EE126: Probability and Random Processes **SP'07**

Problem Set 2 — Due Feb, 1

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Exercise 2.1. *Pick 3 balls from an urn containing 15 balls (7 red balls, 5 blue balls, and 3 greens balls). Specify the probability space for this experiment.*

Exercise 2.2. *A part selected for testing is equally likely to have been produced on any one of six cutting tools.*

- *What is the sample space?*
- *What is the probability that the part is from tool 1?*
- *What is the probability that the part is from tool 1 or tool 3?*
- *What is the probability that the part is not from tool 5?*

Exercise 2.3. *Let A and B be two events. Use the axioms of probability to prove the following:*

1. $P(A \cap B) \geq P(A) + P(B) - 1$
2. *Show that the probability that one and only one of the events A or B occurs is $P(A) + P(B) - 2 \cdot P(A \cap B)$.*

Exercise 2.4. *Measurements of the time needed to complete a chemical reaction might me modeled with the sample space $S = R^+$, the set of positive real numbers. Let*

$$E_1 = \{x | 1 \leq x \leq 10\} \quad \text{and} \quad E_2 = \{x | 3 \leq x \leq 118\}$$

Write the expressions for

$$E_1 \cup E_2, \quad E_1 \cap E_2, \quad E_1 \Delta E_2$$

Exercise 2.5. *Consider two events, X_1 and X_2. Prove the following identities:*

1. $P(X_1 \cap X_2) \leq P(X_1)$
2. $P(X_1) \leq P(X_1 \cup X_2)$
3. $P(X_1 \cup X_2) \leq P(X_1) + P(X_2)$

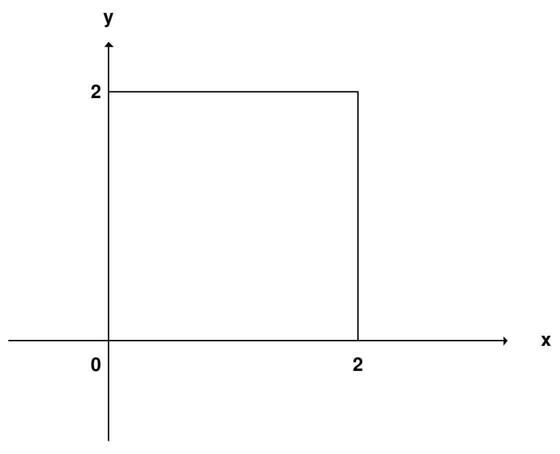
Exercise 2.6. *Twenty distinct cars park in the same parking lot everyday. Ten of these cars are US-made, while the other ten are foreign-made. This parking lot has exactly twenty spaces, and all are in a row, so the cars park side by side each day. The drivers have different schedules on any given day, however, so the position any car might take on a certain day is random.*

1. *In how many different ways can the cars line up?*
2. *What is the probability that on a given day, the cars will park in such a way that they alternate (e.g., US-made, foreign-made, US-made, foreign-made, etc)?*

Exercise 2.7. *Bob has a peculiar pair of four-sided dice. When he rolls the dice, the probability of any particular outcome is proportional to the sum of the outcome of each die. All outcomes that result in a particular sum are equally likely.*

1. *What is the probability of the sum being even?*
2. *What is the probability of Bob rolling a 4 and a 1?*

Exercise 2.8. *A baseball pitcher, Bill, has good control of his pitches. He always throws his pitches inside the "box" which we consider to be a 2 by 2 square. He throws the pitches uniformly over the square (i.e. the probability of a pitch falling within an area of the square is proportional to this area.) Let* $(0,0)$ *and* $(2,2)$ *be the coordinates of the lower-left corner and the upper-right corner of the square, respectively as shown below.*



Two groups A and B of fans are betting on where Bill's next pitch will fall. Among group A,

- *person 1 bets that the pitch is going to be in the left half part of the square, i.e.* $0 \leq x \leq 1$.
- *person 2 bets that it will be in one third of the square from the left, i.e.* $0 \leq x \leq \frac{2}{3}$.

- *and in general, person n makes the bet that the pitch will fall in the area $0 \leq x \leq 2/(n+1)$.*

1. *What is the probability that individual n from group A wins his bet?*
2. *What is the probability that individual n wins but not individual $n+1$?*

Among group B, that fans bet in a similar fashion, but on the height of the pitch, i.e. individual n bets that the next pitch will fall in the area $0 \leq y \leq 2/(n+1)$.

(c) *What is the probability that individuals 1 through n of* both *groups win their bets?*

(d) *When n goes to infinity, what is the probability that all fans of both groups win their bets? Note: Be precise in your derivation.*