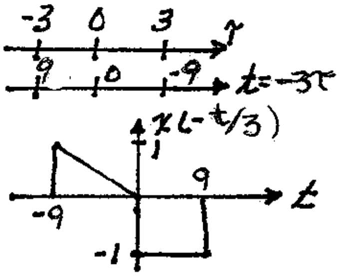
CHAPTER 2

2.1(a)(i)



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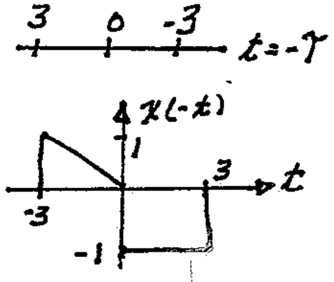

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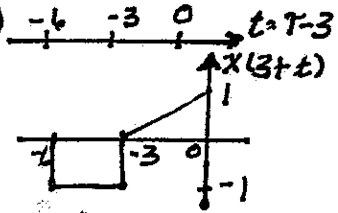

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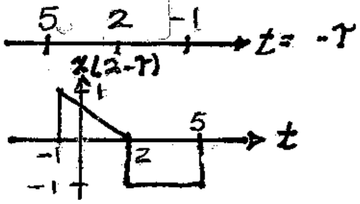

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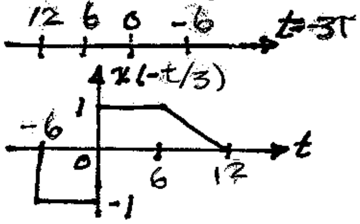

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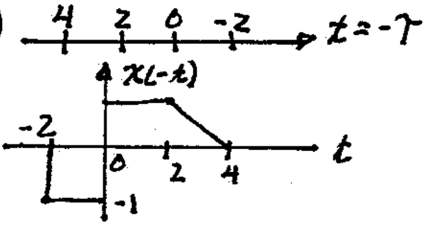

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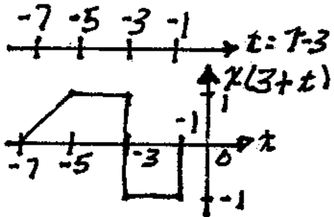

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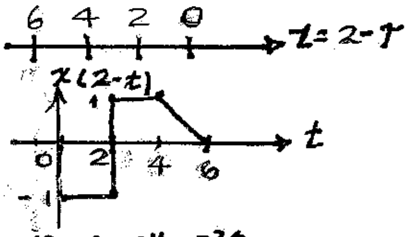
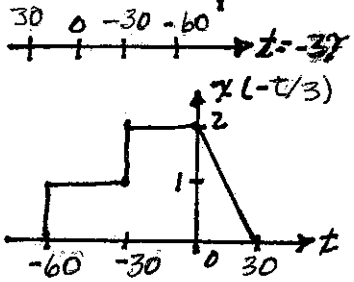


(c) (i)



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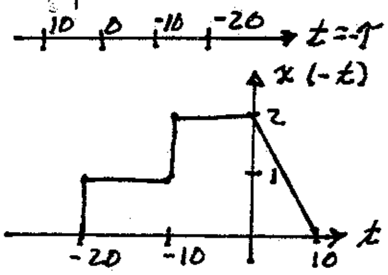

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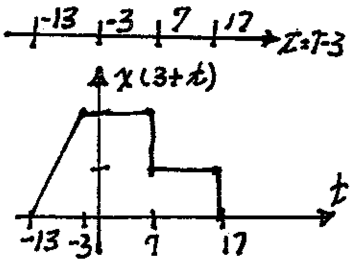

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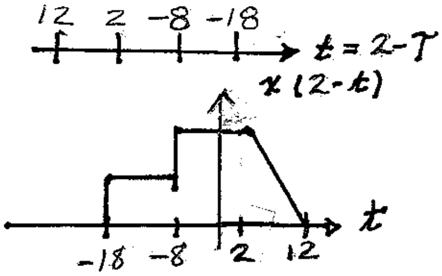
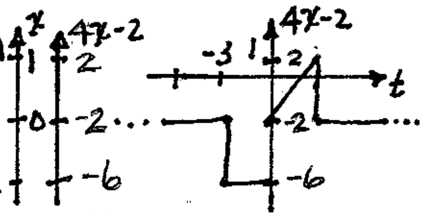


2.2.(a)(i)



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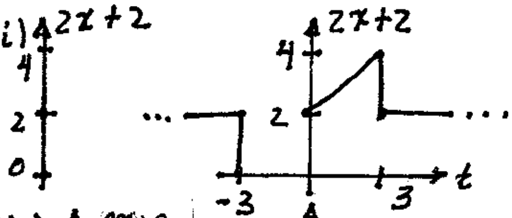

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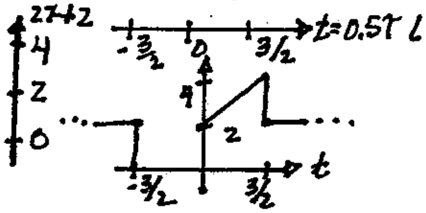

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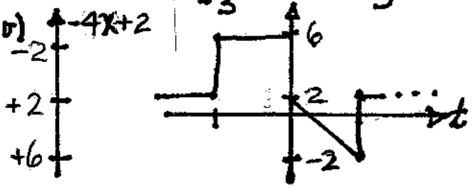
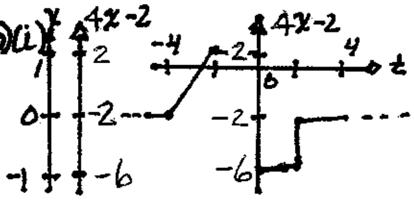


(b)(i)



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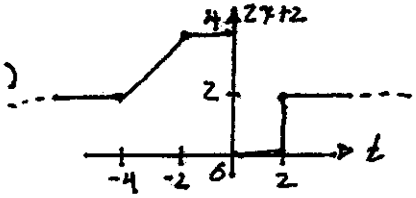

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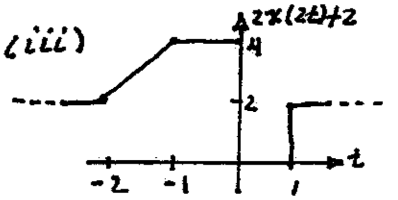

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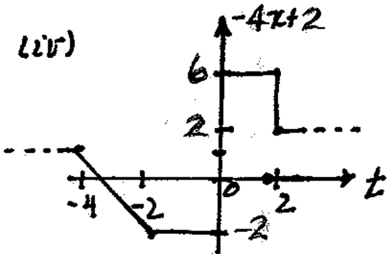
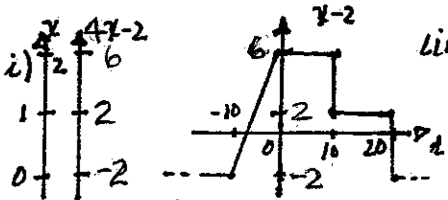


(c)(i)



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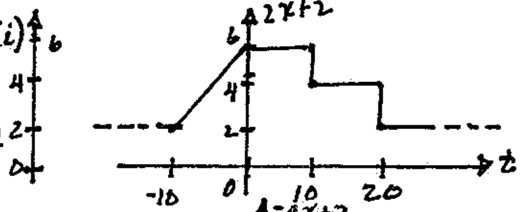

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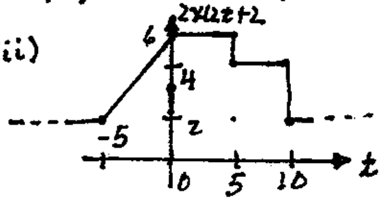

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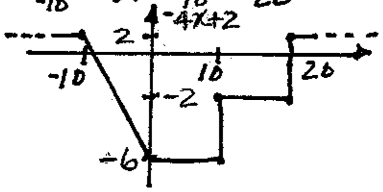
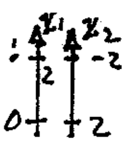


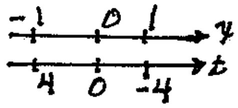
2.3(a) amplitude:



$\Rightarrow \quad x_2 = -Ax_1 + B$

$= -2x_1 + 2$

time:



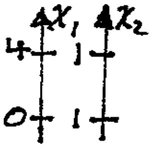
$\Rightarrow \tau = -\frac{1}{4}t$; ∴ $x_2(t) = -2x_1(-\frac{t}{4}) + 2$

(b) $t = -2$, $x_2(-2) = -2x_1(\frac{1}{2}) + 2 = -2(1) + 2 = 0$ ✓

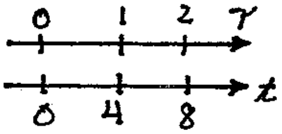
$t = 2$, $x_2(2) = -2x_1(-\frac{1}{2}) + 2 = -2(2) + 2 = -2$ ✓

$t = -1$, $x_2(-1) = -2x_1(\frac{1}{2}) + 2 = -2(\frac{1}{2}) + 2 = 1$ ✓

2.4 (a) at $t=0$:



$\therefore x_2=-\frac{1}{2}x_1+1$



$\therefore t=4\tau$

$\therefore \underline{x_2(t)=-\frac{1}{2}x_1(t/4)+1}$

(b) $t=\frac{1}{2}$: $x_2(\frac{1}{2})=-\frac{1}{2}x_1(\frac{1}{8})+1=-\frac{1}{2}(4)+1=-1$ ✓

$t=-\frac{1}{2}$: $x_2(-\frac{1}{2})=-\frac{1}{2}(-\frac{1}{8})+1=1$ ✓

$t=6$: $x_2(6)=-\frac{1}{2}x_2(\frac{3}{2})+1=-\frac{1}{2}(0)+1=1$ ✓

(c) Let $t=4\tau$: $x_2(4\tau)=-\frac{1}{2}x_1(\tau)+1 \Rightarrow \underline{x_1(t)=-2x_2(4t)+2}$

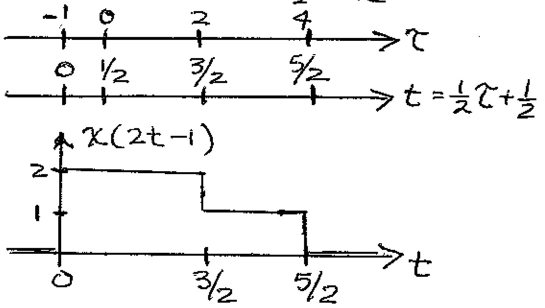
(d) $t=\frac{1}{2}$: $x_1(\frac{1}{2})=-2x_2(2)+2=-2(-1)+2=4$ ✓

$t=-\frac{1}{2}$: $x_1(-\frac{1}{2})=-2x_2(-2)+2=-2(1)+2=0$ ✓

$t=3$: $x_1(3)=-2x_2(12)+2=-2(-1)+2=4$ ✓

2.5 Time transformation

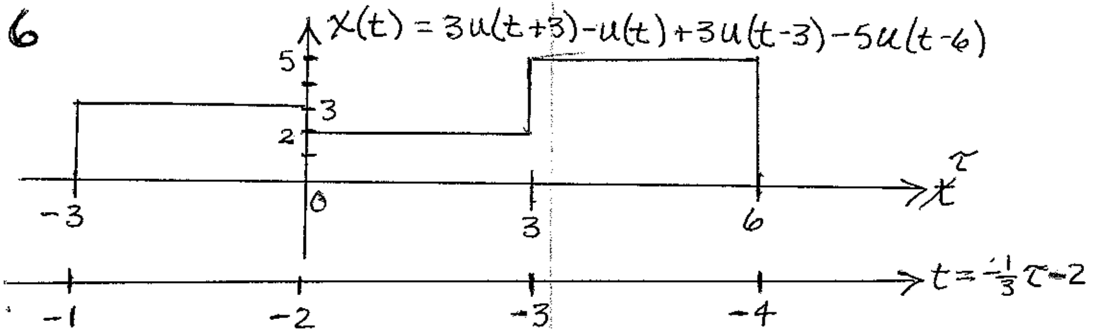
$\tau=2t-1 \Rightarrow t=\frac{1}{2}\tau+\frac{1}{2}$



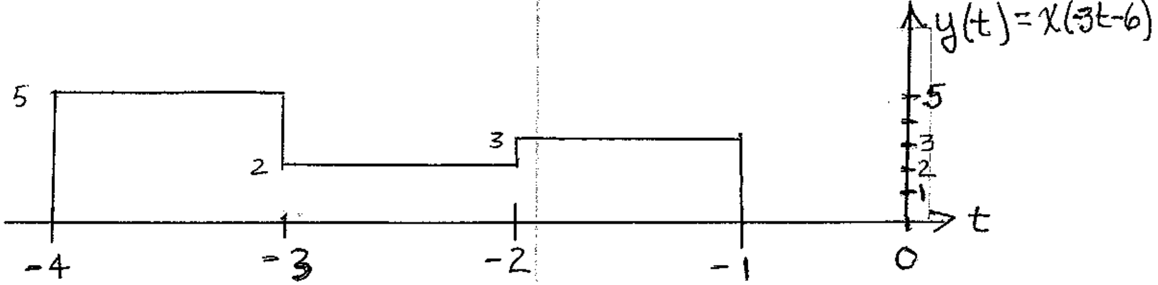
or shift first
$-1 \Rightarrow$ delay
then scale $2 \Rightarrow$ Compression

2.6

$x(t)=3u(t+3)-u(t)+3u(t-3)-5u(t-6)$

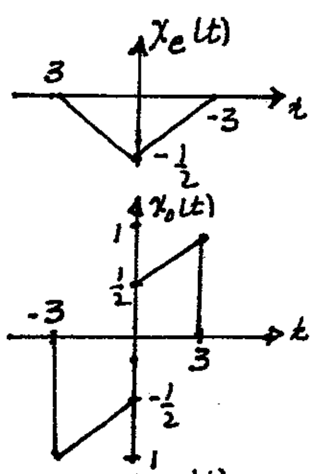


$y(t)=x(-3t-6)$

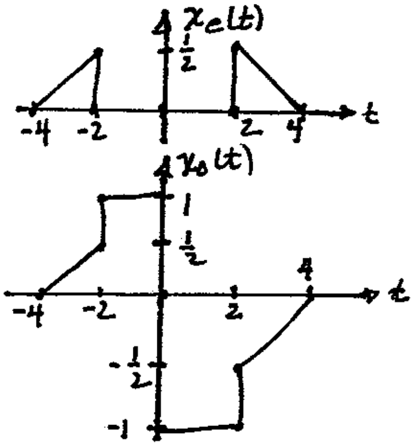


shift first; then scale : $-6 \Rightarrow$ delay (shift to right)
$-3t \Rightarrow$ Compress by 1/3 and reverse

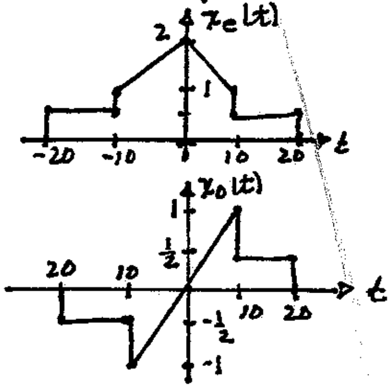
2.7. (a)



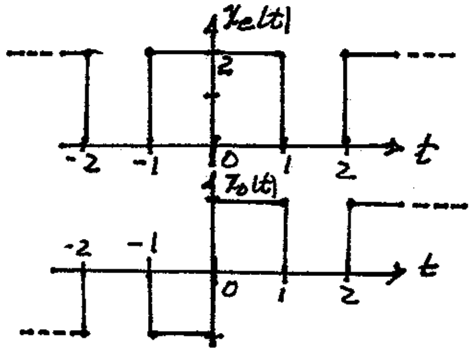
(b)



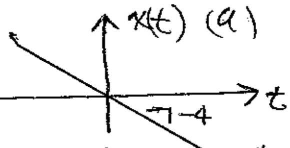
(c)



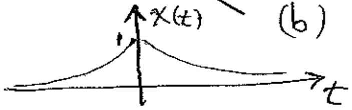
(d)



2.8 (a) $x(-t) = 4t = -x(t)$ $\therefore$ odd



(b) $x(-t) = e^{-|-t|} = e^{-|t|} = x(t)$ $\therefore$ even



(c) $x(-\ ) = 5\cos(-3t) = 5\cos(3t) = x(t)$

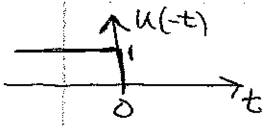
$\therefore$ even

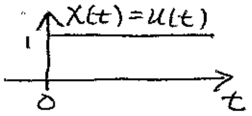
(d) $x(-t) = \sin(-3t - \pi/2) = -\sin(3t + \pi/2) = -\cos(3t)$

$x(t) = \sin(3t - \pi/2) = -\cos(3t)$ $\therefore$ even

(e) $x(-t) = u(-t)$

neither even nor odd





2.9 (a) $\int_{-T}^{T} x_o(t) = \int_{-T}^{0} x_o(t)dt + \int_{0}^{T} x_o(t)dt$; $x_o(t) = -x_o(-t)$

$\therefore \int_{-T}^{0} x_o(t)dt = -\int_{-T}^{0} x_o(-t)dt \Big|_{t=-\tau} = \int_{-T}^{0} x_o(\tau)d\tau = -\int_{0}^{T} x_o(\tau)d\tau$

$\therefore \int_{-T}^{T} x_o(t)dt = 0$

(b) $\int_{-T}^{T} x(t)dt = \int_{-T}^{T}[x_e(t)+x_o(t)]dt = \int_{-T}^{T} x_e(t)dt$

and $A_x = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x_e(t)d\tau$

(c) $x_o(t) = -x_o(t) \Rightarrow x_o(0) = -x_o(0) \Rightarrow x_o(0) = 0$

$\therefore x_o(0) = x_e(0) + x_o(0) = x_e(0)$

2.10. (a) $x_t(t) = x_{e1}(t) + x_{e2}(t)$

$x_t(-t) = x_{e1}(t) + x_{e2}(-t) = x_{e1}(t) + x_{e2}(t) = x_t(t)$, $\therefore$ even

(b) $x_t(t) = x_{o1}(t) + x_{o2}(t)$

$x_t(-t) = x_{o1}(-t) + x_{o2}(-t) = -x_{o1}(t) - x_{o2}(t) = -x_t(t)$, $\therefore$ odd

(c) $x_t(t) = x_e(t) + x_o(t)$

$x_t(-t) = x_e(t) + x_o(-t) = x_e(t) - x_o(t)$, $\therefore$ neither

(d) $x_t(t) = x_{e1}(t)\,x_{e2}(t)$

$x_t(-t) = x_{e1}(t)\,x_{e2}(-t) = x_{e1}(t)\,x_e(t) = x_t(t)$, $\therefore$ even

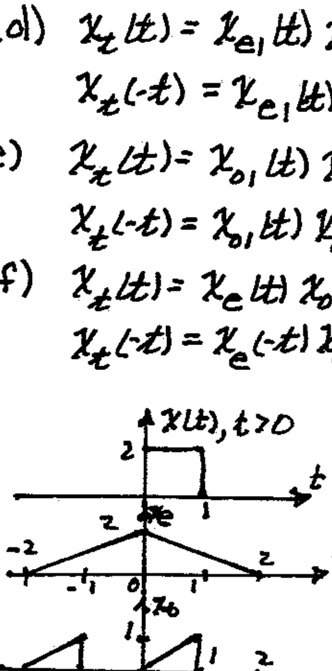
(e) $x_t(t) = x_{o1}(t)\,x_{o2}(t)$

$x_t(-t) = x_{o1}(t)\,x_{o2}(-t) = [-x_{o1}(t)][-x_{o2}(t)] = x_t(t)$, $\therefore$ even

(f) $x_t(t) = x_e(t)\,x_o(t)$

$x_t(-t) = x_e(-t)\,x_o(-t) = x_e(t)[-x_o(t)] = -x_t(t)$, $\therefore$ odd

2.11.



$x_e(t) = x_e(-t)$

$x_o(t) = x(t) - x_e(t)$

$x_o(t) = -x_o(-t)$

$\rightarrow x(t) = x_e(t) + x_o(t)$

2.12(a) $x(t) = 7\sin(3t)$, $\omega_0 = 3$, $\frac{2\pi}{\omega_0} = T_0 = 2\pi/3$

$x(t+T_0) = 7\sin[3(t+2\pi/3)] = 7\ (3t+2\pi) = 7\sin(3t)$ ✓

(b) $x(t) = \sin(8t+30^\circ)$, $\omega_0 = 8$, $T_0 = 2\pi/\omega_0 = \pi/4$

$x(t+T_0) = \sin[8(t+\pi/4)+30^\circ] = \sin(8t+30^\circ+2\pi) = \overset{✓}{\sin(8t+30^\circ)}$

(c) $x(t) = e^{j2t}$, $\omega_0 = 2$, $T_0 = \frac{2\pi}{\omega_0} = \pi$

$x(t+T_0) = e^{j2(t+\pi)} = e^{j(2t+2\pi)} = e^{j2t}$ ✓

(d) $x(t) = \cos(t) + \sin(2t)$; $\omega_0 = 1$, $T_0 = 2\pi$

$x(t+T_0) = \cos(t+2\pi) + \sin(2t+4\pi) = x(t)$ ✓

(e) $x(t) = e^{j(5t+\pi)}$, $\omega_0 = 5$, $T_0 = 2\pi/5$

$x(t+T_0) = e^{j[5(t+2\pi/5)+\pi]} = e^{j(5t+2\pi+\pi)} = e^{j2\pi} e^{j(5t+5\pi)} = x(t)$ ✓

(f) $x(t) = e^{-j10t} + e^{j15t}$, $\omega_0 = 5$, $T_0 = 2\pi/5$

$x(t+T_0) = e^{-j10(t+2\pi/5)} + e^{j15(t+2\pi/5)} = e^{-j10t} e^{-j4\pi} + e^{j15t} e^{j6\pi}$

$x(t+T_0) = x(t)$ ✓

2.13.(a) $x(t+T_0) = 3\cos(15t+15T_0+30^\circ) + \sin(20t+20T_0)$

$\hookrightarrow$ periodic $T = k_1\left(\frac{2\pi}{15}\right)$ $\quad\hookrightarrow$ periodic $T = k_2\left(\frac{2\pi}{20}\right)$

$\therefore \quad 15T_0 = k_1 2\pi \qquad\qquad 20T_0 = k_2 2\pi$

$\therefore \quad \frac{k_1}{k_2} = \frac{15}{20} = \frac{3}{4} \Rightarrow k_1 = 3,\ k_2 = 4$ for smallest values

$\therefore \quad 15T_0 = 3(2\pi) \Rightarrow \underline{T_0 = 2\pi/5}$

for $x(t)$, $\underline{T_0 = 2\pi/5} \Rightarrow \underline{\omega_0} = \frac{2\pi}{T_0} = \underline{5}$

(b) Same development as in (a)

$\therefore \quad 5T_0 = k_1 2\pi \quad ; \quad \pi T = k_2 2\pi$

$\therefore k_1/k_2 = 5/\pi$ $\quad\therefore$ not rational and <u>not periodic</u>

(c) $x(t+T_0) = \cos(5t+5T_0) + 3e^{-j(10t+10T)}$

$\hookrightarrow$ periodic, $T_0 = k_1\left(\frac{2\pi}{5}\right)$ $\quad\hookrightarrow$ periodic, $T_0 = k_2\left(\frac{2\pi}{10}\right)$

$\therefore \quad 5T_0 = k_1 2\pi \qquad\qquad 10T_0 = k_2 2\pi$

$\therefore \quad \frac{k_1}{k_2} = \frac{5}{10} = \frac{1}{2}$, $\quad\therefore 5T_0 = k_1 2\pi = 2\pi$

$T_{0x} = T_0 = \underline{2\pi/5}$; $\omega_{0x} = 2\pi/T_{0x} = \underline{5}$

2.14.(a) $x(t) = x_1(t) + x_2(t)$

$x(t+T) = x_1(t+T) + x_2(t+T)$

$x_1(t+T) = x_1(t + k_1T_1)$; $x_2(t+T) = x_2(t + k_2T_2)$

$\therefore\ T = k_1T_1 = k_2T_2 \Rightarrow \underline{\frac{T_2}{T_1} = \frac{k_1}{k_2}}$

(b) From (a), let T_{0x} be the smallest value of T, and remove any common factors in k_1 and k_2. Then

$T_{0x} = \underline{k_1T_1 = k_2T_2}$

2.15 (a) $x(t) = \cos(3t) + \sin(5t)$; $T_{01} = \frac{2\pi}{3}$, $T_{02} = 2\pi/5$

$T_{01}/T_{02} = 5/3 \therefore T_0 = 3T_{01} = 2\pi \Rightarrow \omega_0 = 1\,(\text{rad/s})$

(b) $x(t) = \cos(6t) + \sin(8t) + e^{j2t}$; $T_{01} = \frac{2\pi}{6}, T_{02} = \frac{2\pi}{8}, T_{03} = \frac{2\pi}{2}$

$\frac{T_{01}}{T_{02}} = 4/3$, $\frac{T_{01}}{T_{03}} = 1/3$ $\therefore T_0 = 3T_{01} = \pi \Rightarrow \omega_0 = 2\,(\text{rad/s})$

(c) $x(t) = \cos(3t) + \sin(\pi t)$; $T_{01} = 2\pi/3$, $T_{02} = 2\pi/\pi$

$\frac{T_{01}}{T_{02}} = 2\pi/6$ – irrational ∴ $x(t)$ is <u>not periodic</u>

(d) $x(t) = \sin(\pi/6\, t) + \sin(\pi/3\, t)$; $T_{01} = \frac{2\pi}{(\pi/6)} = 12$

$T_{02} = 2\pi/(\pi/3) = 6$

$\frac{T_{01}}{T_{02}} = 2/1 \therefore T_0 = T_{01} = 12 \Rightarrow \omega_0 = \pi/6\,(\text{rad/s})$

2.16

$$\int_{-\infty}^{\infty} \delta(at-b)\sin^2(t-4)\,dt = \int_{-\infty}^{\infty} \delta[a(t-\tfrac{b}{a})]\sin^2(t-4)\,dt$$

$$= \int_{-\infty}^{\infty} \delta[a(t-b/a)]\sin^2(\tfrac{b}{a}-4)\,dt \quad (\text{use shifting property of } \delta(t))$$

$$= \frac{1}{a}\sin^2(b/a - 4)\ ,\quad a > 0$$

2.17 -- ONE SOLUTION:

You're given that x(t)=u(t+8)

y(t) = x(-5t+7)= u(-5t+15)=u(-t+3).

therefore, z(t)=x(at+b)=u(at+b+8) =u(-t+3)

Solving for a and b, we get that a=-1 and b+8=3
or b=-5

2.18 By sifting property, y(t) = 1/2 x(2) + 1/2 x(-2)

2.19 (a) $x_1(t) = 2t\,u(t) - 4(t-1)\,u(t-1) + 2(t-2)\,u(t-2)$

(b) $t<0, \quad x_1(t) = 0$ ✓

$0<t<1, \quad x_1(t) = 2t$ ✓

$1<t<2, \quad x_1(t) = 2t - 4t + 4 = 4 - 2t$ ✓

$2<t, \quad x_1(t) = 4 - 2t + 2t - 4 = 0$ ✓

(c) $x(t) = \sum_{k=-\infty}^{\infty} x_1(t - kT_0) = \sum_{k=-\infty}^{\infty} x_1(t - 2k)$

2.20. (a) Let $at = \tau$; $\therefore \int_{-\infty}^{\infty} \delta(at)\,dt = \int_{-\infty}^{\infty} \delta(\tau)\,\frac{d\tau}{a}$

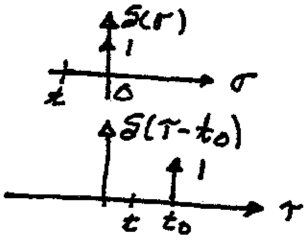
$= \frac{1}{a}\int_{-\infty}^{\infty} \delta(\tau)\,d\tau \Rightarrow \underline{\delta(at) = \frac{1}{a}\delta(t),\ a>0}$

For $a<0$, $at = \tau \Rightarrow -|a|t = \tau$, $dt = \frac{-d\tau}{|a|}$

$\therefore \int_{-\infty}^{\infty} \delta(at)\,dt = \int_{\infty}^{-\infty} \delta(\tau)\,\frac{-d\tau}{|a|} = \frac{1}{|a|}\int_{-\infty}^{\infty} \delta(\tau)\,d\tau$

$\therefore \underline{\delta(at) = \frac{1}{|a|}\delta(t)}$ for the general case.

(b) $\int_{-\infty}^{t} \delta(\sigma)\,d\sigma = u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$



$\therefore \int_{-\infty}^{t} \delta(\tau - t_0)\,d\tau = u(t - t_0)$

(c) $\int_{-\infty}^{\infty} f(t)\,\delta(t - t_0)\,dt = f(t_0)$

Let: $t - t_0 = \tau$, $dt = d\tau$, $t = \tau + t_0$

$\therefore \int_{-\infty}^{\infty} f(t - t_0)\,\delta(t - t_1)\,dt = \int_{-\infty}^{\infty} f(\tau)\,\delta[\tau - (t_1 - t_0)]\,d\tau = f(t_1 - t_0)$

(i) $\int_{-\infty}^{\infty} \sin 3t\;\delta(t)\,dt = \sin(3\times 0) = 1$

(ii) $\int_{-\infty}^{\infty} \sin 3t\;\delta(t-1)\,dt = \sin(3) = 0.1411$

(iii) $\int_{-\infty}^{\infty} \sin[3(t-4)]\,\delta(t-4)\,dt = \sin[3\times 0] = 1$

(iv) $\int_{-\infty}^{\infty} \sin[3(t-1)]\,\delta(t+2)\,dt = \sin[3(-3)] = \sin(-9) = -0.4121$

(v) $\int_{-\infty}^{\infty} \sin[3(t-1)]\,\delta(2t+4)\,dt = \frac{1}{2}\int_{-\infty}^{\infty} \sin(3t-3)\,\delta(t+2)\,dt = \frac{1}{2}\sin(-9)$

$= -.2061$

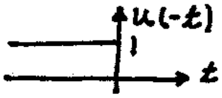
2.21 (a) $u(t/4-4) = u[\frac{1}{4}(t-16)] = u(t-16)$

(b) $u(t/4+4) = u[1/4(t+16) = u(t+16)$

(c) $u(-3t+6) = u[3(-t+2)] = u(-t+2)$

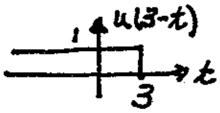
(d) $u(3t+6) = u[3(t+2)] = u(t+2)$

2.22 (a)



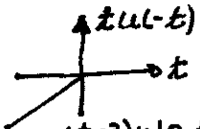
$u(-t) = 1-u(t)$

(b)



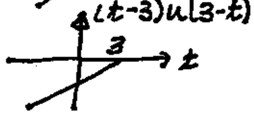
$u(3-t) = 1-u(t-3)$

(c)



$tu(-t) = t[1-u(t)]$

(d)



$(t-3)u(-t) = (t-3)[1-u(t-3)]$

2.23 (a) $y_2(t) = T_2[T_1[x(t)]]$, $y_3(t) = T_3[T_1[x(t)]]$

$y(t) = T_2[T_1[x(t)]] + T_4\{T_3[T_1[x(t)]] + T_5[x(t)]$

(b) $y(t) = T_3\{T_2[T_1[x(t)]]\} + T_4\{T_2[T_1[x(t)]]\} + T_5[T_1[x(t)]]$

(c) $y(t) = T_2[T_1[x(t)] + T_4\{T_3[T_1[x(t)]] \times T_5[x(t)]\}$

(d) $y(t) = T_3\{T_2[T_1[x(t)]]\} \times T_4\{T_2[T_1[x(t)]]\} \times T_5[T_1[x(t)]]$

2.24 $y(t) = T_3[m(t) + T_1[x(t)]]$

$m(t) = T_2[x(t) - T_4[y(t)]]$

$\therefore \underline{y(t) = T_3\{T_2[x(t) - T_4[y(t)] + T_1[x(t)]\}}$

2.25 $m(t) = T_1\{x(t) - T_4[y(t)]\} - T_3[y(t)]$

$y(t) = T_2[m(t)] = T_2\left[T_1\{x(t) - T_4[y(t)]\} - T_3[y(t)]\right]$

2.26 (a) (i) has memory (ii) invertible, (iii) BIBO stable

(iv) $y_d(t) = y(t-t_0) \Rightarrow$ Time invariant.

(v) superposition applies $\Rightarrow$ Linear

(b) <u>For Causality</u>

<u>$y(t_0)$ depends</u> on values of $x(t)|_{t \le t_0} \Rightarrow \begin{matrix} t_0 - \alpha + 1 \le t_0 \\ \alpha \ge 1 \end{matrix}$

2.27 $y(t) = 3x(3t+3) = 3x[3(t+1)]$

(i) $y(0) = 3x(3)$ $\therefore$ not memoryless

(ii) $\left.\begin{array}{l} y(-1) = 3x(0) \Rightarrow x(0) = \frac{1}{3}\,y(-1) \\ y(-\frac{2}{3}) = 3x(1) \Rightarrow x(1) = \frac{1}{3}\,y(-\frac{2}{3}) \\ y(-\frac{4}{3}) = 3x(-1) \Rightarrow x(-1) = \frac{1}{3}\,y(-\frac{4}{3}) \end{array}\right\} \Rightarrow$ invertible

(iii) $|x| < M \Rightarrow |y| < 3M$ $\therefore$ BIBO Stable

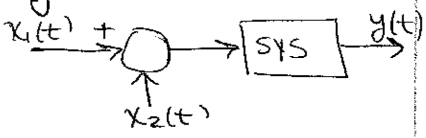
(iv) $y(t-t_0) = 3x(3t - t_0 + 3)$, $y_d(t) = 3x[3(t-t_0)+3)$

$y_d(t) = 3x(3t - 3t_0 + 3)$

$y_d(t) \neq y(t-t_0)$

$\therefore$ not time-invariant

(v) $y_1(t) = 3x_1(3t+3)$, $y_2(t) = 3x_2(3t+3)$



$= 3x_1(3t+3) + 3x_2(3t+3) = y_1(t) + y_2(t)$

$\therefore$ linear

2.28 $x_1(t) = u(t) - u(t-1) \longrightarrow y_1(t)$

$x_2(t) = 2u(t+1) - u(t) - u(t-1) = x_1(t) + 2x_1(t+1)$

$\therefore y_1(t) = y_1(t) + 2y_1(t+1)$

2.29(a) $y(t) = \cos[x(t-1)]$

(i) $y(0) = \cos[x(-1)] \Rightarrow$ not memoryless

(ii) $\cos(-\theta) = \cos(\theta) \Rightarrow$ not invertible

(iii) $y(t_0) = \cos(x(t_0 - 1)] \Rightarrow$ causal

(iv) $|y(t)| < 1$ for all time. $\Rightarrow$ BIBO stable

(v) $y_d(t) = \cos[x(t - t_0 - 1]$

$y(t-t_0) = \cos[x(t - t_0 - 1] = y_d(t) \Rightarrow$ time invariant

(vi) $\cos(\theta_1 + \theta_2) \neq \cos(\theta_1) + \cos(\theta_2) \Rightarrow$ not linear

(b) $y(t) = \ln[x(t)]$

(i) $y(0) = \ln[x(0)] \Rightarrow$ memoryless

(ii) $x(t) = e^{y(t)} \Rightarrow$ invertible

(iii) $y(t_0) = \ln[x(t_0)] \Rightarrow$ causal

(iv) $y(t)$ unbounded for $x(t) = 0$. $\Rightarrow$ not stable

2.29 (b) (v) $y(t-t_0) = \ln[x(t-t_0)] = y_d(t) \Rightarrow$ <u>time-invariant</u>

(vi) $y(t) = \ln[x_1(t)+x_2(t)] \neq \ln[x_1(t)] + \ln[x_2(t)]$

$\therefore$ <u>not linear</u>

(c) $y(t) = e^{tx(t)}$

(i) $y(t_0) = e^{t_0 x(t_0)} \Rightarrow$ <u>memoryless</u>

(ii) $y(0) = 1$, regardless of $x(0) \Rightarrow$ <u>not invertible</u>

(iii) <u>causal</u> (see (i))

(iv) $x(t) = 1 \Rightarrow y(t) = e^t$, unbounded $\Rightarrow$ <u>not stable</u>

(v) $y_d(t) = e^{tx(t-t_0)} \neq y(t-t_0) = e^{(t-t_0)x(t-t_0)} \Rightarrow$ <u>time varying</u>

(vi) $e^{t[x_1+x_2]} \neq e^{tx_1} + e^{tx_2} \Rightarrow$ <u>not linear</u>

(d) $y(t) = 7x(t) + 6$

(i) memorless, $y(t_0) = 7x(t_0) + 6$

(ii) invertible, $x(t) = \frac{1}{7}y(t) - 6/7$

(iii) causal, $y(0) = 7x(t_0) + 6$

(iv) stable, $|x(t)| < M \Rightarrow |y(t)| < 7M + 6$

(v) time-invariant, $y_d(t) = 7x(t-t_0) + 6 = y(t-t_0)$

(vi) <u>not linear</u>, $7(x_1+x_2)+6 \neq (7x_1+6)+(7x_2+6)$

(e) $y(t) = \int_{-\infty}^{t} x(5\tau)\,d\tau$

(i) $y(t)$ depends on all values of $x(\tau)$, $-\infty < \tau \leq t$

$\therefore$ <u>has memory</u>

(ii) $\frac{dy(t)}{dt} = x(5t) \Rightarrow$ <u>invertible</u>

(iii) $y(0) = \int_{-\infty}^{0} x(\tau)\,d\tau \Rightarrow$ <u>not causal</u>

(iv) $x(t) = 1$, $y(t)$ unbounded $\Rightarrow$ <u>not stable</u>

(v) time invariant; $y(t-t_0) = y_d(t)$

(vi) $y(t) = \int_{-\infty}^{t} [x_1(5\tau) + x_2(5\tau)]\,d\tau = \int_{-\infty}^{t} x_1(5\tau)\,d\tau + \int_{-\infty}^{t} x_2(5\tau)\,d\tau$

$\Rightarrow$ <u>linear</u>

2.29 (f) $y(t) = e^{-j\omega t}\int_{-\infty}^{\infty} x(\tau)\, e^{-j\omega\tau}\, d\tau$

(i) not memorless: – integral over all time

(ii) not invertible : (definite integral)

(iii) not causal : – output depends on future values of the input

(iv) stable : $|x(t)| < M \Rightarrow |y(t)| < M$

(v) time varying : $e^{j\omega(t-t_0)}\int_{-\infty}^{\infty} x(\tau)e^{j\omega\tau}d\tau$

$$\neq e^{j\omega t}\int_{-\infty}^{\infty} x(\tau - t_0)e^{-j\omega\tau}d\tau$$

(vi) $y(t) = e^{-j\omega t}\int_{-\infty}^{\infty}[x_1(\tau) + x_2(\tau)]\, e^{-j\omega\tau}d\tau$

$$= e^{-j\omega t}\int_{-\infty}^{\infty} x_1(\tau)e^{-j\omega\tau}d\tau + e^{-j\omega\tau}\int_{-\infty}^{\infty} x_2(\tau)e^{-j\omega\tau}d\tau$$

∴ linear

(g) $y(t) = \int_{t-1}^{t} x(\tau)\, d\tau$

(i) not memoryless – integral over time

(ii) not invertible

(iii) causal – no future inputs required.

(iv) Stable – integrates over a finite window of time.

(v) time invariant – integral is always over one unit of time

(vi) linear – integration is linear.

2.30 $y(t) = x(t - t_0)$

(i) not memoryless

(ii) $x(t) = y(t + t_0)$ ∴ invertible

(iii) Causal

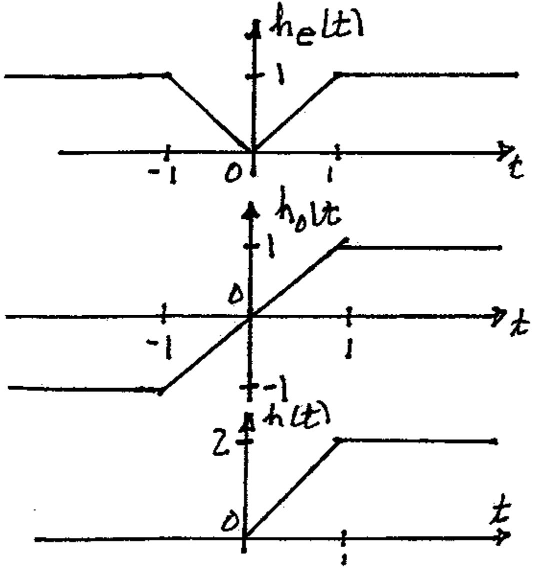
(iv) stable

(v) $y(t - t_1) = x(t - t_1 - t_0) = y_d(t)$ ∴ time invariant

(vi) $y(t) = a_1 x_1(t - t_0) + a_2 x_2(t - t_0) = a_1 y_1(t) + a_2 y_2(t)$

∴ linear

2.31 $h_e(t) = t\,[u(t) - u(t-1)] + u(t-1)$



$h_e(t)$ even

$h_o(t) = h(t) - h_e(t)$

$\therefore h_o(t) = -h_e(t) \quad t<0$

and $h_o(t) = -h_o(-t)$

$\therefore h(t) = 2t\,u(t) - 2(t-1)u(t-1)$

$t<0,\ h(t) = 0$

$0<t<1,\ h(t) = 2t$

$2<t,\ h(t) = 2t - 2t + 2 = 2$

2.32. (a) (i) memoryless

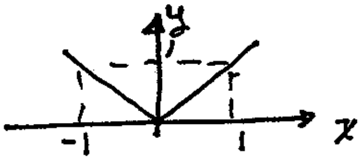
(ii) $y=1$ for $x=\pm1$, not invertible

(iii) causal

(iv) stable

(v) time invariant

(vi) $|x_1+x_2| \neq |x_1|+|x_2|$ not linear



$y = |x|$

(b) (i) memoryless

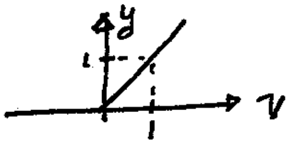
(ii) $y=0$ for $x \le 0$, not invertible

(iii) causal

(iv) stable

(v) time invariant

(vi) $y\Big|_{\substack{x_1=1\\x_2=-1}} \neq y\Big|_{x_1+x_2}$, not linear



2.32 (c) (i) memoryless

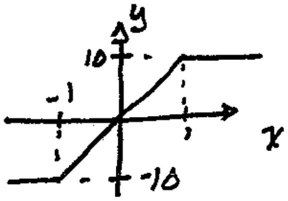
(ii) $y=10$ for $x \geq 1$, not invertible

(iii) causal

(iv) stable

(v) time invariant

(vi) $x_1=x_2=1$, $y_1=y_2=10$, $y|_{x=2} \neq 20$ nonlinear



(d) (i) memoryless

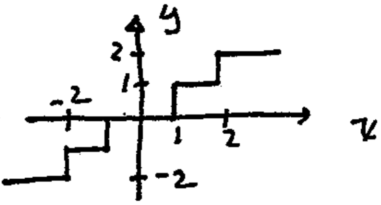
(ii) $y=2$ for $x>2$, not invertible

(iii) causal

(iv) stable

(v) time-invariant

(vi) $x_1=x_2=2 \Rightarrow y_1=y_2=2$; $x=4 \Rightarrow y=2$, nonlinear



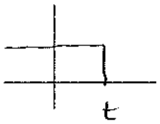
Chapter 3

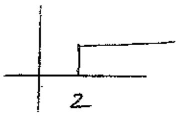
3.1 a) i $x(t) = u(t-2)$

$$y(t) = u(t) * u(t-2) = \int_2^t d\tau = t-2, \quad t>2$$

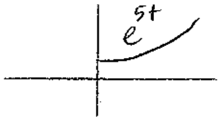
$y(t) = 0, \quad t<2$

$\therefore \; y(t) = (t-2)u(t-2)$

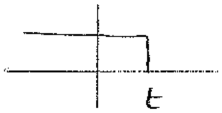




ii $x(t) = e^{5t}u(t)$



$t<0, \quad y(t)=0$



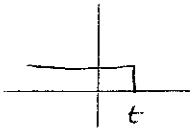
$$t \geq 0, \quad y(t) = \int_0^t e^{5\tau} d\tau$$

$\therefore \; y(t) = 1/5\,(e^{5t} - 1)\,u(t)$

iii $x(t) = u(t)$

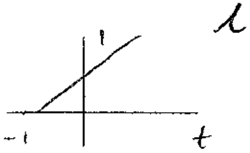
$t<0, \quad y(t) = 0$

$$t \geq 0, \quad y(t) = \int_0^t d\tau = t$$



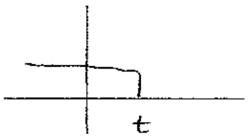
$\therefore \; y(t) = t\,u(t)$

iv $x(t) = (t+1)\,u(t+1)$



$t<-1, \quad y(t) = 0$

$$t \geq -1, \quad y(t) = \int_{-1}^t (\tau+1)\,d\tau = t^2/2 + t + 1/2$$



$\therefore \; y(t) = (t^2/2 + t + 1/2)\,u(t+1)$

3.1 b)

i $y(t)=\int_{-\infty}^{t}u(\tau-2)d\tau=\int_{2}^{t}d\tau=t-2,\quad t>2$

$y(t)=0\quad,\ t<2$

ii $y(t)=\int_{-\infty}^{t}e^{5\tau}u(\tau)d\tau=\int_{0}^{t}e^{5\tau}d\tau=1/5\,e^{5\tau}\Big|_{0}^{t}=1/5(e^{5t}-1),$ $t>0$

$y(t)=0\ ,\ t<0$

iii $y(t)=\int_{-\infty}^{t}u(\tau)d\tau=\int_{0}^{t}d\tau=t\quad,\ t>0$

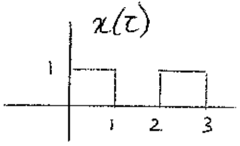
$y(t)=0\ ,\ t<0$

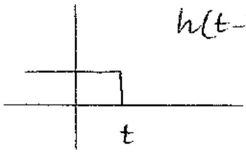
iv $y(t)=\int_{-\infty}^{t}(\tau+1)u(\tau+1)d\tau=\int_{-1}^{t}(\tau+1)d\tau=\tau^2/2+\tau\Big|_{-1}^{t}$

$=t^2/2+t+1/2\quad,\ t>-1$

$=0\quad,\ t<-1$

3.2





$t<0,\quad y(t)=0$

$0\leqslant t\leqslant 1\quad y(t)=\int_{0}^{t}d\tau=t$

$1\leqslant t\leqslant 2\quad y(t)=1$

$2\leqslant t\leqslant 3\quad y(t)=1+\int_{2}^{t}d\tau=1+(t-2)=t-1$

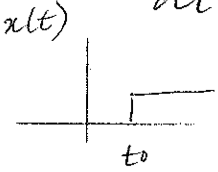
$t\geqslant 3\quad y(t)=2$

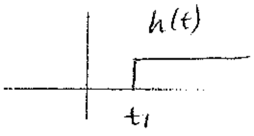
$\therefore\ y(t)=t[u(t)-u(t-1)]+[u(t-1)-u(t-2)]+(t-1)[u(t-2)-u(t-3)]+2u(t-3)$

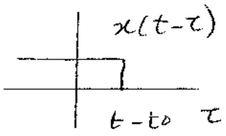
$$\therefore\ y(t) = t\,u(t) + (1-t)u(t-1) + (t-2)u(t-2) + (3-t)u(t-3)$$

3.3 $x(t) = u(t-t_0)$

$h(t) = u(t-t_1)$ $\quad t_1) t_0$



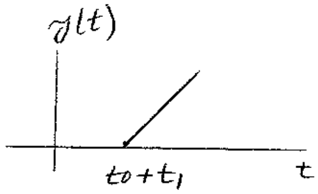




$$y(t) = 0, \quad t - t_0 < t_1$$

$$y(t) = \int_{t_1}^{t-t_0} d\tau = t - t_0 - t_1, \quad t - t_0 \geq t_1$$

$$\therefore\ y(t) = (t - t_0 - t_1)u(t - t_0 - t_1)$$

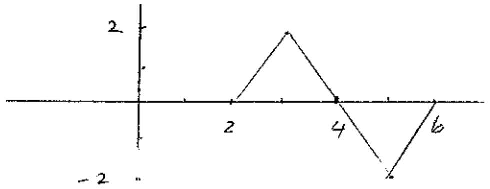


3.4 a) $x(t) = 2[u(t-2) - u(t-4)]$

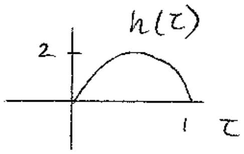
$h(t) = u(t) - 2u(t-1) + u(t-2)$

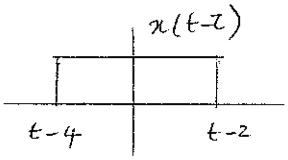
$$y(t) = x(t) * h(t) = 2u(t-2) * u(t) - 4u(t-2) * u(t-1) + 2u(t-2) * u(t-2) - 2u(t-4) * u(t) + 4u(t-4) * u(t-1) - 2u(t-4) * u(t-2)$$

$$y(t) = 2(t-2)u(t-2) - 4(t-3)u(t-3) + 2(t-4)u(t-4) - 2(t-4)u(t-4) + 4(t-5)u(t-5) - 2(t-6)u(t-6)$$



b)



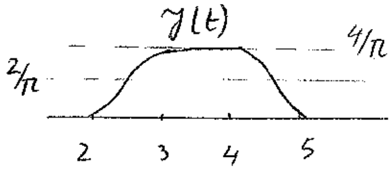


$$y(t) = 2\int_{0}^{t-2} \sin\pi\tau\, d\tau = \frac{-2}{\pi}\cos\pi\tau\Big)_{0}^{t-2} = \frac{2}{\pi}\left[1-\cos\pi(t-2)\right] \quad , 2<t<3$$

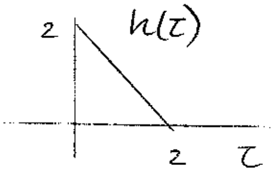
$$y(t) = 4/\pi \quad , \quad 3<t<4$$

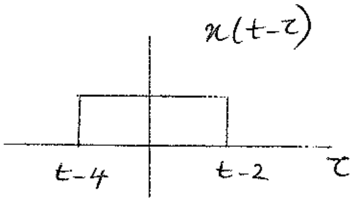
$$y(t) = 2\int_{t-4}^{1} \sin\pi\tau\, d\tau = \frac{2}{\pi}\left[\cos\pi(t-4)+1\right] \quad , 4<t<5$$

$$y(t) = 0 \quad , \quad t<2 \;\&\; t>5$$



c)



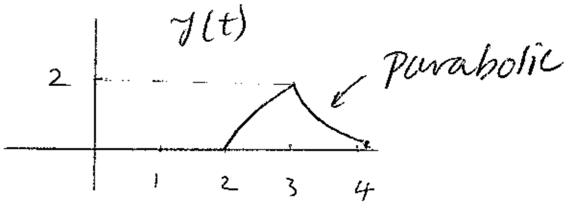


$$y(t) = 0 \quad , \quad t<0 \;\&\; t>6$$

$$y(t) = \int_{0}^{t-2}(-\tau+2)\,d\tau = -\tau^2/2 + 2\tau\Big|_{0}^{t-2} = -t^2/2 + 4t - 6$$

$$2<t<4$$

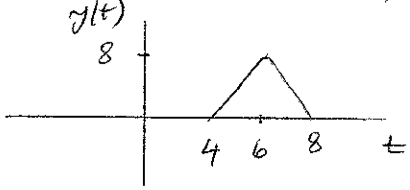
$$y(t) = \int_{t-4}^{2}(-\tau+2)\,d\tau = t^2/2 - 6t + 18 \quad 4<t<6$$



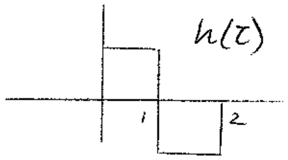
d) $x(t) = h(t) = 2\,(u(t-2) - u(t-4))$

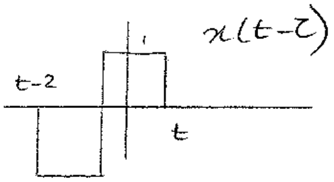
$\therefore\ y(t) = 2u(t-2) * 2u(t-2) - 4u(t-2) * u(t-4)$
$+ 2u(t-4) * 2u(t-4) - 4u(t-4) * u(t-2)$

$y(t) = 4(t-4)u(t-4) - 8(t-6)u(t-6) + 4(t-8)u(t-8)$



3.5





a) $t=0\ ,\ y=0$
$t=1\ ,\ y=1$
$t=2\ ,\ y=2$
$t=2.667\ ,\ y=0$

b) $0<t<1 \qquad y(t) = \int^{t}(1)(1)d\tau = t$

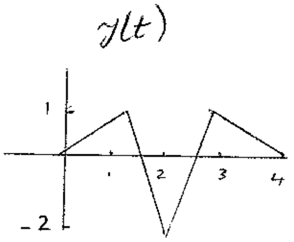
$1<t<2 \qquad y(t) = \int_{0}^{t-1}(1)(-1)d\tau + \int_{t-1}^{1}(1)(1)d\tau + \int_{1}^{t}(1)(-1)d\tau = -t+1+0+1-t+1-t+1$

$= 4-3t$

$2<t<3 \qquad y(t) = \int_{t-2}^{1}(1)(-1)d\tau + \int_{1}^{t-1}(-1)(-1)d\tau + \int_{t-1}^{2}(1)(-1)d\tau$

