needed:
  If $A_1, A_2, \ldots$ are disjoint events, then:

$$\mathbf{P}(A_1 \cup A_2 \cup \cdots) = \mathbf{P}(A_1) + \mathbf{P}(A_2) + \cdots$$