EE126: Probability and Random Processes **SP'07**

Problem Set 7 — Due March, 22

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Problem 7.1. *Let u and v be independent, standard normal random variables (i.e., u and v are independent Gaussian random variables with means of zero and variances of one). Let*

$$x = u + v$$

$$y = u - 2v.$$

1. *Do x and y have a bivariate normal distribution? Explain.*
2. *Provide a formula for $E[x|y]$.*

Problem 7.2. *Let $X = (X_1, X_2, X_3)$ be jointly Gaussian with joint pdf*

$$f_{X_1,X_2,X_3}(x_1, x_2, x_3) = \frac{e^{-(x_1^2+x_2^2-\sqrt{2}x_1x_2+\frac{1}{2}x_3^2)}}{2\pi\sqrt{\pi}}$$

Find a transformation A such that $Y = AX$ consists of independent Gaussian random variables.

Problem 7.3. *A signal of amplitude $s = 2$ is transmitted from a satellite but is corrupted by noise, and the received signal is $Z = s + W$, where W is noise. When the weather is good, W is normal with zero mean and variance 1. When the weather is bad, W is normal with zero mean and variance 4. Good and bad weather are equally likely. In the absence of any weather information:*

1. *Calculate the PDF of Z.*
2. *Calculate the probability that Z is between 1 and 3.*

Problem 7.4. *Suppose X, Y are independent gaussian random variables with the same variance. Show that $X - Y$ and $X + Y$ are independent.*

Problem 7.5. *Steve is trying to decide how to invest his wealth in the stock market. He decides to use a probabilistic model for the shares price changes. He believes that, at the end of the day, the change of price Z_i of a share of a particular company i is the sum of two components: X_i, due solely to the performance of the company, and the other Y due to investors' jitter.*

Assuming that Y is a normal random variable, zero-mean and with variance equal to 1, and independent of X_i. Find the PDF of Z_i under the following circumstances in part a) to c),

1. *X_1 is Gaussian with a mean of 1 dollar and variance equal to 4.*
2. *X_2 is equal to -1 dollars with probability 0.5, and 3 dollars with probability 0.5.*
3. *X_3 is uniformly distributed between -2.5 dollars and 4.5 dollars (No closed form expression is necessary.)*
4. *Being risk averse, Steve now decides to invest only in the first two companies. He uniformly chooses a portion V of his wealth to invest in company 1 (V is uniform between 0 and 1.) Assuming that a share of company 1 or 2 costs 100 dollars, what is the expected value of the relative increase/decrease of his wealth?*

Problem 7.6. *The Binary Phase-shift Keying (BPSK) and Quadrature Phase-shift Keying (QPSK) modulation schemes are shown in figure 7.1. We consider that in both cases, the symbols (S) are sent over an additive gaussian channel with zero mean and variance σ^2. Assuming that the symbols are equally likely, compute the average error probability for each scheme. Which one is better?*

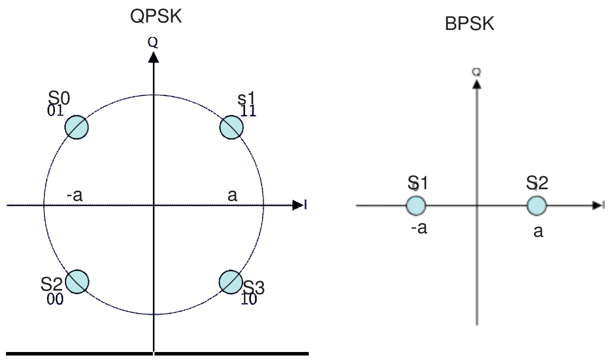

Figure 7.1. BPSK and QPSK modulations

Problem 7.7. *When using a multiple access communication channel, a certain number of users N try to transmit information to a single receiver. If the real-valued random variable X_i represents the signal transmitted by user i, the received signal Y is*

$$Y = X_1 + X_2 + \cdots + X_N + Z,$$

where Z is an additive noise term that is independent of the transmitted signals and is assumed to be a zero-mean Gaussian random variable with variance σ_Z^2. We assume that the signals transmitted by different users are mutually independent and, furthermore, we assume that they are identically distributed, each Gaussian with mean μ and variance σ_X^2.

1. *If N is deterministically equal to 2, find the transform <u>or</u> the PDF of Y.*

2. *In most practical schemes, the number of users N is a random variable. Assume now that N is equally likely to be equal to $0, 1, \ldots, 10$.*
   (a) *Find the transform <u>or</u> the PDF of Y.*
   (b) *Find the mean and variance of Y.*
   (c) *Given that $N \geq 2$, find the transform <u>or</u> PDF of Y.*