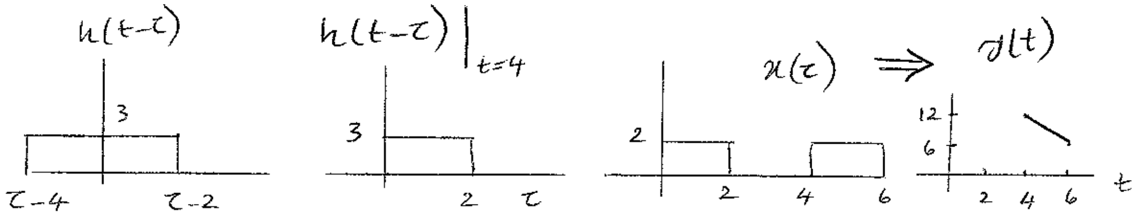
$= -1+t-2+t-1-1-2+t-1 = 3t-8$

$3<t<4 \qquad y(t) = \int_{t-2}^{2}(-1)(-1)d\tau = 4-t$



3.6

a)



b) by inspection $y_{max}=12$

c) $t<2$, $t=6$, $t>10$

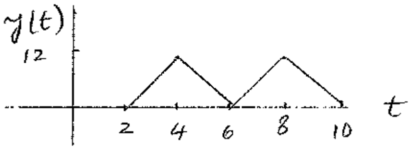
$t-2<0$ $t-2=4$ $t-4>6$

d) $x(t)=2u(t)-2u(t-2)+2u(t-4)-2u(t-6)$

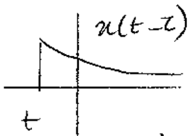
$h(t)=3u(t-2)-3u(t-4)$

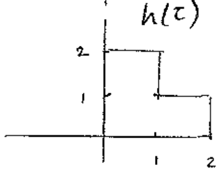
$$\Rightarrow y(t)=6(t-2)u(t-2)-6(t-4)u(t-4)$$
$$+6(t-6)u(t-6)-6(t-8)u(t-8)$$
$$-6(t-4)u(t-4)+6(t-6)u(t-6)$$
$$-6(t-8)u(t-8)+6(t-10)u(t-10)$$

$$\therefore y(t)=6(t-2)u(t-2)-12(t-4)u(t-4)$$
$$+12(t-6)u(t-6)-12(t-8)u(t-8)$$
$$+6(t-10)u(t-10)$$



3.7 a) $x(t)=e^{t}u(-t)$





① $t>2$ no overlap $\therefore y(t)=0$

② $1\leqslant t\leqslant 2$ $y(t)=\int_{t}^{2}e^{t-\tau}d\tau=e^{t}\int_{t}^{2}e^{-\tau}d\tau$

$$y(t)=e^{t}[e^{-t}-e^{-2}]=1-e^{t-2}$$

③ $0 \leqslant t \leqslant 1$, $y(t) = 2\int_t^1 e^{t-\tau} d\tau + \int_1^2 e^{t-\tau} d\tau = 2(1 - e^{t-1})$

$+ e^{t}(e^{-1} - e^{-2}) = 2 - e^{t-1} - e^{t-2}$

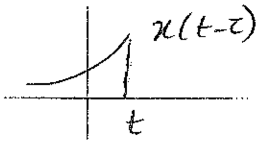
④ $t < 0$, $y(t) = 2\int_0^1 e^{t-\tau} d\tau + \int_1^2 e^{t-\tau} d\tau$

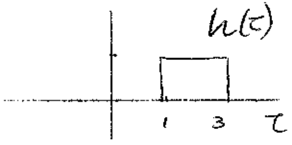
$= 2(e^{t} - e^{t-1}) + e^{t}(e^{-1} - e^{-2}) = 2e^{t} - e^{t-1} - e^{t-2}$

$\therefore \; y(t) = (1 - e^{t-2})[u(t-1) - u(t-2)] + (2 - e^{t-1} - e^{t-2}) \times$

$[u(t) - u(t-1)] + (2e^{t} - e^{t-1} - e^{t-2})u(-t)$

b)





① $t < 1$, $y(t) = 0$

② $1 \leqslant t \leqslant 3$, $y(t) = \int_1^t e^{-(t-\tau)} d\tau = e^{-t}\int_1^t e^{\tau} d\tau = e^{-t}(e^{t} - e)$

$= 1 - e^{-(t-1)}$

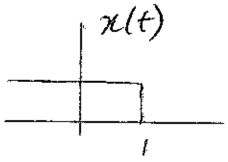
③ $t > 3$, $y(t) = \int_1^3 e^{-(t-\tau)} d\tau = e^{-t}\int_1^3 e^{\tau} d\tau = e^{-t}(e^{3} - e)$

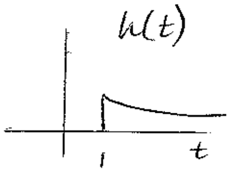
$= e^{-(t-3)} - e^{-(t-1)}$

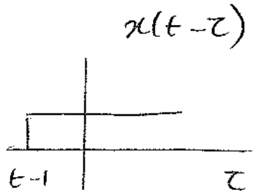
$\therefore \; y(t) = (1 - e^{-(t-1)})[u(t-1) - u(t-3)] +$

$(e^{-(t-3)} - e^{-(t-1)})u(t-3)$

c)





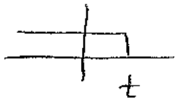


① $t-1<1$ or $t<2$, $y(t)=\int^{\infty} e^{-\tau}d\tau = e^{-1}$

② $t-1>1$ or $t>2$ $y(t)=\int_{t-1}^{\infty} e^{-\tau}d\tau = -e^{-\tau}\Big|_{t-1}^{\infty} = e^{-(t-1)}$

∴ $y(t)=e^{-1}u(2-t)+e^{-(t-1)}u(t-2)$

d) flip h(t)



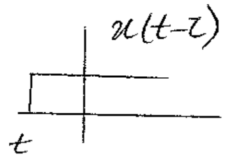
① $t<0$ $y(t)=0$

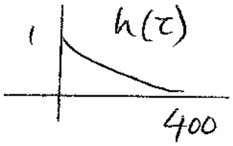
② $0\leqslant t\leqslant 2$, $y(t)=\int_0^t e^{-a\tau}d\tau = \frac{-1}{a}e^{-a\tau}\Big|_0^t = 1/a(1-e^{-at})$

③ $t>2$, $y(t)=\int_0^2 e^{-a\tau}d\tau = \frac{1}{a}(1-e^{-2a})$

∴ $y(t)=1/a(1-e^{-at})[u(t)-u(t-2)]+1/a(1-e^{-2a})u(t-2)$

e) flip x(t)

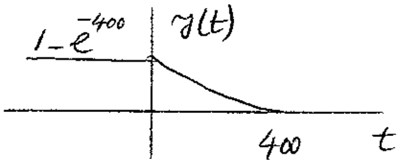




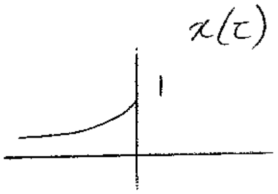
① $t<0$, $y(t)=\int_0^{400} e^{-\tau}d\tau = -e^{-\tau}\Big|_0^{400} = 1-e^{-400}$

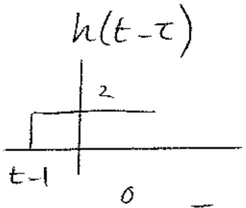
② $t>0$, $y(t)=\int_t^{400} e^{-\tau}d\tau = e^{-t}-e^{-400}$

③ $t>400$, $y(t)=0$



f)





① $t-1<0$ $\quad y(t)=\int_{t-1}^{0} 2e^{\tau}d\tau = 2[1-e^{t-1}]$

② $t-1>0$ $\quad y(t)=0$

$\therefore y(t)=2[1-e^{-(1-t)}]u(1-t)$

3.8 $[f(t)*g(t)]*h(t)=\int_{-\infty}^{\infty} h(t-s)\left[\int_{-\infty}^{\infty} f(s-\tau)g(\tau)d\tau\right]ds$

$= \int_{-\infty}^{\infty} g(\tau)\left[\int_{-\infty}^{\infty} h(t-s)f(s-\tau)ds\right]d\tau$, let $s-\tau=\sigma$

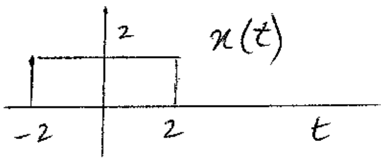
$= \int_{-\infty}^{\infty} g(\tau)\left[\int_{-\infty}^{\infty} h(t-\tau-\sigma)f(\sigma)d\sigma\right]d\tau$, let $s=t-\sigma$
$ds=-d\sigma$

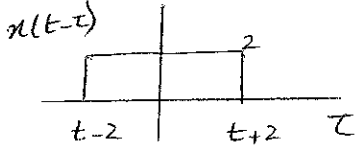
$= \int_{-\infty}^{\infty} g(\tau)\left[\int_{-\infty}^{\infty} h(s-\tau)f(t-s)[-ds]\right]d\tau$

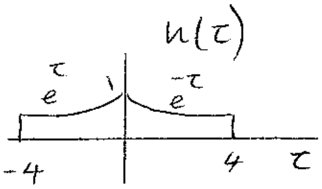
$= \int_{-\infty}^{\infty} f(t-s)\left[\int_{-\infty}^{\infty} h(s-\tau)g(\tau)d\tau\right]ds$

$= f(t)*[g(t)*h(t)]$

3.9 $x_1(t)=2u(t+2)-2u(t-2)$







① $t+2<-4$, $t<-6$, $y(t)=0$

② $-4\leqslant t+2\leqslant 0$, $-6\leqslant t\leqslant -2$

$$y(t)=\int_{-4}^{t+2} 2e^{\tau}d\tau = 2[e^{t+2}-e^{-4}]$$

③ $0 \leqslant t+2 \leqslant 4 \quad , \quad -2 \leqslant t \leqslant 2$

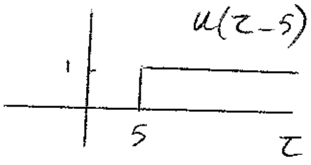
$$y(t) = 2\int_{t-2}^{0} e^{\tau} d\tau + 2\int_{0}^{t+2} e^{-\tau} d\tau = 2\left[1-e^{t-2}\right] + 2\left[1-e^{-(t+2)}\right]$$

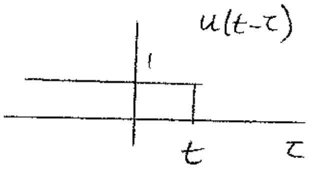
④ $0 \leqslant t-2 \leqslant 4 \quad , \quad 2 \leqslant t \leqslant 6$

$$y(t) = \int_{t-2}^{4} e^{-\tau} d\tau = 2\left[e^{-(t-2)} - e^{-4}\right]$$

⑤ $t \geqslant 6 \quad , \quad y(t) = 0$

3.10

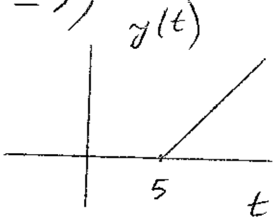




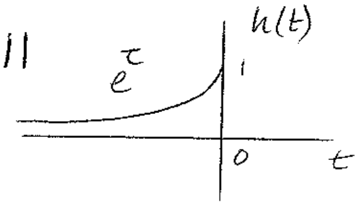
$t < 5 \quad , \quad y(t) = 0$

$t \geqslant 5 \quad , \quad y(t) = \int_{5}^{t} d\tau = (t-5)$

$\therefore \quad y(t) = (t-5)u(t-5)$

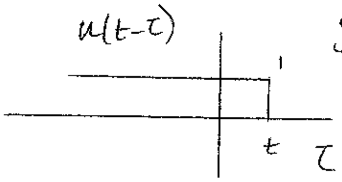


3.11



$x(t) = u(t+3) - u(t+2) + u(t-1) - u(t-2)$

Use superposition



$S(t) = u(t) * h(t)$

$t < 0 \quad , \quad y(t) = \int_{-\infty}^{t} e^{\tau} d\tau = e^{t}$

$t \geqslant 0 \quad , \quad y(t) = \int_{-\infty}^{0} e^{\tau} d\tau = 1$

$\therefore \quad y(t) = S(t+3) - S(t+2) + S(t-1) - S(t-2)$

3.12 a) $h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} e^{-\tau} u(\tau) e^{-(t-\tau)} u(t-\tau) d\tau$

$= e^{-t} \int_{0}^{t} d\tau = t e^{-t} u(t)$

b) $\mathring{u}(t) = \delta(t) * \delta(t) = \int_{-\infty}^{\infty} \delta(\tau)\delta(t-\tau) d\tau = \delta(t)$

c) $h(t) = \delta(t-1) * \delta(t-1) = \delta(t-1-1)$

$= \delta(t-2)$

d) $h(t) = u(t-2) * u(t-2) - 2u(t-2) * u(t-4)$

$+ u(t-4) * u(t-4) = (t-4) u(t-4)$

$- 2(t-6) u(t-6) + (t-8) u(t-8)$

3.13 $z(t) = \int_{-\infty}^{\infty} x(-\tau+a) h(t+\tau) d\tau$

$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(-\tau+t) h(\tau) d\tau$

let $\alpha = t+\tau$, plug in formula for $z(t)$

$z(t) = \int_{-\infty}^{\infty} x(-(\alpha - t) + a) h(\alpha) d\alpha$

$= \int_{-\infty}^{\infty} x(-\alpha + t + a) h(\alpha) d\alpha$

compare with $y(t)$ & see that

$z(t) = y(t+a)$

3.14

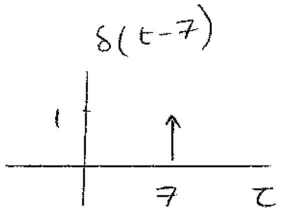
a) $x(t) = \delta(t) \longrightarrow y(t) = h(t)$

$y(t) = x(t-7)$

$h(t) = \delta(t-7)$

b) $y(t) = \int_{-\infty}^{t} x(\tau-7)\,d\tau$

$h(t) = \int_{-\infty}^{t} \delta(\tau-7)\,d\tau$

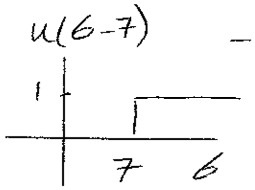


$t<7\ ,\ h(t)=0$

$t>7\ ,\ h(t)=1 \qquad \therefore h(t)=u(t-7)$

c) $y(t) = \int_{-\infty}^{t}\left[\int_{-\infty}^{\sigma} x(\tau-7)\,d\tau\right]d\sigma$ let $x(t)=\delta(t)$

$h(t) = \int_{-\infty}^{t}\left[\int_{-\infty}^{\sigma} \delta(\tau-7)\,d\tau\right]d\sigma = \int_{-\infty}^{t} u(\sigma-7)\,d\sigma$



$t<7\ ,\ h(t)=0$

$t\geq 7\ ,\ h(t)=\int_{7}^{t} d\sigma = (t-7)$

$\therefore h(t) = (t-7)\,u(t-7)$

3.15 let $x(t-\tau) = \begin{cases} 1 & h(\tau)>0 \\ -1 & h(\tau)<0 \end{cases}$ $\therefore x$ is bounded

$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$

$h(\tau)\,x(t-\tau) = \begin{cases} h(\tau)\ , & h(\tau)>0 \\ -h(\tau)\ , & h(\tau)<0 \end{cases}$

$\therefore h(\tau)\,x(t-\tau) = |h(\tau)|$

$\therefore y(t) = \int_{-\infty}^{\infty} |h(\tau)|\,d\tau$ which is assumed unbounded

$\therefore$ system is not BIBO stable

3.16 a) $y_i(t)$ is the output of the ith system

$$y_1(t)=h_1(t)*x(t)$$
$$y_2(t)=h_2(t)*y_1(t)=h_1(t)*h_2(t)*x(t)$$
$$y_3(t)=h_1(t)*h_3(t)*x(t)$$
$$y_5(t)=h_5(t)*x(t)$$
$$y(t)=y_2(t)+y_4(t)$$
$$x(t)*[h_1(t)*h_2(t)+h_1(t)*h_3(t)*h_4(t)+h_4(t)*h_5(t)]$$

b) $h(t)=u(t)*5\delta(t)+u(t)*5\delta(t)*u(t)$
$+u(t)*e^{-2t}u(t)$

now $u(t)*e^{-2t}u(t)=\int_{-\infty}^{\infty}u(\tau)e^{-2(t-\tau)}u(t-\tau)d\tau$

$$=\int_0^t e^{-2(t-\tau)}d\tau = e^{-2t}\int_0^t e^{2\tau}d\tau=\tfrac{1}{2}(1-e^{-2t}]u(t)$$

$$\therefore h(t)=5u(t)+5tu(t)+\tfrac{1}{2}(1-e^{-2t}]u(t)$$

3.17 $y_i(t)$ is the output of the ith system

a) $y_2(t)=h_2(t)*[h_1(t)*x(t)]=h_1(t)*h_2(t)*x(t)$
$y_3(t)=h_3(t)*y_2(t)=h_1(t)*h_2(t)*h_3(t)*x(t)$

in a like manner: $y_4(t)=h_1(t)*h_2(t)*h_4(t)*x(t)$

$$y_5(t)=h_1(t)*h_5(t)*x(t)$$

$$\therefore y(t)=(h_1(t)*h_2(t)*h_3(t)\quad h_1(t)*h_2(t)*h_4(t)+h_1(t)*h_5(t)]*x(t)$$

b) $h(t) = 5\delta(t) * 5\delta(t) * u(t) + 5\delta(t) * 5\delta(t) * u(t)$
$+ 5\delta(t) * u(t) = 25u(t) + 25u(t) + 5u(t) = 55u(t)$

c) blocks 1 and 2 $\longrightarrow$ gains of 5
blocks 3, 4, 5 $\longrightarrow$ integrators

d) block 1 $- 5\delta(t)$ block 4 $- 25u(t)$
block 2 $- 25\delta(t)$ block 5 $- 5u(t)$
block 3 $- 25u(t)$ $\therefore y(t) = 55u(t)$

e) $\delta(t) * 55u(t) = 55u(t)$

3.18 $y(t) = h_1(t) * [x(t) - h_2(t) * y(t)] = h_1(t) * x(t)$
$- h_1(t) * h_2(t) * y(t)$

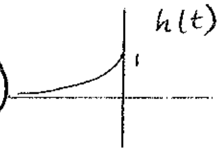
$y(t) = u(t) * x(t) - u(t) * \delta(t) * y(t) = u(t) * x(t)$
$- u(t) * y(t)$

$$= \int_{-\infty}^{\infty} x(\tau)u(t-\tau)d\tau - \int_{-\infty}^{\infty} y(\tau)u(t-\tau)d\tau$$

$$= \int_{-\infty}^{t} x(\tau)d\tau - \int_{-\infty}^{t} y(\tau)d\tau$$

by differentiating:

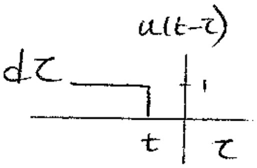
$$dy/dt = x(t) - y(t) \Rightarrow \frac{dy(t)}{dt} + y(t) = x(t)$$

3.19 a) 

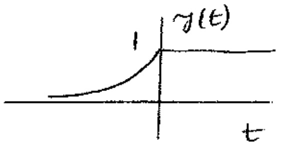
impulse response ≠ 0 for t < 0 ∴ non causal

b) $\int_{-\infty}^{\infty}|h(t)|dt = \int_{-\infty}^{0} e^{t}dt = e^{t}\Big|_{-\infty}^{0} = 1$ ∴ stable

c) $y(t) = \int_{-\infty}^{\infty} e^{\tau}u(-\tau)u(t-\tau)\,d\tau = \int_{-\infty}^{0} e^{\tau}u(t-\tau)\,d\tau$



$$\therefore\ y(t) = \begin{cases} \int_{-\infty}^{t} e^{\tau}d\tau = e^{\tau}\Big|_{-\infty}^{t} = e^{t}, & t<0 \\ \int_{-\infty}^{0} e^{\tau}d\tau = e^{\tau}\Big|_{-\infty}^{0} = 1, & t>0 \end{cases}$$



d) a) causal

b) $\int_{-\infty}^{\infty} h(t)dt = \int_{0}^{\infty} e^{t}\,dt = e^{t}\Big|_{0}^{\infty}$ ∴ unstable

c) $\int_{-\infty}^{\infty} e^{\tau}u(\tau)u(t-\tau)\,d\tau = \int_{0}^{t} e^{\tau}d\tau = (e^{t}-1)\,u(t)$

3.20

a) $[\sin t]x_1(t) + (\sin t]x_2(t) = y_1 + y_2$ ∴ linear

b) $y(t)\Big|_{t-t_0} = [\sin(t-t_0)]\,x(t-t_0)$

$y(t)\Big|_{x(t-t_0)} = [\sin t]\,x(t-t_0)$ ∴ time varying

c) $h(t) = [\sin t]\,\delta(t) = \sin(0)\,\delta(t) = 0$

d) $y(t) = \sin t\,\delta(t-1) = 0.8415\,\delta(t-1)$

3.21

a) $h(t)=e^{-t}u(t-1)$ stable, causal

b) $h(t)=e^{t}u(-t+1)$ Stable, not causal

c) $h(t)=e^{-t}u(-t-1)$ not stable, not causal

d) $h(t)=\sin(5t)\,u(-t)$ not stable, not causal

e) $h(t)=e^{t}u(-t)$ stable, not causal

f) $h(t)=e^{t}\sin(5t)\,u(-t)$

$$\int_{-\infty}^{\infty}|h(t)|\,dt=\int_{-\infty}^{\infty}|e^{t}\sin(5t)\,u(-t)|\,dt=$$

$$\int_{-\infty}^{0}e^{t}|\sin(5t)|\,dt\leqslant\int_{-\infty}^{0}e^{t}\,dt=1$$

∴ Stable, not causal

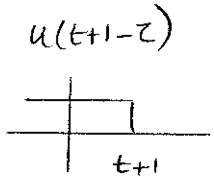
3.22 $y(t)=\int_{0}^{t}e^{-\tau}x_1(t-\tau)\,d\tau=\int_{-\infty}^{t}e^{-\tau}u(\tau)x_1(t-\tau)\,d\tau$

$$=\int_{-\infty}^{\infty}e^{-\tau}u(\tau)x_1(t-\tau)u(t-\tau)\,d\tau$$

a) $h(t)=e^{-t}u(t)$ $\quad x(t)=x_1(t)u(t)$

b) yes, $h(t)=0$ for $t<0$

c) $y(t)=\int_{-\infty}^{\infty}e^{-\tau}u(\tau)u(t+1-\tau)\,d\tau=\int_{0}^{t+1}e^{-\tau}\,d\tau$



$$=-e^{-\tau}\Big|_{0}^{t+1}=\underline{[1-e^{-(t+1)}]\,u(t+1)}$$

d) $h_t(t) = h(t) * \delta(t) - h(t) * \delta(t-1) * \delta(t)$

$= [h(t) - h(t-1)] * \delta(t) = h(t) - h(t-1)$

$y(t) = \underline{e^{-t} u(t) - e^{-(t-1)} u(t-1)}$

e) (i) $y(t) = y_c(t) - y_c(t)\Big|_{t=t+1}$

$= \underline{[1 - e^{-(t+1)}]u(t+1) - [1 - e^{-t}]u(t)}$

(ii) $y(t) = h(t) * u(t+1) = \int_{-\infty}^{\infty} u(t-\tau+1)[e^{-\tau}u(\tau) - e^{-(\tau-1)}u(\tau-1)]d\tau$

$= \int_{0}^{\infty} e^{-\tau}u(t+1-\tau)d\tau - e^{1}\int_{1}^{\infty} e^{-\tau}u(t+1-\tau)d\tau = I_1 - I_2$

$I_1 = \int_{0}^{t+1} e^{-\tau}d\tau = -e^{-\tau}\Big|_{0}^{t+1} = [1 - e^{-(t+1)}]u(t+1)$

$I_2 = e^{1}\int_{1}^{t+1} e^{-\tau}d\tau = e^{1}(-e^{-\tau})\Big|_{1}^{t+1} = e^{1}(e^{-1} - e^{-(t+1)})u(t)$

$= (1 - e^{-t})u(t)$

$\therefore$ $\underline{y(t) = [1 - e^{-(t+1)}]u(t+1) - [1 - e^{-t}]u(t)}$

3.23 a) $y(t) = \int_{-\infty}^{x} e^{-2(t-\tau)} x(\tau-1)d\tau$

(i) $h(t) = \int_{-\infty}^{t} e^{-2(t-\tau)}\delta(\tau-1)d\tau = \underline{e^{-2(t-1)} u(t-1)}$

(ii) $h(t) = 0$ for $t < 0$ $\therefore$ Causal

(iii) $\int_{-\infty}^{\infty} |e^{-2(t-1)}u(t-1)|dt = \int_{1}^{\infty} e^{-2(t-1)}dt = e^{2}\left(e^{-2t}/_{-2}\right)\Big|_{1}^{\infty}$

$= e^{2}(e^{-2}/2) = 1/2$ $\therefore$ Stable

b) $y(t)=\int_{-\infty}^{\infty} e^{-2(t-\tau)} x(\tau-1)\,d\tau$

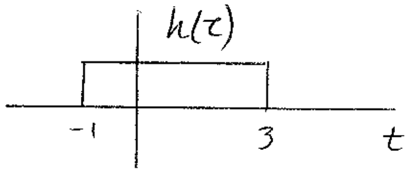
(i) $h(t)=\int_{-\infty}^{\infty} e^{-2(t-\tau)}\,\delta(\tau-1)\,d\tau = \underline{e^{-2(t-1)}}$

(ii) $h(t) \neq 0\ ,\ t<0$ ∴ <u>non causal</u>

(iii) $\int_{-\infty}^{\infty}|e^{-2(t-1)}|\,dt = \int_{-\infty}^{\infty} e^{-2t}e^{2}\,dt = e^{2}\left(\frac{e^{-2t}}{-2}\right)\Big|_{-\infty}^{\infty}$

Unbounded ∴ <u>unstable</u>

3.24

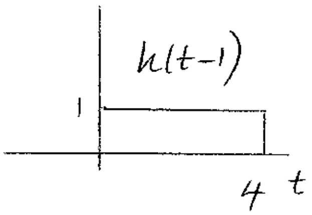


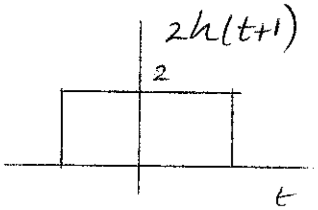
a) System is not causal

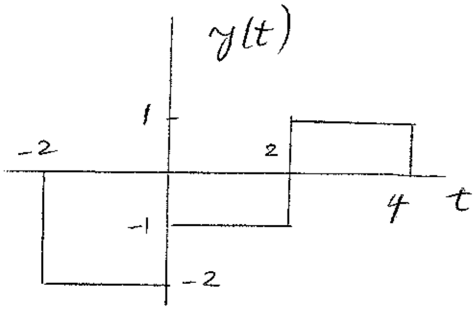
b) yes BIBO, Stable - Integrates over a window of length 4

c) $x(t)=\delta(t-1)-2\delta(t+1)$

$y(t)=h(t)*x(t)=h(t-1)-2h(t+1)$

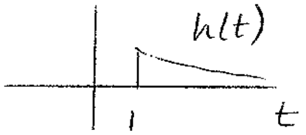






3.25 $h(t)=e^{-at}u(t-1)$

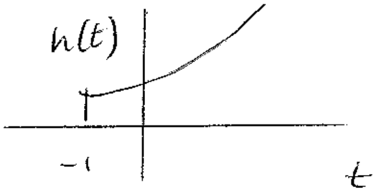
$a>0$



a) clearly system is causal since $h(t)=0,\ t<0$

b) $\int_{-\infty}^{\infty}|h(t)|\,dt=\int_{1}^{\infty}e^{-at}\,dt = 1/a\, e^{-a}$, $a>0$ ∴ stable

c) $h(t) = e^{-at} u(t+1)$, $a < 0$



not causal since $h(t) \neq 0$, $-1 \leq t \leq 0$

$$\int_{-\infty}^{\infty} |h(t)| dt = \int_{-1}^{\infty} e^{-at} dt = -1/a \, e^{-at} \Big|_{-1}^{\infty} = \infty$$ since $a < 0$

∴ not stable

3.26

(i) a) $s+2=0$, $s=-2$ $\quad y_c(t) = Ce^{-2t}$

$y_p(t) = P \Rightarrow \frac{dP}{dt} + 2P = 3 \rightarrow P = 3/2$

∴ $y(t) = [3/2 + Ce^{-2t}]u(t)$, $y(0) = -1 = 3/2 + C$

$\Rightarrow C = -5/2$

∴ $y(t) = 3/2 - 5/2 e^{-2t}$, $t \geq 0$

b) $y + 2y = 5e^{-2t} + 3 - 5e^{-2t} = 3$ ✓

$y(0) = 3/2 - 5/2 = -1$ ✓

(ii) a) from (i) $y_c(t) = Ce^{-2t}$

$y_p(t) = pe^{-t} \Rightarrow -pe^{-t} + 2pe^{-t} = 3e^{-t} \Rightarrow p = 3$

$y(t) = [3e^{-t} + Ce^{-2t}]u(t)$, $y(0) = 1 = 3 + C \Rightarrow C = -2$

∴ $y(t) = 3e^{-t} - 2e^{-2t}$, $t \geq 0$

b) $y + 2y = 3e^{-t} + 4e^{-2t} + 6e^{-t} - 4e^{-2t} = 3e^{-t}$ ✓

$y(0) = 3 - 2 = 1$ ✓

(iii) a) $y_c(t) = Ce^{-2t}$

$y_p(t) = A\sin t + B\cos t$

$A\cos t - B\sin t + 2A\sin t + 2B\cos t = 3\sin t$

$(A+2B)\cos t + (2A-B)\sin t = 3\sin t$

$$\begin{cases} A+2B=0 \\ 2A-B=3 \end{cases}$$

$-4B-B=3 \longrightarrow B=-3/5 \longrightarrow A=6/5$

$$Ce^{-2t} + 6/5\sin t - 3/5\cos t \Big|_{t=0} = 2$$

$C - 3/5 = 2 \Longrightarrow C = 13/5$

$\therefore\ y(t) = 13/5\, e^{-2t} + 6/5\sin t - 3/5\cos t$

(iv) $y_c(t) = Ce^{st}$

$x(t)=0$ $\quad -.7sCe^{st} + Ce^{st} = 0$

$-.7s+1=0 \longrightarrow s = 10/7$

$y_c(t) = Ce^{10/7 t}$

$y_p(t) = pe^{3t}$

$-.7(3)pe^{3t} + pe^{3t} = 6e^{3t}$

$-2.1p+p=6 \Longrightarrow p = \frac{-60}{11}$

$y_p(t) = \frac{-60}{11}e^{3t}$ $\quad y(0)=0 \Longrightarrow C + \frac{-60}{11} = 0$

$\therefore C = \frac{60}{11}$

$y(t) = 60/11\, e^{10/7\, t} - 60/11\, e^{3t}$

(v) $y_c(t) = ce^{st}$

$Sce^{st} + 2ce^{st} = 0, \quad S+2=0 \longrightarrow S=-2$

$y_p(t) = pe^{2t}$

$2pe^{2t} + 2pe^{2t} = 4e^{2t} \longrightarrow 4P=4$

$P=1$

$\Rightarrow y_c(t) = Ce^{-2t}$

$y_p(t) = e^{2t}$

$y(0)=0 \longrightarrow C+1=0 \Longrightarrow C=-1$

$\therefore \ y(t) = -e^{-2t} + e^{2t}$

3.27

a) $H(S) = \frac{7}{S+10}$ Pole at -10, Stable

b) $H(S) = \frac{10S+30}{S^3+7S^2+14S+8}$ Poles at $-1, -2, -4$ Stable

$= \frac{10S+30}{(S+1)(S+2)(S+4)}$

c) $H(S) = \frac{1}{S^2-2.5S+1} = \frac{1}{(S-\frac{1}{2})(S-2)}$ Pole at $\frac{1}{2}, 2$ $\therefore$ unstable

3.28

$S^2-2.5S+1=0 \quad (S-\frac{1}{2})(S-2)=0$

$\therefore$ unstable

3.29

a) mode : e^{-2t}

b) $\tau = 1/2\, s$

c) $\approx 2s = 4\tau = 4(1/2)$

d) (a) e^{-t}, e^{-2t}, e^{-4t}

(b) $\tau_1 = 1s$, $\tau_2 = .5s$, $\tau_3 = .25s$

(c) longest time constant $= 1s \Rightarrow 4s = 4(\tau_1)$

3.30

(a) $.01s^2 + 1 = .01(s^2 + 100) = .01(s + j10)(s - j10)$

$\therefore$ modes : e^{-j10t} , e^{j10t}

(b) $x_c(t) = c_1 e^{j10t} + c_1^* e^{-j10t} \Rightarrow c\cos(10t + \theta)$

(c) $H(s) = \dfrac{100}{s^2 + 100} \Rightarrow \ddot{y} + 100y = 100x$

$\therefore 100y = 100p \Rightarrow p = 1$

3.30(c) (cont) $y(t) = C_1 \cos 10t + C_2 \sin 10t + 1, \; t \geq 0$

$y(0) = 0 = C_1 + 1 \quad \therefore C_1 = \underline{-1}$

$\dot{y}(t) = -10C_1 \sin 10t + 10C_2 \cos 10t = 0, \; \therefore C_2 = \underline{0}$

$\therefore y(t) = \underline{1 - \cos 10t}, \; t \geq 0$

(d) $\dot{y}(t) = 10 \sin 10t; \; \ddot{y} = 100 \cos 10t$

$\therefore \ddot{y} + 100y = 100\cos 10t + 100(1-\cos t) = 100$ ✓

$y(0) = 1 - 1 = 0$ ✓, $\dot{y}(0) = 0$ ✓

3.31. (a)(i) $x(t) = 4e^{(0)t} \Rightarrow \therefore s = 0, \; H(0) = 5/4 = 1.25$

$y_{ss}(t) = H(0)x(t) = (1.25)(4) = \underline{5}$

(ii) $H(0) = 10/10 = 1, \; \therefore y_{ss}(t) = H(0)x(t) = (1)(4) = \underline{4}$

(b)(i) $s = 3, \; H(3) = 5/7, \; y_{ss}(t) = H(3)x(t) = \underline{\frac{20}{7}e^{3t} = 2.857e^{3t}}$

(ii) $H(3) = \frac{6+10}{9+6+10} = \frac{16}{25}, \; y_{ss}(t) = (\frac{16}{25})(4e^{3t}) = \underline{\frac{64}{25}e^{3t} = 2.56e^{3t}}$

(c) (i) $s = j3, \; H(j3) = \frac{5}{4+j3} = 1 \angle -36.87^\circ$

$y_{ss}(t) = |H(j3)| 4\cos(3t + \angle H(j3)) = \underline{4\cos(3t - 36.8^\circ)}$

(ii) $H(j3) = \frac{10+j6}{-9+j6+10} = 1.917 \angle -49.58^\circ$

$\therefore y_{ss}(t) = (1.917)(4))\cos(3t - 49.58^\circ) = \underline{7.668 \cos(3t - 49.56^\circ)}$

```
n=[0 2 10];
d=[1 2 10];
h=polyval(n,3*j)/polyval(d,3*j);
ymag=abs(h)
yphase=angle(h)*180/pi
```

(d) $s = j3$ - use part (c)

(i) $y_{ss}(t) = 4e^{j3t}$ (ii) $y_{ss}(t) = 7.668e^{j(3t - 49.56^\circ)}$

(e) from (c): (i) $y_{ss}(t) = 4\sin(3t - 36.8^\circ)$

(ii) $y_{ss}(t) = 7.668 \sin(3t - 49.56^\circ)$

(f) $\sin 3t = \cos(3t - 90^\circ)$

$\therefore y_{ss}(t)$ in (e) is that of (c) delayed by 90°.

(g) (i) $(s+4) = (s + \frac{1}{\tau}) \Rightarrow \tau = \frac{1}{4}s = \underline{0.25s}$

$s^2 + 2s + 10 = (s+1)^2 + 3^2 \Rightarrow s = -1 \pm j3, \; \therefore \tau = \frac{1}{1}s = \underline{1s}$

3.31.(g) (ii) $\tau = 0.25\,s$, $t > 4\tau = 1\,s$.
(cont) $\tau = 1\,s$, , $t > 4\tau = 4\,s$.

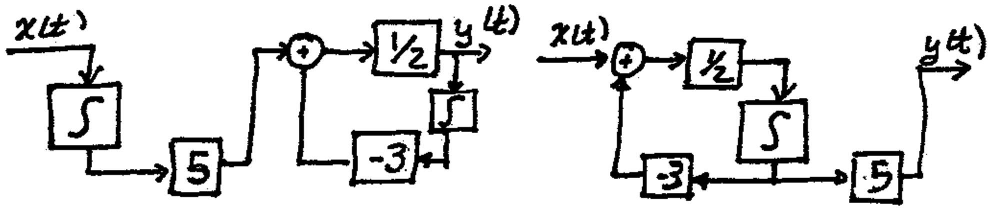
3.32 (a) $H(j\omega) = \frac{5}{2}\angle{-45^\circ} = \frac{K}{a+j\omega} = \frac{K}{a+j4}$, since $\omega = 4$

$\therefore a = \underline{4}$ to yield -45°, $\therefore |H(j4)| = 2.5 = \frac{K}{|4+j4|} = \frac{K}{4\sqrt{2}}$, $\therefore K = \underline{14.14}$

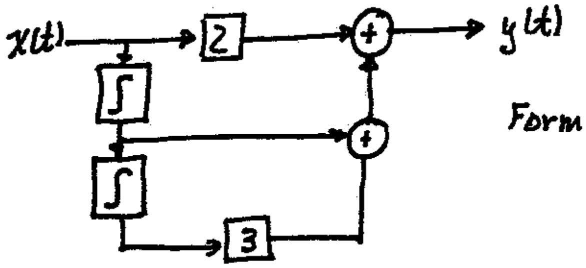
(b) $H(s) = \frac{14.14}{s+4}$;

```
n=[0 14.14];
d=[1 4];
h=polyval(n,4*j)/polyval(d,4*j);
ymag=abs(h)
yphase=angle(h)*180/pi
```

3.33.(a)

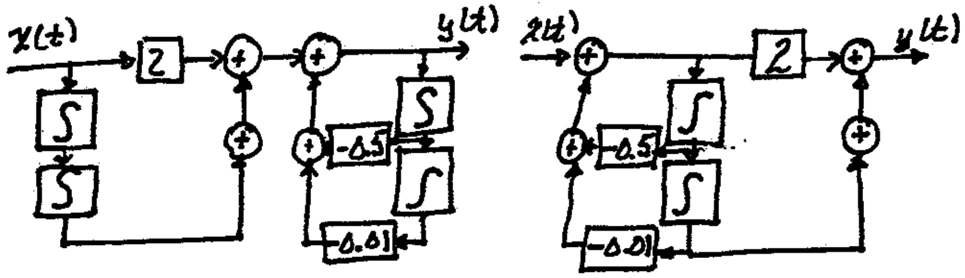


(b) $\frac{d}{dt}\left[\quad\right] \Rightarrow \frac{d^2y}{dt^2} = 2\frac{d^2x}{dt^2} + \frac{dx}{dt} + 3x$

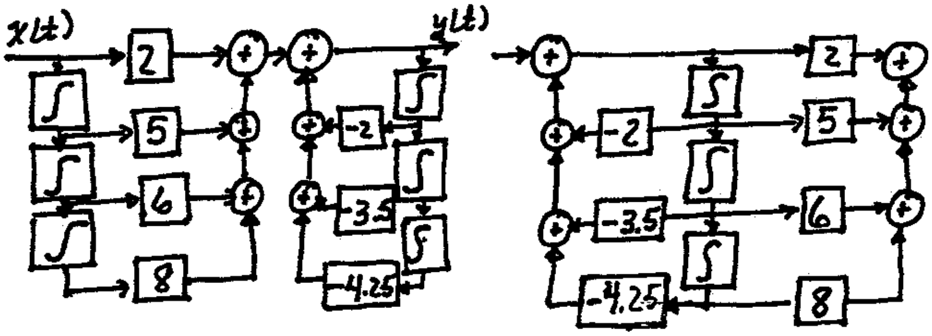


Forms I and II are same.

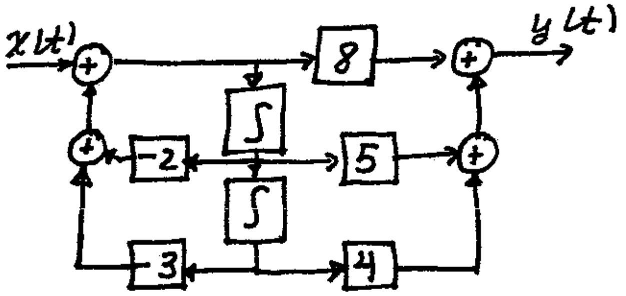
(c)



(d)



3.34 Redraw:



(b) ∴ Form II : (a) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = 8\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x$

3.35 (a) $y_1(t) = x(t) H_1(s)$

$y_2(t) = y_1(t) H_2(s) = x(t) H_1(s) H_2(s) = x(t) H(s)$

∴ $H(s) = \underline{H_1(s) H_2(s)}$

(b) $y(t) = x(t) H_1(s) + x(t) H_2(s) = x(t)[H_1(s) + H_2(s)] = x(t) H(s)$

∴ $H(s) = \underline{H_1(s) + H_2(s)}$

3.36(a) $y(t) = H_2(s)[H_1(s) x(t)] + H_4(s)[H_3(s) H_1(s) x(t) + H_5(s) x(t)]$

$= H(s) x(t)$

∴ $H(s) = \underline{H_1(s) H_2(s) + H_1(s) H_3(s) H_4(s) + H_4(s) H_5(s)}$

(b) $y(t) = H_3(s)[H_2(s)\{H_1(s) x(t)\}] + H_4(s)[H_2(s)\{H_1(s) x(t)\}]$

$+ H_5(s)[x(t) H_1(s)] = H(s) x(t)$

∴ $H(s) = \underline{H_1(s) H_2(s) H_3(s) + H_1(s) H_2(s) H_4(s) + H_1(s) H_5(s)}$

(c) $y(t) = H_1(s)[x(t) - H_2(s) y(t)] = H_1(s) x(t) - H_1(s) H_2(s) y(t)$

$[1 + H_1(s) H_2(s)] y(t) = H_1(s) x(t)$

∴ $y(t) = \frac{H_1(s)}{1 + H_1(s) H_2(s)} x(t) = H(s) x(t)$; ∴ $H(s) = \underline{\frac{H_1(s)}{1 + H_1(s) H_2(s)}}$

3.37(a) $y(t) = H_3(s)[H_1(s) x(t) + H_2(s)\{x(t) - H_4(s) y(t)\}]$

$= [H_1(s) H_3(s) + H_2(s) H_3(s)] x(t) - H_2(s) H_3(s) H_4(s) y(t)$

∴ $y(t) = \underline{\frac{H_1(s) H_3(s) + H_2(s) H_3(s)}{1 + H_2(s) H_3(s) H_4(s)}} x(t) = H(s) x(t)$

(b) $y(t) = H_2(s)[H_1(s)\{x(t) - H_4 y(t)\} - H_3(s) y(t)]$

$= H_1(s) H_2(s) x(t) - [H_1(s) H_2(s) H_4(s) + H_2(s) H_3(s)] y(t)$

∴ $y(t) = \frac{H_1(s) H_2(s)}{1 + H_1(s) H_2(s) H_4(s) + H_2(s) H_3(s)} x(t) = H(s) x(t)$

chapter 4

4.1 $\omega_0 = 2$, $T_0 = 2\pi/\omega_0 = \pi$

a) $C_0 = \frac{1}{T_0}\int_0^{T_0} f(t)\,dt = \frac{1}{\pi}\int_0^{\pi}(\cos 2t + 3\cos 4t)\,dt$

$$= \frac{1}{\pi}\left[\frac{\sin 2t}{2} + 3/4 \sin 4t\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[1/2 \sin 2\pi + 3/4 \sin 4\pi - 1/2 \sin 0 - 3/4 \sin 0\right] = 0$$

$$C_K = \frac{1}{\pi}\int_0^{\pi} e^{-j2Kt}\left[\frac{e^{j2t} + e^{-j2t}}{2} + \frac{3(e^{j4t} + e^{-j4t})}{2}\right] dt$$

$$= \frac{1}{2\pi}\int_0^{\pi}\left[e^{j2(1-K)t} + e^{-j2(1+K)t} + 3e^{j2(2-K)t} + 3e^{-j2(2+K)t}\right]_0^{\pi}$$

$$= \begin{cases} \frac{1}{2\pi}\left[\frac{e^{j(1-K)\pi} - 1}{j2(1-K)}\right], & K = 1 \\ \frac{1}{2\pi}\left[\frac{3e^{j2(2-K)\pi} - 3}{j2(2-K)}\right], & K = 2 \end{cases}$$

$$\therefore C_1 = \underset{K\to 1}{\mathrm{li}} \frac{1}{2\pi}\left[\frac{e^{j(1-K)\pi} - 1}{j2(1-K)}\right] = 1/2\pi\left[\frac{e^{j2\pi} - (j2\pi e^{-j2\pi K})}{j2(-1)}\right] = 1/2$$

$$\therefore C_2 = \underset{K\to 2}{\mathrm{li}} \frac{1}{2\pi}\left[\frac{3e^{j2(2-K)} - 3}{j2(2-K)}\right] = \frac{1}{2\pi}\left[\frac{3e^{j4\pi}(-j2\pi e^{-j2K\pi})}{j2(-1)}\right] = 3/2$$

4.2

a) $x(t) = \sin 4t + \cos 8t + 7$

$\sin 4t$ has $T = 2\pi/4 = \pi/2$

$\cos 8t$ has $T = 2\pi/8 = \pi/4$

$\therefore T_0 = \pi/2 + \omega_0 = 2\pi/T_0 = 4$

$$x(t)=\frac{1}{2j}\left(e^{jw_0t}-e^{-jw_0t}\right)+\frac{1}{2}\left(e^{j2w_0t}+e^{-j2w_0t}\right)+7$$

$C_0=7$ $\quad C_2=1/2$

$C_1=\frac{1}{2j}$ $\quad C_{-2}=1/2$

$C_{-1}=\frac{-1}{2j}$ $\quad C_K=0$, all other K

b) $x(t)=\cos^2 t=\frac{1}{2}(1+\cos 2t)$

$T_0=\frac{2\pi}{2}=\pi$, $w_0=2$

$x(t)=\frac{1}{2}+\frac{1}{4}e^{jw_0t}+\frac{1}{4}e^{-jw_0t}$

$C_0=1/2$, $C_1=1/4$, $C_{-1}=1/4$, $C_K=0$, all other K

c) $x(t)=\cos t+\sin 2t$

$T_0=2\pi$, $w_0=1$

$x(t)=\frac{1}{2}\left(e^{jw_0t}+e^{-jw_0t}\right)+\frac{1}{2j}\left(e^{j2w_0t}-e^{-jw_0t}\right)$

$C_1=1/2$, $C_{-1}=1/2$, $C_2=1/2j$, $C_{-2}=-1/2j$

$C_K=0$ all other K

d) $x(t)=\sin^2 2t+2\cos t$

$=\frac{1}{2}(1-\cos 4t)+2\cos t$

$=\frac{1}{2}-\frac{1}{4}\left(e^{j4t}+e^{-j4t}\right)+\left(e^{jt}+e^{-jt}\right)$

$w_0=1$

$C_0=1/2$, $C_1=1$, $C_{-1}=1$, $C_4=-1/4$, $C_{-4}=-1/4$, $C_K=0$ all other K

e) $x(t) = \cos 2t$

$T_0 = \pi \, , \; \omega_0 = 2$

$x(t) = 1/2 \left(e^{j\omega_0 t} + e^{-j\omega_0 t} \right)$

$C_1 = 1/2 \, , \; C_{-1} = 1/2 \, , \; C_K = 0$ all other K

f) $x(t) = (\cos t)(\sin 2t) = \frac{1}{4j} \left(e^{3jt} + e^{jt} - e^{-jt} - e^{-3jt} \right)$

$C_1 = \frac{1}{4j} \, , \; C_{-1} = \frac{-1}{4j} \, , \; C_3 = \frac{1}{4j} \, , \; C_{-3} = \frac{-1}{4j}$

$C_K = 0$ all other K

4.3

a) $x(t) = \cos(3t) + \sin(5t)$

$\omega_0 = 3 \quad T_0 = \frac{2\pi}{3} \quad \omega_1 = 5 \, , \; T_1 = \frac{2\pi}{5} \quad \rightarrow T = 2\pi$

$\omega = 1$ ✓ yes

b) $x(t) = \cos(6t) + \sin(8t) + e^{j2t}$

$T_1 = \pi/3 \quad T_2 = \pi/4 \quad T_3 = \pi \quad \longrightarrow T = \pi \quad \omega = 2$ ✓ yes

c) aperiodic, NO

d) $x(t) = \sin\left(\frac{\pi t}{6}\right) + \sin\left(\frac{\pi t}{3}\right)$

$T_1 = \frac{2\pi}{\pi/6} = 12 \qquad T_2 = \frac{2\pi}{\pi/3} = 6$

$\longrightarrow T = 12 \, , \; \omega = \pi/6$ ✓ Yes

4.4 let $x(t)$ have a Fourier Series expansion:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

Then,

$$x(t-t_0) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 (t-t_0)} = \sum_{k=-\infty}^{\infty} \underbrace{\left[c_k e^{-jk\omega_0 t_0}\right]}_{\text{call this } \hat{c}_k} e^{jk\omega_0 t}$$

$$|\hat{c}_k| = |c_k e^{-jk\omega_0 t_0}| = |c_k|$$

$$\angle \hat{c}_k = \angle c_k e^{-jk\omega_0 t_0} = \angle c_k - k\omega_0 t_0$$

4.5

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t) + B_k \sin(k\omega_0 t)$$

$$= A_0 + \sum_{k=1}^{\infty} A_k \left[\frac{e^{jk\omega_0 t} + e^{-jk\omega_0 t}}{2}\right] + B_k \left[\frac{e^{jk\omega_0 t} - e^{-jk\omega_0 t}}{2j}\right]$$

$$= A_0 + \sum_{k=1}^{\infty} \left[\frac{A_k}{2} + \frac{B_k}{2j}\right] e^{jk\omega_0 t} + \left[\frac{A_k}{2} - \frac{B_k}{2j}\right) e^{-jk\omega_0 t}$$

Compare to $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$

$$c_k = \begin{cases} A_0 & k=0 \\ \frac{1}{2}\left[A_k - jB_k\right], & k \geq 1 \\ \frac{1}{2}\left[A_k + jB_k\right], & k \leq -1 \end{cases}$$

4.6

a)

$$\int_0^{2\pi} \sin^2(t)\, dt = \int_0^{2\pi} \frac{1}{2}\left[1 - \cos 2t\right] dt$$

$$= \frac{1}{2}\left(t - \frac{1}{2}\sin 2t\right)\Big|_0^{2\pi} = \pi$$

b) $\int_0^{2\pi} \sin^2(2t)\,dt = \int_0^{2\pi} \frac{1}{2}\left[1 - \cos 4t\right] dt$

$= \frac{1}{2}\left(t - \frac{1}{4}\sin 4t\right)\Big|_0^{2\pi} = \pi$

c) $\int_0^{2\pi} \sin(t)\sin(2t)\,dt$

$= 1/2 \int_0^{2\pi} \left[\cos t - \cos 3t\right] dt$

$= 1/2 \left(\sin t - 1/3 \sin 3t\right)\Big|_0^{2\pi}$

$= 0$

4.7. The integral of a sinusoid over an integer number of periods is zero. Orthogonal: $\int_a^b g(t)h(t)dt = 0$

(a) $\cos m\omega_0 t \cos n\omega_0 t$

$$= \frac{1}{2}\cos(m+n)\omega_0 t + \frac{1}{2}\cos(m-n)\omega_0 t$$

$$\therefore \frac{1}{2}\int_0^{T_0}[\cos(m+n)\omega_0 t + \cos(n-m)\omega_0 t]dt = 0, \ m \neq n$$

$$= \frac{1}{2}\int_0^{T_0} dt = \frac{1}{2}t\Big]_0^{T_0}, \ m = n \qquad \therefore \ \underline{m \neq n}$$

(b) $$\cos m\omega_0 t \sin n\omega_0 t = \frac{1}{2}[\sin(m+n)\omega_0 t + \sin(n-m)\omega_0 t]$$

$$\therefore \frac{1}{2}\int_0^{T_0}[\sin(m+n)\omega_0 t + \sin(n-m)\omega_0 t]dt = 0, \ \underline{\text{all } m \,\&\, n}$$

(c) $$\sin m\omega_0 t \sin n\omega_0 t = \frac{1}{2}[\cos(m-n)\omega_0 t - \cos(m+n)\omega_0 t]$$

$$\therefore \frac{1}{2}\int_0^{T_0}[\cos(m-n)\omega_0 t - \cos(m+n)\omega_0 t]dt = \begin{cases} 0, & m \neq n \\ \frac{T_0}{2}, & m = n \end{cases}$$

from(a) →

4.8. (a) $C_k = -j\frac{2X_0}{\pi k}$, $\underline{k \text{ odd}}$ $\quad 2|C_k| = \frac{4X_0}{\pi k}$; $\theta_k = -90^0$

$$\therefore x(t) = \underline{\sum_{\substack{k=1 \\ k \text{ odd}}}^{\infty} \frac{4X_0}{\pi k}\cos(k\omega_0 t - 90^0)}$$

(b) $C_k = j\frac{X_0}{2\pi k}$, $\quad 2|C_k| = \frac{X_0}{\pi k}$, $\quad \theta_k = 90^0$

$$\therefore \underline{x(t) = \frac{X_0}{2} + \sum_{k=1}^{\infty}\frac{X_0}{\pi k}\cos(k\omega_0 t + 90^0)}$$

(c) $C_k = -\frac{2X_0}{(\pi k)^2}$, k odd, $\quad 2|C_k| = \frac{4X_0}{(\pi k)^2}$; $\theta_k = 180^0$

$$\therefore \underline{x(t) = \frac{X_0}{2} + \sum_{\substack{k=1 \\ k \text{ odd}}}^{\infty}\frac{4X_0}{(\pi k)^2}\cos(k\omega_0 t + 180^0)}$$

(d) $C_k = \frac{wX_0}{T_0}\operatorname{sinc}\frac{wk\omega_0}{2}$;

$$\underline{x(t) = \sum_{k=0}^{\infty}\frac{2wX_0}{T_0}\operatorname{sinc}\left(\frac{wk\omega_0}{2}\right)\cos k\omega_0 t}$$

(e) $$\underline{x(t) = \frac{2X_0}{\pi} + \sum_{k=1}^{\infty}\frac{4X_0}{\pi(4k^2-1)}\cos(k\omega_0 t + 180^0)}$$

(f) $$\underline{x(t) = \frac{X_0}{2}\cos(\omega_0 t - 90^0) + \sum_{\substack{k=0 \\ k \text{ even}}}^{\infty}\frac{2X_0}{\pi(k^2-1)}\cos(k\omega_0 t + 180^0)}$$

4.8 (cont) (g) $\underline{x(t) = \sum_{k=0}^{\infty} \frac{2X_0}{T_0} \cos k\omega_0 t}$

4.9. App A used, with $e^{-jk\omega_0 T_0} = e^{-jk2\pi}$

(a) $C_k = \frac{1}{T_0}\int_0^{T_0/2} X_0 e^{-jk\omega_0 t} dt - \frac{1}{T_0}\int_{T_0/2}^{T_0} X_0 e^{-jk\omega_0 t}$

$$= \frac{X_0}{-jk\omega_0 T_0}\left[e^{-jk\omega_0 t}\Big|_0^{T_0/2} - e^{-jk\omega_0 t}\Big|_{T_0/2}^{T_0}\right]$$

$$= \frac{jX_0}{2\pi k}\left[e^{-jk\pi} - 1 - e^{-jk2\pi} + e^{-jk\pi}\right] = \underline{\begin{cases} 0 \,; & k \text{ even} \\ -j\frac{2X_0}{\pi k} \,; & k \text{ odd}\end{cases}}$$

(b) $C_k = \frac{1}{T_0}\int_0^{T_0} \frac{X_0}{T_0} t e^{-jk\omega_0 t} dt = \frac{X_0}{T_0^2}\left[\frac{1}{(-jk\omega_0)^2} e^{-jk\omega_0 t}(-jk\omega_0 t - 1)\right]_0^{T_0}$

$$= \frac{X_0}{-[k2\pi]^2}\left[e^{-jk2\pi}(-jk2\pi - 1) - (-1)\right] = \frac{-X_0}{(2\pi k)^2}(-jk2\pi) = \underline{\frac{jX_0}{2\pi k}}$$

(c) $C_k = \frac{1}{T_0}\int_{-T_0/2}^{0} -\frac{2X_0}{T_0} t e^{-jk\omega_0 t} dt + \frac{1}{T_0}\int_0^{T_0/2} \frac{2X_0}{T_0} t e^{-jk\omega_0 t} dt$

$$= \frac{2X_0}{T_0^2}\frac{1}{(-jk\omega_0)^2}\left[-e^{-jk\omega_0 t}(-jk\omega_0 t - 1)\Big|_{-T_0/2}^{0} + e^{-jk\omega_0 t}(-jk\omega_0 t - 1)\Big|_0^{T_0/2}\right]$$

$$= \frac{2X_0}{-(k2\pi)^2}\left[1 + e^{jk\pi}(jk\pi - 1) + e^{-jk\pi}(-jk\pi - 1) - (-1)\right]$$

Now, $e^{jk\pi} = e^{-jk\pi}$

$$\therefore C_k = \frac{2X_0}{-(k2\pi)^2}\left[-2e^{jk\pi} + 2\right] = \underline{\begin{cases} \frac{-2X_0}{(\pi k)^2} \,; & k \text{ odd} \\ 0 \,; & k \text{ even}\end{cases}}$$

(d) $C_k = \frac{1}{T_0}\int_{-w/2}^{w/2} X_0 e^{-jk\omega_0 t} dt = \frac{X_0}{-jk2\pi} e^{-jk\omega_0 t}\Big|_{-w/2}^{w/2}$

$$= \frac{X_0}{-jk2\pi}\left[e^{-jkw\omega_0/2} - e^{jkw\omega_0/2}\right] = \frac{X_0}{\pi k}\sin(kw\omega_0/2)$$

$$= \frac{X_0}{\pi k}\,\frac{kw\omega_0}{2}\,\frac{\sin(kw\omega_0/2)}{kw\omega_0/2} = \underline{\frac{wX_0}{T_0}\,\text{sinc}(kw\omega_0/2)}$$

(e) $C_k = \frac{1}{T_0}\int_0^{T_0} X_0 \sin(\frac{\omega_0}{2}t) e^{-jk\omega_0} dt$

$$= \frac{X_0}{T_0}\left[\frac{e^{-jk\omega_0 t}(-jk\omega_0 t \sin(\frac{\omega_0 t}{2}) - \frac{\omega_0}{2}\cos(\frac{\omega_0 t}{2}))}{-k^2\omega_0^2 + \omega_0^2/4}\right]_0^{T_0}$$

$$= \frac{X_0}{T_0}\left[\frac{e^{-jk2\pi}(-jk2\pi \sin\pi - (\pi/T_0)\cos\pi) - (1)(-\frac{\pi}{T_0}\cos 0)}{-k^2\omega_0^2 + \omega_0^2/4}\right]$$

4.9 (cont)

$$= \frac{4X_0}{T_0}\left[\frac{(1)\left(\frac{\pi}{T_0}\right)+\frac{\pi}{T_0}}{\omega_0^2(1-4k^2)}\right] = \frac{8X_0}{4\pi(1-4k^2)} = \underline{\frac{-2X_0}{\pi(4k^2-1)}}$$

(f) $$c_k = \frac{1}{T_0}\int_0^{T_0/2} X_0 \sin(\omega_0 t)\, e^{-jk\omega_0 t}\,dt$$

$$= \frac{X_0}{T_0}\left[\frac{e^{-jk\omega_0 t}\left(-jk\omega_0 t \sin(\omega_0 t)-\omega_0 \cos(\omega_0 t)\right)}{-k^2\omega_0^2+\omega_0^2}\right]_0^{T_0/2}$$

$$= \frac{X_0}{T_0}\left[\frac{e^{-j\pi k}(0-\omega_0 \cos\pi)-(-\omega_0)}{\omega_0^2(1-k^2)}\right] = \frac{X_0}{2\pi}\,\frac{[e^{-jk\pi}+1]}{(1-k^2)}$$

$$= \underline{\frac{-X_0}{\pi(k^2-1)},\ k \text{ even}}$$

$$c_1 = \lim_{k\to 1}\frac{X_0}{2\pi}\left[\frac{e^{-jk\pi}+1}{(1-k^2)}\right] = \frac{X_0}{2\pi}\left[\frac{-j\pi e^{-jk\pi}}{-2k}\right]_{k=1} = \underline{\frac{-jX_0}{4}}$$

$$\underline{c_k = 0,\ k \text{ odd and } k\neq 1}$$

(g) $$c_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} X_0\,\delta(t)\,e^{-jk\omega_0 t}\,dt = \frac{1}{T_0}X_0 e^{-j0} = \underline{\frac{X_0}{T_0}}$$

4.10(a) $$c_k = \frac{1}{T_0}\int_0^1 -3e^{-jk\omega_0 t}\,dt + \frac{1}{T_0}\int_{-1}^0 3e^{-jk\omega_0 t}\,dt \quad , \quad \omega_0 = \frac{\pi}{2},\ T_0 = 4$$

$$= \frac{3}{4\left(-j\frac{k\pi}{2}\right)}\left[-e^{-j\frac{k\pi}{2}t}\Big|_0^1 + e^{-jk\pi t/2}\Big|_{-1}^0\right]$$

$$= \frac{j3}{k2\pi}\left[-e^{-jk\pi/2}+1+1-e^{jk\pi/2}\right] = \frac{j3}{2\pi k}\left[2-2\left(\frac{e^{j\frac{k\pi}{2}}+e^{-j\frac{k\pi}{2}}}{2}\right)\right]$$

$$= \underline{\frac{j3}{k\pi}\left[1-\cos\left(\frac{k\pi}{2}\right)\right]}$$

$$c_0 = \lim_{k\to 0} c_k = j\frac{3\left(-\frac{\pi}{2}\sin k\frac{\pi}{2}\right)}{\pi}\Bigg|_{k=0} = \underline{0}$$

(b) $$c_k = \frac{1}{T_0}\int_2^3 e^{-jk\omega_0 t}\,dt + \frac{2}{T_0}\int_3^4 e^{-jk\omega_0 t}\,dt \qquad k\omega_0 = \frac{k\pi}{2},\ T_0 = 4$$

$$= \frac{1}{-jk2\pi}\left[e^{-jk\pi t/2}\Big|_2^3 + 2e^{-jk\pi t/2}\Big|_3^4\right]$$

$$= \frac{j}{2\pi k}\left[e^{-j3k\pi/2} - e^{-jk\pi} + 2e^{-j2\pi k} - 2e^{-j3k\pi/2}\right]$$

$$= \underline{\frac{j}{2\pi k}\left[2e^{-j2\pi k} - e^{-jk\pi} - e^{-j3k\pi/2}\right]}$$

$$c_0 = \lim_{k\to 0} c_k = \frac{j}{2\pi}\left[-4e^{-j2\pi k}(j\pi) + j\pi e^{-jk\pi} + j\frac{3\pi}{2}e^{-j\frac{3k\pi}{2}}\right]_{k=0} = \underline{\frac{3}{4}}$$

4.10 (cont)

(c) $$C_k = \frac{1}{2}\int_0^1 2t\, e^{-jk\pi t} dt = \frac{1}{(-jk\pi)^2}\left[e^{-jk\pi t}(-jk\pi t - 1)\right]_0^1 ; \quad k\omega_0 = k\pi,\ T_0 = 2$$

$$\underline{C_k = \frac{-1}{k^2\pi^2}\left[e^{-jk\pi}(-jk\pi - 1) + 1\right]}$$

$$C_0 = \lim_{k\to 0} C_k = \frac{-1}{2k\pi^2}\left[-j\pi\, e^{-jk\pi}(-jk\pi - 1) - j\pi e^{-jk\pi}\right]_{k=0} = \underline{\frac{1}{2}}$$

(d) $$C_k = \frac{1}{2}\int_0^1 2(1-t)\, e^{-jk\omega_0 t} = \int_0^1 e^{-jk\omega_0 t} dt - \int_0^1 t\, e^{-jk\omega_0 t} dt$$

$$= \frac{e^{-jk\pi t}}{-jk\pi}\Big|_0^1 - \frac{1}{k^2\pi^2}\left[-jk\pi(-1)^k + (-1)^k + 1\right]; \quad \xleftarrow{\text{from (c)}} \quad k\omega_0 = k\pi,\ T_0 = 2$$

$$\underline{= \frac{e^{-jk\pi} - 1}{-jk\pi} - \frac{1}{k^2\pi^2}\left[e^{-jk\pi}(-jk\pi - 1) + 1\right]}$$

$$C_0 = \lim_{k\to 0}\left[\frac{e^{-jk\pi} - 1}{-jk\pi}\right] - \frac{1}{2} \xleftarrow{\text{from (c)}} = \frac{-j\pi e^{-jk\pi}}{-j\pi}\Big|_{k=0} - \frac{1}{2} = \frac{1}{2} \checkmark$$

(e) $$C_k = \frac{1}{4}\int_{-1}^{0} 2\cos\frac{\pi}{2}t\, e^{-jk\frac{\pi}{2}t} dt \ ; \quad \omega_0 = \frac{\pi}{2},\ T_0 = 4$$

$$= \frac{1}{2}\left[\frac{e^{-jk\pi t/2}(-jk\frac{\pi}{2}\cos\frac{\pi}{2}t + \frac{\pi}{2}\sin\frac{\pi}{2}t}{-(k\frac{\pi}{2})^2 + (\frac{\pi}{2})^2}\right]_{-1}^{0}$$

$$= \frac{2}{\pi^2(1-k^2)}\left[-jk\frac{\pi}{2} - e^{j\frac{\pi}{2}k}(-j\frac{\pi}{2}k(0 - \frac{\pi}{2}\sin\frac{\pi}{2})\right]$$

$$\underline{= \frac{1}{\pi(1-k^2)}\left[e^{j\frac{\pi}{2}k} - jk\right]}$$

4.11

a) $x_0 = 4$, $\omega_0 = \frac{2\pi}{.4\pi} = 5$

$c_0 = 0$, $\underset{k\,\text{odd}}{C_k} = \frac{-2(4)}{(\pi k)^2} = \frac{-8}{(\pi k)^2}$ $\quad \underset{k\,\text{even}}{C_k} = 0$

b) $x_0 = 8$, $\omega_0 = \frac{2\pi}{5} = .4\pi$, $\frac{\omega k \omega_0}{2} = \frac{(1)(k)(.4\pi)}{2}$

$= .2\pi k$

$k \neq 0$, $C_k = \frac{(1)(8)}{5}\,\text{sinc}(.2\pi k) = 1.6\,\text{sinc}(.2\pi k)$

$C_0 = \frac{10 + 2(4)}{5} = 3.6$

c) $x_0 = 8$, $\omega_0 = \frac{2\pi}{.2} = 10\pi$, $C_k = \frac{j8}{2\pi k} = j\frac{4}{\pi k}$, $k \neq 0$

$C_0 = 0$

d) $x_0 = 20$, $\omega_0 = \frac{2\pi}{2\pi} = 1$, $\underset{\text{odd}}{C_k} = \frac{2(20)}{(\pi k)^2}$, $\underset{k\,\text{even}}{C_k} = 0$

$C_0 = 10$

e) $\underset{k \neq 0}{2|C_k|} = \frac{24}{\pi(4k^2-1)}$, $\theta_k = 180^\circ$, $c_0 = \frac{12}{\pi}$

4.12. (a) From Prob. 4.10(d); $\omega_0 = 1$, $\underset{k\ \text{odd}}{C_{ka}} = \dfrac{-8}{(\pi k)^2}$; $\underset{k\ \text{even}}{C_{ka}} = 0$

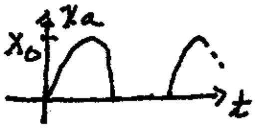
(b) From Prob 4.10(a), $\omega_0 = 1$, $\underset{k\ \text{odd}}{C_{dk}} = \dfrac{40}{(\pi k)^2}$, $\underset{k\ \text{even}}{C_{kd}} = 0$, $C_0 = 10$

(c) $x = a_1 x_a + b_1 x_d = 5x_a + x_d \quad \therefore a_1 = \underline{5}$, ; $b_1 = \underline{1}$

$A = a_1 C_{0a} + b_1 C_{0b} = 5(0) + (1)(10) = \underline{10}$

(d) $5C_{ka} + C_{ka} = \dfrac{-40}{(\pi k)^2} + \dfrac{40}{(\pi k)^2} = 0$, $k \neq 0$

4.13. (a)



$C_0 = \dfrac{X_0}{\pi}$, $\underset{k\ \text{even}}{C_{ka}} = \dfrac{-X_0}{\pi(k-1)}$; $\underset{k\ \text{odd}}{C_{ka}} = 0$, $C_{1a} = -j\dfrac{X_0}{4}$

$$x_b(t) = x_a(\tau)\Big|_{\tau = t - \frac{T_0}{2}} = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0(t - \frac{T_0}{2})} = \sum_{k=-\infty}^{\infty} C_k e^{-j\omega_0 t} e^{-jk\omega_0 \frac{T_0}{2}}$$

$$k\omega_0 \frac{T_0}{2} = k\frac{2\pi}{T_0}\frac{T_0}{2} = k\pi \ ; \therefore e^{-jk\pi} \begin{cases} 1, & k\ \text{even} \\ -1, & k\ \text{odd} \end{cases}$$

$\therefore \underset{k\ \text{even}}{C_{kb}} = \dfrac{-X_0}{\pi(k^2-1)}$, $C_{1b} = j\dfrac{X_0}{4}$

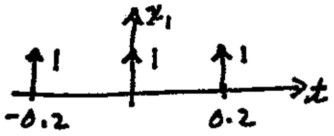
(b) For $x_1 = x_a + x_b$, from (a):

$\underset{k\ \text{even}}{C_{k1}} = \dfrac{-2X_0}{\pi(k_1^2-1)}$; $\underset{k_1 = 1}{C_{k1}} = -j\dfrac{X_0}{4} + j\dfrac{X_0}{4} = 0$, $C_0 = \dfrac{2X_0}{\pi}$

Since k_1 is even, define $k = \dfrac{k_1}{2}$; then $k = 1, 2, 3, \ldots$

$\therefore C_k = \dfrac{-2X_0}{\pi((2k)^2-1)} = \dfrac{-2X_0}{\pi(4k^2-1)}$ ✓ ; $C_0 = \dfrac{2X_0}{\pi}$ ✓

4.14. (a) $x(t) = x_1(t) + x_2(t)$, $\omega_0 = \dfrac{2\pi}{0.2} = 10\pi$



$$x_1(t) = \sum_{k=-\infty}^{\infty} \frac{1}{0.2} e^{jk10\pi t} = \sum_{k=-\infty}^{\infty} 5e^{jk10\pi t}$$

$$x_2(t) = -x_1(t-0.1) = -\sum_{k=-\infty}^{\infty} 5e^{jk10\pi(t-0.1)}$$

$$= -\sum_{k=-\infty}^{\infty} 5e^{-jk\pi} e^{jk10\pi t}$$

$$\therefore x(t) = 5\sum_{k=-\infty}^{\infty} (1 - e^{-jk\pi}) e^{jk10\pi t}$$

4.14(a) (cont)

$$\therefore C_k = \underline{5(1-e^{-jk\pi})} = \begin{cases} 10 \ , & k = \pm 1, \pm 3, \ldots \\ 0 \ , & k = 0, \pm 2, \pm 4, \ldots \end{cases}$$

(b) $$C_k = \frac{1}{T_0}\int_{-T_0/4}^{3T_0/4} [\delta(t) - \delta(t-0.1)] e^{-jk\omega_0 t} dt$$

$$= 5[1 - e^{-jk10\pi(0.1)}] = \underline{5[1-e^{-jk\pi}]}$$

4.15. From Prob P4.8, $x_c(t) \Rightarrow C_{kc} = \frac{-1}{k^2\pi^2}\left[e^{-jk\pi}(-jk\pi - 1) + 1\right]$

$$x_d(t) \Rightarrow C_{kd} = \frac{e^{-jk\pi} - 1}{-jk\pi} + \frac{1}{k^2\pi^2}\left[e^{-jk\pi}(-jk\pi - 1) + 1\right]$$

$$\therefore C_{kc} + C_{kd} = \frac{e^{-jk\pi} - 1}{-jk\pi} = \frac{(-1)^k - 1}{-jk\pi} = \begin{cases} 0^{\checkmark}, & k \text{ even} \\ -j\frac{2}{k\pi} = \frac{2}{k\pi}\angle -90^{\circ\checkmark}, & k \text{ even} \end{cases}$$

4.16. $$C_k = \int_0^{T_0} x(t) e^{-jk\omega_0 t} dt = \int_0^{T_0/2} x(t) e^{-jk\omega_0 t} dt + \int_{T/2}^{T_0} x(t) e^{-jk\omega_0 t} dt$$

$$= I_1 + I_2$$

$$I_2 = \int_{\frac{T_0}{2}}^{T_0} x(\tau) e^{-jk\omega_0 \tau} d\tau \ : \text{let } \tau = t_0 - T_0/2$$

$$\therefore I_2 = \int_0^{T_0/2} x(t - \tfrac{T_0}{2}) e^{-jk\omega_0(t - T_0/2)} dt \quad \text{now } \frac{k\omega_0 T_0}{2} = \frac{k_2}{2}\left(\frac{2\pi}{T_0}\right)T_0 = k\pi$$

$$\therefore I_2 = -e^{jk\pi}\int_0^{\frac{T}{2}} x(t) e^{-jk\omega_0 t} dt = -(-1)^k I_1$$

$$\therefore C_k = I_1 - (-1)^k I_1 = [1 - (-1)^k] = \begin{cases} 2I_1, & k \text{ odd} \\ 0, & k \text{ even} \end{cases}$$

4.17. Property 6, Section 4.4

(a) $\frac{dx_a}{dt}$ discontinuous, $\therefore C_k \to \frac{A}{k^2}$, k large, $\therefore \underline{m=2}$

(b) x_b discontinuous, $\therefore C_k \to \frac{A}{k}$, k large, $\therefore \underline{m=1}$

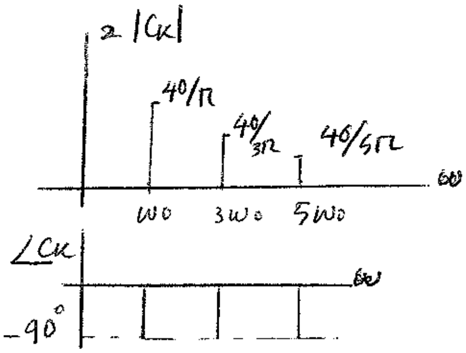
(c) Same as (b) (d) Same as a (e) Same as a

(f) all check

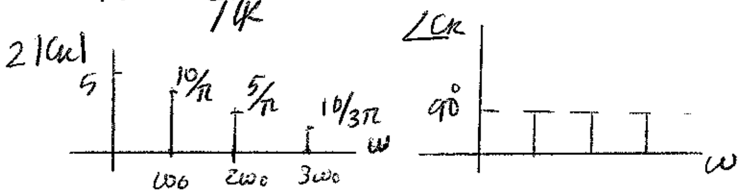
4.18

will use combined trig form with $x_0 = 10$

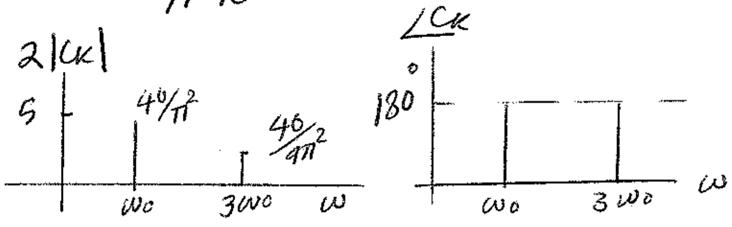
a) $2C_k = \frac{40}{\pi k} \angle -90^\circ$, k odd



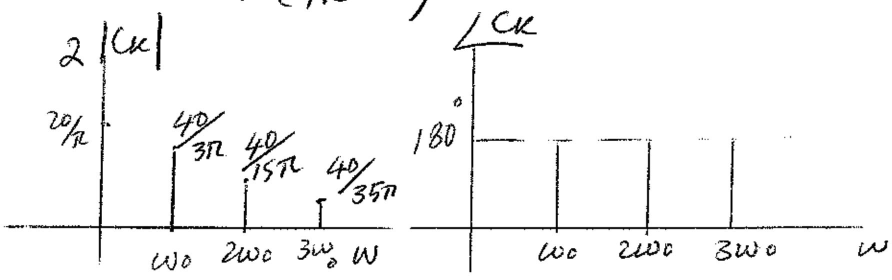
b) $2C_k = \frac{10}{\pi k} \angle 90^\circ$



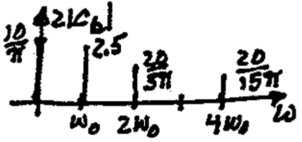
c) $2C_k = \frac{-40}{\pi^2 k^2}$, k odd

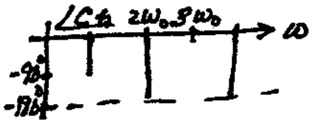


d) $2C_k = \frac{-40}{\pi(4k^2-1)}$



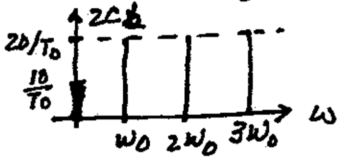
4.18. (e) $2C_k = \frac{-20}{\pi(k^2-1)}$, k even
(cont)
$C_1 = -j\,2.5$





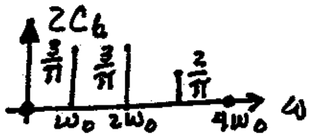
(f) See Figure 4.13 with $\frac{2WX_0}{T_0} = \frac{20W}{T_0}$

(g) $2C_k = \frac{20}{T_0}$



4.19. From Prob 4.7

(a) $C_k = \frac{3}{k\pi}\left(1 - \cos\frac{k\pi}{2}\right), C_0 = 0$



$C_1 = \frac{3}{\pi}$, $C_2 = \frac{6}{2\pi} = \frac{3}{\pi}$, $C_3 = \frac{2}{\pi}$, $C_4 = \frac{(6)(0)}{4\pi} = 0$

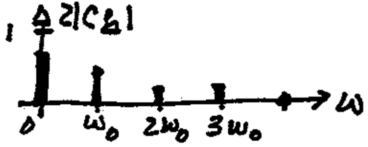
(b) $C_k = \frac{j}{2\pi k}\left[2e^{-j2\pi k} - e^{-jk\pi} - e^{-j3k\pi/2}\right]$, $C_0 = \underline{\frac{3}{4}}$

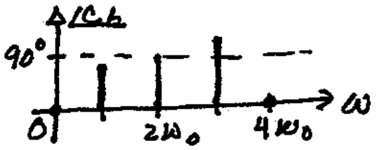
$C_1 = \frac{j}{2\pi}\left[2 - e^{-j\pi} - e^{-j\frac{3\pi}{2}}\right] = \frac{j}{2\pi}\left[2 + 1 - 1\angle{-270^\circ}\right] = \underline{0.503\angle 72.6^\circ}$

$C_2 = \frac{j}{4\pi}\left[2 - e^{-j2\pi} - e^{-j3\pi}\right] = \underline{\frac{j}{2\pi}}$

$C_3 = \frac{j}{6\pi}\left[2 - e^{-j3\pi} - e^{-j9\pi/2}\right] = \frac{j}{6\pi}(3+j) = \underline{0.168\angle 108.4^\circ}$

$C_4 = \frac{j}{4\pi}\left[2 - e^{-j4\pi} - e^{-j6\pi}\right] = \underline{0}$





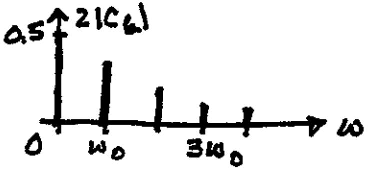
(c) $C_k = -\frac{1}{k^2\pi^2}\left[e^{-jk\pi}(-jk\pi - 1) + 1\right]$; $C_0 = \underline{\frac{1}{2}}$

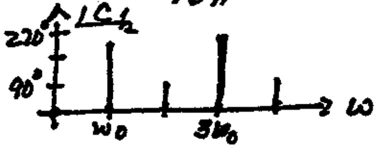
$C_1 = -\frac{1}{\pi^2}\left[(-1)(-j\pi - 1) + 1\right] = -\frac{1}{\pi^2}\left[2 + j\pi\right] = \underline{0.377\angle 237.5^\circ}$

$C_2 = -\frac{1}{4\pi^2}\left[(1)(-2j\pi - 1) + 1\right] = \frac{j2\pi}{4\pi^2} = \underline{0.159\angle 90^\circ}$

$C_3 = -\frac{1}{9\pi^2}\left[(-1)(-j3\pi - 1) + 1\right] = -\frac{1}{9\pi^2}\left[2 + j3\pi\right] = \underline{0.108\angle 258^\circ}$

$C_4 = -\frac{1}{16\pi^2}\left[(1)(-j4\pi - 1) + 1\right] = \frac{j4\pi}{16\pi^2} = \underline{0.080\angle 90^\circ}$





4.19.(d) (cont)

$$C_k = \frac{e^{-jk\pi}-1}{-jk\pi} - \underbrace{\frac{1}{k^2\pi^2}\left[e^{-jk\pi}(-jk\pi-1)+1\right]}_{\text{same as (c)}}, \quad \underline{C_0 = \frac{1}{2}}$$

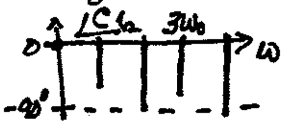
$$C_1 = \frac{-2}{-j\pi} - 0.377\angle 237.5 = \underline{0.377\angle -57.7^\circ}$$

$$C_2 = \frac{1-1}{-j2\pi} - 0.159\angle 90^\circ = \underline{0.159\angle -90^\circ}$$

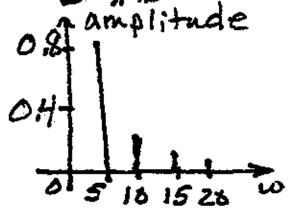
$$C_3 = \frac{-2}{-j3\pi} - 0.108\angle 258^\circ = \underline{0.108\angle -77^\circ}$$

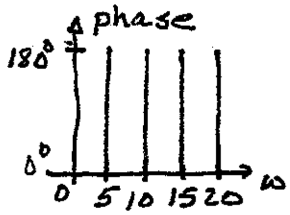
$$C_4 = 0 - 0.080\angle 90^\circ = \underline{0.080\angle -90^\circ}$$

same as <u>mag.</u> spect. in (c)



4.20. (a) $C_k = \frac{8}{\pi^2 k^2}$, ∴ $C_0 = 0$, $C_1 = -0.812$ $C_2 = -0.203$, $C_3 = -0.090$, $C_4 = -0.051$



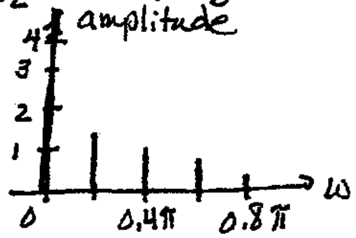


(b) $C_k = \frac{WX_0}{T_0} \text{sinc} \frac{Wk\omega_0}{2} = \frac{(1)(8)}{5} \text{sinc} \frac{(1)(k)0.4\pi}{2} = 1.6 \,\text{sinc}(0.2\pi k)$

$= \frac{1.6}{0.2\pi k} \sin(0.2\pi k) = \frac{8}{\pi k} \sin(0.2\pi k)$

$\omega_0 = \frac{2\pi}{5} = 0.4\pi$, $C_0 = \frac{10+8}{4} = 4.5$, $C_1 = 1.497$

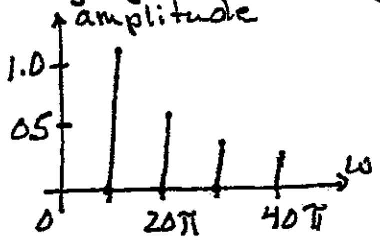
$C_2 = 1.211$, $C_3 = 0.8073$, $C_4 = 0.3742$

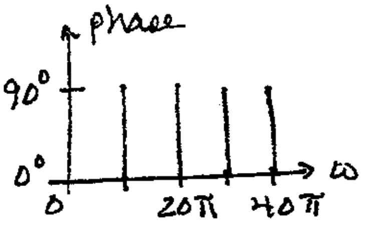


phase $= 0^\circ$

(c) $C_k = \frac{j4}{\pi k}$, $C_0 = 0$, $C_1 = j1.273$, $C_2 = j0.637$

$C_3 = j0.424$, $C_4 = j0.308$, $\omega_0 = \frac{2\pi}{0.2} = 10\pi$





4.21 $\omega_0 = \pi$, $c_0 = 2$, $c_1 = 1$, $c_3 = \frac{1}{2}e^{j\pi/4}$, $c_{-3} = \frac{1}{2}e^{-j\pi/4}$

$$x(t) = 2 + e^{j\pi t} + \frac{1}{2}e^{j\pi/4}e^{j3\pi t} + \frac{1}{2}e^{-j\pi/4}e^{-j3\pi t}$$

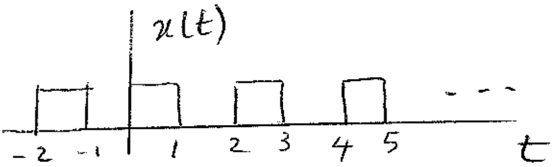
$$= 2 + e^{j\pi t} + \cos(3\pi t + \pi/4)$$

4.22

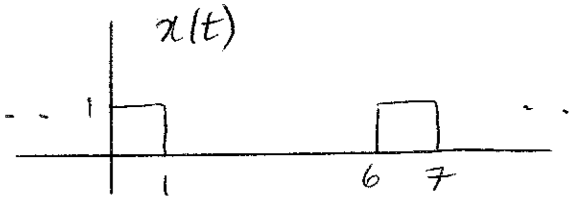
$$c_k = \frac{1}{2}\int_0^1 e^{-jk\omega_0 t}\,dt = \frac{1}{2}\,\frac{1}{-jk\omega_0}\,e^{-jk\omega_0 t}\Big|_0^1$$

$$= \frac{1}{2jk\omega_0}\left[1 - e^{-jk\omega_0}\right], \quad k \neq 0$$

$$c_0 = \frac{1}{2}\int_0^1 dt = \frac{1}{2}$$



4.23



a) $T = 6$ $f = 1/6$ & $\omega_0 = \frac{2\pi}{T} = \pi/3$

b) $c_0 = \frac{1}{T}\int_T x(t)\,dt = 1/6$

$$c_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt = 1/6\int_0^1 e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{jk\omega_0 T}\left(1 - e^{-jk\omega_0}\right), \quad k \neq 0$$

This = 0 when $k \neq 0$, k a multiple of 6

4.24

$H(s) = \frac{10}{s+5}$, $\omega_0 = \frac{2\pi}{3}$, $T_0 = 3$

$H(0) = 10/5 = 2$, $H(j\omega_0) = \frac{10}{5+j2\pi/3} = 1.84\angle{-22.7^\circ}$

$H(j2\omega_0) = \frac{10}{5+j4\pi/3} = 1.533\angle{-40^\circ}$

$H(j3\omega_0) = \frac{10}{5+j2\pi} = 1.245\angle{-51.5^\circ}$

$c_{y_k} = H(jk\omega_0)\, c_{xk}$

a) $x(t) = c_{x_0} = 0$, $c_{xk} = -j\frac{2(20)}{\pi k} = \frac{40}{\pi k}\angle{-90^\circ}$, k odd

$C_{y_0} = 0$

$C_{y_1} = (1.84\angle{-22.7})(12.72\angle{-90^\circ}) = 23.4\angle{-112.7^\circ}$

$C_{y_2} = 0$

$C_{y_3} = (1.245\angle{-51.5^\circ})(4.24\angle{-90^\circ}) = 5.28\angle{-141.5^\circ}$

$y(t) = 46.8\cos(2/3\,\pi t - 112.7^\circ) + 10.56\cos(2\pi t - 141.5^\circ)$
$+ \cdots$

b)
```
w=[.66667*pi  2*pi]; n=[0 10]; d=[1 5];
h=freqs(n,d,w);
hmag=abs(h); hphase=angle(h)*180/pi;
[hmag' hphase']
```

4.24c) (cont) (a) $C_{x0} = \frac{X_0}{2} = 10 ;\ C_{xk} = j\frac{20}{2\pi k}$

$$C_{y0} = (20)(10) = \underline{20}$$

$$C_{y1} = (1.84\angle{-22.7^\circ})(3.18\angle 90^\circ) = \underline{5.86\angle 67.3^\circ}$$

$$C_{y2} = (1.53\angle{-40^\circ})(1.59\angle 90^\circ) = \underline{2.44\angle 50^\circ}$$

$$C_{y3} = (1.245\angle{-51.5^\circ})(1.061\angle 90^\circ) = 1.32\angle 38.5^\circ$$

$$y(t) = \underline{20 + 11.72\cos(\tfrac{2}{3}\pi t + 67.3^\circ) + 4.88\cos(\tfrac{4}{3}\pi t + 50^\circ)}$$

$$\underline{+\,2.64\cos(2\pi t + 38.5^\circ) + \cdots}$$

(d) (a) $C_{x0} = 10 ;\ C_{xk} = -\frac{40}{\pi^2 k^2}$, k odd

$$C_{y0} = 2(10) = \underline{20}$$

$$C_{y1} = (1.84\angle{-22.7^\circ})(4.05\angle 180^\circ) = \underline{7.46\angle 157.3^\circ} ;\ C_{y2} = \underline{0}$$

$$C_{y3} = (1.245\angle{-51.5^\circ})(0.450\angle 180^\circ) = \underline{0.561\angle 128.5^\circ}$$

$$y(t) = \underline{20 + 14.92\cos(\tfrac{2}{3}\pi t + 157.3^\circ) + 1.122\cos(2\pi t + 128.5^\circ)} + \cdots$$

(e) $C_{x0} = \frac{2(20)}{\pi} = \underline{12.73} ,\quad C_{xk} = \frac{-2X_0}{\pi(4k^2-1)} = -\frac{40}{\pi(4k^2-1)}$

$$C_{y0} = (2)(12.73) = \underline{25.46}$$

$$C_{y1} = (1.84\angle{-22.7^\circ})(4.244\angle 180^\circ) = \underline{7.81\angle 157.3^\circ}$$

$$C_{y2} = (1.53\angle{-40^\circ})(0.849\angle 180^\circ) = \underline{1.30\angle 140^\circ}$$

$$C_{y3} = (1.245\angle{-51.5^\circ})(0.364\angle 180^\circ) = \underline{0.453\angle 128.5^\circ}$$

$$y(t) = \underline{25.46 + 15.62\cos(\tfrac{2}{3}\pi t + 157.3^\circ) + 2.60\cos(\tfrac{4}{3}\pi t + 140^\circ)}$$

$$\underline{+\,0.906\cos(2\pi t + 128.5^\circ) + \cdots}$$

(f) $C_{x0} = 20/\pi = 6.367 ;\ C_{x1} = -j\frac{X_0}{4} ,\ C_{x2} = -\frac{X_0}{3\pi} ,\ C_{x3} = 0$

$$C_{y0} = (2)(6.367) = \underline{12.73}$$

$$C_{y1} = (1.84\angle{-22.7^\circ})(5\angle{-90^\circ}) = \underline{9.20\angle{-112.7^\circ}}$$

$$C_{y2} = (1.53\angle{-40^\circ})(2.122\angle 180^\circ) = \underline{3.25\angle 140^\circ} ,\ C_{y3} = \underline{0}$$

$$y(t) = \underline{12.73 + 18.4\cos(\tfrac{2}{3}t - 112.7^\circ) + 3.25\cos(\tfrac{4}{3}t + 140^\circ) + \cdots}$$

(g) $C_{x0} = \frac{wX_0}{T_0} = \frac{(1)(20)}{3} = 6.67 ;\ C_k = \frac{wX_0}{T_0}\,\frac{\sin(\pi wk/T_0)}{\pi wk/T_0} = \frac{20}{\pi k}\sin k\pi/3$

$$C_{y0} = (2)(6.67) = \underline{13.33}$$

$$C_{y1} = (1.84\angle{-22.7^\circ})(5.51) = \underline{10.14\angle{-22.7^\circ}}$$

$$C_{y2} = (1.53\angle{-40^\circ})(2.757) = \underline{4.22\angle{-40^\circ}} ;\ C_{y3} = \underline{0}$$

4. 24. $y(t) = \underline{13.3 + 20.28\cos(\frac{2}{3}\pi t - 22.7^\circ) + 8.44\cos(\frac{4}{3}\pi t - 40^\circ)}$

(cont) (h) $C_k = \frac{20}{3} = 6.67$

$C_{y0} = (2)(6.67) = \underline{13.33}$

$C_{y1} = (1.84\angle -22.7^\circ)(6.67) = \underline{12.3\angle -22.7^\circ}$

$C_{y2} = (1.53\angle -40^\circ)(6.67) = \underline{10.2\angle -40^\circ}$

$C_{y3} = (1.245\angle -51.5^\circ)(6.67) = \underline{8.30\angle -51.5^\circ}$

$\therefore y(t) = \underline{13.33 + 24.6\cos(\frac{2}{3}\pi t - 22.7^\circ) + 20.4\cos(\frac{4}{3}\pi t - 40^\circ)}$

$\underline{+ 16.6\cos(2\pi t - 51.5^\circ)}$

4.25

$H(s) = \frac{20}{s+4}$

(a) $\omega_0 = 2\pi$; $H(j\omega_0) = \frac{20}{4+j2\pi} = 2.685\angle -57.5^\circ$

$\frac{2|C_{y1}|}{2|C_{x1}|} = |H(j\omega_0)| = \underline{2.685}$

(b) $H(j3\omega_0) = \frac{20}{4+j6\pi} = \underline{1.04\angle -78^\circ}$; $C_{xk} = -j\frac{2X_0}{\pi k}$; $\frac{C_{x1}}{C_{x3}} = \frac{3}{1} = \underline{3}$

(C)

```
% problem 4.23
t0=1;
w=[2*pi/t0   6*pi/t0];  n=[0 20];  d=[1  4];
h=freqs(n,d,w);
hmag=abs(h); hphase=angle(h)*180/pi;
[hmag' hphase']
pause
t0=.1;
w=[2*pi/t0   6*pi/t0];  n=[0 20];  d=[1  4];
h=freqs(n,d,w);
hmag=abs(h); hphase=angle(h)*180/pi;
[hmag' hphase']
pause
t0=10;
w=[2*pi/t0   6*pi/t0];  n=[0 20];  d=[1  4];
h=freqs(n,d,w);
hmag=abs(h); hphase=angle(h)*180/pi;
[hmag' hphase']
```

(d) $\omega_0 = 20\pi$; $H(j\omega_0) = \frac{20}{4+j20\pi} = 0.318\angle -86.4^\circ$

$\therefore \frac{2|C_{y1}|}{2|C_{x1}|} = |H(j\omega_0)| = \underline{0.318}$

(e) $H(j3\omega_0) = \frac{20}{4+j60\pi} = 0.106\angle -88.8^\circ$

From (b), $\frac{2|C_{y1}|}{2|C_{y3}|} = \left(\frac{0.318}{0.106}\right)3 = \underline{9}$

(f) $\omega_0 = 0.2\pi$; $H(j\omega_0) = \frac{20}{4+j0.2\pi} = 4.94\angle -8.9^\circ$

4.25 (cont) $\frac{2|C_{y_1}|}{2|C_{x_1}|} = |H(j\omega_0)| = \underline{4.94}$

(g) $H(j3\omega_0) = \frac{20}{4+j0.6\pi} = \frac{20}{4.42\angle 25.2^\circ} = 4.52\angle\underline{-25}.2^\circ$

From (b), $\frac{2|C_{y_1}|}{2|C_{y_3}|} = \left(\frac{4.94}{4.52}\right)3 = \underline{3.28}$

(h)

ω_0	0.2π	2π	20π
ratio	4.94	2.69	0.318

The system is lowpass with a dc gain of 5. At $\omega_0 = 0.2\pi$, almost no filtering occurs, while at $\omega_0 = 20\pi$, the filtering is pronounced.

(i)

ω_0	0.2π	2π	20π
ratio	3.28	7.76	9

The ratio of harmonics in the input is 3. Thus, there in little effect at $\omega_0 = 0.2\pi$, but large effect at $\omega_0 = 20\pi$, due to the low-pass filtering.

4.26 $H(s) = \frac{1}{RCs+1} = \frac{1}{0.5s+1} = \frac{2}{s+2}$

(a) $\omega_0 = 1$, $H(j\omega_0) = \frac{2}{2+j1} = \frac{2}{2.236\angle 26.6^\circ} = 0.8944\underline{\angle -26.6^\circ}$

$H(j3\omega_0) = \frac{2}{2+j3} = \frac{2}{3.606\angle 56.3^\circ} = 0.5547\underline{\angle -56.3^\circ}$

$H(j5\omega_0) = \frac{2}{2+j5} = \frac{2}{5.385\angle 68.2^\circ} = 0.3714\underline{\angle -68.2^\circ}$

$C_{kx} = -j\frac{20}{k\pi}$

$\therefore C_{1x} = -j\frac{20}{\pi}$; $C_{y_1} = (0.8944\underline{\angle -26.6^\circ})(6.3662\underline{\angle -90^\circ}) = 5.6939\underline{\angle -116.6^\circ}$

$C_{3x} = -j\frac{20}{3\pi}$; $C_{3y} = (0.5547\underline{\angle -56.3^\circ})(2.1221\underline{\angle -90^\circ}) = 1.1771\underline{\angle -146.3^\circ}$

$C_{x5} = -j\frac{20}{5\pi}$; $C_{5y} = (0.3714\underline{\angle -68.2^\circ})(1.2732\underline{\angle -90^\circ}) = 0.4729\underline{\angle -158.2^\circ}$

$\therefore y_a(t) = 11.38\cos(t - 116.6^\circ) + 2.35\cos(3t - 146.3^\circ) + 0.95\cos(5t - 158.^\circ) + \cdots$

(b)
```
w=[1 3 5]; n=[0 2]; d=[1 2];
h=freqs(n,d,w);
hmag=abs(h); hphase=angle(h)*180/pi;
[hmag' hphase']
```

(c) $H(0) = 1$ $\therefore$ $C_{y0} = H(0)C_{x0} = (1)(20) = 20$

$y_b(t) = \underline{20 + y_a(t)}$, $y_a(t)$ from (a)

(d) Yes, $|H(jk\omega_0)|$ decreases as k increases.

4.26

(e) $T_0 = \pi$, $\omega_0 = \frac{2\pi}{T_0} = 2$

(a) Since ω_0 is larger, the gain of the circuit is smaller. Hence the amplitude of the harmonics are smaller.

(c) The dc gain is unaffected. Hence the dc component in the output is unchanged.

4.27 $H(s) = \frac{Ls}{R+Ls} = \frac{s}{s+8}$, $\omega_0 = \frac{2\pi}{T_0} = 4$

$$\therefore H(j\omega_0) = \frac{j4}{8+j4} = .4472 \angle 63.43^\circ$$

$$H(j3\omega_0) = \frac{j12}{8+j12} = .8321 \angle 33.69^\circ$$

$$H(j5\omega_0) = \frac{j20}{8+j20} = .9285 \angle 21.80^\circ$$

a) $C_{kx} = \frac{-j2x_0}{\pi k}$

$C_{x_1} = -j6.366$, $C_{1y} = (.447\angle 63.43^\circ)(6.366\angle -90^\circ) = 2.847\angle -26.57^\circ$

$C_{3x} = -j2.122$, $C_{3y} = (.8321\angle 33.69^\circ)(2.122\angle -90^\circ) = 1.766\angle -56.31^\circ$

$C_{5x} = -j1.273$, $C_{5y} = (10.9285\angle 21.8^\circ)(1.273\angle -90^\circ) = 1.182\angle 68.2^\circ$

$$\therefore y_a(t) = 5.694\cos(4t + 26.57^\circ) + 3.5320\cos(12t + 56.31^\circ)$$
$$+ 2.364\cos(20t + 68.2^\circ) + \cdots$$

b)
```
w=[4 12 20] ; n=[1 0] , d=[1 8];
h=freqs(n,d,w);
hmag=abs(h); hphase=angle(h)*180/pi;
[hmag' hphase']
```

c) $H(0)=0 \therefore C_{0y}=0, \therefore y_c(t)=y_a(t)$, from a

d) NO, $H(jk\omega_0)$ increases as k increases, and approaches unity as $k \to \infty$. This circuit is high pass.

e) $\omega_{0a}=\dfrac{2\pi}{T_{0a}}=4 \quad \therefore k\omega_{0a}L=k$

$\omega_{0e}=\dfrac{2\pi}{T_{0e}}=1 \quad \therefore k\omega_{0e}L=k(1)(1/4)=k/4$

(a) ∴ amplitude of harmonics become smaller.

(c) ∴ The dc gain remains at zero.

4.28

$$y(t)=x(\tau)\Big|_{\tau=at+b}=x(at+b)$$

$$\therefore C_{kx}e^{jk\omega_0\tau}\Big|_{\tau=at+b}=C_{kx}e^{jk\omega_0(at+b)}=\left[C_{kx}e^{jk\omega_0 b}\right]e^{jk\omega_0 at}$$

4.28. (cont) $\therefore \omega_{0y} = \frac{2\pi}{T_{0y}} = |a|\,\omega_{0x} = |a| \frac{2\pi}{T_{0x}}$

$\therefore \underline{T_{0y} = \frac{T_{0x}}{|a|}}$ [a can be negative]

for a negative,

$\therefore C_{kx} e^{jk\omega_0 T} \Rightarrow [C_{kx} e^{jk\omega_0 b}] e^{-jk|a|\omega_0 t}$

Since $C_{-k} = C_k^*$

$$C_{ky} = [C_{kx} e^{jk\omega_0 b}]^*, \; a<0$$

$$\therefore C_{ky} = \begin{cases} C_{kx} e^{jk\omega_0 b}, & a>0 \\ [C_{kx} e^{jk\omega_0 b}]^*, & a<0 \end{cases}$$

4.29. (a) $C_k = \frac{-2X_0}{\pi(4k^2-1)} = C_{-k}$

$\therefore [C_k e^{jk\omega_0 t} + C_{-k} e^{-jk\omega_0 t}]_{t=-t} = C_{-k} e^{-jk\omega_0 t} + C_k e^{jk\omega_0 t}$

$\therefore$ no change

(b) For $y(t) = x(t - \frac{T_0}{2})$: $C_{kx} e^{jk\omega_0 t} \Big|_{t \Leftarrow t - T_0/2}$

$= C_{kx} e^{-jk\omega_0 (t - T_0/2)} = [C_{kx} e^{jk\omega_0 t}] e^{jk\omega_0 t}$

$k \frac{\omega_0 T_0}{2} = \frac{k}{2}(\frac{2\pi}{T_0}) T_0 = k\pi$, $\therefore C_{ky} = C_{kx} \underline{\angle -k\pi}$

4.30 $h(t) = e^{-\alpha t} u(t)$

a) $\alpha > 0$

b) $x(t) = \sin(w_0 t) + \cos(3w_0 t) =$

$$\frac{1}{2j}\left(e^{jw_0 t} - e^{-jw_0 t}\right) + \frac{1}{2}\left(e^{j3w_0 t} + e^{-j3w_0 t}\right)$$

$$H(S_k) = \int_{-\infty}^{\infty} h(\tau) e^{-S_k \tau} d\tau = \int_{-\infty}^{\infty} e^{-\alpha\tau} u(\tau) e^{-S_k \tau} d\tau$$

$$= \int_{0}^{\infty} e^{-(\alpha + S_k)\tau} d\tau = \frac{1}{\alpha + SK}$$

$$\phi_k(t) = e^{jKw_0 t} \quad \left(\psi_k(t) = \phi_k(t) * h(t)\right)$$

$$y(t) = \sum_k a_k H(S_k) e^{S_k t}$$

$$x(t) = \frac{1}{2j} e^{jw_0 t} - \frac{1}{2j} e^{-jw_0 t} + \frac{1}{2} e^{j3w_0 t} + \frac{1}{2} e^{-j3w_0 t}$$

$K=1$ $\quad K=-1$ $\quad K=3$ $\quad K=-3$

$$\therefore y(t) = \frac{1}{2j} \frac{1}{\alpha + jw_0} e^{jw_0 t} - \frac{1}{2j} \frac{1}{\alpha - jw_0} e^{-jw_0 t}$$

$$+ \frac{1}{2} e^{j3w_0 t} \frac{1}{\alpha + 3jw_0} + \frac{1}{2} e^{-j3w_0 t} \frac{1}{\alpha - 3jw_0}$$

4.31 $h(t) = \alpha e^{-\alpha t} u(t) \ , \ \alpha > 0$

a) $x(t) = \sin^2 2t = \frac{1}{2}(1 - \cos(4\omega_0 t))$

$= \frac{1}{2}\left(1 - \frac{1}{2}\left(e^{j4\omega_0 t} + e^{-j4\omega_0 t}\right)\right)$

$$H(s_k) = \int_{-\infty}^{\infty} h(\tau) e^{-s_k \tau} d\tau = \int_{-\infty}^{\infty} \alpha e^{-\alpha\tau} u(\tau) e^{-s_k \tau} d\tau$$

$$= \int_{0}^{\infty} \alpha e^{-(\alpha + s_k)\tau} d\tau = \frac{\alpha}{\alpha + s_k}$$

$$y(t) = \sum_k a_k H(s_k) e^{s_k t}$$

$$x(t) = \frac{1}{2} - \frac{1}{4} e^{j4\omega_0 t} - \frac{1}{4} e^{-j4\omega_0 t}$$

$k=0$ $\quad k=1$ $\quad k=-1$

$$\therefore y(t) = \frac{1}{2} - \frac{1}{4} \frac{\alpha}{\alpha + j\ \omega_0} e^{j4t} - \frac{1}{4} \frac{\alpha}{\alpha - j\ \omega_0} e^{-j4t}$$

b) $x(t) = 1 + \cos t + \cos 8t$

$$= 1 + \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) + \frac{1}{2}\left(e^{j8\omega_0 t} + e^{-j8\omega_0 t}\right)$$

$$y(t) = 1 + \frac{1}{2} \frac{\alpha}{\alpha + j\omega_0} e^{jt} + \frac{1}{2} \frac{\alpha}{\alpha - j\omega_0} e^{-jt} +$$

$$\frac{1}{2} \frac{\alpha}{\alpha + j8\omega_0} e^{j8t} + \frac{1}{2} \frac{\alpha}{\alpha - j8\omega_0} e^{-j8t}$$

4.32

$$x(t) = \sum_{k=1}^{\infty} \cos(kt) = 1/2 \sum_{k=-\infty}^{\infty} e^{jkt} - 1/2$$

$$H(jk) = \int_{0}^{\infty} e^{-at} e^{-jkt} dt = \frac{1}{a+jk}$$

$$y(t) = \sum_{k=-\infty}^{\infty} c_k H(jk) e^{jkt}$$

$$= \sum_{k=-\infty}^{\infty} 1/2 \frac{1}{a+jk} e^{jkt} - \frac{1}{(2a)}$$

CHAPTER 5

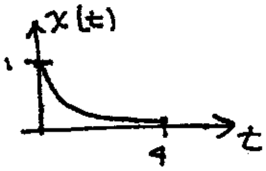
5.1(a) $x(t) = 2[u(t)-u(t-4)]$, $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

$$X(\omega) = 2\int_{-\infty}^{\infty}[u(t)-u(t-4)]e^{-j\omega t}dt = 2\int_{0}^{4}e^{-j\omega t}dt$$

$$= \frac{-2}{j\omega}e^{-j\omega t}\Big|_0^4 = \frac{2(1-e^{-j4\omega})}{j\omega} = \frac{4(e^{j2\omega}-e^{-j2\omega})e^{-j2\omega}}{(j2)(\omega)}$$

$$X(\omega) = \frac{4e^{-j\omega 2}}{\omega}\sin(2\omega) = 8e^{-j2\omega}\text{sinc}(2\omega)$$

(b) $x(t) = e^{-3t}[u(t)-u(t-4)]$

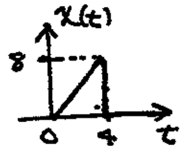


$$X(\omega) = \int_0^4 e^{-3t}e^{-j\omega t}dt = \int_0^4 e^{-(3+j\omega)t}dt$$

$$X(\omega) = \frac{-1}{3+j\omega}e^{-(3+j\omega)t}\Big|_0^4 = \frac{1}{3+j\omega}\left(1-e^{-12}e^{-j4\omega}\right)$$

$$X(\omega) = \frac{1-6.144\times10^{-12}e^{-j4\omega}}{3+j\omega}$$

(c) $x(t) = 2t[u(t)-u(t-4)]$

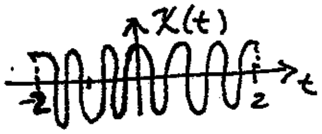


$$X(\omega) = 2\int_0^4 te^{-j\omega t}dt$$

$$\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1) \Rightarrow X(\omega) = \frac{2e^{-j\omega t}}{-\omega^2}(-j\omega t-1)\Big|_0^4$$

$$\therefore X(\omega) = \frac{2(1+j4\omega)e^{-j4\omega}-2}{\omega^2} = \frac{(2-j4\sin4\omega)e^{-j4\omega}}{\omega^2}$$

(d) $x(t) = \cos(4\pi t)[u(t+2)-u(t-2)]$



$$X(\omega) = \int_{-2}^{2}\cos(4\pi t)e^{-j\omega t}dt$$

$$= \int_{-2}^{2}\left(\frac{e^{j4\pi t}+e^{-j4\pi t}}{2}\right)e^{-j\omega t}dt = \int_{-2}^{2}\frac{e^{-j(\omega-4\pi)t}+e^{-j(\omega+4\pi)t}}{2}dt$$

$$X(\omega) = \frac{-e^{-j2(\omega-4\pi)}+e^{j2(\omega-4\pi)}}{2j(\omega-4\pi)} + \frac{e^{j2(\omega+4\pi)}-e^{-j2(\omega+4\pi)}}{2j(\omega+4\pi)}$$

$$X(\omega) = 2\left[\text{sinc}(2\omega-8\pi)+\text{sinc}(2\omega+8\pi)\right]$$

5.2 a) $f(t)=(1-e^{-bt})u(t)$; $F(\omega)=\int_0^\infty (1-e^{-bt})e^{-j\omega t}\,dt$

$$F(\omega)=\int_0^\infty e^{-j\omega t}\,dt-\int_0^\infty e^{-(b+j\omega)t}\,dt=\frac{-e^{-j\omega t}}{j\omega}\Big|_0^\infty+\frac{1}{b+j\omega}e^{-j\omega}\Big|_0^\infty$$

$$F(\omega)=\frac{1}{j\omega}-\frac{1}{b+j\omega}=\frac{-b\omega^2+j\omega b^2}{\omega^4+\omega^2 b}$$

b) $f(t)=A\cos(\omega_0 t+\phi)=\frac{A}{2}e^{j\omega_0 t}e^{j\phi}+\frac{A}{2}e^{-j\omega_0 t}e^{-j\phi}$

$$F(\omega)=\frac{Ae^{j\phi}}{2}\int_{\infty}^{\infty}e^{-j(\omega-\omega_0)t}\,dt+\frac{Ae^{-j\phi}}{2}\int_{-\infty}^{\infty}e^{-j(\omega+\omega_0)t}\,dt$$

aside : $\frac{1}{2\pi}\int_{-\infty}^{\infty}\delta(\omega-\omega_0)e^{j\omega t}\,d\omega=\frac{1}{2\pi}e^{j\omega_0 t}$

$\Longrightarrow \mathcal{F}\{e^{j\omega_0 t}\}=2\pi\delta(\omega-\omega_0)$

Similarly, $\mathcal{F}\{e^{-j\omega_0 t}\}=2\pi\delta(\omega+\omega_0)$

c) $F(\omega)=\int_{-\infty}^{\infty}e^{at}u(-t)e^{-j\omega t}\,dt=\int_{-\infty}e^{(a-j\omega)t}\,dt$

$$\frac{1}{a-j\omega}e^{(a-j\omega)t}\Big|_{-\infty}^{0}=\frac{1}{a-j\omega}\qquad\left(|F(\omega)|^2=\frac{1}{a^2+\omega^2}\right)$$

d) $F(\omega)=\int_{-\infty}^{\infty}C\delta(t+t_0)e^{-j\omega t}\,dt=Ce^{-j\omega(-t_0)}=Ce^{j\omega t_0}$

b) Final answer:

$$F(\omega)=A\pi e^{j\phi}\delta(\omega-\omega_0)+A\pi e^{-j\phi}\delta(\omega+\omega_0)$$

5.3 a) $x(t) = 2[u(t) - u(t-4)] = 2\,\text{rect}\left(\frac{t-2}{4}\right)$

$\text{rect}\left(t/\tau\right) \xleftrightarrow{\mathcal{F}} \tau\, \text{sinc}\left(\frac{\omega\tau}{2}\right)$

$f(t-\tau) \xleftrightarrow{\mathcal{F}} F(\omega) e^{-j\omega\tau}$

$\therefore\ X(\omega) = 8\, \text{sinc}(2\omega) e^{-j2\omega}$

b) $x(t) = e^{-3t}[u(t) - u(t-4)]$

$x(t) = e^{-3t} u(t) - e^{-12} e^{-3(t-4)} u(t-4)$

$e^{-at} u(t) \xleftrightarrow{\mathcal{F}} \frac{1}{a + j\omega}$

$\therefore\ X(\omega) = \frac{1}{3 + j\omega} - \frac{e^{-12}}{3 + j\omega} e^{-j4\omega} = \frac{1 - e^{-4(3+j\omega)}}{3 + j\omega}$

c) $x(t) = 2t[u(t) - u(t-4)] = t\left(2\,\text{rect}\left(\frac{t-2}{4}\right)\right)$

$-jtf(t) \xleftrightarrow{\mathcal{F}} \frac{dF(\omega)}{d\omega} \Rightarrow tf(t) \longleftrightarrow j\frac{dF(\omega)}{d\omega}$

let $f(t) = 2\,\text{rect}\left(\frac{t-2}{4}\right)$, then $F(\omega) = 8\, \text{sinc}(2\omega) e^{-j2\omega}$

then $X(\omega) = j\frac{d}{d\omega}\left(8\, \text{sinc}(2\omega) e^{-j2\omega}\right) = j\frac{d}{d\omega}\left(\frac{8 \sin(2\omega) e^{-j2\omega}}{2\omega}\right)$

$X(\omega) = \frac{2(1 + j4\omega) e^{-j4\omega} - 2}{\omega^2} = \frac{[2 - j4 \sin(4\omega)] e^{-j4\omega}}{\omega^2}$

d) $x(t) = \cos(4\pi t)[u(t+2) - u(t-2)] = \cos(4\pi t)\,\mathrm{rect}(t/4)$

$$f_1(t)f_2(t) \xleftrightarrow{\mathcal{F}} \frac{1}{2\pi}F_1(\omega)F_2(\omega)$$

$$\cos(\omega_0 t) \xleftrightarrow{\mathcal{F}} \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$$

$$\mathrm{rect}(t/\tau) \xleftrightarrow{\mathcal{F}} \tau\,\mathrm{sinc}(\omega\tau/2)$$

$$\therefore X(\omega) = \frac{1}{2\pi}\pi[\delta(\omega-4\pi)+\delta(\omega+4\pi)] * 4\,\mathrm{Sinc}(2\omega)$$

$$X(\omega) = 2[\mathrm{sinc}(2\omega-8\pi)+\mathrm{Sinc}(2\omega+8\pi)]$$

5.4

a) $\mathcal{F}[af_1(t)+bf_2(t)] = \int_{-\infty}^{\infty}[af_1(t)+bf_2(t)]e^{-j\omega t}\,dt =$

$a\int_{-\infty}^{\infty} f_1(t)e^{-j\omega t} + b\int_{-\infty}^{\infty} f_2(t)e^{-j\omega t}\,dt = aF_1(\omega)+bF_2(\omega)$

b) time shift

$\int_{-\infty}^{\infty} f(t-t_0)e^{-j\omega t}\,dt$ let $u = t-t_0$

$= \int_{-\infty}^{\infty} f(u)e^{-j\omega(v+t_0)}\,du = e^{-j\omega t_0}\int_{-\infty}^{\infty} f(u)e^{-j\omega u}\,du$

$= F(\omega)e^{-j\omega t_0}$

c) Duality

$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{+j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}F(a)e^{jat}\,da$

$f(-\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}F(a)e^{-ja\omega}\,da\,, \quad 2\pi f(-\omega) = \int_{-\infty}^{\infty}F(a)e^{-ja\omega}$

d) Frequency shifting

$$\int_{-\infty}^{\infty} f(t) e^{j\omega_0 t} e^{-j\omega t} dt = \int_{-\infty}^{\infty} f(t) e^{-j(\omega-\omega_0)t} dt = F(\omega-\omega_0)$$

e) Time Differentiation

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

$$\frac{d}{dt} f(t) = \frac{d}{dt}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) j\omega e^{j\omega t} d\omega$$

$$\therefore \quad \frac{d}{dt} f(t) \longleftrightarrow j\omega F(\omega)$$

f) time Convolution

$$\int_{-\infty}^{\infty} x(t) * h(t) e^{-j\omega t} dt = \underset{dt}{\int_{-\infty}^{\infty}} \underset{d\tau}{\int_{-\infty}^{\infty}} x(\tau) h(t-\tau) d\tau e^{-j\omega t} dt$$

$$= \underset{dt}{\int_{-\infty}^{\infty}} x(\tau) \underset{d\tau}{\int_{-\infty}^{\infty}} h(t-\tau) e^{-j\omega t} dt \, d\tau \qquad \text{let } u = t-\tau$$

$$= \underset{d\tau}{\int_{-\infty}^{\infty}} x(\tau) \underset{du}{\int_{-\infty}^{\infty}} h(u) e^{-j\omega(u+\tau)} du \, d\tau =$$

$$\int_{-\infty}^{\infty} x(\tau) e^{-j\omega\tau} d\tau \int_{-\infty}^{\infty} h(u) e^{-j\omega u} du$$

$$= X(\omega) H(\omega)$$

g) prove the time scale property

$$\mathcal{F}[x(at)] = \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

$$\mathcal{F}[x(at)] = \int_{-\infty}^{\infty} x(at) e^{-j\omega t}\, dt \quad \text{let } u = at$$

$$du = a\, dt$$

$$= \int_{-\infty}^{\infty} x(u) e^{-j\omega \frac{u}{a}} \frac{du}{a}, \text{ if } a > 0$$

$$= \frac{1}{a} X(\omega/a)$$

if $a < 0$, then

$$= \int_{+\infty}^{-\infty} x(u) e^{-j\omega \frac{u}{a}} \frac{du}{a} = -\int_{-\infty}^{\infty} x(u) e^{\frac{-j\omega u}{a}} \frac{du}{a}$$

$$= \frac{-1}{a} X(\omega/a)$$

$$\therefore \mathcal{F}[x(at)] = \frac{1}{|a|} X(\omega/a)$$

5.5 $\mathcal{F}[\sin \omega_0 t] = \frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$

ⓐ Differentiation Property

$$\frac{d}{dt} f(t) \longleftrightarrow j\omega F(\omega)$$

$$d/dt\, [\sin \omega_0 t] = \omega_0 \cos \omega_0 t$$

$$\omega_0 \cos \omega_0 t \longleftrightarrow \omega_0 \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

Show this is equal to $j\omega \mathcal{F}[\sin \omega_0 t] = \frac{j\omega\pi}{j}(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$

$= \pi\omega[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$

$= \pi\omega_0(\delta(\omega-\omega_0) + \delta(\omega+\omega_0))$, by shifting property

ⓑ time shift property

$\sin\omega_0 t = \cos(\omega_0 t - \pi/2) = \cos\omega_0(t - \pi/2\omega_0)$

$f(t-t_0) \longleftrightarrow F(\omega)e^{-j\omega t_0}$

$\cos\omega_0(t - \frac{\pi}{2\omega_0}) \longleftrightarrow \pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]e^{\frac{-j\omega\pi}{2\omega_0}}$

$= \pi\delta(\omega+\omega_0)e^{\frac{j\omega_0\pi}{2\omega_0}} + \pi\delta(\omega-\omega_0)e^{\frac{-j\omega_0\pi}{2\omega_0}}$

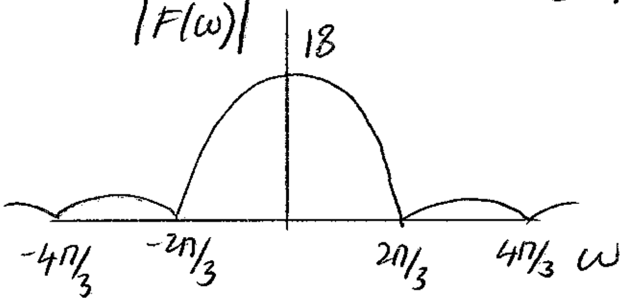
$= \pi\delta(\omega+\omega_0)e^{j\pi/2} + \pi\delta(\omega-\omega_0)e^{-j\pi/2}$

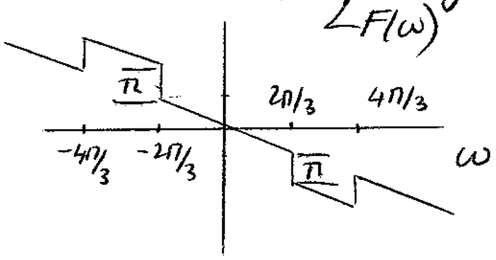
$= \pi j\delta(\omega+\omega_0) - j\pi\delta(\omega-\omega_0) = \frac{\pi}{j}[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$

5.6 parts a, b, c next page

d) $f(t) = 6\,\text{rect}\left[\frac{(t-4)}{3}\right] \xleftrightarrow{\mathcal{F}} 18\,\text{sinc}\left(\frac{3\omega}{2}\right)e^{-j4\omega}$

use the time shift property





5.6 (a) $f(t) = Ae^{-\beta t}\cos(\omega_0 t)u(t) = f_1(t)\,f_2(t)$

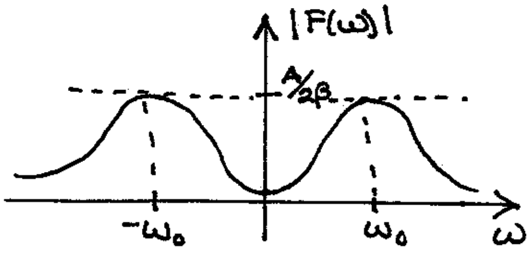
$f_1(t) = Ae^{-\beta t}u(t)$, $f_2(t) = \cos\omega_0 t$

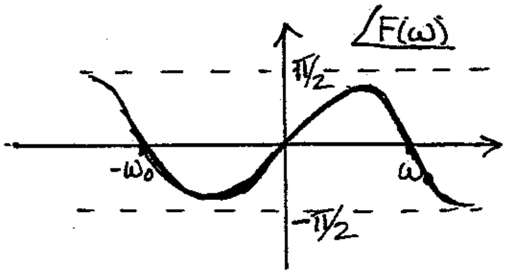
$F_1(\omega) = \frac{A}{\beta+j\omega}$, $F_2(\omega) = \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$

use frequency convolution

$F(\omega) = \frac{1}{2\pi}F_1(\omega)*F_2(\omega) = \frac{1}{2}\left(\frac{A}{\beta+j\omega}\right)*[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$

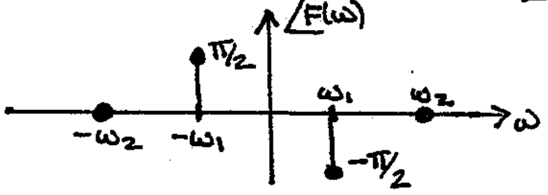
$= \frac{A/2}{\beta+j(\omega-\omega_0)} + \frac{A/2}{\beta-j(\omega+\omega_0)}$





(b) $f(t) = A\sin(\omega_1 t) + B\cos(\omega_2 t)$ ⇒ use the linearity Property: $F(\omega) = A\,\mathcal{F}\{\sin(\omega_1 t)\} + B\,\mathcal{F}\{\cos(\omega_2 t)\}$

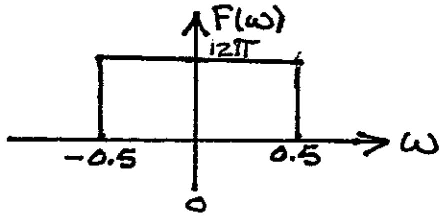
$F(\omega) = \frac{A\pi}{j}[\delta(\omega-\omega_1)-\delta(\omega+\omega_1)] + B\pi[\delta(\omega-\omega_2)+\delta(\omega+\omega_2)]$



(c) $f(t) = 6\,\text{sinc}(0.5t)$, from Table 5.2 $\frac{\beta}{\pi}\,\text{sinc}(\beta t) \overset{\mathcal{F}}{\Leftrightarrow} \text{rect}\left(\frac{\omega}{2\beta}\right)$

$\beta = 0.5 \quad \therefore 6 = 12\beta$

$F(\omega) = 12\pi\,\text{rect}(\omega)$



5.7(a) $g_4(t) = \text{rect}(t/0.1) + \text{rect}(t/0.2)$

$G_4(\omega) = 0.1\ \text{sinc}(0.05\omega) + 0.2\ \text{sinc}(0.1\omega)$

(b) $g_5(t) = 2.5\ \text{rect}(t/0.1) - 0.5\ \text{rect}(t/0.2)$

$G_5(\omega) = 0.25\ \text{sinc}(0.05\omega) - 0.1\ \text{sinc}(0.1\omega)$

(c) $g_6(t) = 5 g_4(10t) \Rightarrow G_6(\omega) = \frac{5}{10} G_4(\omega/10)$

$\therefore G_6(\omega) = 0.05\ \text{sinc}(0.005\omega) + 0.1\ \text{sinc}(0.01\omega)$

(d) $g_7(t) = 10 g_5(20(t-0.1))$

use time-transformation (5. 14)

$G_7(\omega) = \frac{10}{20} G_5(\frac{\omega}{20}) e^{-j(0.1)(\frac{\omega}{20})} = \frac{1}{2} G_5(\frac{\omega}{20}) e^{-j0.005\omega}$

$G_7(\omega) = [0.125\ \text{sinc}(0.0025\omega) - 0.05\ \text{sinc}(0.0005\omega)] e^{-j0.005\omega}$

5.8 (a) use the derivate property

$\frac{d}{dt}(e^{-|t|}) \overset{\mathcal{F}}{\longleftrightarrow} j\omega\left(\frac{2}{\omega^2+1}\right) = \frac{j2\omega}{\omega^2+1}$

(b) $\frac{1}{2\pi(t^2+1)}$, from Table 5.1 $F(t) \overset{\mathcal{F}}{\longleftrightarrow} 2\pi f(-\omega)$

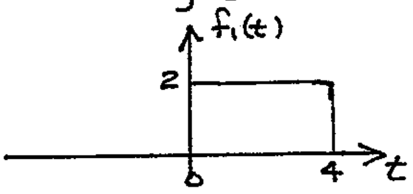
$\frac{1}{4\pi}\left(\frac{2}{t^2+1}\right) \overset{\mathcal{F}}{\longleftrightarrow} \left(\frac{1}{4\pi}\right) 2\pi e^{-|-\omega|} = \frac{1}{2} e^{-|\omega|}$

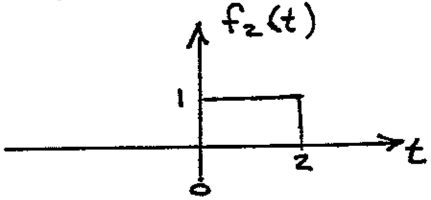
(c) $\frac{4\cos(2t)}{t^2+1} = \frac{2[e^{j2t} + e^{-j2t}]}{t^2+1} = \frac{2e^{j2t}}{t^2+1} + \frac{2e^{-j2t}}{t^2+1}$

use frequency-shift and duality properties

$F(\omega) = 2\pi\left[e^{-|\omega-2|} + e^{-|\omega+2|}\right]$

5.9 (a) f(t) can be recognized to be the result of convolving two rectangular pulses.





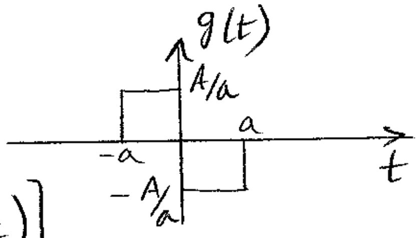
5.9 (cont) $f(t) = f_1(t) * f_2(t) \xrightarrow{\mathcal{F}} F_1(\omega)F_2(\omega)$

$$F_1(\omega) = \mathcal{F}\left\{2\,\text{rect}\left(\frac{t-2}{4}\right)\right\} = 8\,\text{sinc}(2\omega)e^{-j2\omega}$$

$$F_2(\omega) = \mathcal{F}\left\{\text{rect}\left(\frac{t-1}{2}\right)\right\} = 2\,\text{sinc}(\omega)e^{-j\omega}$$

$$F(\omega) = F_1(\omega)F_2(\omega) = 16\,\text{sinc}(2\omega)\text{sinc}(\omega)\,e^{-j3\omega}$$

5.10 let $g(t) = \frac{df(t)}{dt}$



$$g(t) = \frac{A}{a}\left[\text{rect}\left(\frac{t+a/2}{a}\right) - \text{rect}\left(\frac{t-a/2}{a}\right)\right]$$

Use the linearity property and the time shift

$$G(\omega) = A\,\text{sinc}(a\omega/2)\left[e^{ja\omega/2} - e^{-ja\omega/2}\right]$$

To find $F(\omega)$ use time integration property

$$F(\omega) = \frac{1}{j\omega}G(\omega) + \pi G(0)\delta(\omega)$$

$$G(0) = 0$$

$$\therefore F(\omega) = aA\,\text{sinc}\left(\frac{a\omega}{2}\right)\left[\frac{e^{\frac{j\omega a}{2}} - e^{-\frac{j\omega a}{2}}}{2j\left(\frac{a\omega}{2}\right)}\right]$$

$$F(\omega) = aA\,\text{sinc}^2\left(\frac{a\omega}{2}\right)$$

5.11 $x(t) = \cos(t) + \sin(3t)$

$$h(t) = \frac{.5\sin(2t)}{t} = \text{sinc}(2t) \longleftrightarrow \pi/2\,\text{rect}(\omega/4)$$

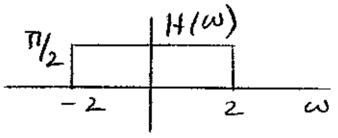
$$X(\omega) = \pi[\delta(\omega-1) + \delta(\omega+1)] + \pi/j\,[\delta(\omega-3) - \delta(\omega+3)]$$

$$Y(\omega) = H(\omega)X(\omega)$$

$$Y(\omega) = \pi^2/2\,[\delta(\omega-1) + \delta(\omega+1)]$$

impulses ∓3 will not pass the filter

$$y(t) = \pi/2\cos(t)$$

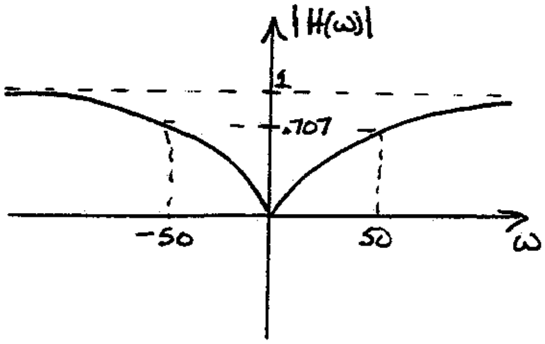


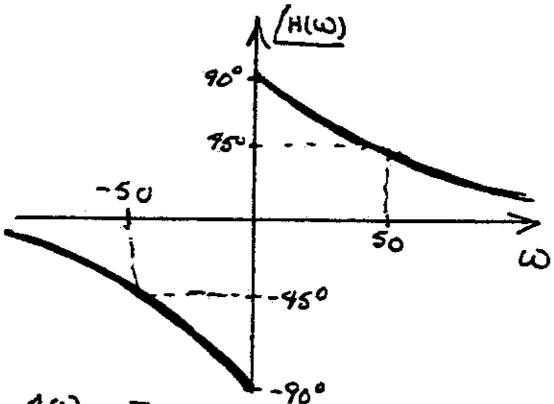
5.12 (a) $v_1(t) = 10i(t) + 0.2\frac{di(t)}{dt}$, $v_2(t) = 0.2\frac{di(t)}{dt}$

$V_1(\omega) = 10I(\omega) + 0.2j\omega I(\omega)$, $V_2(\omega) = 0.2j\omega I(\omega)$

$V_1(\omega) \rightarrow \boxed{H(\omega)} \rightarrow V_2(\omega)$ $\quad H(\omega) = \frac{V_2(\omega)}{V_1(\omega)} = \frac{0.2j\omega}{10+0.2j\omega} = \frac{j\omega}{50+j\omega}$

(b) $H(\omega) = \frac{1}{\sqrt{1+(\frac{50}{\omega})^2}} \underline{/90^\circ sgn(\omega) - \tan^{-1}(\frac{\omega}{50})}$





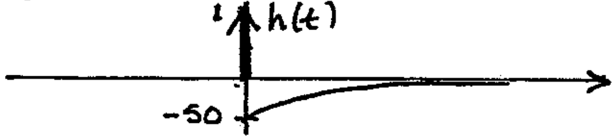
(c) $h(t) = \mathcal{F}^{-1}\{H(\omega)\} = \mathcal{F}^{-1}\{\frac{j\omega}{50+j\omega}\}$, let $F(\omega) = \frac{1}{50+j\omega}$

then $H(\omega) = j\omega F(\omega)$. From the differentiation Property,

$h(t) = \frac{df(t)}{dt}$.

$f(t) = \mathcal{F}^{-1}\{\frac{1}{50+j\omega}\} = e^{-50t}u(t) \therefore h(t) = \frac{d}{dt}\left(e^{-50t}u(t)\right)$

$h(t) = e^{-50t}\delta(t) - 50e^{-50t}u(t) = \delta(t) - 50e^{-50t}u(t)$



5.13 $F(\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

$\mathcal{F}\{f(at\} = \int_{-\infty}^{\infty} f(at) e^{-j\omega t} dt$, let $\tau = at$

$d\tau = a\,dt$

$\mathcal{F}\{f(at)\} = \mathcal{F}\{f(\tau)\} = \int_{-\infty}^{\infty} f(\tau) e^{-j\omega\tau/a} \frac{1}{a} d\tau$

$= \frac{1}{a} \int_{-\infty}^{\infty} f(\tau) e^{-j\omega/a\,\tau} d\tau$

$\mathcal{F}\{f(at)\} = \frac{1}{a} F(\omega/a)$, $a > 0$

5.14 a) $g_1(t) = 4\cos(100\pi t) rect(t/10^{-2}) = 2[e^{j100\pi t} + e^{-j100\pi t}] \times rect(t/10^{-2})$

$g_1(t) = 2e^{j100\pi t} rect(t/10^{-2}) + 2e^{-j100\pi t} rect(t/10^{-2})$

Use the frequency-shift property & linearity

$G_1(\omega) = 2\times10^{-2}[sinc(5\times10^{-3}(\omega+100\pi)) + sinc(5\times10^{-3}(\omega-100\pi))]$

b) $g_2(t) = -1\, g_1(t - 5\times10^{-3})$, use the time-shift property

$G_2(\omega) = -.02 e^{-j0.005\omega}[sinc(5\times10^{-3}(\omega+100\pi)) +$
$sinc(5\times10^{-3}(\omega-100\pi))]$

c) $g_3(t) = g_1(10t + 5\times10^{-4})$, use the time transform

$G_3(\omega) = \frac{1}{10} G_1(\omega/10) e^{j5\times10^{-5}\omega}$

$G_3(\omega) = 2\times10^{-3}[sinc(5\times10^{-4}(\omega+100\pi)) +$
$sinc(5\times10^{-4}(\omega-100\pi))] e^{j5\times10^{-5}\omega}$

5.15

a) $G(\omega) = 5\,\mathrm{rect}(\omega/20)$

$\beta/\pi\,\mathrm{sinc}(\beta t) \xleftrightarrow{F} \mathrm{rect}(\omega/2\beta)$, $\beta=10$

$g(t) = \frac{50}{\pi}\,\mathrm{sinc}(10t)$

b) $G(\omega) = 5\cos(\frac{\pi\omega}{20})\mathrm{rect}(\omega/20)$

$= 2.5\left[e^{\frac{j\omega\pi}{20}} + e^{\frac{-j\omega\pi}{20}}\right]\mathrm{rect}(\omega/20)$

$= 2.5\,\mathrm{rect}(\omega/20)e^{j\pi\omega/20} + 2.5\,\mathrm{rect}(\omega/20)e^{-j\omega\pi/20}$

Use the time shift & Linearity properties on the result of (a)

$g(t) = \frac{25}{\pi}[\mathrm{sinc}(10t+5\pi) + \mathrm{sinc}(10t-5\pi))]$

5.16 $G(\omega) = \frac{j\omega}{-\omega^2+7j\omega+6}$

a) $g(4t) \longleftrightarrow \frac{1}{4}G(\omega/4) = \frac{\frac{1}{4}\,j\omega/4}{-(\omega/4)^2+7j(\omega/4)+6} = \frac{j\omega}{-\omega^2+28j\omega+96}$

b) $g(6t-12) \longleftrightarrow 1/6\,G(\omega/6)e^{-\frac{j12\omega}{6}}$

$= 1/6\,G(\omega/6)e^{-2j\omega}$

c) $$\frac{dg(t)}{dt} \longleftrightarrow j\omega G(\omega) = \frac{-\omega^2}{-\omega^2+7j\omega+6}$$

d) $$g(-t) \longleftrightarrow G(-\omega) = \frac{-j\omega}{-\omega^2-7j\omega+6} = \frac{j\omega}{\omega^2+7j\omega-6}$$

e) $$e^{-j200t} g(t) \longleftrightarrow G(\omega-200) = \frac{j(\omega-200)}{-(\omega-200)^2+7j(\omega-200)+6}$$

f) $$\int_{-\infty}^{t} g(\tau)d\tau \Longleftrightarrow \frac{1}{j\omega}G(\omega) + \pi G(0)\delta(\omega)$$

$$= \frac{1}{-\omega^2+7j\omega+6} + \pi \cdot 0 = \frac{1}{-\omega^2+7j\omega+6}$$

5.17 (a) $f_1(t) = \sum_{n=-\infty}^{\infty} g_1(t - n\,8\times10^{-3})$, $T_0 = 8\times10^{-3}$, $\omega_0 = 250\pi$

$$g_1(t) = 8\cos\left(\frac{2\pi t}{8\times10^{-3}}\right)\mathrm{rect}\left(t/_{4\times10^{-3}}\right)$$

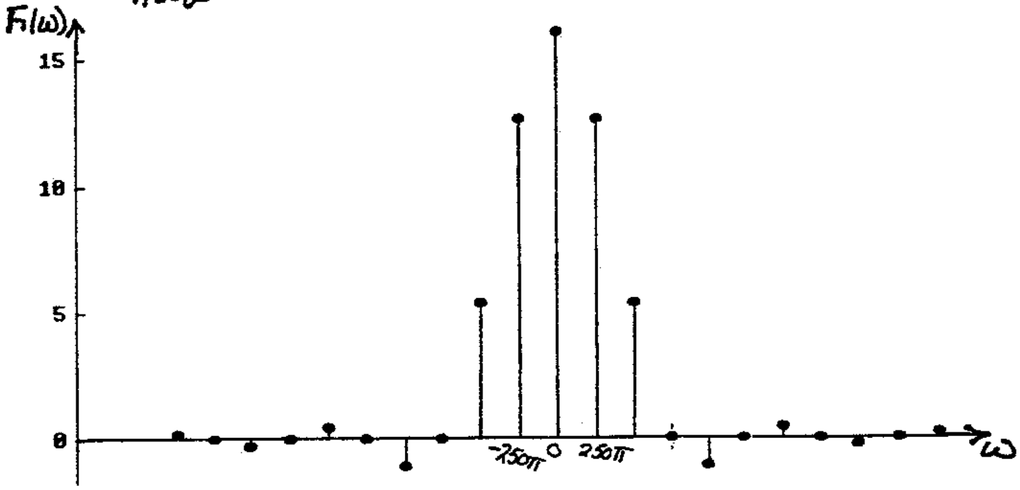
$$= 8\cos(250\pi t)\,\mathrm{rect}\left(\frac{t}{4\times10^{-3}}\right)$$

$$= 4\,\mathrm{rect}\left(\frac{t}{4\times10^{-3}}\right)e^{j250\pi t} + 4\,\mathrm{rect}\left(\frac{t}{4\times10^{-3}}\right)e^{-j250\pi t}$$

$$G_1(\omega) = 16\times10^{-3}\left[\mathrm{sinc}(2\times10^{-3}(\omega-250\pi) + \mathrm{sinc}(2\times10^{-3}(\omega+250\pi)\right]$$

$$F_1(\omega) = \sum_{n=-\infty}^{\infty} \omega_0 G_1(n\omega_0)\,\delta(\omega - n\omega_0)$$

$$F_1(\omega) = \sum_{n=-\infty}^{\infty} 4\pi\left[\mathrm{sinc}\left(\frac{(n-1)\pi}{2}\right) + \mathrm{sinc}\left(\frac{(n+1)\pi}{2}\right)\right]\delta(\omega - n\,250\pi)$$

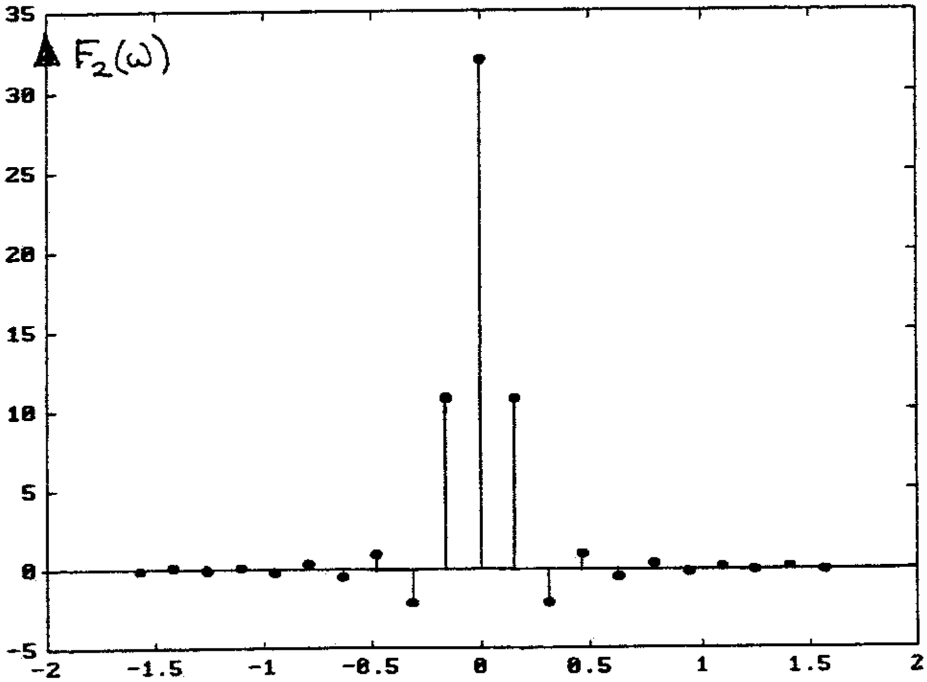


5.17(b) $f_2(t)=\sum_{n=-\infty}^{\infty} g_2(t-n\,4\times10^{-3})$, $T_0=4\times10^{-3}$, $\omega_0=500\pi$

$g_2(t)=8\cos(250\pi t)\,rect\left(\frac{t}{4\times10^{-3}}\right)=g_1(t)$ from (a)

$$F_2(\omega)=\sum_{n=-\infty}^{\infty} 500\pi\,G_1(n500\pi)\,\delta(\omega-n500\pi)$$

$$F_2(\omega)=\sum_{n=-\infty}^{\infty} 8\pi\left[sinc[(2n-1)\pi/2]+sinc[(2n+1)\pi/2]\right]\delta(\omega-n500\pi)$$



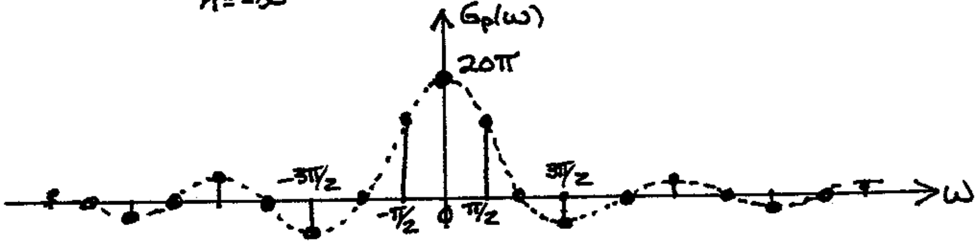
(c) The zero-frequency component of $F_2(\omega)$, $F_2(0)=2F_1(0)$. The impulses in frequency are more widely separated.

(d) If the period is halved, the frequency ω_0 is doubled. Therefore, the separation between frequency components in both frequency spectra will be doubled.

5.18 $g_p(t)=\sum_{n=-\infty}^{\infty} g(t-nT_0)$, $T_0=4s$, $\omega_0=\pi/2$ (rad/s)

$g(t)=20\,rect(t/2)\xleftrightarrow{\mathcal{F}}40\,sinc(\omega)=G(\omega)$

$$G_p(\omega)=\sum_{n=-\infty}^{\infty} 20\pi\,sinc\left(\frac{n\pi}{2}\right)\delta(\omega-n\pi/2)$$



5.19 a) Duality

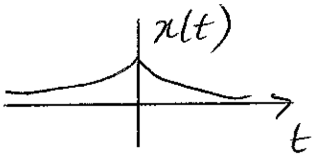
$$x(t) = \frac{1}{2\pi}\frac{1}{(a-jt)^2}$$

we know $te^{-at}u(t) \longleftrightarrow \frac{1}{(a+j\omega)^2}$

$a > 0$

So $\frac{1}{2\pi}\frac{1}{(a-jt)^2} \longleftrightarrow \omega e^{-a\omega}u(\omega)$

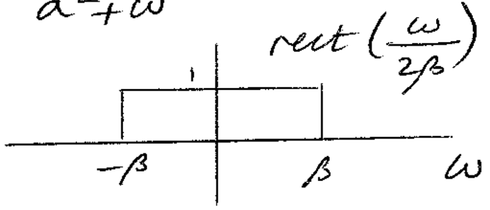
b)



$$X(\omega) = \int_{-\infty}^{0} e^{at}e^{-j\omega t}dt + \int_{0}^{\infty} e^{-at}e^{-j\omega t}dt$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2+\omega^2}$$

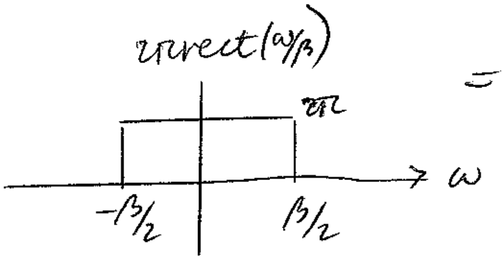
c) $\beta/\pi \, \text{Sinc}(\beta t) \longleftrightarrow$



By time scale, $f(at) \longleftrightarrow \frac{1}{|a|}F(\omega/a)$

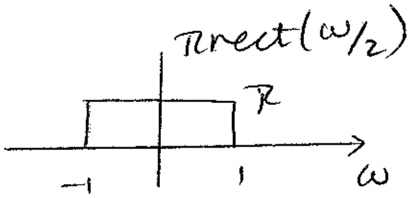
$$\therefore \beta\,\text{Sinc}(\beta t/2) \longleftrightarrow \pi\frac{1}{1/2}\text{rect}\left(\frac{\omega}{\frac{1}{2}2\beta}\right)$$

$$= 2\pi\,\text{rect}(\omega/\beta)$$

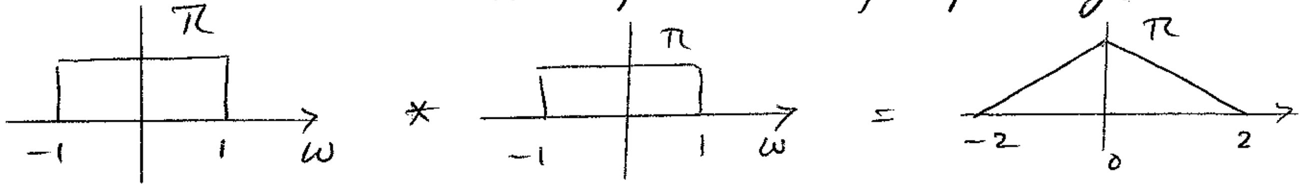


d) $\mathcal{F}[\text{sinc}^2 t]$

$\mathcal{F}[\text{sinc } t] =$



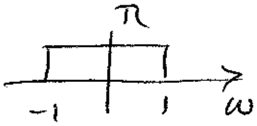
$\mathcal{F}[\text{sinc}^2 t] = \frac{1}{2\pi}(\pi\text{rect}(\omega/2) * \pi\text{rect}(\omega/2))$ by moltiplication property



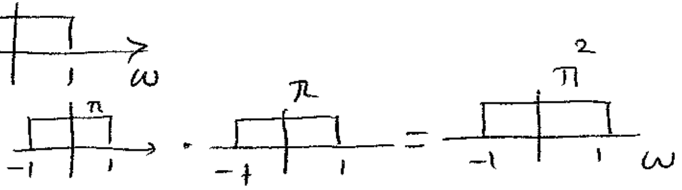
$= \pi\text{tri}(\omega/2)$

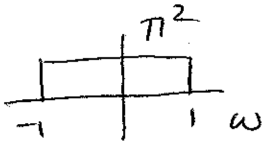
5.20

a) $\text{sinc } t \longleftrightarrow$



$\text{sinc } t * \text{sinc } t \longleftrightarrow$



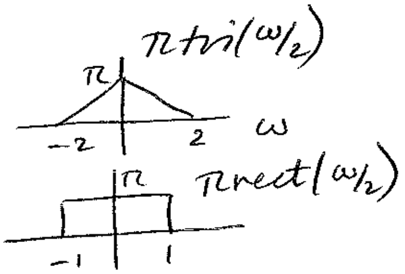


$\longleftrightarrow \pi\,\text{sinc } t$

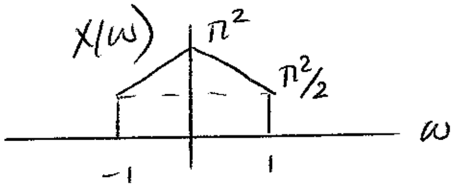
b) $\text{sinc}^2 t * \text{sinc } t$

$\text{sinc}^2 t \longleftrightarrow \pi\text{tri}(\omega/2)$

$\text{sinc } t \longleftrightarrow \pi\text{rect}(\omega/2)$

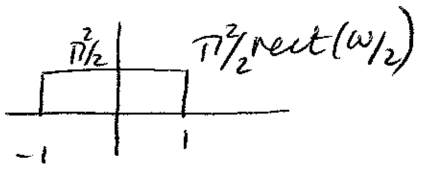


$\text{sinc}^2 t * \text{sinc } t \longleftrightarrow \pi\text{tri}(\omega/2)\cdot\pi\text{rect}(\omega/2) = X(\omega)$

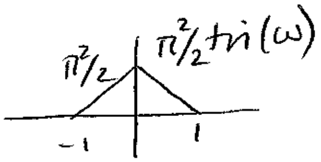


Now take the inverse Fourier transform of $X(\omega)$)

$X(\omega)$



$+$



$\therefore\ x(t) = \pi/2\ \text{sinc}\,t + \mathcal{F}^{-1}\left[\pi^2/2\ \text{tri}(\omega)\right]$

Since $\text{sinc}^2 t \longleftrightarrow \pi\,\text{tri}(\omega/2)$

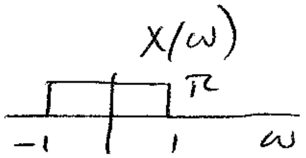
$\text{sinc}^2\, t/2 \longleftrightarrow 2\pi\,\text{tri}(\omega)$

And $\pi/4\ \text{sinc}^2(t/2) \longleftrightarrow \pi^2/2\,\text{tri}(\omega)$

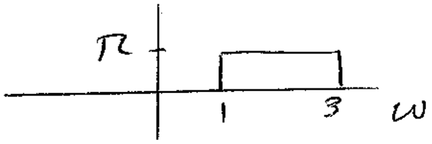
$\therefore\ X(t) = \pi/2\ \text{sinc}\,t + \pi/4\ \text{sinc}^2(t/2)$

c) $\text{sinc}\,t * e^{j2t}\,\text{sinc}\,t$

$x(t) = \text{sinc}\,t \longleftrightarrow$



$e^{j2t}\,\text{sinc}\,t \longleftrightarrow X(\omega-2)$

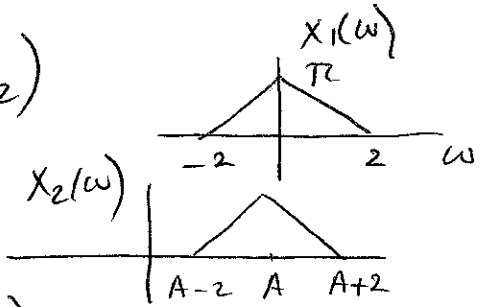


by modulation property

Multiply in frequency to get 0

$\therefore\ \text{sinc}\,t * e^{j2t}\,\text{sinc}\,t = 0$

5.21 $X_1(\omega) = \pi \, tri(\omega/2)$



$X_2(\omega) = X_1(\omega - A)$

$X_1(\omega) * X_2(\omega) \longleftrightarrow X_1(\omega) X_1(\omega - A)$

$x_1(t) * x_2(t)$ are nonzero for

$A-2 < 2$ & $A+2 > -2$

$\therefore$ the range is $-4 < A < 4$

5.22 $v_2(t) = v_1(t) * h(t) \therefore V_2(\omega) = V_1(\omega) H(\omega)$

$$H(\omega) = \frac{j\omega}{50 + j\omega}$$

$$V_1(\omega) = \mathcal{F}\{\sin(50t)\} = \frac{\pi}{j}\left[\delta(\omega - 50) - \delta(\omega + 50)\right]$$

$$V_2(\omega) = \frac{\pi}{j}\left(\frac{j\omega}{50 + j\omega}\right)\delta(\omega - 50) + j\pi\left(\frac{j\omega}{50 + j\omega}\right)\delta(\omega + 50)$$

$$= \left(\frac{50\pi}{50 + j50}\right)\delta(\omega - 50) + \left(\frac{50\pi}{50 - j50}\right)\delta(\omega + 50)$$

$$= \left(\frac{\pi}{1 + j}\right)\delta(\omega - 50) + \left(\frac{\pi}{1 - j}\right)\delta(\omega + 50)$$

$$V_2(\omega) = \frac{\pi}{\sqrt{2}}\left[e^{-j\pi/4}\delta(\omega - 50) + e^{j\pi/4}\delta(\omega + 50)\right]$$

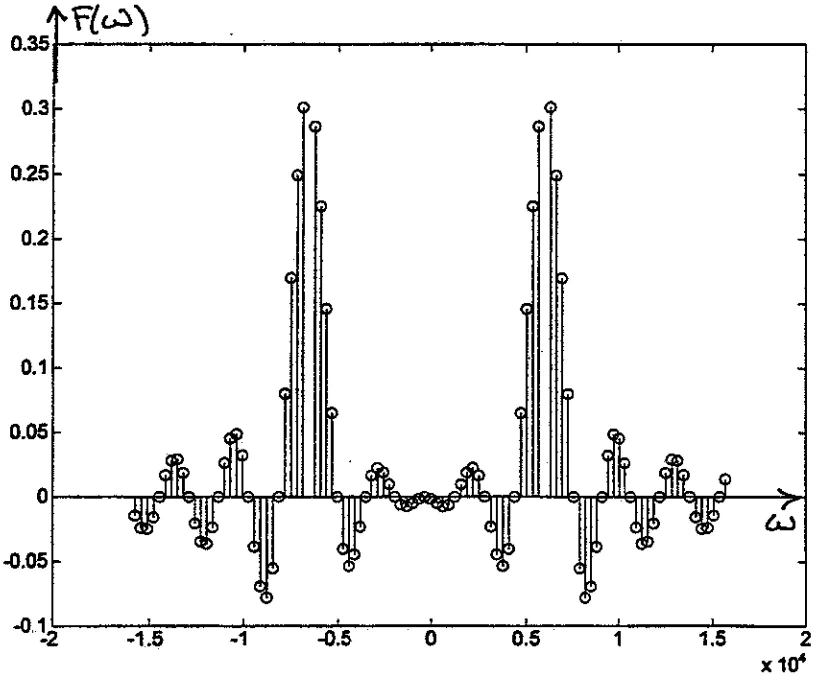
5.23 $f(t)=\sum_{n=-\infty} g(t-nT_0)$, $T_0 = 20(ms)$, $\omega_0 = 100\pi(rad/s)$

$$g(t) = 1 \cos(2000\pi t)\, rect\left(t/_{2\times10^{-3}}\right)$$

$$G(\omega) = 1\times10^{-3}\left[sinc\left(10^{-3}(\omega-2000\pi)\right)+sinc\left(10^{-3}(\omega+2000\pi)\right)\right]$$

$$F(\omega) = \sum_{n=-\infty}^{\infty} \omega_0\, G(n\omega_0)\, \delta(\omega-n\omega_0)$$

$$F(\omega) = \sum_{n=-\infty}^{\infty} \frac{\pi}{10}\left[sinc\left(\tfrac{2\pi}{10}(n-20)\right)+sinc\left(\tfrac{2\pi}{10}(n+20)\right)\right]\delta(\omega-n100\pi)$$



5.24 in the next page

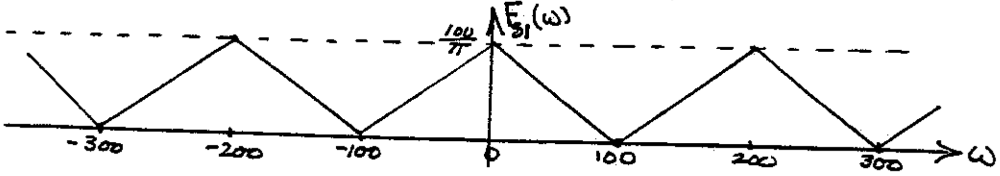
5.25.(a) THE SAMPLED SIGNAL CAN BE WRITTEN AS

$$f_{s_1}(t) = f_1(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\ ,\ T_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{200} = \pi/100$$

$$F_{s_1}(\omega) = \frac{1}{2\pi}F_1(\omega) * \sum_{n=-\infty}^{\infty}\omega_s\delta(\omega-n\omega_s)$$

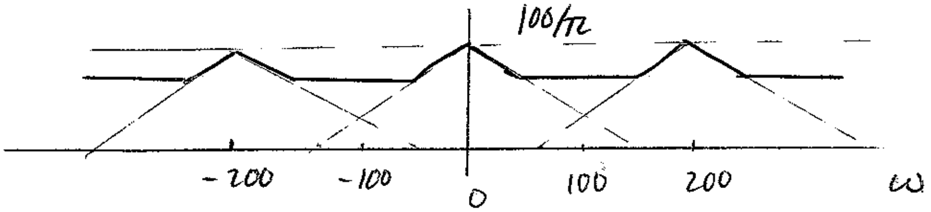
$$= \frac{\omega_s}{2\pi}\sum_{n=-\infty}^{\infty}F_1(\omega)*\delta(\omega-n\omega_s)$$

$$= \frac{1}{T_s}\sum_{n=-\infty}^{\infty}F_1(\omega-n\omega_s) = \frac{100}{\pi}\sum_{n=-\infty}^{\infty}F_1(\omega-n\,200)$$



5.25 (cont) a)

$$F_{S2}(\omega) = \frac{100}{\pi} \sum_{-\infty}^{\infty} F_2(\omega - n200)$$



b) $\omega_S = 200$ (rad/s) is the Nyquist frequency for $f_1(t)$. $\omega_S \geqslant 300$ (rad/s) is necessary for proper sampling of $f_2(t)$.

5.24

$$X(\omega) = \sum_{-\infty}^{\infty} 2\pi C_k \, \delta(\omega - k\omega_0)$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} 2\pi C_k \, \delta(\omega - k\omega_0) e^{j\omega t} \, d\omega$$

$$= \sum_{k=-\infty}^{\infty} C_k \int_{-\infty}^{\infty} \delta(\omega - k\omega_0) e^{j\omega t} \, d\omega$$

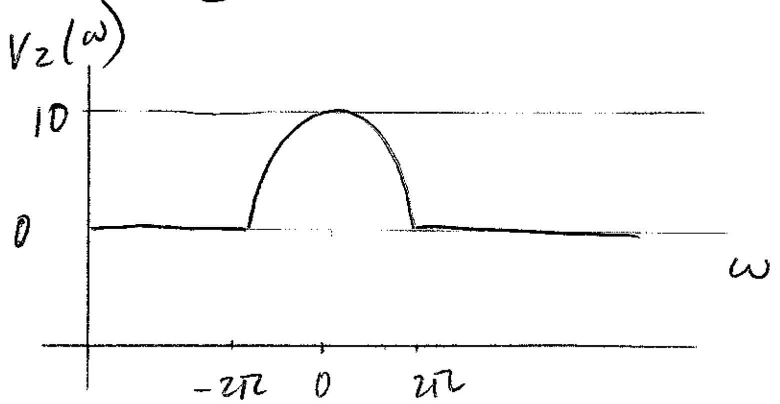
$$= \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$ by sifting property

5.26 $V_2(\omega) = H(\omega) V_1(\omega)$

$H(\omega) = \text{rect}(\omega/4\pi)$

$V_1(\omega) = \mathcal{F}\{10 \,\text{rect}(t)\} = 10 \,\text{sinc}(\omega/2)$

$$V_2(\omega) = \begin{cases} 10 \,\text{sinc}(\omega/2), & |\omega| \leq 2\pi \\ 0, & |\omega| > 2\pi \end{cases}$$



5.27 $f(t) = e^{-t} u(t) \xleftrightarrow{\mathcal{F}} \frac{1}{1+j\omega}$

$$|F(\omega)|^2 = \frac{1}{1+\omega^2}$$

$$E_T = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{1+\omega^2} d\omega = \frac{1}{\pi}\int_0^{\infty} \frac{1}{\omega^2+1} d\omega$$

$$E_T = \frac{1}{\pi} \tan^{-1}(\omega)\Big|_0^{\infty} = 1/2 \, J$$

in the frequency band $-7 \leq \omega \leq 7$ (rad/s)

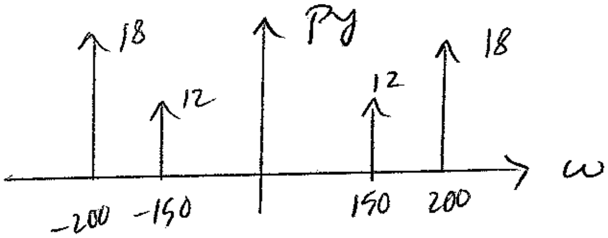
$$E_7 = \frac{1}{\pi} \tan^{-1}(\omega)\Big|_0^{7} = \frac{1}{\pi}(\tan^{-1}(7)) = .455 \, J$$

$$E7/E_T \times 100\% = \frac{.455}{.5} \times 100\% = 91\%$$

5.28 $\xrightarrow{x(t)} \boxed{\text{sys}} \xrightarrow{y(t)}$ $P_y(\omega) = |H(\omega)|^2 P_X(\omega)$

$P_x(150) = 3$, $|H(150)|^2 = 2^2 = 4 \Rightarrow P_y(150) = 3\times 4 = 12$

$P_x(200) = 2$ $|H(200)|^2 = 3^2 = 9 \Rightarrow P_y(200) = 2\times 3^2 = 18$



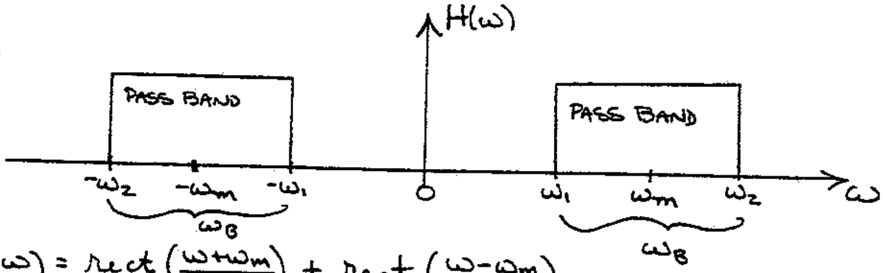
$$P_y = \frac{1}{\pi}\int_0^\infty P_y(\omega)\,d\omega = \frac{1}{\pi}(12+18) = \frac{30}{\pi} = 9.55\ \omega$$

CHAPTER 6

6.1 $H(\omega) = 1 - rect(\omega/2\omega_c) \xleftrightarrow{\mathcal{F}} \delta(t) - \frac{\omega_c}{\pi} sinc(\omega_c t) = h(t)$

h(t) is non-causat ∴ not physically realizable.

6.2.



$$H(\omega) = rect\left(\frac{\omega+\omega_m}{2\omega_B}\right) + rect\left(\frac{\omega-\omega_m}{2\omega_B}\right)$$

$$h(t) = \mathcal{F}^{-1}\{H(\omega)\} = \frac{\omega_B}{2\pi} \underbrace{sinc\left(\frac{\omega_B t}{2}\right)}_{\text{NON-CAUSAL}} e^{j\omega_m t} + \frac{\omega_B}{2\pi} \underbrace{sinc\left(\frac{\omega_B t}{2}\right)}_{\text{NON CAUSAL}} e^{-j\omega_m t}$$

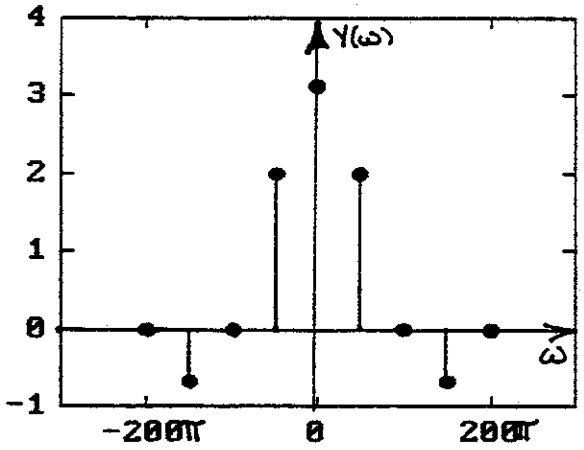
6.3(a) $T = 40(ms)$, $\omega_0 = 50\pi$, $x(t) = \sum_{k=-\infty}^{\infty} rect\left(\frac{t - k40\times10^{-3}}{20\times10^{-3}}\right)$

$$x(t) = \sum_{n=-\infty}^{\infty} g(t-nT), \quad g(t) = rect(t/20\times10^{-3})$$

$$G(\omega) = 20\times10^{-3} sinc(10^{-2}\omega)$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} \omega_0 G(n\omega_0)\delta(\omega - n\omega_0) = \pi \sum_{n=-\infty}^{\infty} sinc\left(\frac{n\pi}{2}\right)\delta(\omega - n50\pi)$$

$$Y(\omega) = X(\omega)H(\omega) = \begin{cases} X(\omega), & |\omega| \le 200\pi \\ 0, & |\omega| > 200\pi \end{cases} = \pi \sum_{n=-4}^{4} sinc\left(\frac{n\pi}{2}\right)\delta(\omega - n50\pi)$$



6.3(b) $T = 20(ms)$, $\omega_0 = 100\pi$, $x(t) = \sum_{n=-\infty}^{\infty} g(t - n20\times10^{-3})$

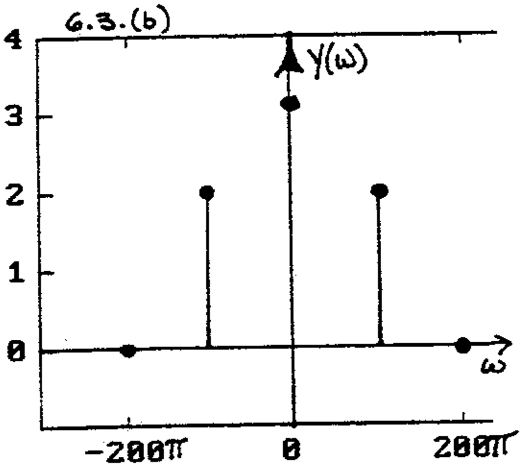
$$g(t) = rect(t/10^{-2}) \xleftrightarrow{\mathcal{F}} 10^{-2} sinc(\omega/200) = G(\omega)$$

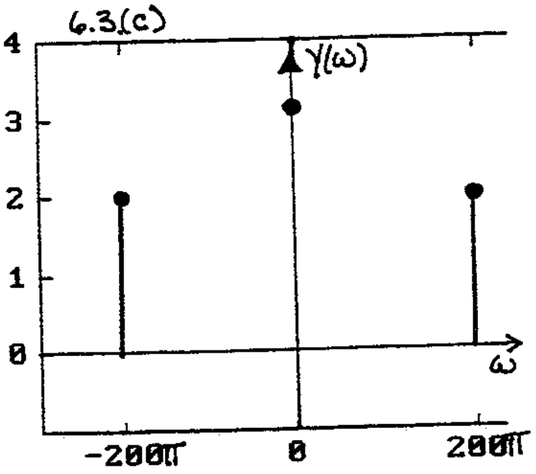
$$X(\omega) = \pi \sum_{n=-\infty}^{\infty} sinc\left(\frac{n\pi}{2}\right)\delta(\omega - n200\pi)$$

$$Y(\omega) = \pi \sum_{n=-1}^{1} sinc\left(\frac{n\pi}{2}\right)\delta(\omega - n200\pi)$$

6.3(c) $T = 10(ms),\ \omega_0 = 200\pi,\quad g(t) = rect\left(t/5\times10^{-3}\right)$

$$G(\omega) = 5\times10^{-3}\, sinc\left(\omega/400\right)$$
$$X(\omega) = \pi\sum_{n=-\infty}^{\infty} sinc\left(\frac{n\pi}{2}\right)\delta(\omega - n200\pi)$$
$$Y(\omega) = \pi\sum_{n=-1}^{1} sinc\left(\frac{n\pi}{2}\right)\delta(\omega - n200\pi)$$





6.4 FOLLOW EXAMPLE 6.7. USE A FIRST-ORDER BUTTERWORTH LOWPASS FILTER WITH $\omega_c = 200\pi$ rad/s

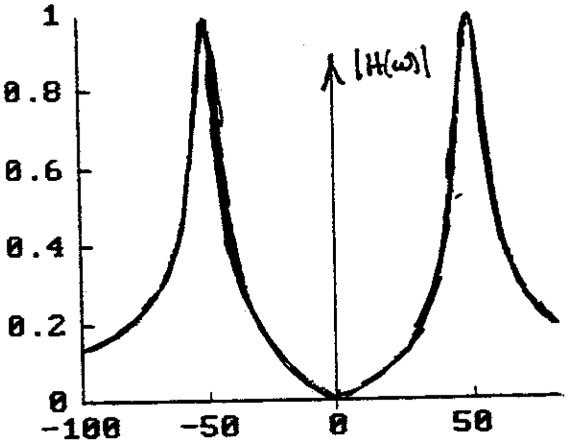
6.5 $v_i(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau \quad , v_o(t) = Ri(t)$

$$V_i(\omega) = RI(\omega) + j\omega LI(\omega) + \frac{1}{j\omega C}I(\omega) + \frac{\pi}{C}\cancelto{0}{I(0)}\delta(\omega)$$
$$V_o(\omega) = RI(\omega)$$
$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)}$$

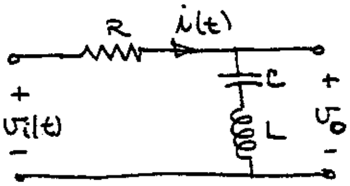
$$H(\omega_m) = 1 \Rightarrow \frac{\omega_m L}{R} = \frac{1}{\omega_m RC} \Rightarrow \omega_m = \pm\frac{1}{\sqrt{LC}}$$
$$H(\omega_c) = \frac{1}{1 \pm j1} \Rightarrow \frac{\omega_c L}{R} - \frac{1}{\omega_c RC} = \pm 1$$
$$\omega_{c_{1,2}} = \frac{R}{2L} \pm \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}$$
$$\omega_{c_{3,4}} = -\frac{R}{2L} \pm \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}$$

THIS IS A BANDPASS FILTER.

THE FIGURE SHOWS A PLOT OF $|H(\omega)|$ WHEN $R = 1\,\Omega$
$L = 0.1\,H$
$C = 4\times10^{-3}F$



6.6



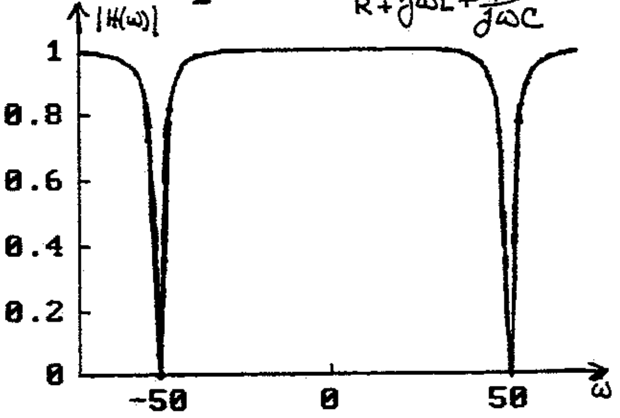
$$v_i(t) = Ri(t) + v_o(t)$$

$$v_o(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

$$V_i(\omega) = RI(\omega) + j\omega L I(\omega) + \frac{1}{j\omega C} I(\omega) + \frac{\pi}{C}\cancel{I(0)}\delta(\omega) \to 0$$

$$V_o(\omega) = j\omega L + \frac{1}{j\omega C} + \frac{\pi}{C}\overset{0}{\cancel{I(0)}}\delta(\omega)$$

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{1 + j\left(\frac{\omega RC}{1-\omega^2 LC}\right)}$$



THIS IS A BANDSTOP OR "NOTCH" FILTER FREQUENCY RESPONSE

6.7

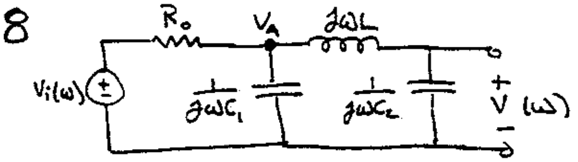
$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{\frac{1}{j\left(\frac{\sqrt{2}}{R_o\omega_c}\right)\omega}}{R_o + j\frac{R_o\omega}{\sqrt{2}\,\omega_c} + \frac{1}{j\frac{\sqrt{2}\,\omega}{R_o\omega_c}}} = \frac{1}{1 - \frac{\omega^2}{\omega_c^2} + j\frac{\sqrt{2}\,\omega}{\omega_c}}$$

$$|H(\omega)| = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_c^2}\right)^2 + \frac{2\omega^2}{\omega_c^2}}} = \frac{1}{\sqrt{1 - \frac{2\omega^2}{\omega_c^2} + \frac{\omega^4}{\omega_c^4} + \frac{2\omega^2}{\omega_c^2}}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^4}} = \frac{1}{\sqrt{1+\left[\left(\frac{\omega}{\omega_1}\right)^2\right]^2}} \leftarrow$$

2nd ORDER BUTTERWORTH FREQUENCY RESPONSE FUNCTION

6.8



KCL EQUATIONS

$$\frac{V_A - V_i}{R_o} + V_A j\omega C_1 + \frac{V_A - V_o}{j\omega L} = 0$$

$$\frac{V_o - V_A}{j\omega L} + j\omega C_2 V_o = 0$$

KCL (IN MATRIX FORM)

$$\begin{bmatrix} \frac{1}{R_o} + j\omega C_1 + \frac{1}{j\omega L} & \frac{-1}{j\omega L} \\ \frac{-1}{j\omega L} & j\omega C_2 + \frac{1}{j\omega L} \end{bmatrix} \begin{bmatrix} V_A \\ V_o \end{bmatrix} = \begin{bmatrix} \frac{V_i}{R_o} \\ 0 \end{bmatrix}$$

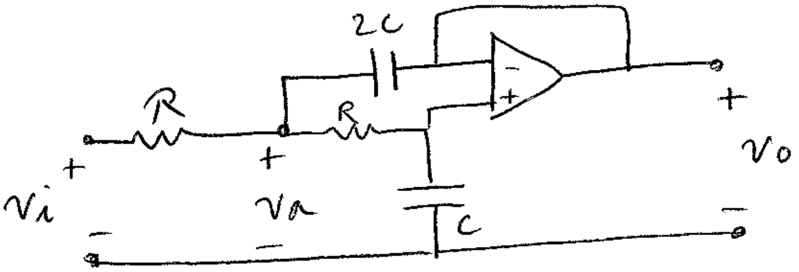
6.8 $\omega_c = 2\pi \times 10 \text{ KHZ} = 20\pi \, (\text{K-rad/s})$

eg. choose $R_0 = 1 k\Omega$, then $L = \frac{R_0}{\sqrt{2}\,\omega_c} = .01125 \text{ H}$

$$C = \frac{\sqrt{2}}{R_0 \omega_c} = .0225 \, \mu F$$

6.9

a)



$$\begin{bmatrix} 2/R + j\omega 2C & -1/R - j\omega 2C \\ -1/R & 1/R + j\omega C \end{bmatrix} \begin{bmatrix} V_a(\omega) \\ V_o(\omega) \end{bmatrix} = \begin{bmatrix} V_i(\omega)/R \\ 0 \end{bmatrix}$$

from KCL:

$$1/R\,(v_i(t) - v_a(t)) + 2C\frac{d}{dt}(v_o(t) - v_a(t)) + 1/R\,(v_o(t) - v_a(t)) = 0$$

$$1/R\,(v_o(t) - v_a(t)) + C\,\frac{dv_o(t)}{dt} = 0$$

find Fourier Transform

$$1/R\,[V_i(\omega) - V_a(\omega)] + 2Cj\omega[V_o(\omega) - V_a(\omega)] + 1/R[V_o(\omega) - V_a(\omega)] = 0$$

$$1/R\,[V_o(\omega) - V_a(\omega)] + Cj\omega V_o(\omega) = 0$$

6.9(a) Continued

$$V_o(\omega) = \frac{\begin{vmatrix} \frac{2}{R}+j\omega 2C & \frac{V_i(\omega)}{R} \\ -\frac{1}{R} & 0 \end{vmatrix}}{\begin{vmatrix} \frac{2}{R}+j\omega 2C & -\frac{1}{R}-j\omega 2C \\ -\frac{1}{R} & \frac{1}{R}+j\omega C \end{vmatrix}} = \frac{V_i(\omega)}{R^2\left(\frac{}{R^2}+\frac{j\omega 2C}{R}-\omega^2 2C^2\right)}$$

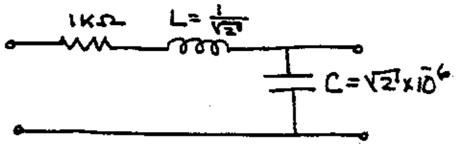
$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{1}{1-\omega^2 2R^2C^2+j\omega 2RC}$$

(b) $$|H(\omega)| = \frac{1}{\sqrt{1+4\omega^4R^4C^4}} = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^4}} \leftarrow$$ 2nd ORDER BUTTERWORTH FILTER

(c) $$\omega_c = \frac{1}{\sqrt{2}\,RC} = \frac{1}{\sqrt{2}\,RC} = \frac{1}{\sqrt{2}(1000)(35\times10^{-9})} = 20{,}203 \ (rad/s)$$

6.10 $\omega_c = 1000 (rad/s)$, let $R_o = 1000\Omega$

THE BUTTERWORTH LOW-PASS FILTER IS (FROM FIGURE 6.13):

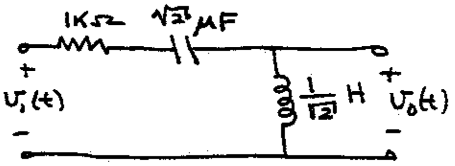


FOR HIGHPASS FILTER

$$L = \frac{1}{C\,10^6} = \frac{1}{\sqrt{2}}$$

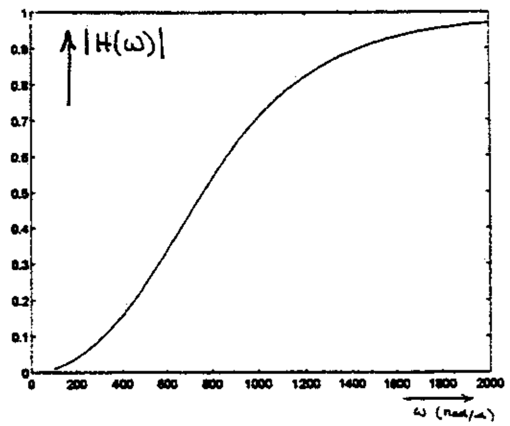
$$C = \frac{1}{L\times10^6} = \sqrt{2}\times10^{-6}$$

∴ THE HIGH PASS FILTER IS:

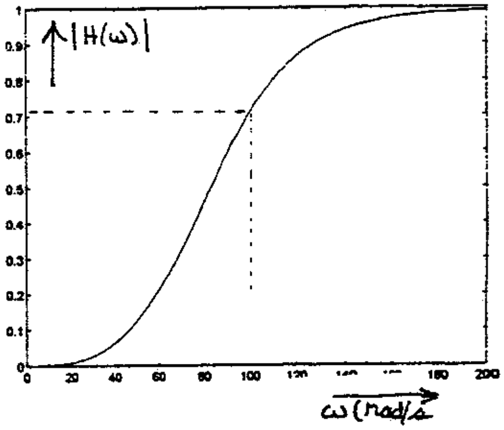


$$H(\omega) = \frac{j\omega}{(\sqrt{2})(1000)+j\left(\omega-\frac{10^6}{\omega}\right)}$$

THE MAGNITUDE FREQUENCY RESPONSE OF THE HIGHPASS FILTER IS SHOWN BELOW.



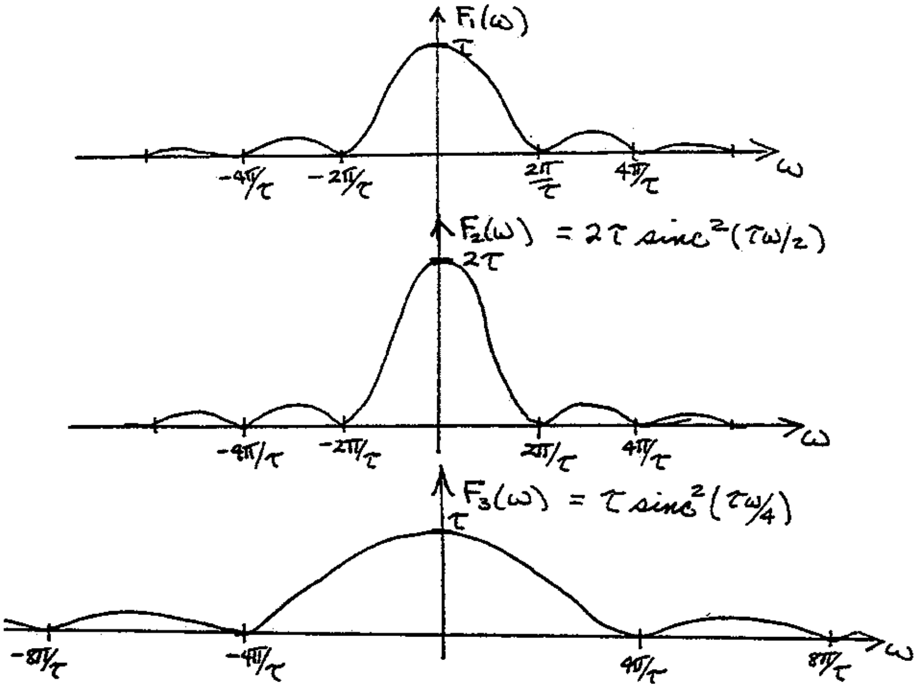
PLOT FOR 6.12



PLOT FOR 6.13

6.11 Set up the System Shown in Figure P6.14 in SIMULINK. Choose a Suitable cutoff frequency for the filters, so that AC components are minimal. (An m.file "Fwrectbw.m" with this simulation is included in the accompanying Software.)

6.12(a) $f_1(t) = tri(t/\tau) \xleftrightarrow{\mathcal{F}} \tau\, sinc^2(\tau\omega/2) = F_1(\omega)$



(b) Shorter time duration results in wider bandwidth.

6.13 (a) $V(\omega) = \frac{\pi}{j}[\delta(\omega-200) - \delta(\omega+200)$

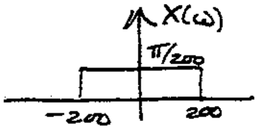
THE HIGHEST FREQUENCY COMPONENT IS $|\omega| = 200$ rad/s

$\therefore\ \omega_s > 2\omega_M \Rightarrow \underline{\omega_s > 400}$ rad/s

(b) $W(\omega) = \frac{\pi}{j}[\delta(\omega-100) - \delta(\omega+100)] - 4\pi[\delta(\omega-100\pi)+\delta(\omega+100\pi)]$
$+ 30\pi[\delta(\omega-200)+\delta(\omega+200)]$

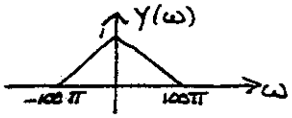
$\Rightarrow \underline{\omega_s > 200\pi}$ rad/s

(c) $X(\omega) = \frac{\pi}{200}\,\mathrm{rect}\left(\frac{\omega}{400}\right)$



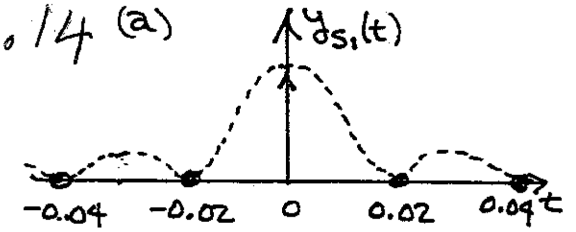
$\underline{\omega_s > 2(200) = 400 \text{ rad/s}}$

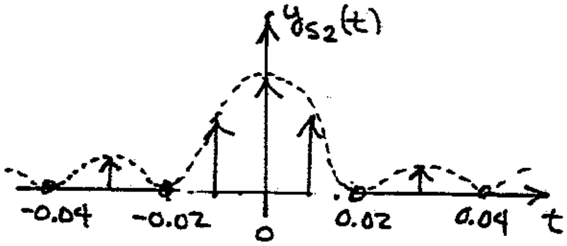
(d) $Y(\omega) = \mathrm{tri}(\omega/100\pi)$

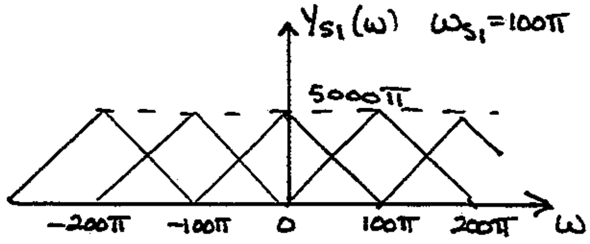


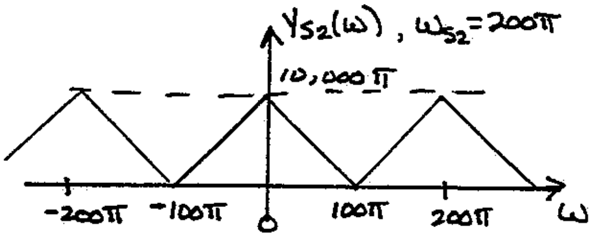
$\underline{\omega_s > 200\pi}$

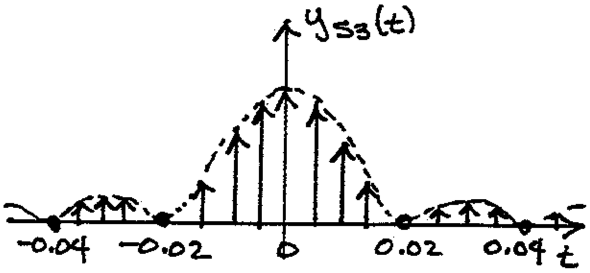
6.14 (a)

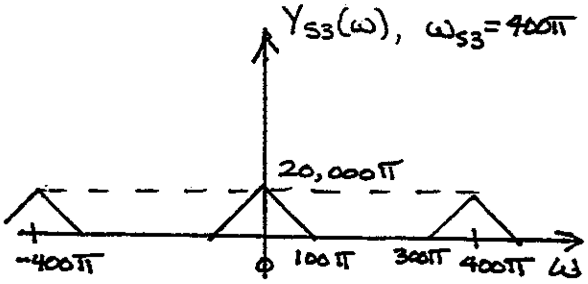










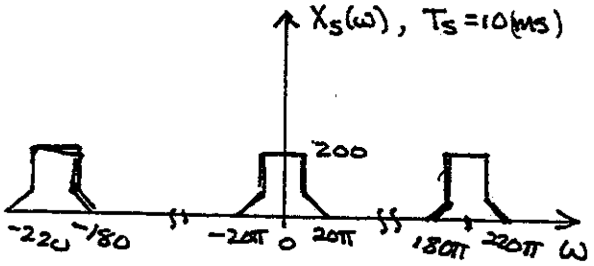


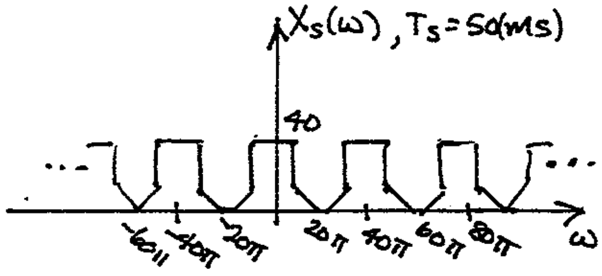
6.14 (b) $f_s = 50Hz$ is not a suitable sampling frequency for this signal. $f_s = 50Hz$ is one-half the Nyquist rate for the signal. Aliasing is seen in the frequency spectrum.

$f_s = 100Hz$ is a satisfactory sampling frequency. This is the Nyquist rate for the signal.

6.15 (a) $T_s = 10(ms) \Rightarrow f_s = 100Hz \Rightarrow \omega_s = 200\pi\ (rad/s)$

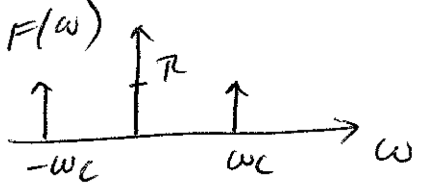
$T_s = 50(ms) \Rightarrow f_s = 20Hz \Rightarrow \omega_s = 40\pi\ (rad/s)$



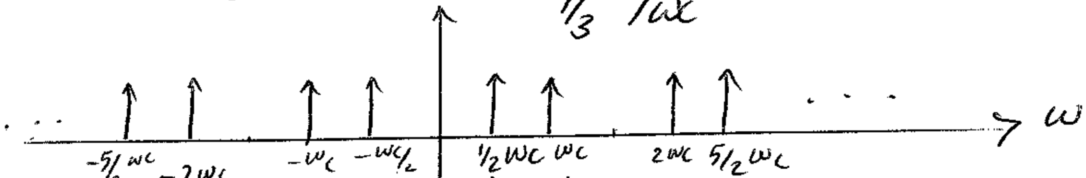


(b) Both are theoretically acceptable sampling frequencies. $T_s = 10(ms)$ would be preferable in most practical applications.

6.16 $f(t) = \cos\omega_c t$

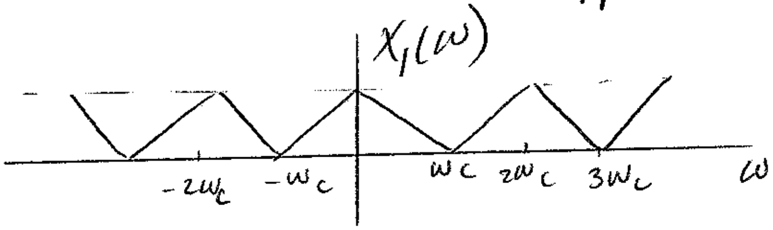


$T = \frac{4}{3}\frac{\pi}{\omega_c}$, $\omega_s = \frac{2\pi}{T} = \frac{2\pi}{\frac{4}{3}\pi/\omega_c} = \frac{3}{2}\omega_c$

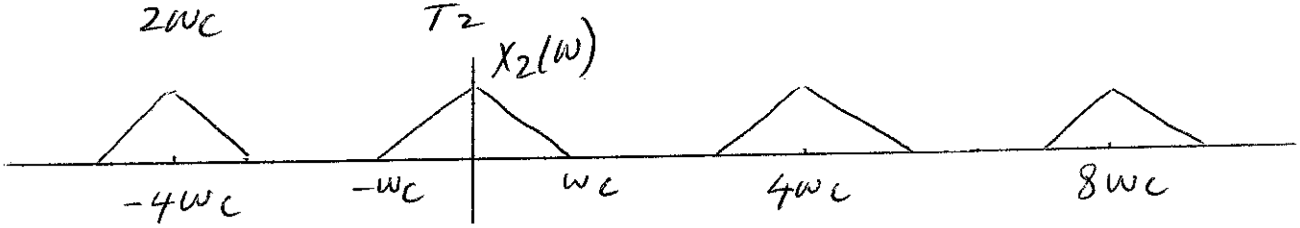


Sampled Spectrum

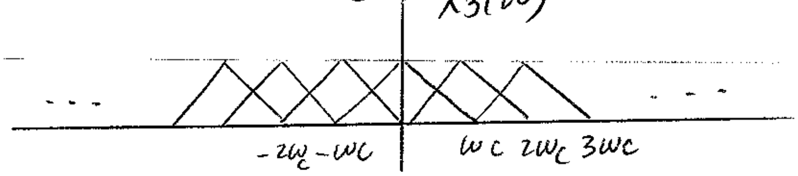
6.17 $T = \frac{\pi}{\omega_c}$ $\omega_1 = \frac{2\pi}{T_1} = 2\omega_c$



$T_2 = \frac{\pi}{2\omega_c}$ $\omega_2 = \frac{2\pi}{T_2} = 4\omega_c$



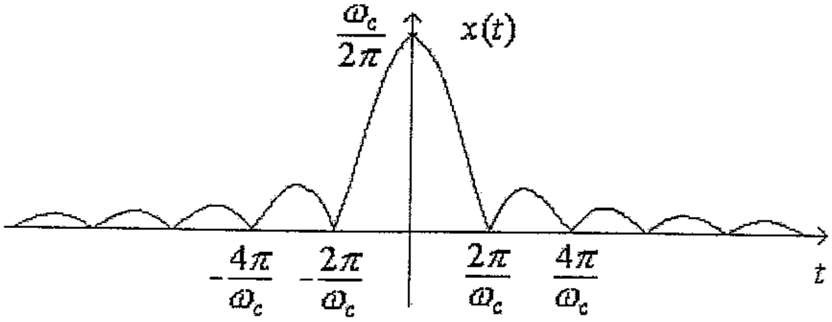
$T_3 = \frac{2\pi}{\omega_c}$ $\omega_3 = \frac{2\pi}{T_3} = \omega_c$



Aliasing example

6.18

$$x(t) = \frac{\omega_c}{2\pi}\text{sinc}^2\left(\frac{\omega_c t}{2}\right)$$

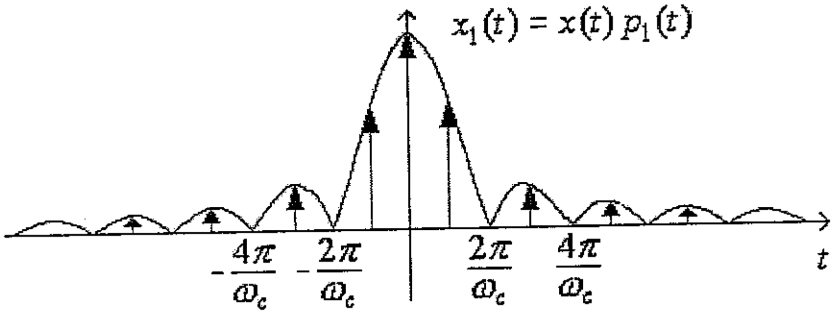


Draw the sampled signals using the sampling trains of the previous example

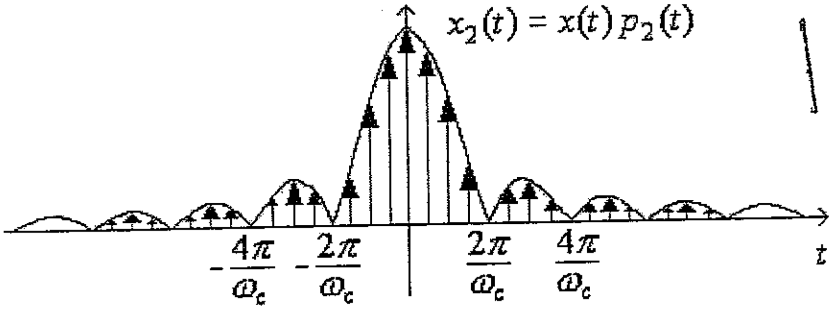
$$\left(T_1 = \frac{\pi}{\omega_c}\ ,\ T_2 = \frac{\pi}{2\omega_c}\ ,\ \text{and } T_3 = \frac{2\pi}{\omega_c}\ \right).$$

Notice how aliasing looks in the time domain.

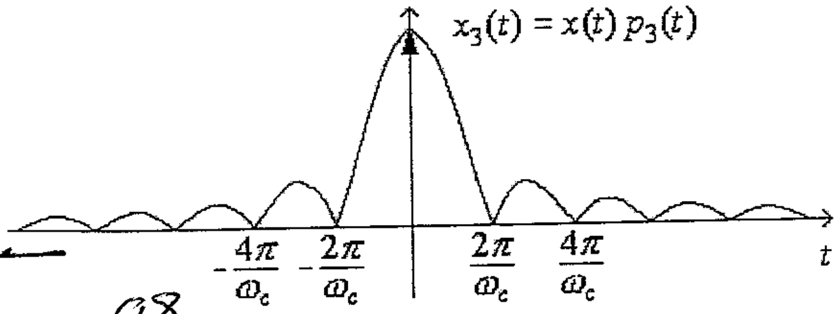
$T_1 = \frac{\pi}{\omega_c}$



$T_2 = \frac{\pi}{2\omega_c}$



$T_3 = \frac{2\pi}{\omega_c}$



6.19 $\omega_0 = \frac{3\pi}{4}$ or $\omega_s \geq \frac{6\pi}{4} = 3/_2\pi$

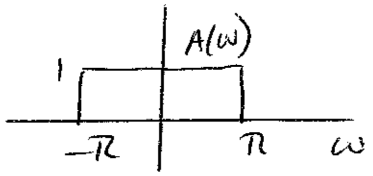
require $\omega_s \geq 2\omega_0$

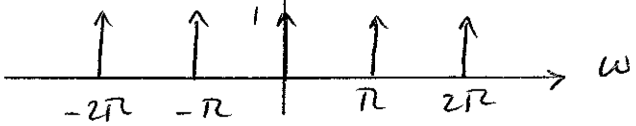
The given r(t) has $T = 2$

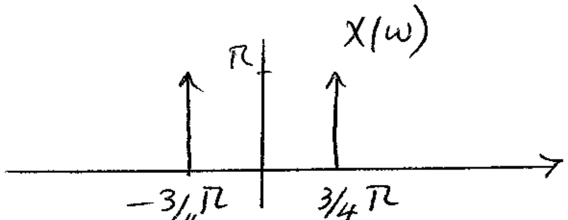
a) $\omega_s \geq 3/_2\pi \rightarrow 2\pi/_T \geq 3/_2\pi$ or $T \leq 4/3$

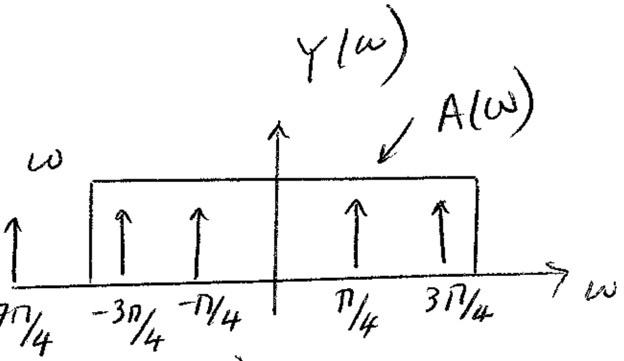
∴ Sampling Therorem is violated

b) $\hat{Y}(\omega) = Y(\omega)A(\omega)$







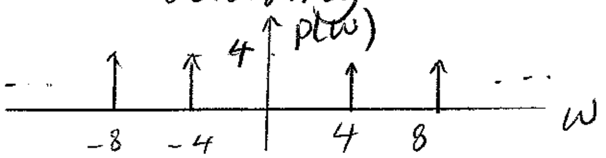


$Y(\omega) = \frac{1}{2\pi} X(\omega) * p(\omega)$

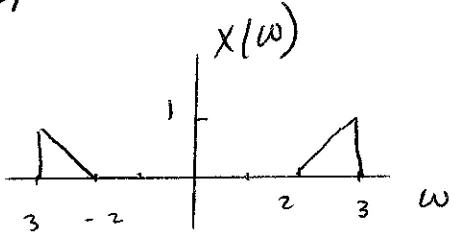
only 4 impulses pass through $A(\omega)$

$∴ \hat{y}(t) = \underbrace{\frac{1}{2\pi}\cos \pi/_4 t}_{\text{aliasing}} + 1/_{2\pi}\cos\frac{3\pi}{4}t$

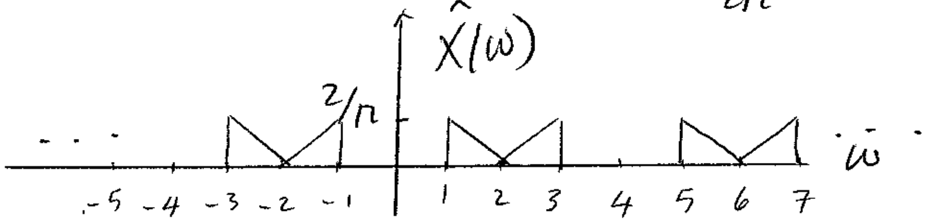
6.20



$*$



$\hat{x}(t) = x(t)p(t) \longleftrightarrow \hat{X}(\omega) = \frac{1}{2\pi} X(\omega) * p(\omega)$

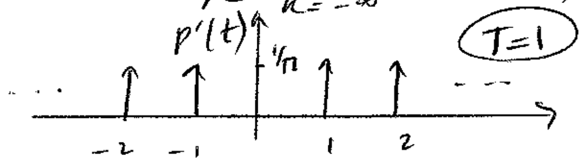


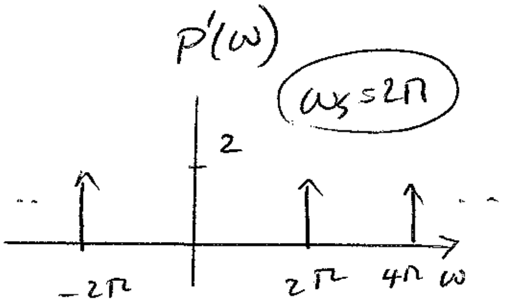
6.21 $x(t) = \cos\frac{2\pi}{4}t$ $\omega_0 = \pi/2$

a) require : $\omega_s \geq 2\omega_0 = \pi$

$T_s = \frac{2\pi}{\omega_s}$ ∴ $T_s \leq 2$

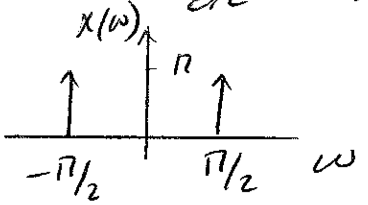
b) $p'(t) = \frac{1}{\pi}\sum_{k=-\infty}^{\infty}\delta(t-k)$

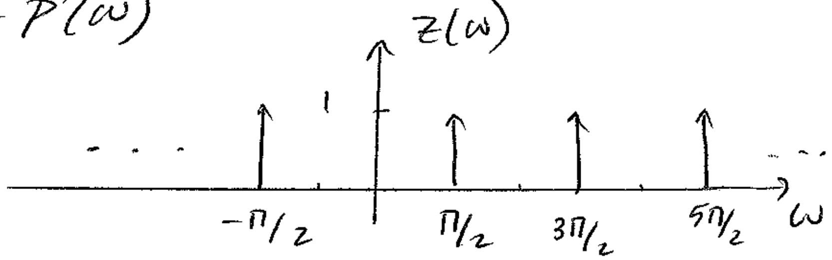




This satisfies sampling criterion of part a) so no aliasing occurs

$Z(\omega) = \frac{1}{2\pi}X(\omega) * P'(\omega)$



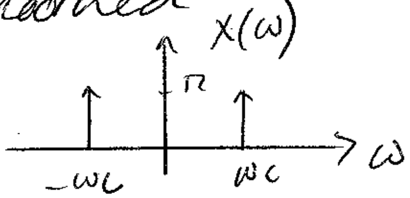


6.22

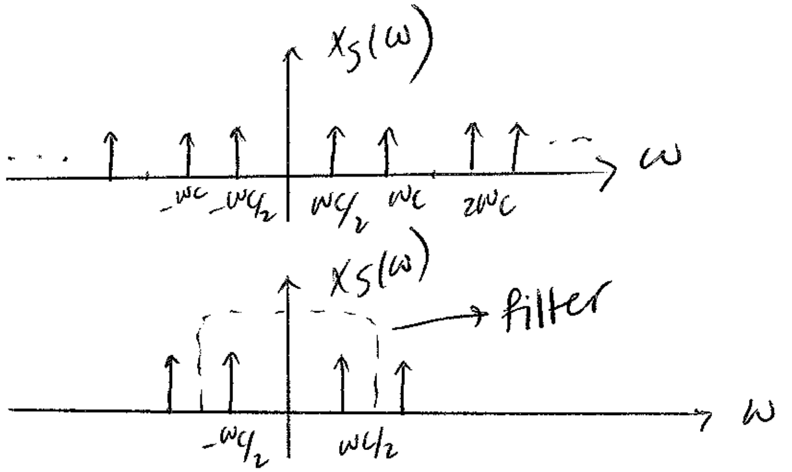
a) Filter A is a High pass filter since the DC effect was removed

b) Filter B is a Low pass filter since the edges were smoothed

6.23 $\omega_s = 3/2\,\omega_c$



a)



$\therefore$ $y(t) = \cos \omega c/_2 t$ and aliasing has occured

6.24

a) 40 Hz Sampled @ 60 Hz

looks like 20 Hz due to aliasing

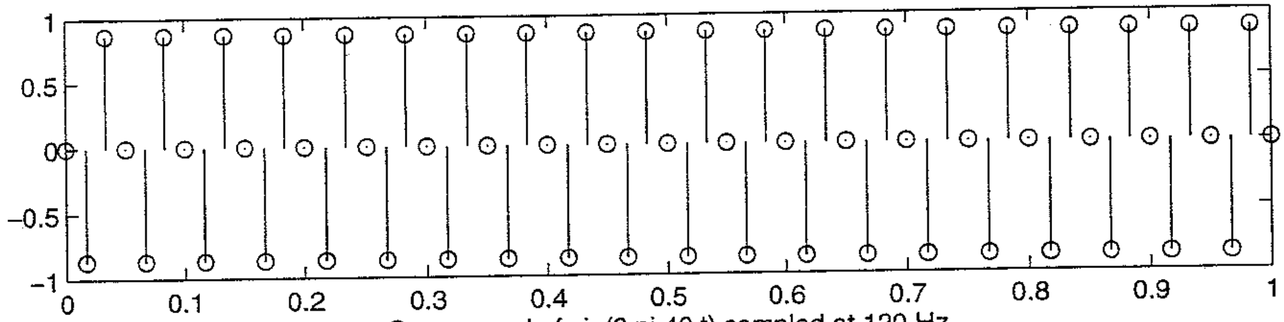
b) 40 Hz Sampled @ 120 Hz

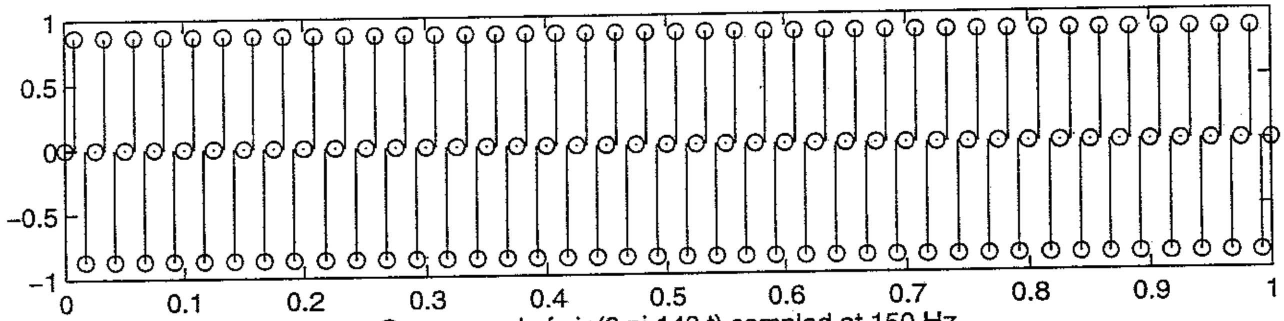
No aliasing so looks like 40 Hz

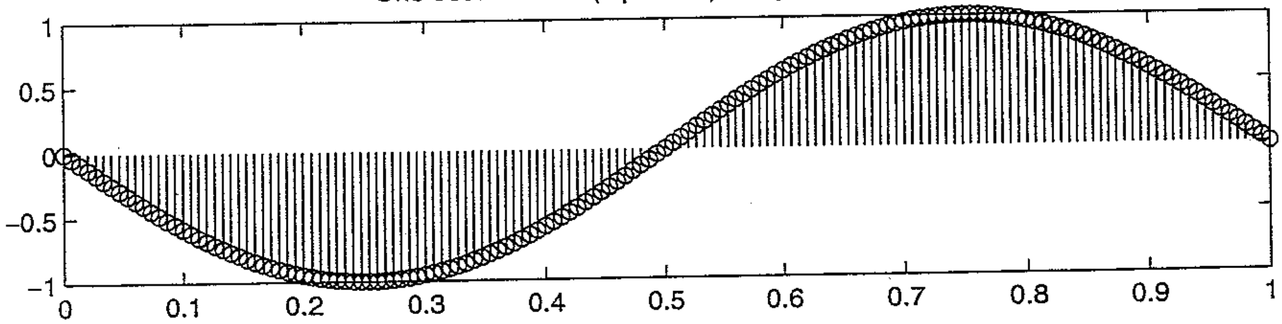
c) 149 Hz Sampled @ 150 Hz

looks like 1 Hz due to aliasing

Matlab is on next page


One second of sin(2 pi 40 t) sampled at 60 Hz.

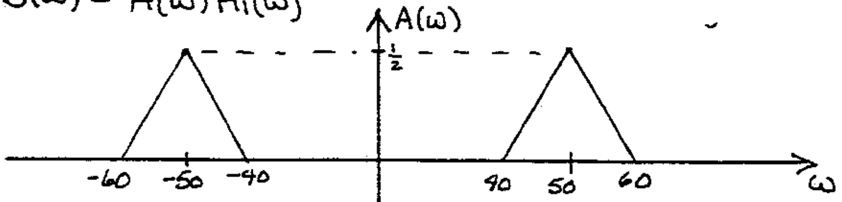

One second of sin(2 pi 40 t) sampled at 120 Hz.

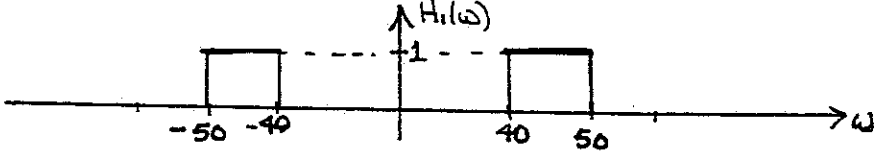

One second of sin(2 pi 149 t) sampled at 150 Hz.

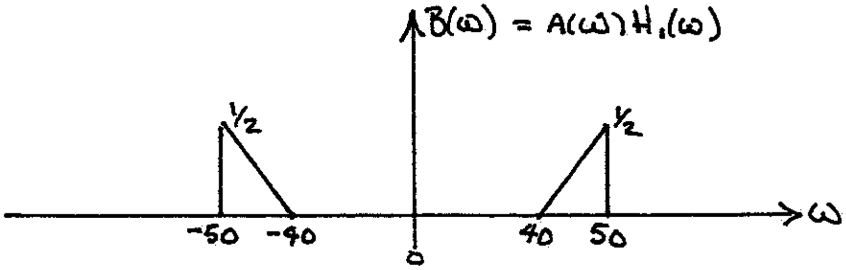
6.25 $a(t) = x(t)\cos 50t \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi}X(\omega) * \pi[\delta(\omega-50)+\delta(\omega+50)]$

$A(\omega) = \frac{1}{2}X(\omega-50)+\frac{1}{2}X(\omega+50)$

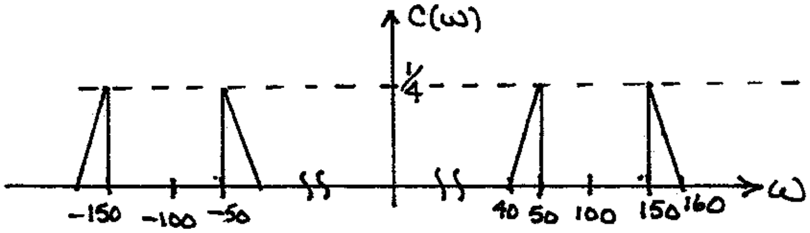
$B(\omega) = A(\omega)H_1(\omega)$

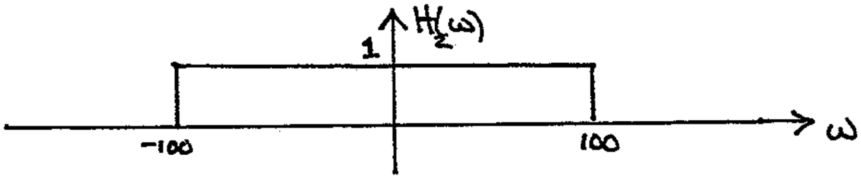


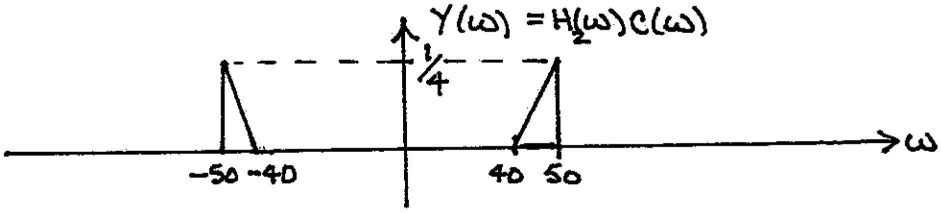




$c(t) = b(t)\cos(100t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}B(\omega-100)+\frac{1}{2}B(\omega+100)$







6.26 $x(t) = m(t)C_1(t) = m(t)\cos(\omega_c t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}\left[M(\omega-\omega_c)+M(\omega+\omega_c)\right]$

$$y(t) = x(t)C_2(t) = x(t)\cos(\omega_c t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}\left[X(\omega-\omega_c)+X(\omega+\omega_c)\right]$$

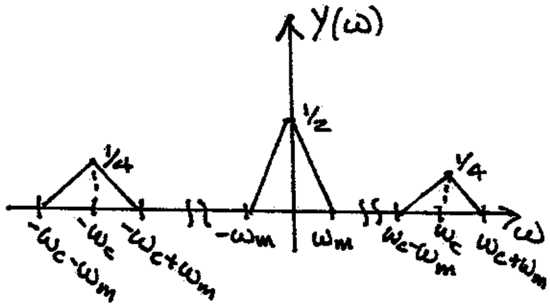
$$X(\omega+\omega_c) = \frac{1}{2}\left[M(\omega+2\omega_c)+M(\omega)\right]$$

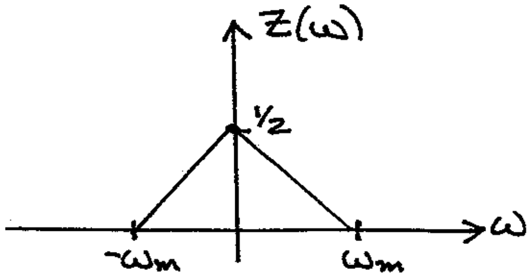
$$X(\omega-\omega_c) = \frac{1}{2}\left[M(\omega-2\omega_c)+M(\omega)\right]$$

$$\therefore Y(\omega) = 1/4\left[2M(\omega)+M(\omega-2\omega_c)+M(\omega+2\omega_c)\right]$$

$$Z(\omega) = Y(\omega)H(\omega) = \begin{cases} Y(\omega), & |\omega| \le \omega_m \\ 0, & |\omega| > \omega_m \end{cases}$$

$$\therefore Z(\omega) = \frac{1}{2}M(\omega)$$





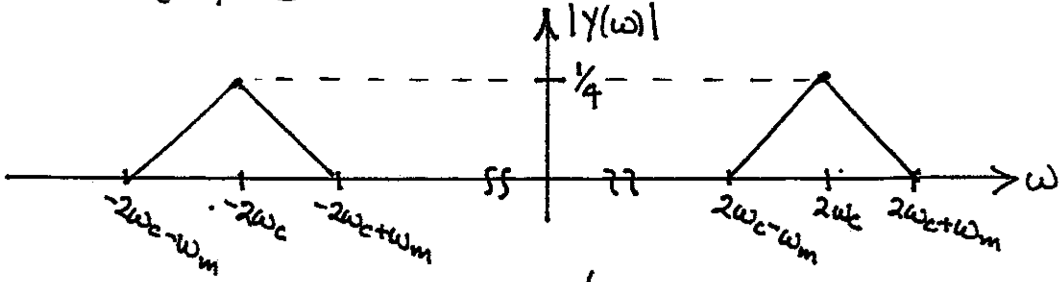
6.27 from 6.17, $X(\omega) = \frac{1}{2}\left[M(\omega-\omega_c)+M(\omega+\omega_c)\right]$

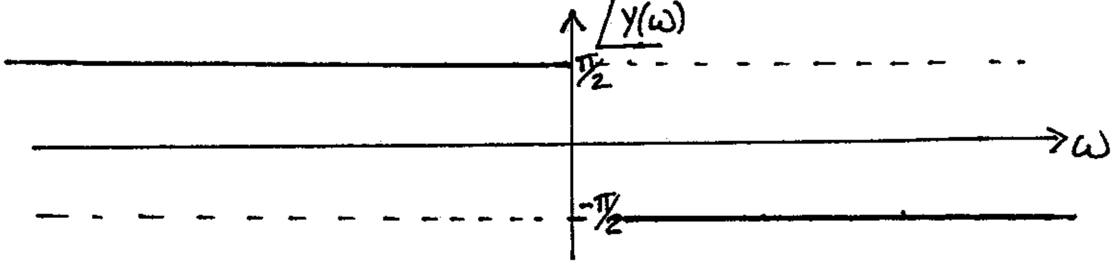
$$y(t) = x(t)\sin(\omega_c t) \overset{\mathcal{F}}{\longleftrightarrow} Y(\omega) = \frac{1}{2j}\left[X(\omega-\omega_c)-X(\omega+\omega_c)\right]$$

$$Y(\omega) = \frac{1}{4j}\left[M(\omega-2\omega_c)+M(\omega+2\omega_c)\right]$$

$$Z(\omega) = Y(\omega)H(\omega) = \begin{cases} Y(\omega), & |\omega| \le \omega_m \\ 0, & |\omega| > \omega_m \end{cases}$$

$$\therefore Z(\omega) = 0$$



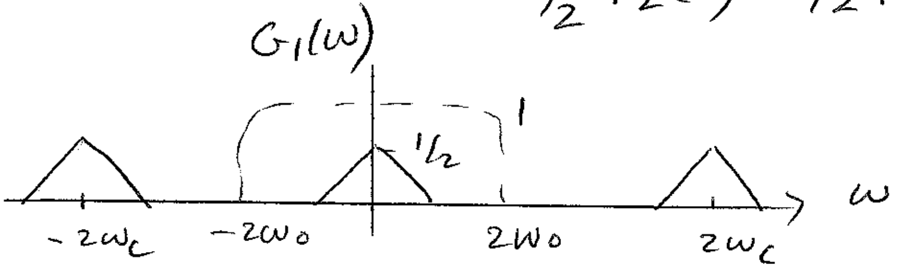


6.28

a) $g_1(t) = f_1(t)\cos^2 \omega_c t + f_2(t)\cos\omega_c t \sin\omega_c t$

$= \frac{1}{2} f_1(t) + \frac{1}{2} f_1(t)\cos 2\omega_c t + \frac{1}{2} f_2(t)\sin 2\omega_c t$

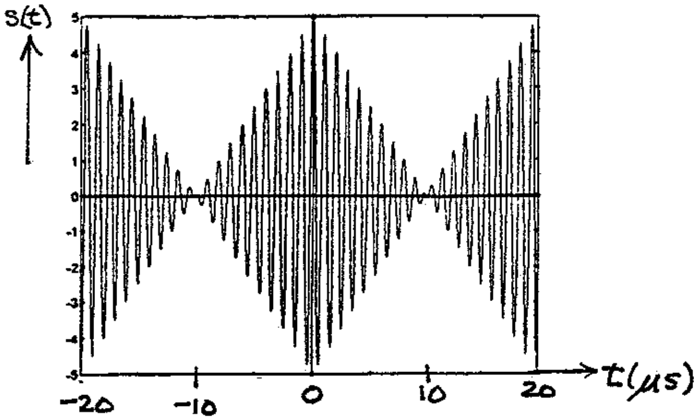
b) $g_2(t) = \phi(t)\sin W_c t = \frac{1}{2} f_1(t)\sin 2\omega_c t + \frac{1}{2} f_2(t) - \frac{1}{2} f_2(t)\cos 2\omega_c t$



$e_1(t) = \frac{1}{2} f_1(t)$

$e_2(t) = \frac{1}{2} f_2(t)$

6.29 (a)



(b) $m(t) = -5 + \sum_{n=-\infty}^{\infty} 10\,\mathrm{tri}\left(\frac{t - n40\times10^{-6}}{40\times10^{-6}}\right) = -5 + \sum_{n=-\infty}^{\infty} g(t - nT_0)$

$g(t) = 10\,\mathrm{tri}\left(\frac{t}{40\times10^{-6}}\right) \xleftrightarrow{\mathcal{F}} 4\times10^{-4}\,\mathrm{sinc}^2(10^{-5}\omega)$

$M(\omega) = -10\pi\,\delta(\omega) + \frac{2\pi}{40\times10^{-6}} \sum_{n=-\infty}^{\infty} 4\times10^{-4}\,\mathrm{sinc}^2\left(\frac{10^{-5}n\pi}{20\times10^{-6}}\right)\delta\left(\omega - \frac{n\pi}{20\times10^{-6}}\right)$

$= -10\pi\,\delta(\omega) + 20\pi \sum_{n=-\infty}^{\infty} \mathrm{sinc}^2\left(\frac{n\pi}{2}\right)\delta\left(\omega - \frac{n\pi}{2\times10^{-5}}\right)$

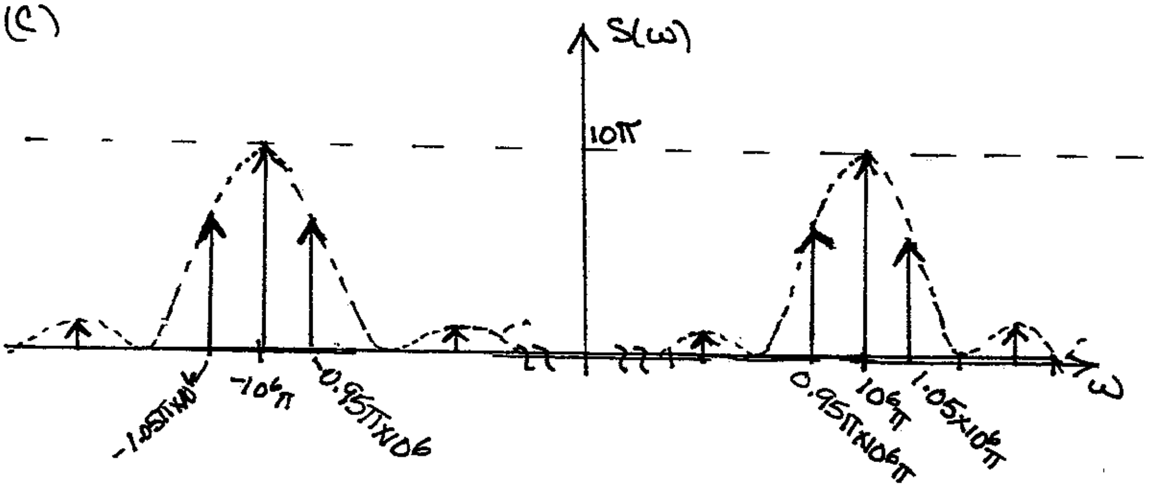
$s(t) = m(t)\cos(10^6\pi t) \xleftrightarrow{\mathcal{F}} \frac{1}{2\pi} M(\omega) * \pi\left[\delta(\omega - 10^6\pi) + \delta(\omega - 10^6\pi)\right]$

$S(\omega) = \frac{1}{2} M(\omega - 10^6\pi) + \frac{1}{2} M(\omega + 10^6\pi)$

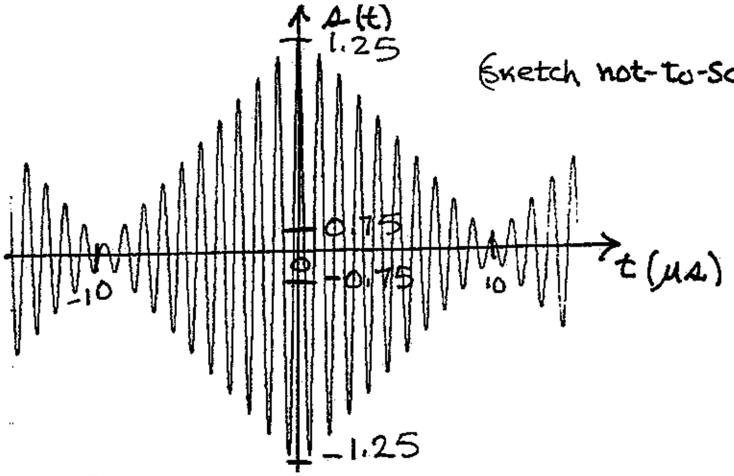
$= 5\pi\,\delta(\omega - 10^6\pi) + 10\pi \sum_{n=-\infty}^{\infty} \mathrm{sinc}^2\left(\frac{n\pi}{2}\right)\delta\left(\omega - \left(1 + \frac{n}{20}\right)10^6\pi\right)$

$-5\pi\,\delta(\omega + 10^6\pi) + 10\pi \sum_{n=-\infty}^{\infty} \mathrm{sinc}^2\left(\frac{n\pi}{2}\right)\delta\left(\omega + \left(1 - \frac{n}{20}\right)10^6\pi\right)$

(c)



6.30 (a)



(b) $m(t) = -5 + \sum_{n=-\infty}^{\infty} 10\, \text{tri}\left(\frac{t - n40\times10^{-6}}{20\times10^{-6}}\right)$

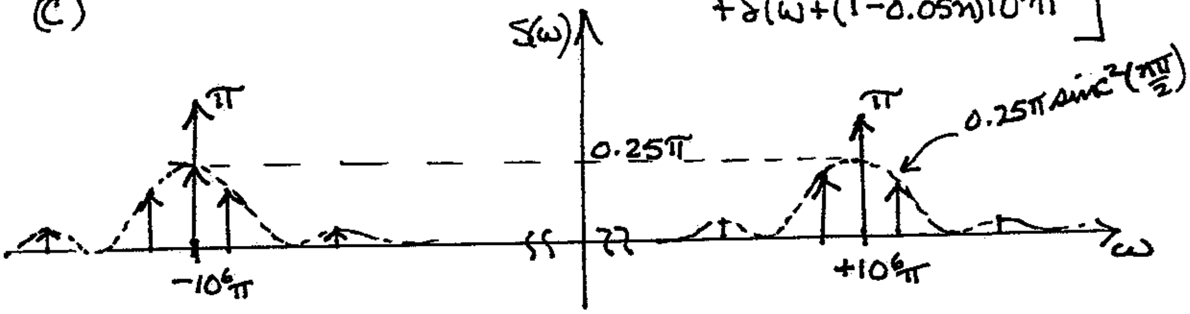
$m_2(t) = 1 + k_a m(t) = 1 - 5k_a + 10k_a \sum_{n=-\infty}^{\infty} \text{tri}\left(\frac{t - n40\times10^{-6}}{20\times10^{-6}}\right)$

$M_2(\omega) = (1-5k_a)2\pi\delta(\omega) + 10\pi k_a \sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{n\pi}{2}\right)\delta\left(\omega - \frac{n\pi}{20\times10^{-6}}\right)$

$s(t) = m_2(t)\cos 10^6\pi t \xleftrightarrow{\mathcal{F}} \frac{1}{2}M_2(\omega - 10^6\pi) + \frac{1}{2}M_2(\omega + 10^6\pi)$

$$S(\omega) = 0.75\pi\big[\delta(\omega - 10^6\pi) + \delta(\omega + 10^6\pi)$$
$$+ 0.25\pi \sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{n\pi}{2}\right)\big[\delta(\omega - (1+0.05n)10^6\pi)$$
$$+ \delta(\omega + (1-0.05n)10^6\pi \big]$$

(c)



6.31 $s(t) = m(t)p(t) \xleftrightarrow{\mathcal{F}} \frac{1}{2\pi}M(\omega) * P(\omega) = S(\omega)$

$P(\omega) = \sum_{k=-\infty}^{\infty} 2\pi C_k \delta(\omega - k\omega_c)$, $C_k = \frac{\Delta}{T_0}\text{sinc}(k\omega_c\Delta/2)$ (6.19)

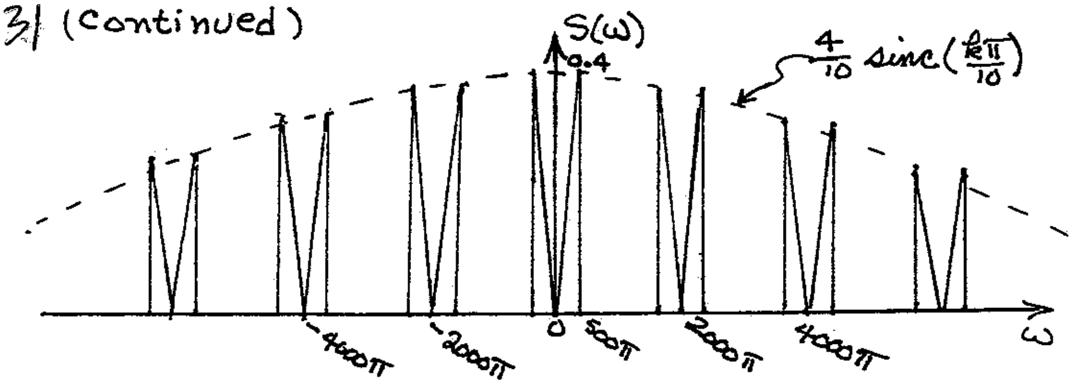
$C_k = \frac{1\times10^{-4}}{1\times10^{-3}}\text{sinc}\left(k\left(\frac{2\pi}{10^{-3}}\right)\times\frac{10^{-4}}{2}\right) = \frac{1}{10}\text{sinc}\left(k\pi/10\right)$

$P(\omega) = \frac{2\pi}{10}\sum_{k=-\infty}^{\infty}\text{sinc}\left(\frac{k\pi}{10}\right)\delta(\omega - k\,2000\pi)$

$S(\omega) = \frac{1}{10}M(\omega) * \sum_{k=-\infty}^{\infty}\text{sinc}\left(\frac{k\pi}{10}\right)\delta(\omega - k\,2000\pi)$

$S(\omega) = \frac{1}{10}\sum_{k=-\infty}^{\infty}\text{sinc}\left(\frac{k\pi}{10}\right)M(\omega - k\,2000\pi)$

6.31 (Continued)



6.32 $f_s = 2.4\,MHz \Rightarrow 2.4 \times 10^6$ PULSES/s.

(a) $\tau = 8 \times 10^{-6}$ (s/pulse) $\Rightarrow R_{max} = \frac{1}{\tau} = 0.125 \times 10^6$ (pulses/s./SIGNAL)

$$\frac{2.4 \times 10^6 \text{ (PULSES/s.)}}{0.125 \times 10^6 \text{(PULSES/s/SIGNAL)}} = 19.2 \Rightarrow 19 \text{ SIGNALS CAN BE MULTIPLEXED}$$

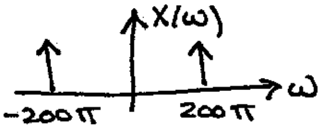
(b) 1st Null bandwidth of a rectangular pulse = $\frac{2\pi}{\tau}$

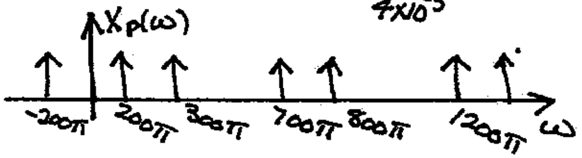
$$\omega_c = \frac{2\pi}{8 \times 10^{-6}} = 785.4 (K\text{-rad/s})$$

6.33 (a) $x(t)$ is bandlimited signal, so that its frequency components above some finite frequency, ω_m, are negligible. Then $\omega_s > 2\omega_m \Rightarrow T_s < \frac{\pi}{\omega_m}$

(b) To recover the original signal from $x_p(t)$, pass the signal through a lowpass filter so that all frequency components $|\omega| > \frac{\omega_s}{2}$ are eliminated.

(c) $x(t) = Cos(200\pi t)$, $T_s = 0.004\,s \Rightarrow \omega_s = \frac{2\pi}{4 \times 10^{-3}} = 500\pi$





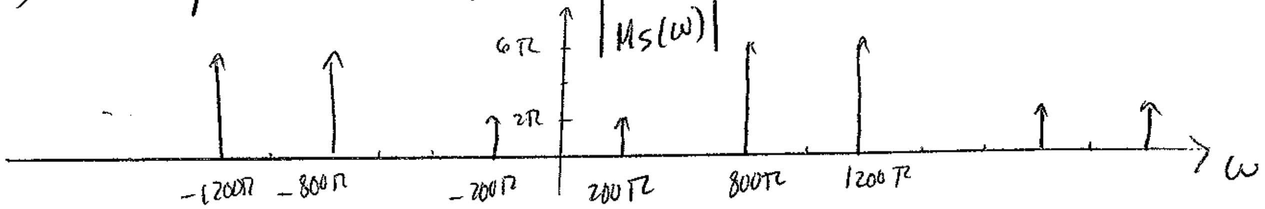
frequency Components less than 700 Hz (1400π rad/s) in $x_p(t)$ are: $\pm 200\pi$, $\pm 300\pi$, $\pm 700\pi$, $\pm 800\pi$, $\pm 1200\pi$, $\pm 1300\pi$

(d) $f_x = 100\,Hz \Rightarrow \omega_x = 300\pi \;\therefore\; x(t) = Cos(300\pi t)$

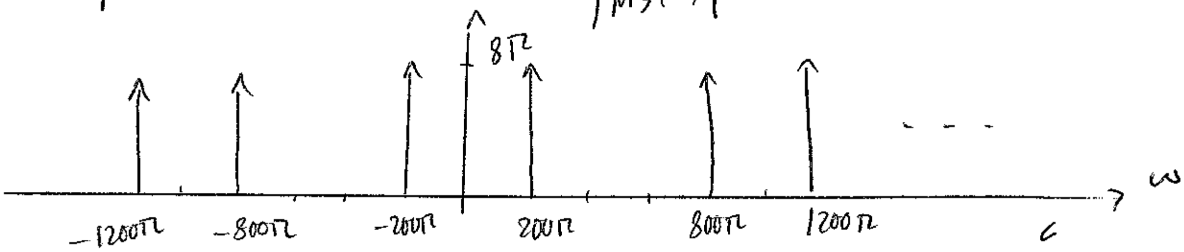
6.34 $m(t) = 2\cos(200\pi t) + 6\cos(800\pi t)$

$$M(\omega) = 2\pi\left[\delta(\omega - 200\pi) + \delta(\omega + 200\pi)\right] + 6\pi\left[\delta(\omega - 800\pi) + \delta(\omega + 800\pi)\right]$$

a) Sampled at $\omega_s = 2000\pi$



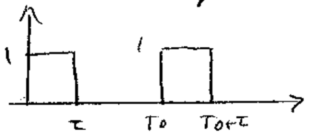
b) Sampled at $\omega_s = 1000\pi$



c) $\omega_s > 2\omega_c$, $\omega_s > 2(800\pi) = 1600\pi$ rad/sec

$\omega_s > 1600\pi$ rad/sec

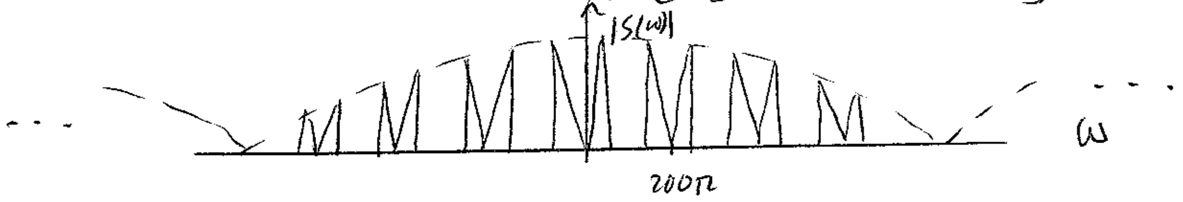
6.35



$$S(\omega) = \frac{\tau}{T_0}\,\text{sinc}\left(\frac{\omega\tau}{2}\right)\left[\sum_{-\infty}^{\infty} M(\omega - n\omega_s)\right] e^{\frac{-j\omega t}{2}}$$

$T_0 = 1.0$ ms, $\omega_s = \frac{2\pi}{T_0} = 2000\pi$ rad/sec, $\tau = .1$ ms

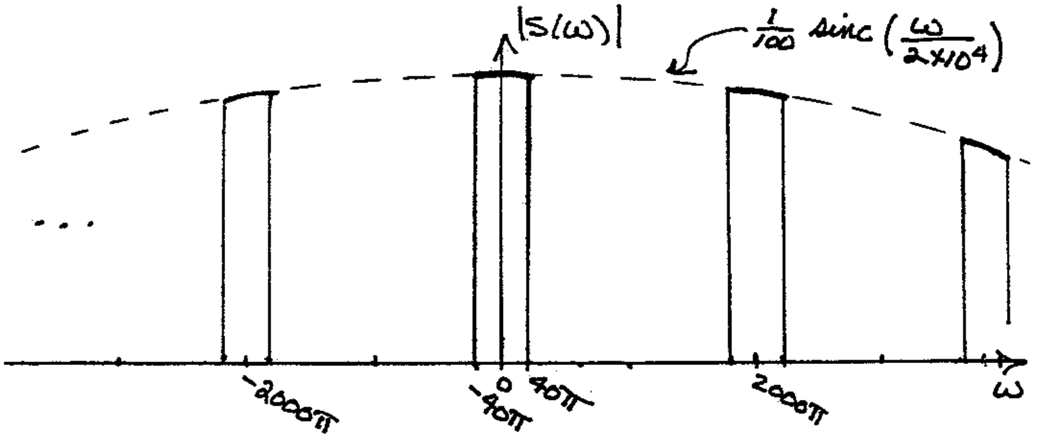
$$S(\omega) = 1/10\ \text{sinc}\left(\frac{\omega}{2\times 10^4}\right)\left[\sum_{-\infty}^{\infty} M(\omega - 2000\,\pi n)\right] e^{-j\omega/2\times10^4}$$



6.36 $m(t) = 4\,\text{sinc}(40\pi t) \xleftrightarrow{\mathcal{F}} M(\omega) = \frac{1}{10}\,\text{rect}\left(\frac{\omega}{80\pi}\right)$

$$S(\omega) = \frac{\tau}{T_0}\,\text{sinc}\left(\frac{\omega\tau}{2}\right)\left[\sum_{n=-\infty}^{\infty} M(\omega - n\omega_s)\right] e^{-j\omega\tau/2}$$

$$= \frac{1}{10}\,\text{sinc}\left(\frac{\omega}{2\times10^4}\right)\left[\sum_{n=-\infty}^{\infty} M(\omega - n2000\pi)\right] e^{-j\frac{\omega}{2\times10^4}}$$



6.37

a) $s_0(t) = A\phi_0(t)$

$$\left.\begin{aligned} r_0 &= \int_0^T s_0(t)\phi_0(t)\,dt = A\int_0^T \phi_0^2(t)\,dt = A \\ r_1 &= \int_0^T s_0(t)\phi_1(t)\,dt = A\int_0^T \phi_0(t)\phi_1(t)\,dt = 0 \end{aligned}\right\}$$

means a 0 bit was sent

b) $s_1(t) = A\phi_1(t)$

$$\left.\begin{aligned} r_0 &= \int_0^T s_1(t)\phi_0(t)\,dt = A\int_0 \phi_1(t)\phi_0(t)\,dt = 0 \\ r_1 &= \int_0^T s_1(t)\phi_1(t)\,dt = A\int_0^T \phi_1(t)^2\,dt = A \end{aligned}\right\}$$

means a 1 bit was sent

6.38

a) Digital 0 : $s(t) = -\phi(t)$

$$r = \int_0^T s(t)\phi(t)\,dt = -\int_0^T \phi^2(t)\,dt = -1$$

a digital <u>0</u> was sent

b) Digital 1 : $s(t) = \phi(t)$

$$r = \int_0^T s(t)\phi(t)\,dt = \int_0^T \phi^2(t)\,dt = 1$$

a digital <u>1</u> was sent

chapter 7

7.1 a) $$\mathcal{L}\left[t \sin bt\right] = \int_0^\infty t \sin bt\, e^{-st}\, dt = \int_0^\infty t\left[\frac{1}{2j}\left(e^{jbt} - e^{-jbt}\right)\right] e^{-st}\, dt$$

$$= \frac{+1}{2j}\int_0^\infty t\, e^{-(s-jb)t}\, dt - \frac{1}{2j}\int_0^\infty t\, e^{-(s+jb)t}\, dt$$

$$= \frac{1}{2j}\,\frac{e^{-(s-jb)t}}{(s-jb)^2}\left((s-jb)t-1\right)\Big|_0^\infty - \frac{1}{2j}\,\frac{e^{-(s+jb)t}}{(s+jb)^2}\left((s+jb)t-1\right)\Big|_0^\infty$$

$$= \frac{-1}{2j}\,\frac{(-1)}{(s-jb)^2} + \frac{1}{2j}\,\frac{(-1)}{(s+jb)^2} = \frac{1}{2j}\left[\frac{1}{(s-jb)^2} - \frac{1}{(s+jb)^2}\right]$$

$$= \frac{1}{2j}\,\frac{(s+jb)^2 - (s-jb)^2}{(s-jb)^2(s+jb)^2} = \boxed{\frac{2sb}{(s^2+b^2)^2}}$$

b) $$\mathcal{L}\left[\cos bt\right] = \int_0^\infty \cos bt\, e^{-st}\, dt = \frac{1}{2}\int_0^\infty e^{jbt} e^{-st}\, dt + \frac{1}{2}\int_0^\infty e^{-jbt} e^{-st}\, dt = \frac{1}{2}\int_0^\infty e^{-(s-jb)t}\, dt + \frac{1}{2}\int_0^\infty e^{-(s+jb)t}\, dt = \frac{1}{2}\,\frac{1}{s-jb} + \frac{1}{2}\,\frac{1}{s+jb} = \frac{2s}{2(s^2+b^2)}$$

$$= \boxed{\frac{s}{s^2+b^2}}$$

c) $$F(s) = \int_0^\infty e^{at} e^{-st}\, dt = \int_0^\infty e^{(a-st)}\, dt = \frac{1}{a-s}\, e^{(a-s)t}\Big|_0^\infty = \boxed{\frac{1}{s-a}}$$

d) $$F(s) = \int_0^\infty t\, e^{at} e^{-st}\, dt = \int_0^\infty t\, e^{(a-s)t}\, dt =$$

$$\frac{1}{(a-s)^2}\left[e^{(a-s)t}\left[at - st - 1\right]\right]_0^\infty = \frac{1}{(a-s)^2}\left[0-(-1)\right] = \boxed{\frac{1}{(s-a)^2}}$$

e) $\int u e^{u} du = e^{u}(u-1) + C$; $u = -st$

$$\int_0^\infty t e^{-st} dt = \int_0^\infty \frac{-st}{-s} e^{-st} \frac{d(-st)}{-s} = \frac{1}{s^2} e^{-st}(st-1)\Big|_0^\infty$$

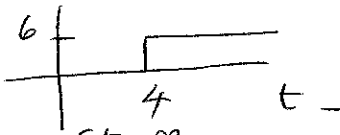
$$\therefore F(s) = 0 - \frac{1}{s^2}(-1) = \boxed{\frac{1}{s^2}} \quad , \quad Re(s) > 0$$

f) $$\int_0^\infty t e^{-(s+a)t} dt = \int_0^\infty \frac{-(s+a)t}{(-s+a)} e^{-(s+a)t} \frac{d[-(s+a)t]}{-(s+a)}$$

$$= \frac{1}{s+a} e^{-(s+a)t}\left[-(s+a)t-1\right]\Big|_0^\infty$$

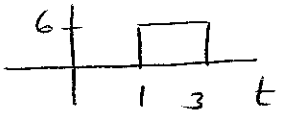
$$\therefore F(s) = 0 - \frac{1}{(s+a)^2}(-1) = \boxed{\frac{1}{(s+a)^2}} \quad , \quad Re(s) > -a$$

7.2 a) $6u(t-4)$



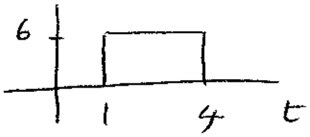
$$F(s) = \int_4^\infty 6 e^{-st} dt = \frac{-6}{s} e^{-st}\Big|_4^\infty = \boxed{\frac{6}{s} e^{-4s}}$$

b) $6[u(t-1) - u(t-3)]$



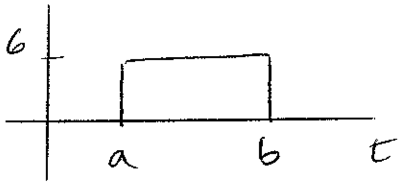
$$F(s) = \int_1^3 6e^{-st} dt = \frac{-6}{s} e^{-st}\Big|_1^3 = \boxed{\frac{6}{s}\left[e^{-s} - e^{-3s}\right]}$$

c) $6u(t-1)u(4-t)$



$$F(s) = 6\int_1^4 e^{-st} dt = \frac{-6}{s} e^{-st}\Big|_1^4 = \boxed{\frac{6}{s}\left[e^{-s} - e^{-4s}\right]}$$

d) $6u(t-a)u(b-t)$



clearly from part c
it gives $F(s) = \frac{6}{s}\left[e^{-as} - e^{-bs}\right]$

7.3 a) $f(t) = 5t\,u(t) - 5(t-2)u(t-2) - 15u(t-2) + 5u(t-4))$

b) $F(s) = \frac{5}{s^2} - \frac{5}{s^2}e^{-2s} - \frac{15}{s}e^{-2s} + \frac{5}{s}e^{-4s}$

7.4 a) $\omega = \frac{2\pi}{\pi} = 2$, $\therefore f(t) = 10\sin(2t)[u(t) - u(t-\pi)]$

b) $F(s) = \int_0^\pi 10\sin 2t\, e^{-st}\,dt = \frac{10e^{-st}}{s^2+(2)^2}(-s\sin 2t - 2\cos 2t)\Big|_0^\pi$

$= \frac{10}{s^2+4}\left[e^{-\pi s}(-2) - (-2)\right] = \frac{20(1-e^{-\pi s})}{s^2+4}$

c) $f(t) = 10\sin 2t\,u(t) - 10\sin[2(t-\pi)]u(t-\pi)$

$\therefore F(s) = \frac{20}{s^2+4} - \frac{20e^{-\pi s}}{s^2+4} = \frac{20(1-e^{-\pi s})}{s^2+4}$

7.5 a) $f(t) = \cosh at = \frac{1}{2}(e^{at} + e^{-at})$

$\therefore F(s) = \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right] = \frac{s}{s^2-a^2}$

b) $\cos bt\Big|_{b=ja} = \frac{e^{jbt}+e^{-jbt}}{2}\Big|_{b=ja} = \frac{e^{-at}+e^{at}}{2} = \cosh at$

$\therefore \mathcal{L}[\cos bt]\Big|_{b=ja} = \frac{s}{s^2+b^2}\Big|_{b=ja} = \frac{s}{s^2-a^2}$ ✓

c) $f(t) = \sinh at = \frac{1}{2}(e^{at} - e^{-at})$

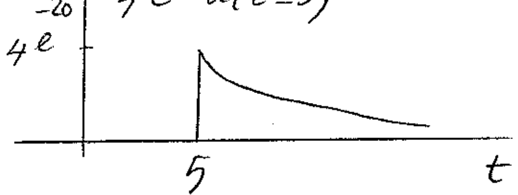
$\therefore\ F(S) = \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right] = \frac{a}{s^2 - a^2}$

$\sin bt\Big|_{b=ja} = \frac{e^{jbt} - e^{-jbt}}{2j}\Big|_{b=ja} = \frac{e^{-at} - e^{at}}{2j} = j\sinh t$

$\therefore\ \mathcal{L}[-j\sin bt] = \frac{-jb}{s^2+b^2}\Big|_{b=ja} = \frac{a}{s^2 - a^2}$

7.6 (i) $4e^{-4t}u(t-5)$

a)



b) $\int_5^{\infty} 4e^{-4t}e^{-st}\,dt = 4\int_5^{\infty} e^{-(s+4)t}\,dt = \frac{4}{-(s+4)}e^{-(s+4)t}\Big|_5^{\infty}$

$= \frac{4}{s+4}e^{-20}e^{-5s}$

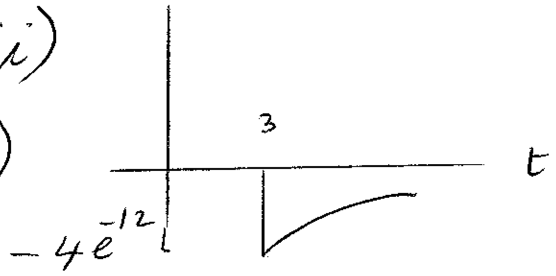
c) $f(t) = 4e^{-4t}u(t-5) = 4e^{-20}e^{-4(t-5)}u(t-5)$

$F(s) = \frac{4e^{-20}e^{-5s}}{s+4}$

d) equal

(ii)

a)



b) $-4\int_3^\infty e^{-4t} e^{-st}\,dt = -4\int_3^\infty e^{-(s+4)t}\,dt = \frac{-4}{s+4} e^{-(s+4)t}\Big|_3^\infty$

$= \frac{-4}{s+4} e^{-(s+4)3} = \frac{-4}{s+4} e^{-12} e^{-3s}$

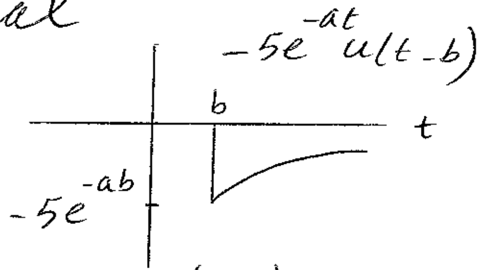
c) $f(t) = -4e^{-4t}u(t-3) = -4e^{-12}e^{-4(t-3)}u(t-3)$

$F(s) = \frac{-4e^{-12}}{s+4} e^{-3s}$

d) equal

(iii)

a)



b) $-5\int_b^\infty e^{-(s+a)t}\,dt = \frac{-5}{-(s+a)} e^{-(s+a)t}\Big|_b^\infty = \frac{-5}{s+a} e^{-(s+a)b}$

$= \left(\frac{-5e^{-ab}}{s+a}\right) e^{-bs}$

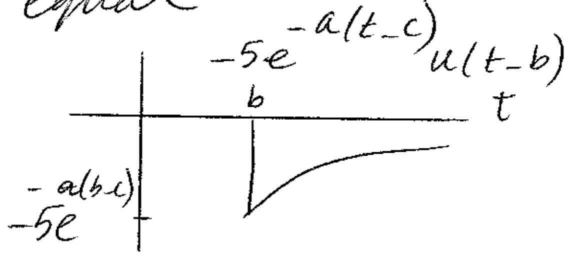
c) $-5e^{-at}u(t-b) = -5e^{-ab}e^{-a(t-b)}u(t-b) = f(t)$

$F(s) = \left(\frac{-5e^{-ab}}{s+a}\right) e^{-bs}$

d) equal

(iv)

a)



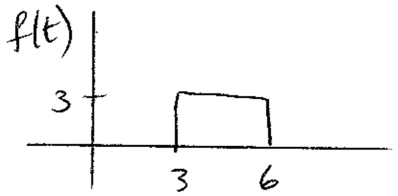
b) $-5\int_b^\infty e^{ac} e^{-(s+a)t}\,dt = \frac{-5e^{ac}}{s+a} e^{-(s+a)b} = \left(\frac{-5e^{a(c-b)}}{s+a}\right) e^{-bs} = F(s)$

c) $f(t) = -5e^{ac}e^{-at}u(t-b) = -5e^{ac}e^{-ab}e^{-a(t-b)}u(t-b)$

$$F(s) = \left(\frac{-5e^{a(c-b)}}{s+a}\right)e^{-bs}$$

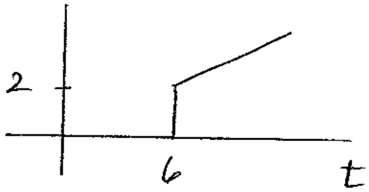
d) equal

7.7 a) $3u(t-3)u(6-t) = 3[u(t-3) - u(t-6))]$



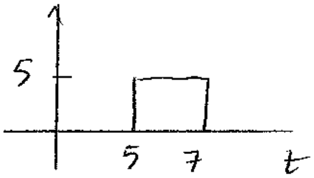
$$F(s) = \frac{3}{s}\left[e^{-3s} - e^{-6s}\right]$$

b) $(t-4)u(t-6) = (t-6)u(t-6) + 2u(t-6)$



$$F(s) = \frac{e^{-6s}}{s^2} + \frac{2e^{-6s}}{s}$$

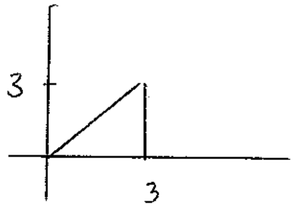
c) $5u(t-5)u(7-t) = 5[u(t-5)-u(t-7))]$



$$F(s) = \frac{5}{s}\left[e^{-5s} - e^{-7s}\right]$$

d) $t[u(t)-u(t-3)] = tu(t) - (t-3)u(t-3) - 3u(t-3)$

$$F(s) = \frac{1}{s^2} - \frac{e^{-3s}}{s^2} - \frac{3e^{-3s}}{s}$$



e) $f(t)=3t[u(t-a)-u(t-b)]=[3(t-a)u(t-a)+3a]u(t-a)$

$-[3(t-b)+3b]u(t-b)$

$\therefore F(s)=\left(\frac{3}{s^2}+\frac{3a}{s}\right)e^{-as}-\left(\frac{3}{s^2}+\frac{3b}{s}\right)e^{-bs}$

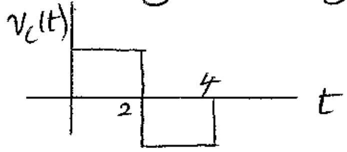
f) $f(t)=3e^{-bt}u(t-a)=3e^{-b(t-a)}e^{-ab}u(t-a)$

$F(s)=\frac{3e^{-ab}e^{-as}}{s+b}$

7.8 a) $v(t)=\frac{5}{2}tu(t)-5(t-2)u(t-2)+5/2(t-4)u(t-4)$

b) $V(s)=\frac{5/2}{s^2}-\frac{5e^{-2s}}{s^2}+\frac{5/2e^{-4s}}{s^2}$

c)



$v_c(t)=5/2u(t)-5u(t-2)+5/2u(t-4)$

d) $V_c(s)=1/s\left(5/2-5e^{-2s}+5/2e^{-4s}\right)$

e) $\int_0^t v_c(\tau)d\tau=v(t)$ $\therefore$ $V(s)=1/sV_c(s)$

$\therefore V(s)=\frac{1}{s^2}\left(5/2-5e^{-2s}+5/2e^{-4s}\right)$ ✓

f) $v_c(t)=\frac{dv(t)}{dt}$; $V_c(s)=sV(s)-v(0^+)$

$\therefore V_c(s)=s\left[\frac{1}{s^2}\left(5/2-5e^{-2s}+5/2e^{-4s}\right)\right]-0=$

$\frac{1}{s}\left(5/2-5e^{-2s}+5/2e^{-4s}\right)$

7.9 $V(s) = \frac{s}{(s+2)(s+4)} = \frac{-1}{s+2} + \frac{2}{s+4}$

a) (i) $v(0^+) = \lim_{s \to \infty} sV(s) = \lim_{s \to \infty} \frac{s^2}{s^2+6s+8} = 1$

(ii) $v(t) = -e^{-2t}u(t) + 2e^{-4t}u(t) \Rightarrow v(0^+) = -1+2=1$ ✓

b) (i) $v(\infty) = \lim_{s \to 0} sV(s) = \lim_{s \to 0} \frac{s^2}{s^2+6s+8} = 0$

(ii) $v(\infty) = \lim_{t \to \infty} \left(-e^{-2t}u(t) + 2e^{-4t}u(t)\right) = 0$ ✓

c)

```
n = [0 1 0];
d = [1 6 8];        [r,p,k] = residue(n,d)
```

7.10 $v(s) = \frac{2s+1}{s^2+4} = \frac{2s}{s^2+4} + \frac{1}{s^2+4}$

a) (i) $v(0^+) = \lim_{s \to \infty} sV(s) = \lim_{s \to \infty} \frac{2s^2+s}{s^2+4} = 2$

(ii) $v(t) = [2\cos 2t + \frac{1}{2} \sin 2t]u(t)$,

$\therefore V(0^+) = 2 + \frac{1}{2}(0) = 2$ ✓

b) (i) $v(\infty) = \lim_{s \to 0} sV(s) = \lim_{s \to 0} \frac{2s^2+s}{s^2+4} = 0$ [in error]

(ii) $v(\infty) = \lim_{t \to \infty} (2\cos 2t + \frac{1}{2} \sin 2t) \Rightarrow$ undefined

7.11 a) $\mathcal{L}[tu(t)] = \frac{-d}{ds}\mathcal{L}[u(t)] = \frac{-d}{ds}(1/s) = \underline{1/s^2}$

b) $\mathcal{L}[\cos bt] = \frac{s}{s^2+b^2}$

$$\mathcal{L}[t\cos bt] = \frac{-d}{ds}\left[\frac{s}{s^2+b^2}\right] = \frac{-1}{s^2+b^2} + \frac{s\cdot 2s}{(s^2+b^2)^2}$$

$$= \underline{\frac{s^2-b^2}{(s^2+b^2)^2}}$$

c) $\mathcal{L}[t\,t^{n-1}] = \frac{-d}{ds}\mathcal{L}[t^{n-1}] = \frac{-d}{ds}\left[\frac{(n-1)!}{s^n}\right] = \underline{\frac{n!}{s^{n+1}}}$

7.12 $f(t) = \frac{d}{dt}[\sin bt] = b\cos bt$

$$F(s) = s\mathcal{L}[\sin bt] - \sin(0^+) = s\left[\frac{b}{s^2+b^2}\right] = b\mathcal{L}[\cos bt]$$

$$\therefore \mathcal{L}[\cos bt] = \underline{\frac{s}{s^2+b^2}}$$

7.13

a) $F(s) = \frac{5}{s(s+2)} = \frac{2.5}{s} + \frac{-2.5}{s+2} \Rightarrow f(t) = 2.5(1-e^{-2t})u(t)$

b) $F(s) = \frac{s+3}{s(s+1)(s+2)} = \frac{1.5}{s} + \frac{-2}{s+1} + \frac{.5}{s+2} \Rightarrow f(t) = (1.5 - 2e^{-t} + .5e^{-2t})u(t)$

c) $F(s) = \frac{10(s+3)}{s^2+25} = \frac{K_1}{s+j5} + \frac{K_1^*}{s-j5}$; $K_1 = \frac{10(3+j5)}{-j5-j5}$

$$\therefore K_1 = 5.831 \angle 149°$$

d) $F(s) = \frac{3}{s[(s+1)^2+(2^2)]} = \frac{3/5}{s} + \frac{k_1}{s+1+j2} + \frac{k_1^*}{s+1-j2}$

$$k_1 = \frac{3}{s(s+1-j2)}\Big|_{s=-1-j2} = \frac{3}{(-1-j2)(-j4)} = .335\angle -153.4^\circ$$

```
n=[0 0 5]; d=[1 2 0];[r,p,k]=residue(n,d)
n=[0 0 1 3]; d=[1 3 2 0];[r,p,k]=residue(n,d),pause
n=[0 10 30]; d=[1 0 25];[r,p,k]=residue(n,d),pause
n=[0 0 0 3]; d=[1 2 5 0];[r,p,k]=residue(n,d)
```

7.14

a) $F(s) = \frac{10}{s(s+1)^2} = \frac{10}{s} + \frac{-10}{(s+1)^2} + \frac{-10}{s+1}$

$$\frac{d}{ds}\left(\frac{10}{s}\right)_{s=-1} = \frac{-10}{s^2}\Big|_{s=-1} = -10$$

∴ $f(t) = 10(1-e^{-t}-te^{-t})u(t)$

b) $F(s) = \frac{s+2}{s^2(s+1)} = \frac{2}{s^2} + \frac{-1}{s} + \frac{1}{s+1}$

$$\frac{d}{ds}\left(\frac{s+2}{s+1}\right)_{s=0} = \frac{s+1-(s+2)}{(s+1)^2}\Big|_{s=0} = -1$$

∴ $f(t) = (-1+2t+e^{-t})u(t)$

c) $F(s) = \frac{1}{s^2(s^2+4)} = \frac{1/4}{s^2} + \frac{k_1}{s} + \frac{k_2}{s+j2} + \frac{k_2^*}{s-j2}$

$$k_1 = \frac{d}{ds}\left[\frac{1}{s^2+4}\right]_{s=0} = \frac{-2s}{(s^2+4)^2}\Big|_{s=0} = 0$$

$$k_2 = \frac{1}{s^2(s-j2)}\Big|_{s=-j2} = \frac{1}{(-4)(-j4)} = \frac{1}{16}\angle -90^\circ$$

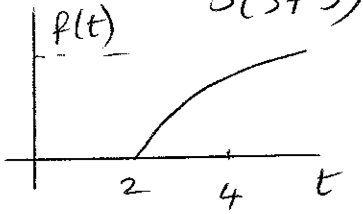
d) $F(S) = \frac{30}{(S+1)^2[(S+3)^2+4^2]} = \frac{3/2}{(S+1)^2} + \frac{K_1}{(S+1)} + \frac{K_2}{S+3+j4} + \frac{K_2^*}{S+3-j4}$

$K_1 = \frac{d}{dS}\left[\frac{30}{S^2+6S+25}\right]_{S=-1} = \left.\frac{-30(2S+6)}{(S^2+6S+25)^2}\right|_{S=-1} = \frac{(-30)(4)}{400} = \frac{-.3}{30}$

$K_2 = \left.\frac{30}{(S+1)^2(S+3-j4)}\right|_{S=-3-j4} = \frac{30}{(-2-j4)^2(-j8)} = .1875\angle{-36.8^\circ}$

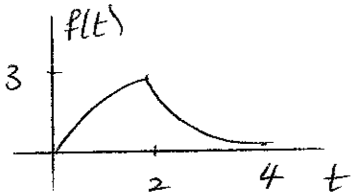
7.15

a) $F(S) = \frac{3e^{-2S}}{S(S+3)} = e^{-2S}\left(\frac{1}{S} + \frac{-1}{S+3}\right) \Rightarrow f(t) = (1-e^{-3(t-2)})u(t-2)$



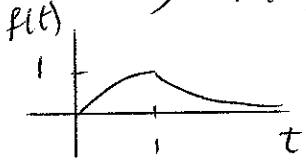
$1-e^{-3(t-2)}\Big|_{t=4} = .0025$

b) $F(S) = \left(\frac{1}{S} + \frac{-1}{S+3}\right)(1-e^{-2S}) \Rightarrow f(t) = (1-e^{-3t})u(t)$
$-(1-e^{-3(t-2)})u(t-2)$

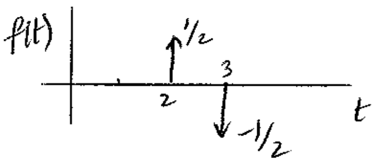


$\tau = \frac{1}{3}S;\ 1-e^{-3t}\Big|_{t=2} = .0025$

7.16 a) $f(t) = [1-e^{-t}]u(t) - [1-e^{-(t-1)}]u(t-1),\ \tau = 1S$



b) $\mathcal{L}^{-1}[1/2] = 1/2\,\delta(t) \Longrightarrow f(t) = 1/2\,[\delta(t-2) - \delta(t-3)]$



7.17

(i) a) $s^2Y(s) + 6sY(s) + 5Y(s) = 4X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{4}{s^2+6s+5} = \frac{4}{(s+5)(s+1)} = \frac{-1}{s+5} + \frac{1}{s+1}$$

$$h(t) = (-e^{-5t} + e^{-t})\,u(t)$$

b) $$S(s) = \frac{4}{s(s+5)(s+1)} = \frac{4/5}{s} + \frac{1/5}{s+5} + \frac{-1}{s+1}$$

$$s(t) = \underline{4} + \tfrac{1}{5}e^{-5t} - e^{-t}, \quad t>0$$

c) $$\frac{d}{dt}s(t) = -e^{-5t} + e^{-t}, \quad t>0$$

(ii) a) $s^2Y(s) + 6sY(s) + 5Y(s) = 4sX(s) + 6X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{4s+6}{s^2+6s+5} = \frac{7/2}{s+5} + \frac{1/2}{s+1}$$

$$h(t) = \left(\tfrac{7}{2}e^{-5t} + \tfrac{1}{2}e^{-t}\right)u(t)$$

b) $$S(s) = \frac{4s+6}{s(s+5)(s+1)} = \frac{6/5}{s} + \frac{-7/10}{s+5} + \frac{-1/2}{s+1}$$

$$s(t) = 6/5 - 7/10\,e^{-5t} - 1/2\,e^{-t}, \quad t>0$$

c) $$\frac{d}{dt}s(t) = 3.5e^{-5t} + \tfrac{1}{2}e^{-t}, \quad t>0$$

(iii) a) $s^2Y(s) + 4Y(s) = 2X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s^2+4} = \frac{2}{(s+2j)(s-2j)} = \frac{-1/2j}{s+2j} + \frac{1/2j}{s-2j}$$

$h(t) = \sin 2t\, u(t)$

b) $$S(s) = \frac{2}{s(s^2+4)} = \frac{1/2}{s} + \frac{-1/4}{s+2j} + \frac{-1/4}{s-2j}$$

$$S(t) = 1/2 - 1/4\, e^{-2jt} - 1/4\, e^{2jt}, \quad t>0$$

$$= 1/2 - 1/2 \cos 2t, \quad t>0$$

c) $\frac{d}{dt} S(t) = -\frac{1}{2}(-1)2\sin 2t = \sin 2t, \quad t>0$ ✓

(iv) a) $$H(s) = \frac{2s-6}{(s+1)[(s-1)^2+1]} = \frac{-8/5}{s+1} + \frac{K_1}{s-1+j} + \frac{K_1^*}{s-1-j}$$

$$K_1 = \frac{2s-6}{(s+1)(s-1-j)}\Bigg|_{s=1-j} = \frac{-4-j2}{(2-j)(-2j)} = 1 \angle -36.9^\circ$$

$$\therefore h(t) = -8/5\, e^{-t} + 2e^{-t}\cos(t+36.9^\circ)$$

b) $$S(s) = \frac{2s-6}{s(s+1)(s^2-2s+2)} = \frac{-3}{s} + \frac{8/5}{s+1} + \frac{K_1}{s-1+j} + \frac{K_1^*}{s-1-j}$$

$$K_1 = \frac{2s-6}{s(s+1)(s-1-j)}\Bigg|_{s=1-j} = .707 \angle 8.14^\circ$$

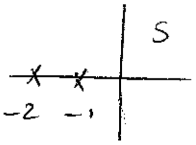
$$\therefore S(t) = -3 + 8/5\, e^{t} + 1.414 e^{t}\cos(t-8.14^\circ)$$

c) $h(t) = \frac{ds(t)}{dt} = \frac{-8}{5}e^{-t} + 1.414e^{t}\cos(t-8.14^\circ)$

$= -1.414e^{t}\sin(t-8.14^\circ)$

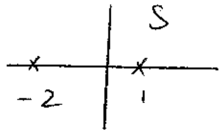
$= -8/5\, e^{-t} + 2e^{t}\cos(t-8.14^\circ+45^\circ) = -8/5\, e^{-t} + 2e^{t}\cos(t+36.9^\circ)$ ✓

7.18 a) $H(s) = \frac{1}{(s+1)(s+2)}$



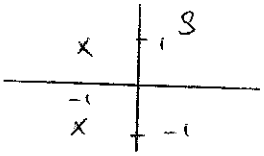
$, e^{-t}, e^{-2t}$

b) $H(s) = \frac{1}{(s-1)(s+2)}$



$, e^{t}, e^{-2t}$

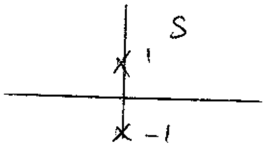
c) $H(s) = \frac{1}{(s+1)^2+1}$



$, e^{-t}\cos(t+\theta)$

d) Same as ⓐ

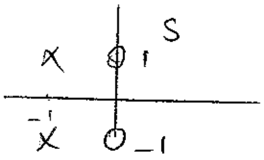
e) $H(s) = \frac{1}{s^2+1}$



$\cos(t+\theta)$

f) Same as ⓐ

g) $H(s) = \frac{s^2+1}{s^2+2s+2}$



$e^{-t}\cos(t+\theta)$

7.19

(i) a) poles of $H(s)$: $s=-1,-2$ ∴ stable

b) $H_i(s) = 1/2\,(s^2+3s+2)$

c) e^{-t}, e^{-2t}

(ii) a) poles of $H(s)$: $s=-1,-2$ ∴ stable

b) $H_i(s) = (s^2+3s+2)/(2s+6)$

c) e^{-t}, e^{-2t}

(iii) a) poles of $H(s)$: $s=-1, -1\mp j$ ∴ stable

b) $H_i(s) = \frac{1}{2}(s^3+3s^2+4s+2)$

c) $e^{-t}, e^{-t}e^{j}, e^{-t}e^{-j}$ or $e^{-t}, e^{-t}\cos(t+\theta)$

(iv) a) poles of $H(s)$: $s=-1, 1\mp j$ ∴ unstable

b) $H_i(s) = \frac{s^3-s^2+2}{2s-6}$

c) $e^{-t}, e^{t}e^{j}, e^{t}e^{-j}$, or $e^{-t}, e^{t}\cos(t+\theta)$

```
d=[1 3 4 2];
 roots(d)
pause
 d=[1 -1 0 2];
 roots(d)
```

7.20 use inverse convolution

$Y(s) = \frac{1}{s+b}$ $\quad X(s) = \frac{1}{s+a}$

$H(s) = \frac{Y(s)}{X(s)} = \frac{s+a}{s+b} = \frac{a}{s+b} + \frac{s}{s+b}$

$h(t) = ae^{-bt}u(t) + \frac{d}{dt}(e^{-bt}u(t)) = ae^{-bt}u(t) + -be^{-bt}u(t)$
$+ e^{-bt}\delta(t)$

$\therefore \; h(t) = \delta(t) + (a-b)e^{-bt}u(t)$

7.21

a) $\int_{-\infty}^{\infty} e^{-2t}u(t)e^{-st}\,dt = \int_{0}^{\infty} e^{-(2+s)t}\,dt = \frac{1}{s+2}$, $Re(s) > -2$

b) $\int_{-\infty}^{\infty} e^{-2t}u(t-4)e^{-st}\,dt = \int_{4}^{\infty} e^{-(s+2)t}\,dt = \frac{e^{-4s}e^{-8}}{s+2}$, $Re(s) > -2$

c) $-\int_{-\infty}^{\infty} e^{2t}u(-t)e^{-st}dt = -\int_{-\infty}^{0} e^{(2-s)t}dt = \frac{-1}{2-s} = \frac{1}{s-2}$

$Re(s)<2$

d) $-\int_{-\infty}^{\infty} e^{2t}u(-t-4)e^{-st}dt = -\int_{-\infty}^{-4} e^{2t}e^{-st}dt = -\int_{-\infty}^{-4} e^{(2-s)t}dt$

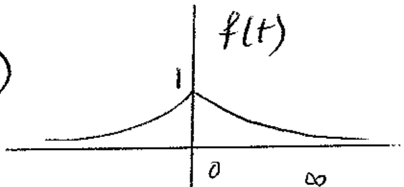
$= \frac{-1}{2-s}e^{(2-s)(-4)} = \frac{e^{4(s-2)}}{s-2}, \quad Re(s)<2$

e) $\int_{-\infty}^{\infty} e^{-2t}u(t+4)e^{-st}dt = \int_{-4}^{\infty} e^{-(s+2)t}dt = \frac{e^{(s+2)4}}{(s+2)}, \quad Re(s)>-2$

f) $-\int_{-\infty}^{\infty} e^{2t}u(-t+4)e^{-st}dt = -\int_{\infty}^{4} e^{(2-s)t}dt = \frac{1}{s-2}e^{(2-s)4},$

$Re(s)<2$

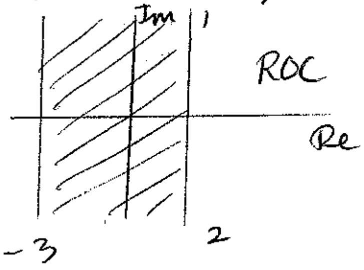
7.22 a)



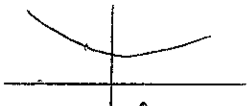
$F(s) = \int_{-\infty}^{0} e^{2t}e^{-st}dt + \int_{0}^{\infty} e^{-3t}e^{-st}dt = \frac{1}{-(s-2)} + \frac{1}{s+3}$

$Re(s)<2 \quad Re(s)>-3$

$F(s) = \frac{1}{-(s-2)} + \frac{1}{s+3} \quad -3<Re(s)<2$



b)



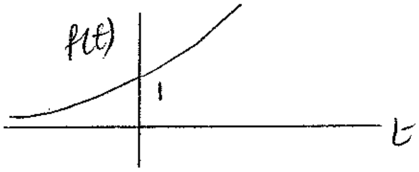
$F(s) = \int_{-\infty}^{0} e^{-2t}e^{-st}dt + \int_{0}^{\infty} e^{3t}e^{-st}dt$

$= \int_{-\infty}^{0} e^{-(2+s)t}dt + \int_{0}^{\infty} e^{(3-s)t}dt = \frac{-1}{s+2} + \frac{-1}{3-s}$

$Re(s)<-2 \wedge Re(s)>3 \neq \phi$

$\therefore F(s)$ does not exist

c)



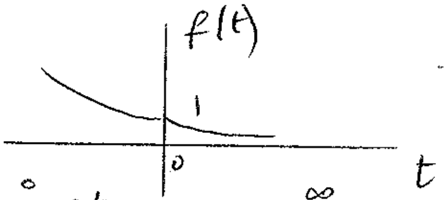
$$F(S) = \int_0^\infty e^{3t} e^{-st}\,dt + \int_{-\infty}^0 e^{2t} e^{-st}\,dt$$

$$= \frac{1}{S-3} + \frac{-1}{S-2}$$

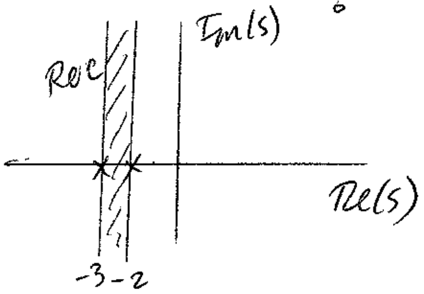
$Re(S) > 3$ $\quad$ $Re(S) < 2$

∴ $F(S)$ does not exist

d)



$$F(S) = \int_{-\infty}^0 e^{-2t} e^{-st}\,dt + \int_0^\infty e^{-3t} e^{-st}\,dt = \int_{-\infty}^0 e^{-(S+2)t}\,dt + \int_0^\infty e^{-(S+3)t}\,dt$$



$$= \frac{-1}{S+2} + \frac{1}{S+3}$$

$Re(S) < -2$ $\quad$ $Re(S) > -3$

$$\therefore\ F(S) = \frac{-1}{S+2} + \frac{1}{S+3}$$

$$-3 < Re(S) < -2$$

7.23 a) $F(S) = \int_{-5}^{4} e^{-3t} e^{-st}\,dt = \int_{-5}^{4} e^{-(S+3)t}\,dt$

$$= \frac{-1}{S+3} e^{-(S+3)t}\Big|_{-5}^{4} = \frac{1}{S+3}\left[e^{5(S+3)} - e^{-4(S+3)}\right]$$ ROC : entire S-plane

b) $f(t) = e^{-3t}\left[u(t+5) - u(t-4)\right] = e^{15} e^{-3(t+5)} u(t+5) - e^{-12} e^{-3(t-4)} u(t-4)$

$$F(S) = \frac{e^{15} e^{5S}}{S+3} - \frac{e^{-12} e^{-4S}}{S+3} = \frac{e^{15} e^{5S} - e^{-12} e^{-4S}}{S+3}$$

at $S = -3$, the numerator $= 0$, ∴ ROC is entire S-plane

c) $f(t) = e^{-3t}[u(4-t) - u(-5-t)] =$

$$e^{-12} e^{3(4-t)} u(4-t) - e^{15(-5-t)} u(-5-t)$$

$$F(s) = \frac{-e^{-12} e^{-4s}}{s+3} - \left(\frac{-e^{15} e^{5s}}{s+3}\right) = \frac{e^{15} e^{5s} - e^{-12} e^{-4s}}{s+3}$$

again at $s = -3$

numerator $= 0$, So ROC is the entire s-plane

7.24 (a) Left-sided function

$$F_b(s) = \frac{s+9}{s(s+1)} = \frac{9}{s} + \frac{-8}{s+1}$$

From (7.83), $\underline{f(t) = -9u(-t) + 8e^{-t}u(-t)}$

(b) Right-sided function

$\underline{f(t) = 9u(t) - 8e^{-t}u(t)}$

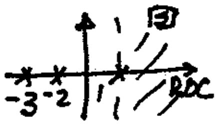
(c) $\frac{9}{s}$ left sided; $\frac{-8}{s+1}$ right sided

$\therefore\ \underline{f(t) = -9u(-t) - 8e^{-t}u(t)}$

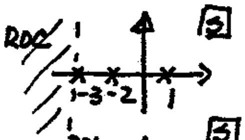
(d) (a) $f(\infty) = 0$ (b) $f(\infty) = 9$ (c) $f(\infty) = 0$

$$F_b(s) = \frac{2}{s-1} + \frac{4}{s+2} - \frac{1}{s+3}$$

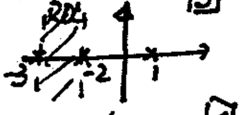
(a)



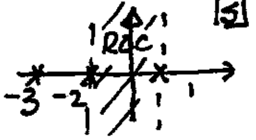
(b) $f(t) = [2e^{t} + 4e^{-2t} - e^{-3t}]u(t)$



$f(t) = [-2e^{t} - 4e^{-2t} + e^{-3t}]u(-t)$



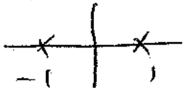
$f(t) = [-2e^{t} - 4e^{-2t}]u(-t) - e^{-3t}u(t)$



$f(t) = -2e^{t}u(-t) + [4e^{-2t} - e^{-3t}]u(t)$

7.25 $X(s) = \frac{s+3}{(s+1)(s-1)} = \frac{-1}{s+1} + \frac{2}{s-1}$

poles at $-1, 1$

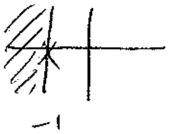


a) $Re(s) < -1$, $x(t) = e^{-t}u(-t) - 2e^{t}u(-t)$

$-1 \leq Re(s) \leq 1$, $x(t) = -e^{-t}u(t) - 2e^{t}u(-t)$

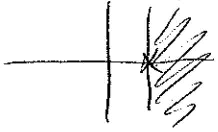
$Re(s) \geq 1$, $x(t) = -e^{-t}u(t) + 2e^{t}u(t)$

b) $Re(s) < -1$



$-1 \leq Re(s) \leq 1$

$Re(s) \geq 1$



c) for $Re(s) < -1$, $x(t)$ is noncausal

for $-1 \leq Re(s) \leq 1$, $x(t)$ is 2-sided

for $Re(s) \geq 1$, $x(t)$ is causal

d) for $Re(s) < -1$, $x(t)$ is Not BIBO Stable

for $-1 \leq Re(s) \leq 1$, $x(t)$ is BIBO Stable

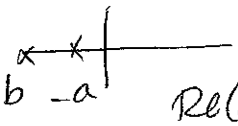
for $Re(s) \geq 1$, $x(t)$ is Not BIBO stable

e) for $Re(s) < -1$, Final value is 0

for $-1 \leq Re(s) \leq 1$, Final value is 0

for $Re(s) \geq 1$, Final value does not exist

7.26 a) $h(t)$ causal $\Rightarrow$ both functions are right-Sided

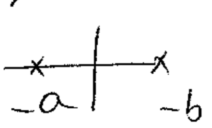


$Re(a) > 0$ & $Re(b) > 0$

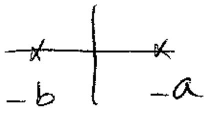
b) 2 sided $\Rightarrow$ one is left-Sided & one is right-Sided

either $Re(b) < 0$ and $Re(a) > 0$

or $Re(a) < 0 < Re(b)$

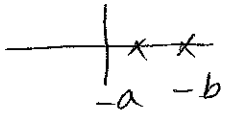


or



c) Both left-Sided

$Re(a) < 0$ & $Re(b) < 0$

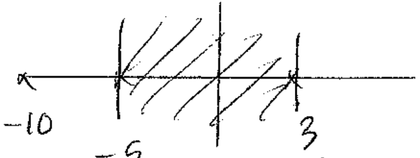


7.27

$$H(s) = \frac{s+1}{(s+4)(s+2)} = \underset{\substack{\swarrow \\ \text{Right}}}{\frac{3/2}{s+4}} + \underset{\substack{\swarrow \\ \text{Left}}}{\frac{-1/2}{s+2}}$$

$$\therefore \quad h(t) = \tfrac{3}{2} e^{-4t} u(t) + \tfrac{1}{2} e^{-2t} u(-t)$$

7.28 $H(s) = \frac{1}{(s+10)(s+5)(s-3)}$ Poles at $-10, -5, 3$



Converges to the right of -10 & -5 $\Rightarrow \therefore$ these are right-Sided time functions

Converges to the left of 3 $\Rightarrow \therefore$ This is left-Sided time function

7.29

a) $x(t) = e^{5t}u(t)$, $X(s) = \frac{1}{s-5}$, $Re(s) > 5$

$h(t) = u(t)$, $H(s) = \frac{1}{s}$, $Re(s) > 0$

$Y(s) = H(s)X(s) = \frac{1}{s(s-5)}$, $Re(s) > 5$

$Y(s) = \frac{-1/5}{s} + \frac{1/5}{s-5} \Rightarrow y(t) = -1/5\, u(t) + 1/5\, e^{5t}u(t)$

$= 1/5 \left[e^{5t} - 1 \right] u(t)$

b) $x(t) = e^{t}u(-t)$

$X(s) = \frac{-1}{s-1}$, $Re(s) < 1$

$h(t) = 2u(1-t)$, $H(s) = \frac{-2e^{-s}}{s}$, $Re(s) < 0$

$Y(s) = \frac{2e^{-s}}{s(s-1)} = e^{-s}Z(s)$ where $Z(s) = \frac{2}{s(s-1)} = \frac{-2}{s} + \frac{2}{s-1}$

$z(t) = 2u(-t) - 2e^{t}u(-t)$. Now $y(t) = z(t-1)$ by delay property

so $y(t) = 2(1 - e^{t-1})u(-(t-1)) = 2(1 - e^{t-1})u(t-1)$

7.30 $h(t) = e^{t}u(t)$

a) $H(s) = \frac{1}{s-1}$, $Re(s) > 1$

NOT BIBO Stable

b) $w(t) = x(t) - Ay(t)$, $W(s) = X(s) - AY(s)$

$y(t) = w(t) * h(t)$, $Y(s) = W(s)H(s)$

$$\frac{Y(s)}{H(s)} = W(s) = X(s) - AY(s)$$

$$Y(s)\left[\frac{1}{H(s)} + A\right] = X(s) \quad , \quad \frac{Y(s)}{X(s)} = \frac{H(s)}{1+AH(s)}$$

c) For stability, examine $\frac{H(s)}{1+AH(s)} = \frac{\frac{1}{s-1}}{1+\frac{A}{s-1}} \quad \frac{\frac{1}{s-1}}{\frac{s+A-1}{s-1}}$

$$= \frac{1}{s+A-1}$$

As long as $A-1>0$, then the pole at $A-1$ will be in the left half-plane and the system will be stable.

$\therefore$ we require $A>1$

CHAPTER 8

8.1. $L\frac{di}{dt} + Ri = v_i \Rightarrow \frac{di}{dt} = -\frac{R}{L} i + \frac{1}{L} v_i \; , v_R = Ri$

(a) $x_1 = i \; , u(t) = v_i \; , y = v_R$

$$\dot{x} = -\frac{R}{L} x + \frac{1}{L} u$$
$$y = Rx$$

(b) $x = v_R = Ri \; , i = \frac{1}{R} x$

$$\frac{1}{R} \dot{x} = -\frac{1}{L} x + \frac{1}{L} u \Rightarrow \begin{array}{l} \dot{x} = -\frac{R}{L} x + \frac{R}{L} u \\ y = x \end{array}$$

8.2. (a) $v_i = L\frac{di}{dt} + v_c \Rightarrow \frac{di}{dt} = -\frac{1}{L} v_c + \frac{1}{L} v_c$

$$v_c = \frac{1}{C}\int_0^t i \, d\tau \Rightarrow \frac{dv_c}{dt} = \frac{1}{C} i$$

$$\therefore \begin{bmatrix} di/dt \\ dv_c/dt \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} 1/L \\ 0 \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} v_i \Rightarrow \underline{\dot{x}} = \begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} u$$

$$v_c = [0 \quad 1] \begin{bmatrix} i \\ v_c \end{bmatrix} \qquad \Rightarrow y = [0 \quad 1] \underline{x}$$

(b) Same state equation, with

$$i = [1 \quad 0] \begin{bmatrix} i \\ v_c \end{bmatrix} \qquad \Rightarrow y = [1 \quad 0] \underline{x}$$

8.3. (a) $x = y \quad ; \quad \dot{x} = -x + u$

(b) $x_1 = y$, $x_2 = \dot{y}$; $\quad \underline{\dot{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u \; ; \; y = [1 \quad 0] \underline{x}$

(c) $x_1 = y$, $x_2 = \dot{y}$; $\quad \underline{\dot{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{5} & -\frac{4}{5} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ \frac{3}{5} \end{bmatrix} u \; ; \quad y = [1 \quad 0] \underline{x}$

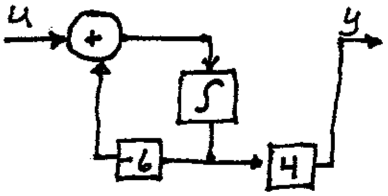
(d) $x_1 = y_1 \; , \; x_2 = \dot{y}_1 \; ; \; x_2 = y_2$

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -6 & -5 & 1 \\ -8 & 0 & -1 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 & 0 \\ 4 & -1 \\ 1 & -3 \end{bmatrix} \underline{u} \; ; \; \underline{y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{x}$$

8.3. (e) $x_1 = y_1$, $x_2 = \dot{y}_1$, $x_3 = y_2$, $x_4 = \dot{y}_2$
(cont)

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -16 & 9 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 & 0 \\ 1 & -5 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} \underline{u} \; ; \; y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underline{x}$$

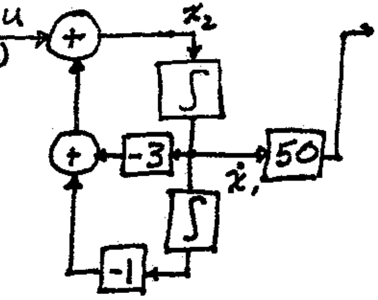
8.4. (a)



(b) $\dot{x} = -6x + u$

$\dot{y} + 6y = 4u$

(d) (a)



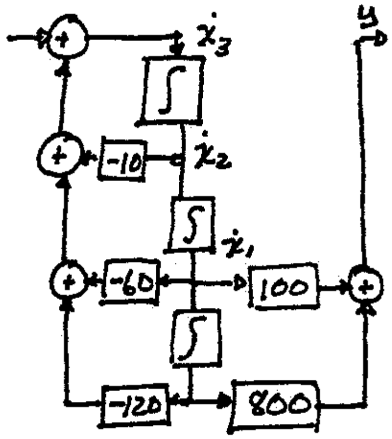
(b)

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [0 \quad 50] \underline{x}$$

(c) $\ddot{y} + 3\dot{y} + y = 50\dot{u}$

(e) (a)



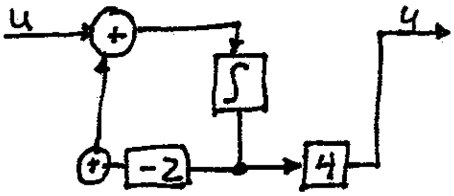
(b)

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -120 & -60 & -10 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \underline{u}$$

$$y = [800 \; 100 \; 0] \underline{x}$$

(c) $\dddot{y} + 10\ddot{y} + 60\dot{y} + 120y = 100\dot{u} + 800u$

8.5. (a) $\dot{y} = -2y + 4u$



(b) $\dot{x} = -2x + u$

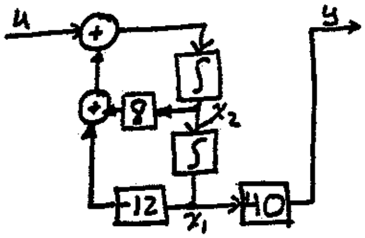
$y = 4x$

(c) $\dfrac{Y(s)}{U(s)} = \dfrac{4}{s+2}$

8.5 (d)
(cont)

```
A=[-2];B=[1];C=[4];D=0;
[n,d]=ss2tf(A,B,C,D), pause,
A=[0 1;-12 8];B=[0;1];C=[40 0];D=0;
[n,d]=ss2tf(A,B,C,D), pause
A=[0 1 0;0 0 1;-15 -10 -20];B=[0;0;1];C=[50 0 0];D=0;
[n,d]=ss2tf(A,B,C,D)
```

(e)
(a)

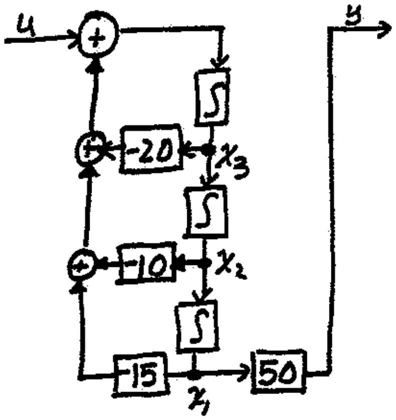


(b) $\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -12 & 8 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = [40 \quad 0] \underline{x}$

(c) $H(s) = \dfrac{40}{s^2 - 8s + 12}$

(f)(a)



(b) $\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -15 & -10 & -20 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$

$y = [50 \quad 0 \quad 0] \underline{x}$

(C) $H(s) = \dfrac{50}{s^3 + 20s^2 + 10s + 15}$

8.6. (a) $\dot{x} = -3x + 6u$

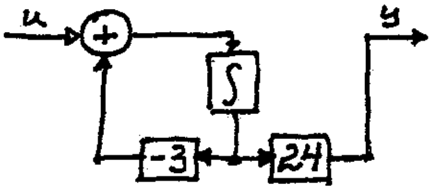
$y = 4x$

(b) $sI - A = s + 3$

(c)

```
A=[-3];B=[6];C=[4];D=0;
[n,d]=ss2tf(A,B,C,D)
```

(d)



8.7. (a) $\dot{\underline{x}} = \begin{bmatrix} -6 & 2 \\ -5 & 1 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} y$

$y = [5 \quad 6] \underline{x}$

(b) $sI - A = \begin{bmatrix} s+6 & -2 \\ 5 & s-1 \end{bmatrix}$; $|sI - A| = s^2 + 5s - 6 + 10 = s^2 + 5s + 4$

$\text{Adj}(sI - A) = \begin{bmatrix} s-1 & 2 \\ -5 & s+6 \end{bmatrix}$

8.7. (b) $H(s) = [5 \quad 6]\frac{1}{|sI-A|}\begin{bmatrix} s-1 & 2 \\ -5 & s+6 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \frac{1}{|sI-A|}[5 \quad 6]\begin{bmatrix} s+5 \\ 3s+13 \end{bmatrix}$

(cont)

$$= \frac{23s+103}{s^2+5s+4}$$

(c)

```
A=[-6 2;-5 1];B=[1;3];C=[5 6];D=0;
 [n,d]=ss2tf(A,B,C,D), pause
A=[0 1;-4 -5];B=[0;1];C=[103 23];D=0;
 [n,d]=ss2tf(A,B,C,D)
```

8.8. (a) $\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -4 & -3 \end{bmatrix}\underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$ (b) $H_p(s) = \frac{5s+2}{s^2+3s+4} = \frac{Y(s)}{M(s)}$

$y = [2 \quad 5]\underline{x}$

(c) $\ddot{y} + 3\dot{y} + 4y = 5\dot{m} + 2m$

(d) $\dot{x}_3 = -4x_3 + e$ (e) $H_c(s) = \frac{2}{s+4} = \frac{M(s)}{E(s)}$

$m = 2x_3$

(f) $\dot{m} + 4m = 2e$

(g) $\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -3 & 2 \\ -2 & -5 & -4 \end{bmatrix}\underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u$

$y = [2 \quad 5 \quad 0]\underline{x}$

(h) $sI-A = \begin{bmatrix} s & -1 & 0 \\ 4 & s+3 & -2 \\ 2 & 5 & s+4 \end{bmatrix}$; $|sI-A| = s(s^2+7s+12+10)+(4s+16+4)$

$= \underline{s^3+7s^2+26s+20}$

$$\text{cof}(sI-A) = \begin{bmatrix} s^2+7s+12 & -4s-20 & -2s+14 \\ s+4 & s^2+4s & -5s+2 \\ 2 & 2s & s^2+3s+4 \end{bmatrix}$$

$$\text{Adj } sI-A = \begin{bmatrix} s^2+7s+12 & s+4 & 2 \\ -4s-20 & s^2+4s & 2s \\ -2s+14 & -5s+2 & s^2+3s+4 \end{bmatrix}$$

$$\therefore H(s) = C[sI-A]^{-1}B = [2 \quad 5 \quad 0]\frac{1}{|sI-A|}\begin{bmatrix} - & - & 2 \\ - & - & 2s \\ - & - & s^2+3s+4 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{|sI-A|}[2 \quad 5 \quad 0]\begin{bmatrix} 2 \\ 2s \\ - \end{bmatrix} = \frac{10s+4}{s^3+7s^2+26s+20}$$

(i) $\dddot{y} + 7\ddot{y} + 26\dot{y} + 20y = 10\dot{u} + 4u$

(j)

```
A=[0 1 0;-4 -3 2;-2 -5 -4];B=[0;0;1];C=[2 5 0];D=0;
 [n,d]=ss2tf(A,B,C,D)
```

(k) $H_c H_p = \frac{5s+2}{s^2+3s+4} \cdot \frac{2}{s+4} = \frac{10s+4}{s^3+7s^2+16s+16}$

8.9. (a) $\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}\underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} m$ (b) $H_p(s) = \dfrac{2s+5}{s^2+4s+3}$

(c) $\ddot{y} + 4\dot{y} + 3y = 2\dot{m} + 5m$

(d) $m = 2e$ (e) $H_c = 2$ (f) $m = 2e$

(g) $\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -8 & -13 \end{bmatrix}\underline{x} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$y = [2 \quad 5]\,\underline{x}$

(h) $sI - A = \begin{bmatrix} s & -1 \\ 8 & s+13 \end{bmatrix}$; $|sI-A| = s^2 + 13s + 8$

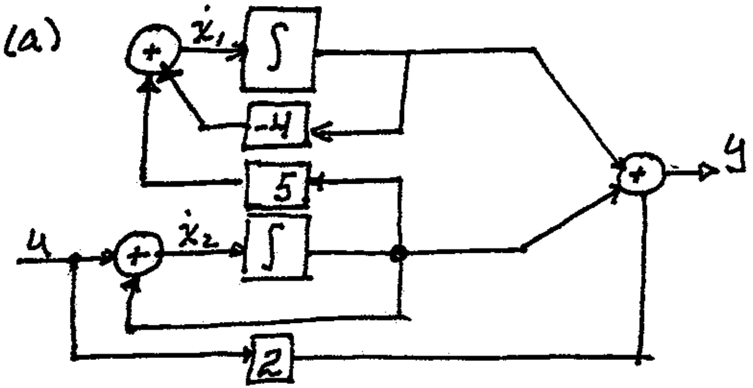
$\mathrm{Adj}\,(sI-A) = \begin{bmatrix} s+13 & 1 \\ -8 & s \end{bmatrix}$

$H(s) = [2 \quad 5]\dfrac{1}{|sI-A|} = \begin{bmatrix} - & 1 \\ - & s \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \dfrac{1}{|sI-A|}[2 \quad 5]\begin{bmatrix} 2 \\ 2s \end{bmatrix}$

$= \dfrac{10s+4}{s^2+13s+8}$

(i) $\ddot{y} + 13\dot{y} + 8y = 5\dot{u} + 2u$

8.10 (a)



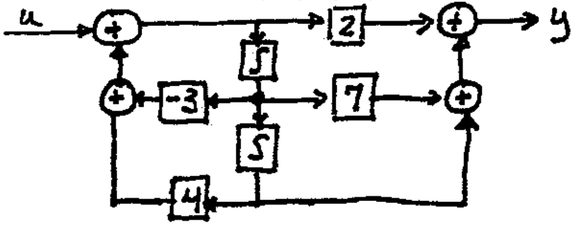
(b) $|sI - A| = \begin{vmatrix} s+4 & -5 \\ 0 & s-1 \end{vmatrix} = s^2 + 3s - 4$

$H(s) = C(sI-A)^{-1}B + D = \dfrac{1}{|sI-A|}[1 \quad 1]\begin{bmatrix} s-1 & 5 \\ 0 & s+4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2$

$= \dfrac{1}{|sI-A|}[1 \quad 1]\begin{bmatrix} 5 \\ s+4 \end{bmatrix} + 2 = \dfrac{s+9}{s^2+3s-4} + 2 = \dfrac{2s^2+7s+1}{s^2+3s-4}$

(c)
```
A=[0 1;-4 -5];B=[0;1];C=[103 23];D=0;
[n,d]=ss2tf(A,B,C,D)
```

(d)



8.10. (cont) (e) $\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ 4 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$; $|sI-A| = \begin{vmatrix} s & -1 \\ -4 & s+3 \end{vmatrix} = s^2+3s-4$

$y = [9 \quad 1] \underline{x}$

(f) $H(s) = \frac{1}{|sI-A|} [9 \quad 1] \begin{bmatrix} s+3 & 1 \\ 4 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2 = \frac{1}{|sI-A|} [9 \quad 1] \begin{bmatrix} 1 \\ s \end{bmatrix} + 2$

$= \frac{s+9}{s^2+3s-4} + 2 = \frac{2s^2+7s+1}{s^2+3s-4}$

8.11. (a) From Problem 8.1

$\dot{x} = -\frac{R}{L} x + \frac{R}{L} u$

$y = x$

(b) $H(s) = C(sI-A)^{-1}B = (1)\left(\frac{1}{s+R/L}\right)\left(\frac{R}{L}\right)$

$= \underline{\frac{R/L}{s+R/L}}$

(c) $\frac{V_R(s)}{V_i(s)} = \frac{R}{sL+R} = \underline{\frac{R/L}{s+R/L}} = H(s)$

8.12. (a) From Prob 8.1: $\dot{x} = -\frac{R}{L} x + \frac{1}{L} u$

$y = Rx$

(b) $H(s) = C(sI-A)^{-1} B = R\left(\frac{1}{s+R/L}\right)\frac{1}{L} = \frac{R/L}{s+R/L}$

(c) $\frac{V_R(s)}{V_i(s)} = \frac{R}{sL+R} = \frac{R/L}{s+R/L}$

8.13. (a) See problem 8.2.

(b) $|sI-A| = \begin{vmatrix} s & \frac{1}{L} \\ -\frac{1}{C} & s \end{vmatrix} = s^2 + \frac{1}{LC}$

$H(s) = C(sI-A)^{-1}B = \frac{1}{|sI-A|} [0 \quad 1] \begin{bmatrix} s & -\frac{1}{L} \\ \frac{1}{C} & s \end{bmatrix} \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} = \frac{1}{|sI-A|} [\frac{1}{C} \quad s] \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$

$= \frac{\frac{1}{LC}}{s^2+\frac{1}{LC}}$

(c) $H(s) = \frac{\frac{1}{sC}}{sL+\frac{1}{sC}} = \frac{1}{s^2LC+1} = \frac{\frac{1}{LC}}{s^2+\frac{1}{LC}}$

8.14. (a) See Problem 8.2.

(b) From Problem 8.13(b)

$H(s) = \frac{1}{|sI-A|} [1 \quad 0] \begin{bmatrix} s & -\frac{1}{L} \\ \frac{1}{C} & s \end{bmatrix} \begin{bmatrix} 1/L \\ 0 \end{bmatrix} = \frac{1}{|sI-A|} [s \quad -\frac{1}{L}] \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$

$= \frac{\frac{1}{L}s}{s^2+\frac{1}{LC}}$

8.14.(c) (cont) $Z(s) = Ls + \frac{1}{Cs}$

$$\frac{I(s)}{V_i(s)} = \frac{1}{Z(s)} = \frac{1}{Ls+\frac{1}{Cs}} = \frac{Cs}{LCs^2+1} = \frac{\frac{1}{L}s}{s^2+\frac{1}{LC}}$$

8.15.(a) $\dot{x} = -3x + 6u$

$y = 4x$

(b) $\Phi(s) = (sI-A)^{-1} = \frac{1}{s+3}$; $\Phi(t) = \underline{e^{-3t}}$

(c) $y_c(t) = C\Phi(t)x(0) = \underline{8e^{-3t}}$, $t > 0$

(d) $X(s) = \Phi(s)BU(s) = \frac{1}{s+3}\cdot 6 \cdot \frac{1}{s} = \frac{6}{s(s+3)} = \frac{2}{s} + \frac{-2}{s+3}$

$\therefore x(t) = 2(1-e^{-3t}),\ t>0 \Rightarrow y_p(t) = 4x(t) = \underline{8(1-e^{-3t})},\ t>0$

(e) From Problem 8.6, $H(s) = \frac{24}{s+3}$

$\therefore Y_p(s) = H(s)\cdot\frac{1}{s} = \frac{24}{s(s+3)} = \frac{8}{s} - \frac{8}{s+3} \Rightarrow y_p(t) = \underline{8(1-e^{-3t})},\ t>0$

(f) $y(t) = y_c(t) + y_p(t) = 8e^{-3t} + 8 - 8e^{-3t} = \underline{8}$, $t > 0$

8.16.(a) From Problem 8.7, $\dot{\underline{x}} = \begin{bmatrix} -6 & 2 \\ -5 & 1 \end{bmatrix}\underline{x} + \begin{bmatrix} 1 \\ 3 \end{bmatrix}u$

$y = [5 \quad 6]\underline{x}$

(b) $$\Phi(s) = (sI-A)^{-1} = \begin{bmatrix} \frac{s-1}{(s+1)(s+4)} & \frac{2}{(s+1)(s+4)} \\ \frac{-5}{(s+1)(s+4)} & \frac{s+6}{(s+1)(s+4)} \end{bmatrix} = \begin{bmatrix} \frac{-2/3}{s+1} + \frac{5/3}{s+4} & \frac{2/3}{s+1} + \frac{-2/3}{s+4} \\ \frac{-5/3}{s+1} + \frac{5/3}{s+4} & \frac{5/3}{s+1} + \frac{-2/3}{s+4} \end{bmatrix}$$

$$\therefore \Phi(t) = \begin{bmatrix} -\frac{2}{3}e^{-t} + \frac{5}{3}e^{-4t} & \frac{2}{3}e^{-t} - \frac{2}{3}e^{-4t} \\ -\frac{5}{3}e^{-t} + \frac{5}{3}e^{-4t} & \frac{5}{3}e^{-t} - \frac{2}{3}e^{-4t} \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \underline{x}(0)$$

(c) $$\underline{x}(t) = \Phi(t)\underline{x}(0) = \begin{bmatrix} -\frac{2}{3}e^{-t} + \frac{5}{3}e^{-4t} \\ -\frac{5}{3}e^{-t} + \frac{5}{3}e^{-4t} \end{bmatrix}$$

$y_c(t) = C\underline{x}(t) = [5 \quad 6]\underline{x}(t) = \underline{-\frac{40}{3}e^{-t} + \frac{55}{3}e^{-4t}}$, $t>0$

(d) $$\underline{X}(s) = \Phi(s)BU(s) = \Phi(s)\begin{bmatrix} 1 \\ 3 \end{bmatrix}\frac{1}{s} = \begin{bmatrix} \frac{s+5}{s(s+1)(s+4)} \\ \frac{3s+13}{s(s+1)(s+4)} \end{bmatrix} = \begin{bmatrix} \frac{5/4}{s} + \frac{-4/3}{s+1} + \frac{1/12}{s+4} \\ \frac{13/4}{s} + \frac{-10/3}{s+1} + \frac{1/12}{s+4} \end{bmatrix}$$

$\therefore y_p(t) = [5 \quad 6]\underline{x}(t) = \underline{\frac{103}{4} - \frac{80}{3}e^{-t} + \frac{11}{12}e^{-4t}}$, $t>0$

(e) From Problem 8.7,

$Y_p(s) = H(s)\cdot\frac{1}{s} = \frac{23s+103}{s(s+1)(s+4)} = \frac{103/4}{s} + \frac{-80/3}{s+1} + \frac{11/12}{s+4}$

$\therefore y_p(t) = \frac{103}{4} - \frac{80}{3}e^{-t} + \frac{11}{12}e^{-4t}$, $t>0$

8.16. (f) $y(t) = y_c(t) + y_p(t) = \underline{\frac{103}{4} - \frac{120}{3}e^{-t} + \frac{231}{12}e^{-4t}}, \ t > 0$

8.17. (a) From Problem 8.10,

$$\Phi(s) = (sI - A)^{-1} = \begin{bmatrix} \frac{s-1}{(s-1)(s+4)} & \frac{5}{(s-1)(s+4)} \\ 0 & \frac{s+4}{(s-1)(s+4)} \end{bmatrix} = \begin{bmatrix} \frac{1}{s+4} & \frac{1}{s-1} + \frac{-1}{s+4} \\ 0 & \frac{1}{s-1} \end{bmatrix}$$

$$\therefore \Phi(t) = \begin{bmatrix} e^{-4t} & e^{t} - e^{-4t} \\ 0 & e^{t} \end{bmatrix}$$

$$x(t) = \Phi(t)\underline{x}(0) = \Phi(t)\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^{-4t} \\ 0 \end{bmatrix}$$

$$\therefore y_c(t) = C\underline{x}(t) = [1 \quad 1]\underline{x}(t) = \underline{e^{-4t}}, \ t > 0$$

(b) $$\Phi(t)BU(s) = \Phi(s)\begin{bmatrix} 0 \\ 1 \end{bmatrix}\frac{1}{s} = \begin{bmatrix} \frac{5}{s(s-1)(s+4)} \\ \frac{1}{s(s-1)} \end{bmatrix} = \begin{bmatrix} \frac{-5/4}{s} + \frac{1}{s-1} + \frac{1/4}{s+4} \\ -\frac{1}{s} + \frac{1}{s-1} \end{bmatrix}$$

$$\therefore \underline{x}(t) = \begin{bmatrix} -\frac{5}{4} + e^{t} + \frac{1}{4}e^{-4t} \\ -1 + e^{t} \end{bmatrix}$$

$$y_p(t) = [1 \quad 1]\underline{x}(t) + 2 = -\frac{9}{4} + 2e^{t} + \frac{1}{4}e^{-4t} + 2$$
$$= \underline{-\frac{1}{4} + 2e^{t} + \frac{1}{4}e^{-4t}}, \ t > 0$$

(c) From Problem 8.10,

$$Y_p(s) = H(s)\cdot\frac{1}{s} = \frac{2s^2 + 7s + 1}{s(s-1)(s+4)} = \frac{-1/4}{s} + \frac{10/5}{s-1} + \frac{1/4}{s+4}$$

$$\therefore y_p(t) = \underline{-\frac{1}{4} + 2e^{t} + \frac{1}{4}e^{-4t}}, \ t > 0$$

(d) $\ddot{y} + 3\dot{y} - 4y = 2\ddot{u} + 7\dot{u} + 1$

(e) $\dot{u} = \ddot{u} = 0, \ \dot{y} = 2e^{t} - e^{-4t}$

$\therefore (2e^{t} + 4e^{-4t}) + (6e^{t} - 3e^{-4t}) - (-1 + 8e^{t} + e^{-4t}) = 1$

$\therefore 1 = 1$

(f) $y(t) = y_c(t) + y_p(t) = -\frac{1}{4} + 2e^{t} + \frac{5}{4}e^{-4t}, \ t > 0$

$y(0) = C\underline{x}(0) + 2u(0) = [1 \quad 1]\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 = 3$ ✓

8.18. $(sI - A) = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}, \ |sI - A| = s^2$

$$\Phi(s) = (sI - A)^{-1} = \begin{bmatrix} 1/s & 1/s^2 \\ 0 & 1/s \end{bmatrix} \Rightarrow \Phi(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

8.19.(a) $(sI-A) = \begin{bmatrix} s & 0 \\ -1 & s \end{bmatrix}$, $|sI-A| = s^2$

$$\Phi(s) = (sI-A)^{-1} = \begin{bmatrix} \frac{1}{s} & 0 \\ \frac{1}{s^2} & \frac{1}{s} \end{bmatrix} \Rightarrow \underline{\Phi(t) = \begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}}$$

(b) $\Phi(t) = I + At$; since $A^2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\therefore \Phi(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ t & 0 \end{bmatrix} = \underline{\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}}$$

(c) $\underline{x}(t) = \Phi(t)\,\underline{x}(0) = \begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2+t \end{bmatrix}$

$$y(t) = C\underline{x}(t) = [0 \quad 1]\begin{bmatrix} 1 \\ 2+t \end{bmatrix} = \underline{2+t}, \ t>0$$

(d) $\dot{\underline{x}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = A\underline{x} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2+t \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(e) $\underline{X}(s) = \Phi(s)BU(s) = \Phi(s)\begin{bmatrix} 1 \\ 1 \end{bmatrix}\frac{1}{s} = \begin{bmatrix} \frac{1}{s^2} \\ \frac{1}{s^2} + \frac{1}{s^3} \end{bmatrix}$

$$\therefore x(t) = \begin{bmatrix} t \\ t + t^2/2 \end{bmatrix} \Rightarrow y(t) = C\underline{x}(t) = [0 \quad 1]\,\underline{x}(t) = \underline{t + \frac{t^2}{2}}, \ t>0$$

(f) $H(s) = C(sI-A)^{-1}B = [0 \quad 1]\begin{bmatrix} \frac{1}{s} & 0 \\ \frac{1}{s^2} & \frac{1}{s} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \left[\frac{1}{s^2} \quad \frac{1}{s}\right]\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$H(s) = \frac{1}{s} + \frac{1}{s^2}$$

$$Y(s) = H(s)U(s) = \frac{1}{s^2} + \frac{1}{s^3} \Rightarrow y(t) = \underline{t + \frac{t^2}{2}}, \ t>0$$

$$y_t(t) = \underline{2 + 2t + \frac{t^2}{2}}, \ t>0$$

8.20. (a) $\dot{x} = -2x + 4$

$$\Phi(s) = (sI - A)^{-1} = \frac{1}{s+2} \ ; \ \Phi(t) = \underline{e^{-2t}}, \ t>0$$

(b) $\Phi(t) = I + At + \frac{A^2t^2}{2!} + \cdots] = 1 + (-2t) + \frac{(-2t)^2}{2!} + \cdots = \underline{e^{-2t}}, \ t>0$

(c) $x_c(t) = \Phi(t)x(0) = \underline{e^{-2t}}, \ t>0$

(d) $\dot{x}_c = -2x_c$

$$-2e^{-2t} = - \ e^{-2t}$$

(e) $X_p(s) = \frac{20}{s+2} \Rightarrow x_p(t) = 20e^{-2t}, \ t>0$

8.21. (a) From Problem 8.10(b), $H(s) = \dfrac{2s^2+2s+1}{s^2+3s-4}$

8.21.(b) (cont) Let $Q = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$, $\therefore P = Q^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

$$A_v = P^{-1}AP = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -4 & 5 \\ 0 & 1 \end{bmatrix} P = \begin{bmatrix} -8 & 11 \\ -4 & 6 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -19 & 30 \\ -10 & 16 \end{bmatrix}$$

$$B_v = P^{-1}B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C_v = CP = [1 \quad 1]\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = [0 \quad 1]; \quad D_v = D = 2$$

$$\therefore \dot{\underline{v}} = \begin{bmatrix} -19 & 30 \\ -10 & 16 \end{bmatrix}\underline{v} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u$$

$$y = [0 \quad 1]\underline{v} + 2u$$

(c)

```
A=[-4 5;0 1];B=[0;1];C=[1 1];D=2;Q=[2 1;1 1];
P=inv(Q);
Av=Q*A*P
Bv=Q*B
Cv=C*P
Dv=D
pause
[n,d]=ss2tf(Av,Bv,Cv,Dv)
```

(d) $|sI - A_v| = \begin{vmatrix} s+19 & -30 \\ 10 & s-16 \end{vmatrix} = s^2+3s-304+200 = s^2+3s-4$ ✓

$$C_v(sI-A_v)^{-1}B_v = [0 \quad 1]\frac{1}{|sI-A_v|}\begin{bmatrix} s-16 & 30 \\ -10 & s+19 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{|sI-A_v|}[-10 \quad s+19]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{s+9}{s^2+3s-4}$$

$$\therefore H(s) = \frac{s+9}{s^2+3s-4} + 2 = \frac{2s^2+7s+1}{s^2+3s-4}$$ ✓

(e) See (c)

(f) $|sI-A| = |sI-A_v| = s^2+3s-4 = (s-1)(s+4) = (s-\lambda_1)(s-\lambda_2)$ ✓

$|A| = -4$; $|A_v| = -304+300 = -4 = \lambda_1\lambda_2 = (1)(-4) = -4$ ✓

$\text{tr}\, A = -4+1 = -3$; $\text{tr}\, A_v = -19+16 = -3 = \lambda_1+\lambda_2 = 1-4 = -3$ ✓

8.22.(a) $sI - A = \begin{bmatrix} s & 0 \\ -1 & s \end{bmatrix}$; $\det sI-A = s^2$

$$(sI-A)^{-1} = \begin{bmatrix} \frac{1}{s} & 0 \\ \frac{1}{s^2} & \frac{1}{s} \end{bmatrix}$$

$$H(s) = C(sI-A)^{-1}B = [0 \quad 1]\begin{bmatrix} \frac{1}{s} & 0 \\ \frac{1}{s^2} & \frac{1}{s} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = [0 \quad 1]\begin{bmatrix} \frac{1}{s} \\ \frac{1}{s}+\frac{1}{s^2} \end{bmatrix}$$

$$\therefore H(s) = \frac{1}{s} + \frac{1}{s^2}$$

(b) Choose $Q = P^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, $P = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

8.22. (cont) $\therefore A_v = P^{-1}AP = \begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\2 & 0\end{bmatrix}\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}$

$$= \begin{bmatrix}2 & -1\\4 & -2\end{bmatrix}$$

$B_v = P^{-1}B = \begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$; $C_v = CP = [0 \quad 1]\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix} = [-1 \quad 1]$

$$\dot{\underline{v}} = \begin{bmatrix}2 & -1\\4 & -2\end{bmatrix}\underline{v} + \begin{bmatrix}2\\3\end{bmatrix}u$$

$$y = [-1 \quad 1]\underline{v}$$

(d) $sI - A_v = \begin{bmatrix}s-2 & 1\\-4 & s+2\end{bmatrix}$; $|sI - A_v| = s^2 - 4 + 4 = s^2$

$$(sI - A_v)^{-1} = \begin{bmatrix}(s+2)/s^2 & -1/s^2\\4/s^2 & (s-2)/s^2\end{bmatrix}$$

$$H(s) = [-1 \quad 1]\left[\quad \downarrow \quad\right]\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}\frac{-s+2}{s^2} & \frac{s-1}{s^2}\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix} = \frac{-2s+4+3s-3}{s^2}$$

$$= \frac{s+1}{s^2} = \underline{\frac{1}{s} + \frac{1}{s^2}}$$

(f) $s^2 = 0 \Rightarrow \lambda_1 = 0,\ \lambda_2 = 0$

(8.74) $s^2 = s^2$

(8.75) $\det A = 0 = \det A_v = \lambda_1\lambda_2$

(8.76) $\mathrm{tr}\, A = 0 = \mathrm{tr}\, A_v = 0 = \lambda_1 + \lambda_2$

(c)(e)

```
A=[0 0;1 0];B=[1;1];C=[0 1];D=0;Q=[1 1;1 2];
P=inv(Q);
Av=Q*A*P
Bv=Q*B
Cv=C*P
Dv=D
pause
[n,d]=ss2tf(Av,Bv,Cv,Dv)
```

8.23. (a) $H(s) = C(sI - A)^{-1}B = 5 \cdot \frac{1}{s+2} \cdot 4 = \frac{20}{s+2}$

(b) $Q = 2$; $P = Q^{-1} = \frac{1}{2}$

$A_v = P^{-1}AP = 2(-2)\frac{1}{2} = -2$

$B_v = P^{-1}B = (2)(4) = 8$

$C_v = CP = 5(\frac{1}{2}) = \frac{5}{2}$

$\therefore \dot{v} = -2v + 8u$

$y = \frac{5}{2}v$

(c) see (e)

(d) $H(s) = C_v(sI - A_v)^{-1}B_v = (\frac{5}{2})(\frac{1}{s+2})(8) = \frac{20}{s+2}$ ✓

8.23. (e)
(cont)

```
A=[-2];B=[4];C=[5];D=0;Q=[2];
P=inv(Q);
Av=Q*A*P
Bv=Q*B
Cv=C*P
Dv=D
pause
[n,d]=ss2tf(Av,Bv,Cv,Dv)
```

(f) $|sI-A| = |sI-A_v| = s+2\,;\ \lambda=-2$

$|A|=|A_v|=|-2|=-2=\lambda\,;\ tr\,A = tr\,A_v = -2 = \lambda$

8.24. (a) From Problem 8.4(g): $H(s)=\dfrac{2s}{s^3-s^2-1}$

(b) $P=\begin{bmatrix}1&1&0\\0&0&1\\1&0&0\end{bmatrix},\ P^{-1}=\begin{bmatrix}0&0&1\\1&0&-1\\0&1&0\end{bmatrix}$

$$A_v = P^{-1}AP = \begin{bmatrix}0&0&1\\1&0&-1\\0&1&0\end{bmatrix}\begin{bmatrix}0&1&0\\0&0&1\\1&0&1\end{bmatrix}\begin{bmatrix}1&1&0\\0&0&1\\1&0&0\end{bmatrix}$$

$$= \begin{bmatrix}0&0&1\\1&0&-1\\0&1&0\end{bmatrix}\begin{bmatrix}0&0&1\\1&0&0\\2&1&0\end{bmatrix} = \begin{bmatrix}2&1&0\\-2&-1&1\\1&0&0\end{bmatrix}$$

$$B_v = P^{-1}B = \begin{bmatrix}0&0&1\\1&0&-1\\0&1&0\end{bmatrix}\begin{bmatrix}2\\0\\0\end{bmatrix} = \begin{bmatrix}0\\2\\0\end{bmatrix}$$

$$C_v = CP = \begin{bmatrix}0&0&1\end{bmatrix}\begin{bmatrix}1&1&0\\0&0&1\\1&0&0\end{bmatrix} = \begin{bmatrix}1&0&0\end{bmatrix}$$

$$\therefore\ \dot{\underline{v}} = \begin{bmatrix}2&1&0\\-2&-1&1\\1&0&0\end{bmatrix}\underline{v} + \begin{bmatrix}0\\2\\0\end{bmatrix}u$$

$$y = \begin{bmatrix}1&0&0\end{bmatrix}\underline{v}$$

(d) $sI-A_v = \begin{bmatrix}s-2&-1&0\\2&s+1&-1\\-1&0&s\end{bmatrix};\ Adj(sI-A_v)^T = \begin{bmatrix}s^2+s&-2s+1&s+1\\s&s^2-2s&1\\1&s-2&s^2-s\end{bmatrix}$

$$|sI-A_v| = s^3-s^2-2s-1-(-2s) = s^3-s^2-1=\Delta$$

$$H(s) = C_v(sI-A_v)^{-1}B_v = \begin{bmatrix}1&0&0\end{bmatrix}\begin{bmatrix}s^2+s&s&1\\ \vdots&\vdots&\vdots\end{bmatrix}\begin{bmatrix}0\\2\\0\end{bmatrix}\frac{1}{\Delta}$$

$$= \begin{bmatrix}s^2+s&s&1\end{bmatrix}\begin{bmatrix}0\\2\\0\end{bmatrix}\frac{1}{\Delta} = \underline{\frac{2s}{s^3-s^2-1}}$$

(e) (8.74) $s^3-s-1 = s^3-s-1$

(8.75) $\det A = 1 = \det A_v = 1$

(8.76) $tr\,A = 1 = tr\,A_v = 1$

8.25. (c)
(cont)

```
a=[0 1 0;0 0 1;1 0 1]; b=[2;0;0]; c=[0 0 1]; d=0;
p=[1 1 0;0 0 1;1 0 0];
q=inv(p);
av=q*a*p
bv=q*b
cv=c*p
pause
[n,d1]=ss2tf(av,bv,cv,d)
```

8.25. $C_v(sI-A_v)^{-1}B_v + D_v = CP(sI - P^{-1}AP)^{-1}P^{-1}B$

$= CP(s\,P^{-1}IP - P^{-1}AP)^{-1}P^{-1}B = CP(P^{-1}(sI-A)P)^{-1}P^{-1}B$

$= CPP^{-1}(sI-A)^{-1}PP^{-1}B = C(sI-A)B$, since $(AB)^{-1} = B^{-1}A^{-1}$

8.26. (a) $|sI-A| = \begin{vmatrix} s+4 & -5 \\ 0 & s+1 \end{vmatrix} = (s+4)(s+1)$, ∴ roots: $-4, 1$, ∴ <u>not stable</u>

(b) e^{-4t}, e^{-t}

(c)

```
A=[-4 5;0 1];
eig(A)
```

8.27. (a) $|sI-A| = s+2$; root: -2, ∴ stable

(b) e^{-2t}

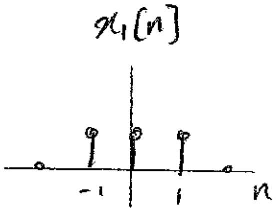
8.28. (a) From Prob 8.23, C.E.: $s^3-s-1=0$; roots $\begin{cases} s = 1.4656 & \underline{\text{unstable}} \\ s = -0.2328 \pm j0.7926 \end{cases}$

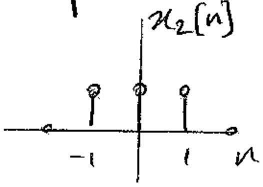
(b) $e^{1.4656t}, e^{-0.2328t}\cos(0.7926t+\theta)$ (c)

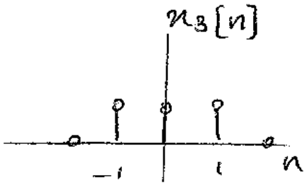
```
a=[1 1 0;0 0 1;1 0 0];
eig(a)
```

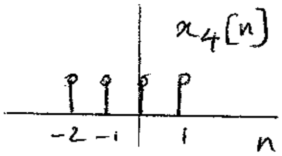
Chapter 9

9.1





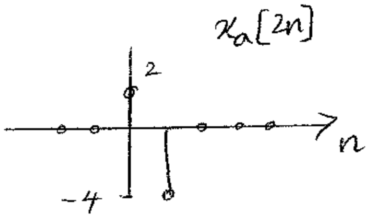




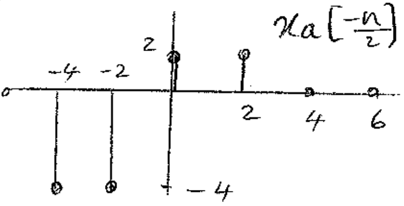
$x_4[n]$ is different

9.2

(a) (i) Take even points



(ii)



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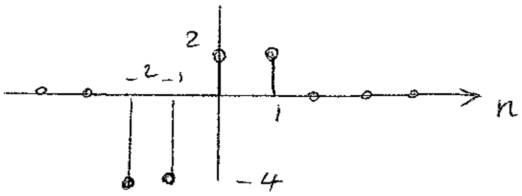
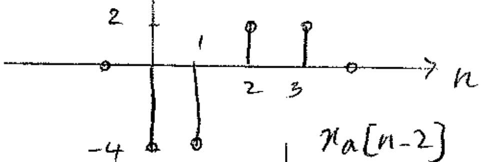


(iv)



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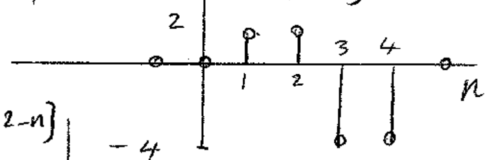
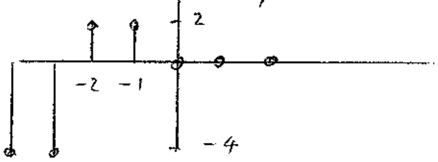
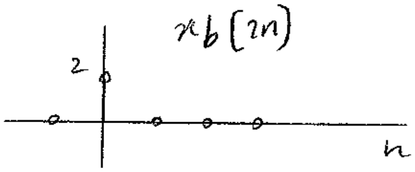


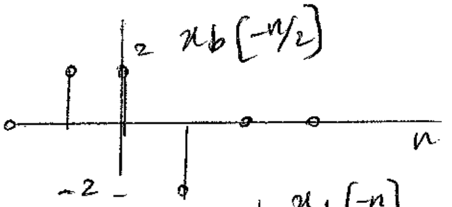
(vi)



(b) (i)



(ii)



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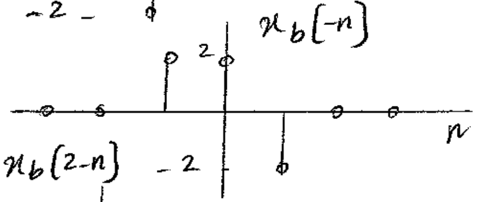

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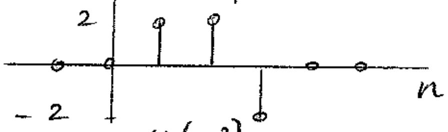
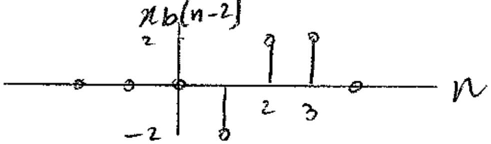
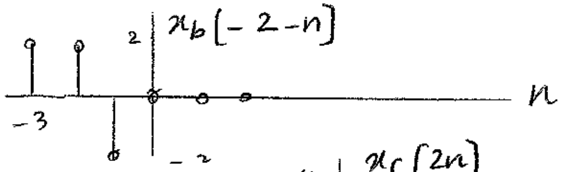


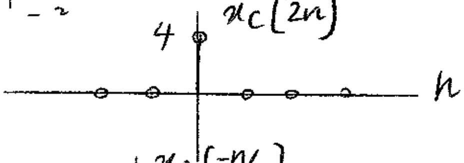
(v)



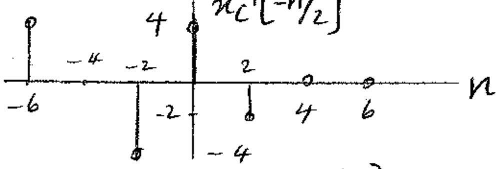
(vi)



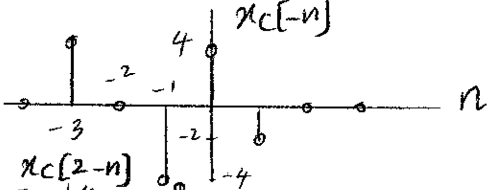
(c) (i)



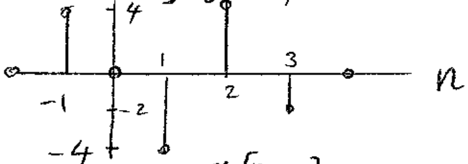
(ii)



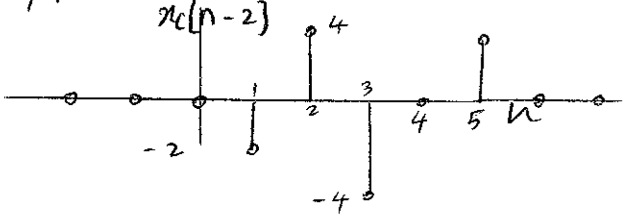
(iii)



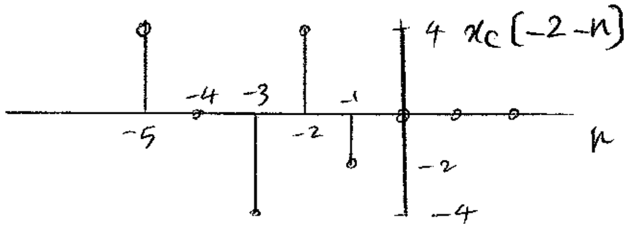
(iv)



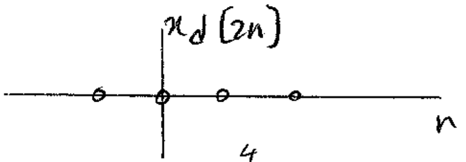
(v)



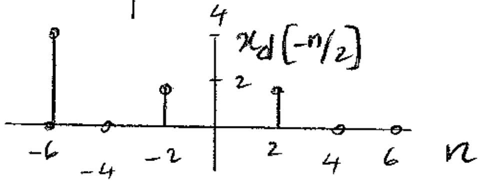
(vi)



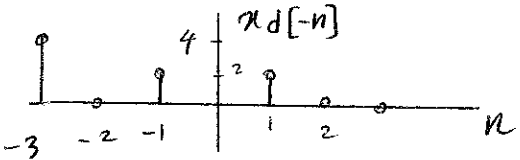
(d) (i)



(ii)



(iii)



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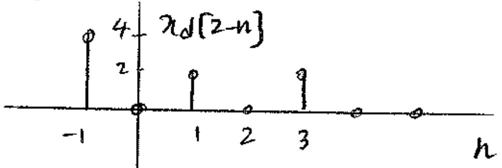
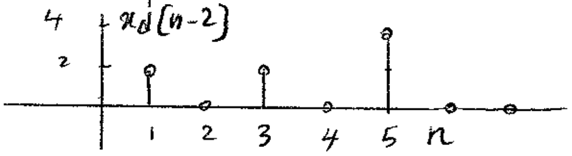


(v)



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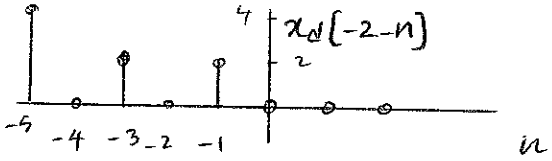


9.3 a)

(i)

n	$\leqslant -2$	-1	0	1	2	$\geqslant 3$
$2-3x_a[n]$	2	-4	-4	14	14	2

(ii)

n	$\leqslant -3$	-2	-1	0	1	$\geqslant 2$
$2x_a[-n]$	0	-8	-8	4	4	0

(iii)

n	$\leqslant 0$	1	2	3	4	$\geqslant 5$
$3x_a[n-2]$	0	6	6	-12	-12	0

(iv)

n	$\leqslant -2$	-1	0	1	2	$\geqslant 3$
$3-x_a[n]$	3	1	1	7	7	3

(v)

n	$\leqslant 0$	1	2	3	4	$\geqslant 5$
$1+2x_a(-2+n]$	1	5	5	-7	-7	1

(vi)

n	$\leqslant -3$	-2	-1	0	1	$\geqslant 2$
$2x_a[-n]-4$	-4	-12	-12	0	0	-4

(b) (i)

n	$\leqslant -2$	-1	0	1	$\geqslant 2$
$2-3x_b[n]$	2	8	-4	-4	2

(ii)

n	$\leqslant -2$	-1	0	1	$\geqslant 2$
$2x_b(-n]$	0	4	4	-4	0

(iii)

n	$\leqslant 0$	1	2	3	$\geqslant 4$
$3x_b[n-2]$	0	-6	6	6	0

(iv)

n	$\leqslant -2$	-1	0	1	$\geqslant 2$
$3 - x_b[n]$	3	5	1	1	3

(v)

n	$\leqslant 0$	1	2	3	$\geqslant 4$
$1+2x_b[-2+n]$	1	-3	5	5	1

(vi)

n	$\leqslant -2$	-1	0	1	$\geqslant 2$
$2x_b[-n]-4$	-4	0	0	-8	-4

(c) (i)

n	$\leqslant -2$	-1	0	1	2	3	$\geqslant 4$
$2-3x_c[n]$	2	8	-10	14	2	-10	2

(ii)

n	$\leqslant -4$	-3	-2	-1	0	1	$\geqslant 2$
$2x_c[-n]$	0	8	0	-8	8	-4	0

(iii)

n	$\leqslant 0$	1	2	3	4	5	$\geqslant 6$
$3x_c[n-2]$	0	-6	12	-12	0	12	0

(iv)

n	$\leqslant -2$	-1	0	1	2	3	$\geqslant 4$
$3-x_c[n]$	3	5	-1	7	3	-1	3

(v)

n	$\leqslant 0$	1	2	3	4	5	$\geqslant 6$
$1+2x_c[-2+n]$	1	-3	9	-7	1	9	1

(vi)

n	≤ -4	-3	-2	-1	0	1	≥ 2
$2x_c[-n]-4$	-4	4	-4	-12	4	-8	-4

d) (i)

n	≤ -2	-1	0	1	2	3	≥ 4
$2-3x_d[n]$	2	4	2	-4	2	-10	2

(ii)

n	≤ -4	-3	-2	-1	0	1	≥ 2
$2x_d[-n]$	0	8	0	4	0	4	0

(iii)

n	≤ 0	1	2	3	4	5	≥ 6
$3x_d[n-2]$	0	6	0	6	0	12	0

(iv)

n	≤ -2	-1	0	1	2	3	≥ 4
$3-x_d[n]$	3	1	3	1	3	-1	3

(v)

n	≤ 0	1	2	3	4	5	≥ 6
$1+2x_d[-2+n]$	1	5	1	5	1	9	1

(vi)

n	≤ -4	-3	-2	-1	0	1	≥ 2
$2x_d[-n]-4$	-4	-12	-4	0	-4	0	-4

9.4

a) (i) $2(\delta[n]+\delta[n-1])$

(ii) $2(\delta[n]+\delta[n+1])$

(iii) $x_a[n]$

(iv) $-4\delta[n+2]$

(v) $-4\delta[n-2]$

(vi) $2\delta[n+1]-4\delta[n-1]$

b) (i) $2\delta[n]-2\delta[n-1]$

(ii) $-2\delta[n+1]+2\delta[n]$

(iii) $x_b[n]$

(iv) $x_a[-n]u[-2-n]=0, \forall n$

(v) $0, \forall n$

(vi) $-2\delta[n+1]+2\delta[n-1]$

c) (i) $4\delta[n]-2\delta[n-1]$

(ii) $-2\delta[n+1]+4\delta[n]$

(iii) $x_c[n]$

(iv) $4\delta[n+3]$

(v) $0, \forall n$

(vi) $-2\delta[n+1]-4\delta[n-1]$

(d) (i) $2\delta[n-1]$

(ii) $2\delta[n+1]$

(iii) $x_d[n]$

(iv) $0, \forall n$

(v) $0, \forall n$

(vi) $2\delta[n+1] + 2\delta[n-1]$

9.5

a) $y_1[n] = x[3n]$

b) $y_2[n] = x[2n+1]$

c) $y_3[n] = x[3n-1]$ or $x[-3n+11]$

9.6

a) $x_t[n] = Ax[an+n_0] + B$

let $an+n_0 = m \Longrightarrow n = \frac{1}{a}m - \frac{1}{a}n_0$

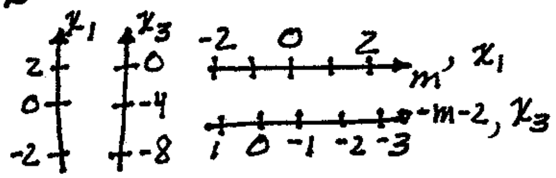
$Ax[m] = x_t\left[\frac{1}{a}m - \frac{1}{a}n_0\right] - B$

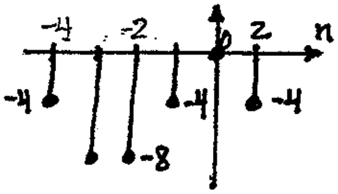
let $m \leftarrow n$

$x[n] = \frac{1}{A}x_t\left[\frac{1}{a}n - \frac{1}{a}n_0\right] - B/A$

9.6.(b) $x_1[n] = 0.5\, x_3[-n-1] + 2$
(cont)

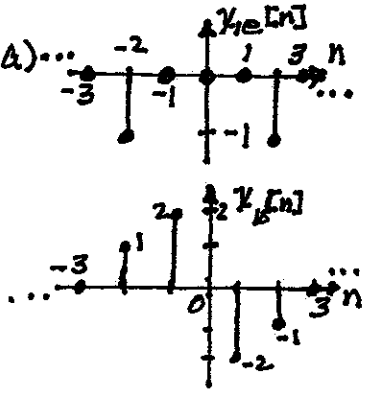
$\therefore A = 0.5$, $a = -1$
$B = 2$, $n_0 = -1$
$\therefore x_3[n] = \underline{2x_1[-n-1] - 4}$



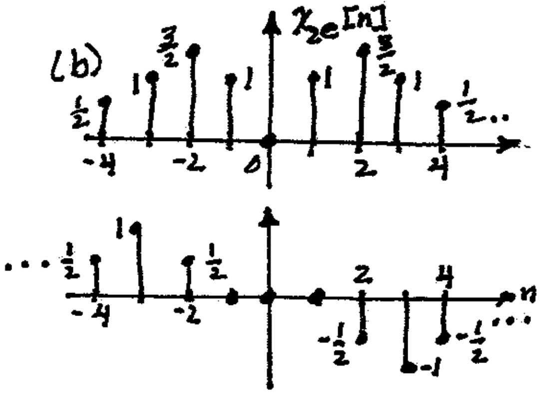


(c) $n = -2$, $x_3[-2] = 2x_1[1] - 4 = 2(-2) - 4 = -8$ ✓
$n = -1$, $x_3[-1] = 2x_1[0] - 4 = 2(0) - 4 = -4$ ✓
$n = 0$, $x_3[0] = 2x_1[-1] - 4 = 2(2) - 4 = 0$ ✓

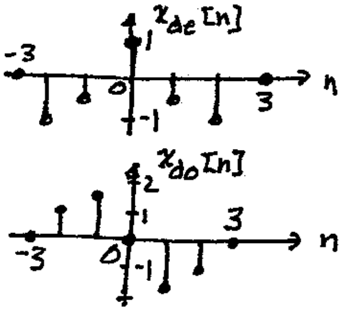
9.7.(a)



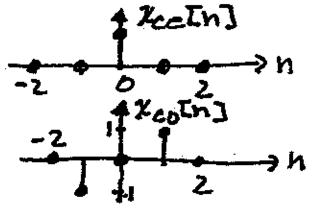
(b)



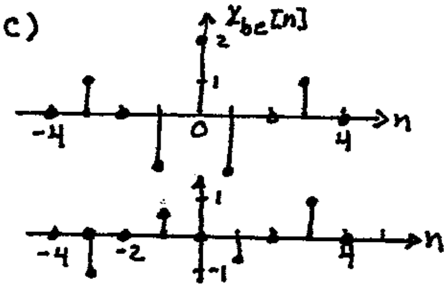
9.8. (a)



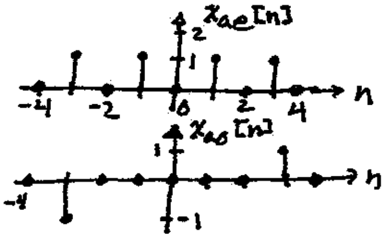
(b)



(c)



(d)



9.9

(a) (i) $x[n] = 3u[n-2]$ Neither

(ii) $x[n] = n$ odd

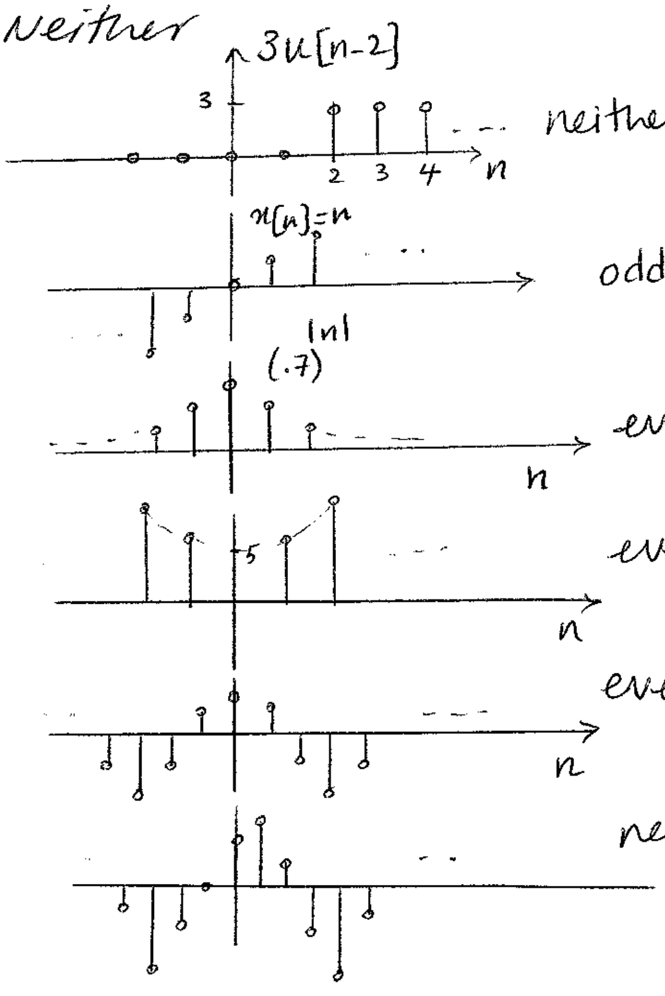
(iii) even

(iv) even

(v) even

(vi) Neither

(b)



(i) neither

(ii) odd

(iii) even

(iv) even

(v) even

(vi) neither

(c) (i) $x_e[n] = 3/2\, u[n-2] + 3/2\, u[-n-2]$

$x_o[n] = 3/2\, u[n-2] - 3/2\, u[-n-2]$

(ii) $x_e[n] = 0$

$x_o[n] = n$

(iii) $x_e[n] = x[n]$

$x_o[n] = 0$

(iv) $x_e[n] = x[n]$

$x_o[n] = 0$

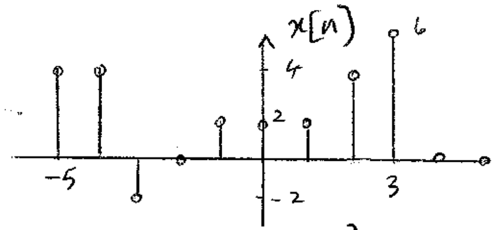
(v) $x_e[n] = x[n]$

$x_o[n] = 0$

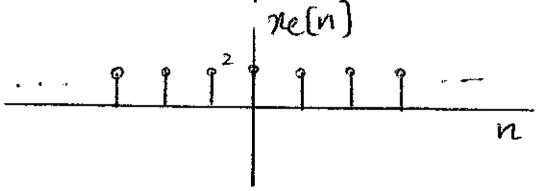
(vi) $x_e[n] = \frac{1}{2}\cos(n - \pi/6) + \frac{1}{2}\cos(-n - \pi/6)$

$x_o[n] = \frac{1}{2}\cos(n - \pi/6) - \frac{1}{2}\cos(-n - \pi/6)$

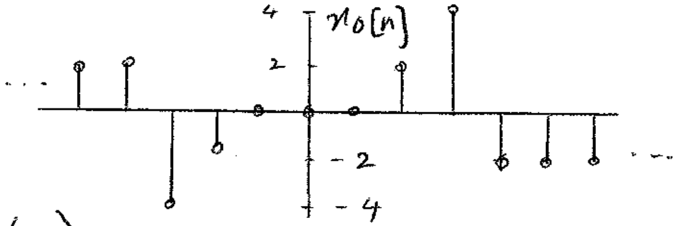
9.10



$x_e[n] = x_e$



$x_e[n] = x_e[-n]$



$x_o[n] = x[n] - x_e[n] \quad n \geq 0$

$x_o[n] = -x_o[-n] \quad n \leq 0$

(b) $x_o[0] = 0$ means that $x_e[0] = 0$ with no other changes.

9.11 (a) $x_o[n] = -x_o[-n] \Rightarrow x_o[0] = -x_o[0]$, $\therefore x_o[0] = 0$

$x_e[0] = x_o[0] - x_o[0] = x[0]$

(b) $\sum_{-\infty}^{\infty} x_o[n] = \sum_{-\infty}^{0} x_o[n] + \sum_{0}^{\infty} x[n] = \sum_{-\infty}^{0} -x_o[-n] + \sum_{0}^{\infty} x_o[n]$

9.11. Let $n \to -n$ in the first summation:
(cont)

$$\Rightarrow -\sum_{n=\infty}^{0} x[n] + \sum_{n=0}^{\infty} x[n] = 0$$

(c) $$\therefore \sum_{n=-\infty}^{\infty} x[n] = \sum_{n=-\infty}^{\infty} x_e[n] + \sum_{n=-\infty}^{\infty} x_o[n] = \sum_{n=-\infty}^{\infty} x_e[n]$$, from (a)

(d) Development in (b) also applies for $\sum_{k=-n}^{n} x_o[k] = 0$
Thus, the given sums are equal only for $n_1 = n_2$, and do not apply for $n_1 \neq n_2$.

9.12.(a) $x_t[n] = x_{e_1}[n] + x_{e_2}[n]$

$x_t[-n] = x_{e_1}[-n] + x_{e_2}[-n] = x_{e_1}[n] + x_{e_2}[n] = x_t[n]$, $\therefore$ even

(b) $x_t[n] = x_{o_1}[n] + x_{o_2}[n]$

$x_t[-n] = x_{o_1}[-n] + x_{o_2}[-n] = -x_{o_1}[n] - x_{o_2}[n] = -x_t[n]$, $\therefore$ odd

(c) $x_t[n] = x_e[n] + x_o[n]$

$x_t[-n] = x_e[-n] + x_o[-n] = x_e[n] - x_o[n]$, $\therefore$ neither

(d) $x_t[n] = x_{e_1}[n] x_{e_2}[n]$

$x_t[-n] = x_{e_1}[-n] x_{e_2}[-n] = x_{e_1}[n] x_{e_2}[n] = x_t[n]$, $\therefore$ even

(e) $x_t[n] = x_{o_1}[n] x_{o_2}[n]$

$x_t[-n] = x_{o_1}[-n] x_{o_2}[-n] = \left[-x_{o_1}[n]\right]\left[-x_{o_2}[n]\right] = x_t[n]$, $\therefore$ even

(f) $x_t[n] = x_e[n] x_o[n]$

$x_t[-n] = x_e[-n] x_o[-n] = x_e[n]\left[-x_o[n]\right] = -x_t[n]$, $\therefore$ odd

9.13

a) $x_1[n] = \cos(.2\pi n)$

Test : $\Omega_0 N = 2\pi k$, $\frac{\Omega_0}{2\pi} = \frac{k}{N} = \frac{.2\pi}{2\pi} = \frac{1}{10}$

$\therefore$ 1 cycle in 10 points , periodic

b) $x_2[n] = \cos(.125\pi n)$

$\frac{\Omega_0}{2\pi} = \frac{.125\pi}{2\pi} = \frac{1}{16}$

$\therefore$ 1 cycle in 16 points, periodic

c) $x_3[n] = \cos(.4\pi n)$

$\frac{\Omega_0}{2\pi} = \frac{.4\pi}{2\pi} = 1/5$

$\therefore$ 1 cycle in 5 points , periodic

d) $x_1[n] + x_2[n] + x_3[n]$

Take LCM of 5, 10, 16 = 80

$\therefore$ periodic with $N = 80$

9.14 a) $x[n+N] = e^{j5\pi(n+N)/7} = e^{j5\pi n/7} e^{j5\pi N/7} = e^{j5\pi n/7} e^{j2\pi k}$

$\therefore 5\pi N/7 = 2\pi k \Rightarrow N = \frac{14k}{5}$; $k = 5$, $\underline{N_0 = 14}$

b) $x[n+N] = e^{j5n} e^{j5N}$ $\therefore$ $5N = 2\pi k$, Not periodic

c) $x[n+N] = e^{j2\pi n} e^{j2\pi N}$ $\therefore$ $2\pi N = 2\pi k$, $\underline{N_0 = 1}$

d) $x[n+N] = e^{j.3n/\pi} e^{j.3N/\pi}$ $\therefore$ $\frac{.3N}{\pi} = 2\pi k$ $\therefore$ not periodic

e) $x[n+N] = \cos(3\pi n/7 + 3\pi N/7)$ $\therefore$ $\frac{3\pi N}{7} = 2\pi k$

$N = \frac{14k}{3}$, $\underline{N_0 = 14}$

f) $x[n+N] = e^{j\cdot 3n} e^{j\cdot 3N}$ $\therefore .3N = 2\pi K$, not periodic

g) $x(n+N) = e^{j5\pi n/7} e^{j5\pi N/7}$, $e^{j2\pi n}$ is Constant

So just period of $e^{j5\pi n/7}$

$\frac{5\pi N}{7} = 2\pi K$ $\therefore$ $\frac{N}{K} = \frac{14}{5}$, $\underline{N_0 = 14}$

h) from part g $N_{0_1} = 14$

from part e $N_{0_2} = 14$ $\therefore$ $\underline{N_0 = 14}$

i) $x[n+N] = e^{j\cdot 3n} e^{j\cdot 3N}$ $\therefore .3N = 2\pi K$, aperiodic

9.14.(a) $x[n+N] = e^{j5\pi(n+N)/7} = e^{j5\pi n/7} e^{j5\pi N/7} = e^{j5\pi n/7} e^{j2\pi k}$

$\therefore 5\pi N/7 = 2\pi k \Rightarrow N = \frac{14k}{5}$; $k = 5$, $N_0 = \underline{14}$

(b) $x[n+N] = e^{j5n} e^{j5N}$ $\therefore 5N = 2\pi k$ $\underline{\text{not periodic}}$

(c) $x[n+N] = e^{j2\pi n} e^{j2\pi N}$ $\therefore$ $2\pi N = 2\pi k$, $\underline{N_0 = 1}$ $(x[n] = 1)$

(d) $x[n+N] = e^{j0.3n/\pi} e^{j\frac{0.3N}{\pi}}$ $\therefore$ $\frac{0.3N}{\pi} = 2\pi k$ $\therefore$ $\underline{\text{not periodic}}$

(e) $x[n+N] = \cos(3\pi n/7 + 3\pi N/7)$, $\therefore \frac{3\pi N}{7} = 2\pi k$, $N = \frac{14k}{3}$, $\underline{N_0 = 14}$

(f) $x[n+N] = e^{j0.3n\pi} e^{j0.3N}$, $\therefore 0.3N = 2\pi k$, not periodic

→ 9.15. $t = nT$, $x[n] = \cos(2\pi nT)$, $\omega_0 = 2\pi$, $\therefore T_0 = 1$

N_0 = # of samples in the fundamental period.

(a) (i) $x[n] = \cos(2\pi nT)$

$x[n+N_0] = \cos(2\pi n + 2\pi N_0)$, $\therefore$ $2\pi N_0 = 2\pi k \Rightarrow k = \underline{1}$

$\therefore$ $\underline{\text{periodic}}$ with $N_0 = \underline{1}$ (constant signal)

(ii) $x[n] = \cos(0.2\pi n) \overset{?}{=} \cos(0.2\pi n + 0.2\pi N_0)$

$\therefore 0.2\pi N_0 = 2\pi k \Rightarrow N_0 = \frac{2k}{0.2} \Rightarrow N_0 = \underline{10}, k = \underline{1}$, $\underline{\text{periodic}}$

(iii) $x[n] = \cos(0.25\pi n) \overset{?}{=} \cos(0.25\pi n + 0.25\pi N_0)$

$\therefore 0.25\pi N_0 = 2\pi k \Rightarrow N_0 = \frac{2k}{0.25} \Rightarrow N_0 = \underline{8}, k = \underline{1}$ $\therefore$ $\underline{\text{periodic}}$

(iv) $x[n] = \cos(0.26\pi n) \overset{?}{=} \cos(0.26\pi n + 0.26\pi N_0)$

$\therefore 0.26\pi N_0 = 2\pi k \Rightarrow N_0 = \frac{2k}{0.26} = \frac{200}{26}k = \frac{100}{13}k$

$\therefore$ $N_0 = \underline{100}$, $k = \underline{13}$ $\underline{\text{periodic}}$

(v) $x[n] = \cos(10\pi n) \overset{?}{=} \cos(10\pi n + 10\pi N_0)$

$\therefore 10\pi N_0 = 2\pi k$, $N_0 \overset{?}{=} \frac{k}{5} \Rightarrow N_0 = \underline{1}, k = \underline{5}$ $\therefore$ $\underline{\text{periodic}}$ (constant)

(vi) $x[n] = \cos(\frac{8}{3}\pi n) \overset{?}{=} \cos(\frac{8}{3}\pi n + \frac{8}{3}\pi N_0)$

$\therefore$ $\frac{8}{3}\pi N_0 = 2\pi k \Rightarrow N_0 = \frac{6k}{8} \Rightarrow N_0 = \underline{3}$, $k = \underline{4}$ $\underline{\text{periodic}}$

(b) (i) $k = \underline{1}$ (ii) $k = \underline{1}$ (iii) $k = \underline{1}$ (c) (i) $N_0 = 1$ (ii) $N_0 = 10$ (iii) $N_0 = 8$

(iv) $k = \underline{13}$ (v) $k = \underline{5}$ (vi) $k = \underline{4}$ (iv) $N_0 = 100$ (v) $N_0 = 1$ (vi) $N_0 = 3$

9.16

(a) $T = .1s$, $x(t)|_{t=nT} = x(nT) = x[n]$

(1) $e^{-at}|_{t=nT} = e^{-anT} = (e^{-aT})^n$, where $\tau = 1/a$

(2) $\cos \omega t|_{t=nT} = \cos \omega_n T = \cos(\omega T)n = \cos bn$

a) from (1), $e^{-aT} = .3$ $\omega = \frac{b}{T}$

$aT = 1.204$ $a = 12.04$

b) $e^{-aT} = .3 \Rightarrow \tau = .8s$ $\tau = 1/a = .8 s$

$\omega = b/T = 1/.1 = 10$

c) $(-.3)^n = (.3)^n(-1)^n = (.3)^n \cos \pi n$

from (a), $\tau = .8s$ & $\omega = \frac{b}{T} = \frac{\pi}{.1} = 10\pi$

d) $e^{-aT} = .3 \Rightarrow \tau = .8s$

$\omega = \frac{b}{T} = \frac{1}{.1} = 10$ rad/sec

9.17

$x[n+N_0] = x[n]$, No # of samples in a fundamental period

a) (i) $\cos(\pi n + \pi N) = \cos(\pi n + 2\pi)$

$\therefore N = 2K$, $\underline{N_0 = 2}$ periodic

(ii) $-3\sin(.01\pi n + .01\pi N) = -3\sin(.01\pi n + 2\pi k)$

$\therefore .01N = 2K$, $\underline{N_0 = 20}$, periodic

(iii) $\cos(3\pi n/2 + 3\pi N/2 + \pi) = \cos(3\pi n/2 + 2\pi k + \pi)$

$\therefore 3N/2 = 2K$, $\underline{N_0 = 4}$, $K = 3$ periodic

(iv) $\cos(3.15n+3.15N)=\cos(3.15n+2\pi k)$

$\therefore 3.15N = 2\pi k$, $\frac{N}{k}=\frac{2\pi}{3.15}$, not periodic since not rational

(v) $1+\cos(.5\pi n+.5\pi N)=1+\cos(.5\pi n+2\pi k)$

$\therefore .5N=2k$, $\underline{N_0=4}$ periodic

(vi) $\sin(3.15\pi n+3.15\pi N)=\sin(3.15\pi n+2\pi k)$

$\therefore 3.15N=2k$ $\frac{k}{N}=\frac{2}{3.15}=\frac{200}{315}=\frac{40}{63}$

$\therefore \underline{N_0=40}$ periodic

b) (i) $N_0=2$ (iii) $N_0=4$ (v) $N_0=4$

(ii) $N_0=20$ (iv) Not periodic (vi) $N_0=40$

9.18

a) Two functions are equal

b) $u[n]=\begin{cases}1 & n\geq 0\\ 0 & n<0\end{cases}$

$u[n/3]=\begin{cases}1, & n\geq 0 \text{ and } n\in[0,3,6,9,\dots)\\ 0, & n<0 \text{ and } n\in[-3,-6,-9,\dots)\\ \text{undefined}, & \text{otherwise}\end{cases}$

$\therefore$ Two functions are not equal

9.19 a) $x_a[n]=2\delta[n+1]+2\delta[n]-4\delta[n-1]-4\delta[n-2]$

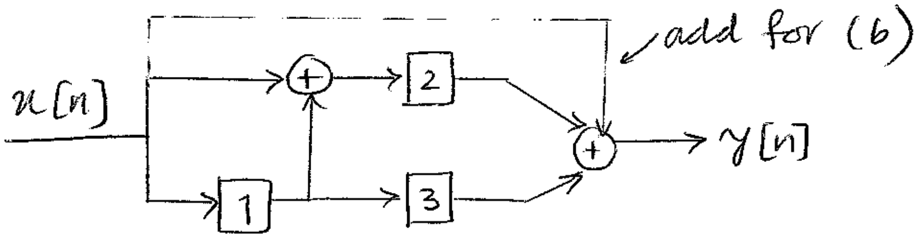
b) $x_b[n]=-2\delta[n+1]+2\delta[n]+2\delta[n-1]$

c) $x_c[n]=-2\delta[n+1]+4\delta[n]-4\delta[n-1]+4\delta[n-3]$

d) $x_d[n]=2\delta[n+1]+2\delta[n-1]+4\delta[n-3]$

9.20

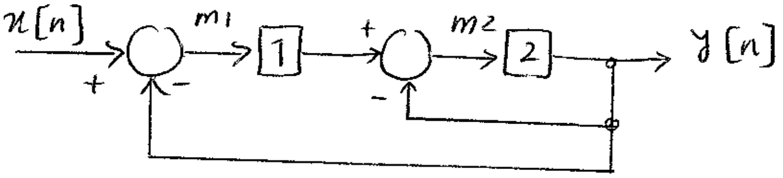
a&b)



9.21 a) $m[n] = T_3[T_2\{x[n] - T_4(y[n])\}]$

$y[n] = T_1(x[n]) + T_3[T_2\{x[n] - T_4(y[n])\}]$

b) $y[n] = T_2(m_2[n]) = T_2(T_1(x[n] - y[n] - y[n])$



$m_1[n] = x[n] - y[n]$

$m_2[n] = T_1(x[n] - y[n]) - y[n]$

9.22 a) $\therefore\ y[k] = y[k-1] + T/2\,[x[k] + x[k-1]]$

b)
```
y(1)=0, T=.1;
for n=1:51
        y(n+1)=y(n)+T/2*(exp(-n*T)+exp((1-n)*T));
end
y
```

c) Result : $y = .9941$

$\int_0^5 e^{-t}dt = e^{-t}\Big|_0^5 = 1 - e^{-5} = .9933$

9.23 a) $y[n] = \cos(x[n+2])$

(i) has memory

(ii) not invertible since $y = 0$ at $x = 0, \pm\pi, \ldots$

(iii) Not causal

(iv) Stable since $|y[n]| \leqslant 1$

(v) $y[n-n_0] = \cos(x[n-n_0+2])$ ∴ Time-Invariant

(vi) nonlinear $\cos(x_1+x_2) \neq \cos(x_1)+\cos(x_2)$

b) $y[n] = x[-n]$

(i) has memory

(ii) invertible (just do time reversal)

(iii) Not causal, $y[-1]=x[1]$

(iv) Stable

(v) Time-Varying

$y[n-n_0] = x[-(n-n_0)] = x[-n+n_0]$

$S[x[n-n_0]] = x[-n-n_0] \neq$

(vi) Linear

c) $$y[n] = \frac{\sin x[n]}{x[n]}, \lim_{x\to 0} \frac{\sin x}{x} = 1$$

(i) memoryless

(ii) $y=0$ for $x = \mp\pi, \mp 2\pi, \ldots$ ∴ not invertible

(iii) causal

(iv) stable $|y| \leqslant 1$ for all x

(v) $y[n-n_0] = \dfrac{\sin[x[n-n_0]]}{x[n-n_0]}$ ∴ time invariant

(vi) $\dfrac{\sin(x_1+x_2)}{x_1+x_2} \neq \dfrac{\sin x_1}{x_1} + \dfrac{\sin x_2}{x_2}$ ∴ not linear

d) $y[n] = e^{x[n]}$

(i) memoryless

(ii) $x[n] = \ln y[n]$ ∴ invertible

(iii) causal

(iv) e^{x} bounded for x bounded ∴ stable

(v) $y[n-n_0] = e^{x[n-n_0]}$ ∴ time invariant

(vi) $e^{x_1+x_2} \neq e^{x_1} + e^{x_2}$ ∴ not linear

e) $y[n] = e^{n x[n]}$

(i) memoryless

(ii) $x[n] = \frac{1}{n} \ln y[n]$ ∴ not invertible at $n=0$

(iii) causal

(iv) $x[n] = 1$, e^{n} unbounded for increasing n ∴ not stable

(v) $y[n]\Big|_{n \leftarrow n-n_0} \neq y[n]\Big|_{x[n] \leftarrow x[n-n_0]}$ ∴ time varying

(vi) not linear

f) $y[n] = 4x[n] - 2$

(i) memoryless

(ii) Invertible

(iii) causal

(iv) stable

(v) time invariant

(vi) $4[x_1 + x_2] - 2 \neq [4x_1 - 2] + [4x_2 - 2]$ ∴ not linear

g) $y[n] = \sum_{-\infty}^{n-3} \sin(x[k])$

(i) has memory

(ii) Not invertible due to sin function

(iii) causal

(iv) not stable

(v) Time invaiant

(vi) not linear

9.24 $y[n] = 2y[n-1] - y[n-2] + x[n]$

a) has memory

b) $y[n-n_0] = 2y[n-n_0-1] - y[n-n_0-2] + x[n-n_0]$

$\therefore$ time invaiant

c) $a_1 y_1[n] + a_2 y_2[n] - 2[a_1 y_1[n-1] + a_2 y_2[n-1]]$
$+ a_1 y_1[n-2] + a_2 y_2[n-2] = a_1 x_1[n] + a_2 x_2[n]$

$\therefore a_1\{y_1[n] - 2y_1[n-1] + y_1[n-2] - x_1[n]\} + a_2\{y_2[n]$
$- 2y_2[n-1] + y_2[n-2] - x_2[n]\} = 0$

$\Rightarrow 0 + 0 = 0$, $\therefore$ linear

9.25 a) $y[n] = \sum_{-n}^{n} x[k+a]$

(i) has memory

(ii) not invertible

(iii) not causal , whether or not it looks at future depends on $\underline{\underline{a}}$ & we don't know

(iv) stable

(v) Time varying (vi) linear

b) $y[n] = \frac{1}{2}\left[x[n] + x[n-1]\right]$

(i) has memory

(ii) $x[n] = 2y[n] - x[n-1]$ invertible

$= 2y[n] - 2y[n-1] + x[n-2] = 2y[n] - \cdots$

(iii) causal

(iv) Stable

(v) Time invariant

(vi) linear

c) (i) has memory

(ii) Invertible

(iii) Causal

(iv) Stable

(v) Time invariant

(vi) linear

9.26 $y[n] = K_n x[n]$ with $K_n = \left[\frac{n+2.5}{n+1.5}\right]^2$

as $n \to \infty$ & as $n \to -\infty$, $K_n \to 1$

$\therefore$ K_n is max for $n=-1$ & $|y[-1]| = 9|x[-1]|$

n	K_n
2	1.65
1	1.96
0	1.67
−1	9.0
−2	1
−3	.111

9.27

a) $y[n] = -3|x[n]|$

(i) memoryless

(ii) not invertible

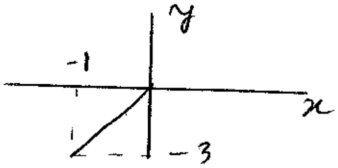
(iii) Causal

(iv) Stable

(v) Time-Invariant

(vi) $|x_1| + |x_2| \neq |x_1 + x_2|$ ∴ Not linear

b) $y[n] = \begin{cases} 3x[n] & x<0 \\ 0 & x \geq 0 \end{cases}$



(i) memoryless

(ii) not invertible ; $y=0$, $x \geq 0$

(iii) Causal

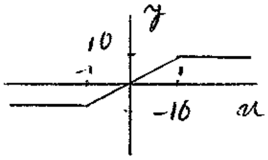
(iv) Stable

(v) time invariant

(vi) let $x_1=1$, $x_2=1 \Rightarrow y_1=0$, $y_2=-1$

$$\therefore -1 = y[n]\Big|_{\substack{x_1=1 \\ x_2=-1}} \neq y[n]\Big|_{x=1-1=0} = 0 \quad \text{not linear}$$

c)



(i) memoryless

(ii) $y=10$ for $x \geq 1$, not invertible

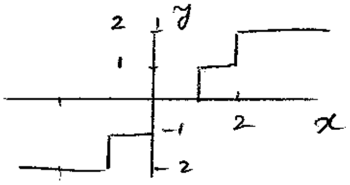
(iii) Causal

(iv) Stable

(v) time-invariant

(vi) $x_1 = x_2 = 1 \Rightarrow y_1 = y_2 = 10$, $y\big|_{x=2} \neq 20$ ∴ not linear

d)



(i) memoryless

(ii) $y = 2$ for $x \geqslant 2$, not invertible

(iii) causal

(iv) Stable

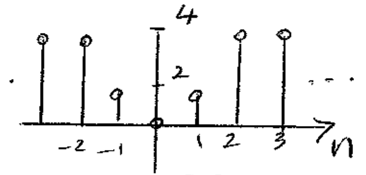
(v) time-invariant

(vi) $y\big|_{x_1 = x_2 = 2} = 2 \neq y\big|_{x_1 = 2} + y\big|_{x_2 = 2}$ ∴ nonlinear

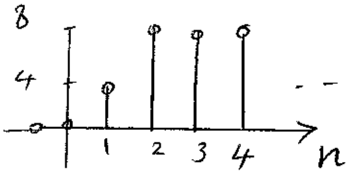
9.28 $he[n] = \begin{cases} 0 & , n = 0 \\ 2 & , n = 1 \\ 4 & , n \geqslant 2 \end{cases}$

$h[n] = 0, \quad n < 0$

$he[n] = he[-n]$



he[n]



h[n]

$n < 0, \quad ho[n] = -he[n]$

$n > 0, \quad ho[n] = he[n] \quad \Rightarrow \quad h[n] = \begin{cases} 2he[n] & n \geqslant 0 \\ 0 & n < 0 \end{cases}$

Chapter 10

10.1 $\sum_{k=-\infty}^{\infty} x[k]h[n-k]$ - replace k with $(n-k_1)$, n constant

$$\Rightarrow \sum_{k=-\infty}^{\infty} x[n-k_1]h[k_1] = \sum_{-\infty}^{\infty} h[k_1]x[n-k_1]$$

10.2 $$g[n]*\delta[n] = \sum_{k=-\infty}^{\infty} g[k]\delta[n-k]$$

$$\delta[n-k] = \begin{cases} 1 & , k=n \\ 0 & , \text{otherwise} \end{cases}$$

$$\therefore \quad g[n]*\delta[n] = g[n](1) = g[n]$$

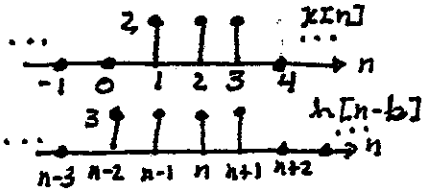
10.3 a) $$y[5] = x[1]h[4] + x_2[2]h[3] + x[3]h[2]$$
$$= (2)(0) + (2)(0) + (2)(3) = \underline{6}$$

b) $$y_{max}[n] = x[1]h[2] + x[2]h[1] + x[3]h[0]$$
$$= 6+6+6 = 18 \quad \therefore y[3] = 18$$

c) $$y_{max}[n] = x[1]h[1] + x[2]h[0] + x[3]h[-1] = \underline{18}$$
$$\underline{n = 2, 3}$$

d) next page

10.3, (d)
(cont)



$n<0,\ y[n]=0$
$y[0]=6$
$y[1]=12$
$y[2]=18$
$y[3]=18$
$y[4]=12$
$y[5]=6$
$y[n]=0,$
$n\geq 6$

(e)

```
x = [0 0 2 2 2]; h = [3 3 3 3 0]; y=conv(x,h)
```

10.4. $h[n]=\alpha^n u[n]$, $x[n]=\beta^n u[n]$, $\alpha\neq\beta$

(a) $$y[n]=\sum_{k=-\infty}^{\infty}\alpha^k u[k]\beta^{n-k}u[n-k]=\sum_{k=0}^{n}\alpha^k\beta^{n-k}$$

$$=\beta^n\Big[\sum_{k=0}^{n}(\alpha\beta^{-1})^k\Big]u[n]=\beta^n\left[\frac{1-\alpha^{n+1}\beta^{-(n+1)}}{1-\alpha\beta^{-1}}\right]$$

$$=\frac{\beta^n-\alpha^{n+1}\beta^{-1}}{1-\alpha\beta^{-1}}=\frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}$$

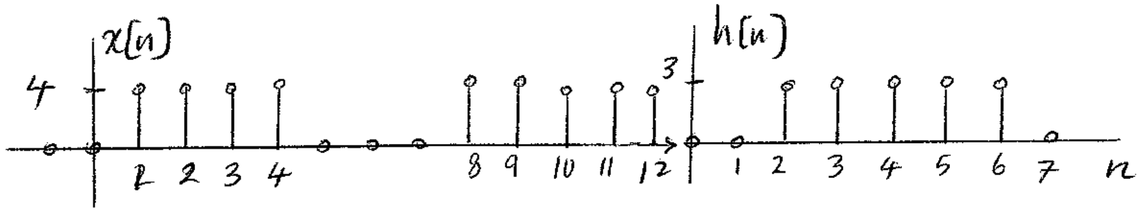
(b) $$y[4]=\frac{\beta^5-\alpha^5}{\beta-\alpha}\Rightarrow\ \begin{array}{r|l} & \beta^4+\alpha\beta^3+\alpha^2\beta^2+\alpha^3\beta+\alpha^4 \\ \hline \beta-\alpha & \beta^5 \qquad\qquad\qquad\qquad\qquad -\alpha^5 \\ & \underline{\beta^5-\alpha\beta^4} \\ & \quad\alpha\beta^4 \\ & \quad\underline{\alpha\beta^4-\alpha^2\beta^3} \\ & \qquad\cdots \end{array}$$

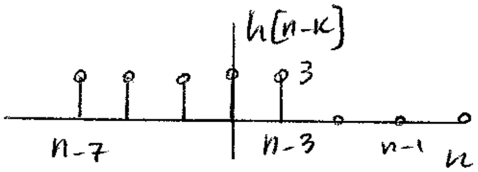
$\therefore y[4]=\underline{\beta^4+\alpha\beta^3+\alpha^2\beta^2+\alpha^3\beta+\alpha^4}$

(c) $$y[4]=\sum_{k=0}^{4}\alpha^k\beta^{4-k}=\alpha^0\beta^4+\alpha^1\beta^3+\alpha^2\beta^2+\alpha^3\beta^1+\alpha^4\beta^0$$

$$=\underline{\beta^4+\alpha\beta^3+\alpha^2\beta^2+\alpha^3\beta+\alpha^4}$$

10.5





a) $y[8] = (3 \cdot 4) \cdot 4 = 48$

b) max value is $n-3=12$ or $n=15$

here $y[15] = 3 \cdot 4 \cdot 5 = 60$

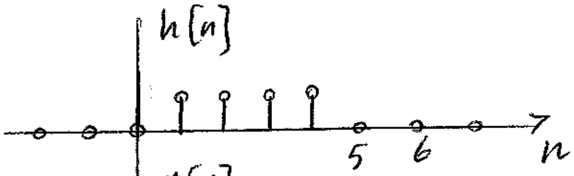
c) 15

d) $y(n) = [\overset{n=0}{0}\ \overset{n=1}{0}\ 0\ 0\ 12\ 24\ 36\ 48\ 48\ 36\ 24\ 24\ 24$

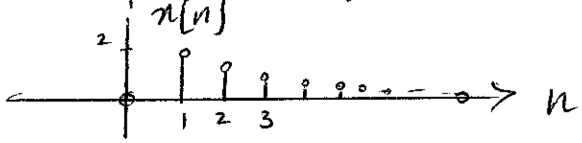
$\underset{n=13}{36}\ 48\ 60\ 48\ 36\ 24\ \underset{n=19}{12}]$, $y(n)=0$, else wher

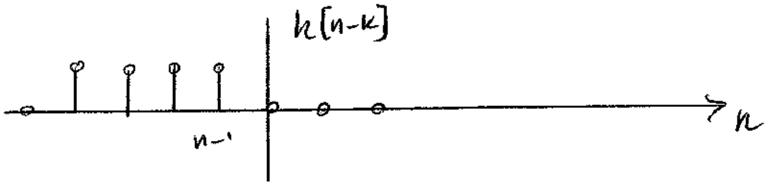
e) $x=[0\,4\,4\,4\,4\,0\,0\,0\,4\,4\,4\,4\,4]$, $h=[0\,0\,0\,3\,3\,3\,3\,3]$, Conv$(x,h)$

10.6 a)



b)





h	-1	0	1	2	3	4	5	6	7	8
y(n)	0	0	1	1.2	1.24	1.248	.0496	.0096	.0016	0

10.7(a) $y[n]=\sum_{k=-\infty}^{\infty} h[k]x[n-k]=\sum_{k=0}^{1} x[n-k]+\sum_{k=4}^{5} x[n-k]$

$x[n-k]=u[n-k]$

$\therefore\ y[n]=u[n]+u[n-1]+u[n-4]+u[n-5]$

$\therefore\ y[n]=0\ ,\ n<0$

$y[0]=1$

$y[1]=2$

$y[2]=2$

$y[3]=2$

$y[4]=3$

$y[n]=4\ ,\ n\geq 5$

(b) $y[n]=\sum_{k=-\infty}^{\infty} h[k]x[n-k]=\sum_{k=0}^{1}\left(u[n-k]-u[n-2-k]\right)+\sum_{k=4}^{5}\left(\quad\right)$

$=u[n]-u[n-2]+u[n-1]-u[n-3]$

$+u[n-4]-u[n-6]+u[n-5]-u[n-7]$

$y[n]=0,\ n<5$

$y[0]=1$

$y[1]=2$

$y[2]=1$

$y[3]=0$

$y[4]=1$

$y[5]=2$

$y[6]=1$

$y[n]=0 \quad n\geq 7$

(c)

```
x=[1 1 0 0 0 0];h=[1 1 0 0 1 1];
y=conv(x,h)
```

(d) $y[n]=\sum_{k=-\infty}^{\infty} h[k]x[n-k]=\sum_{k=0}^{1}\left(u[n-k]-u[n-6-k]\right)+\sum_{k=4}^{5}\left(\quad\right)$

$=u[n]-u[n-6]+u[n-1]-u[n-7]$

$+u[n-4]-u[n-10]+u[n-5]-u[n-11]$

10.7 (cont) (d) $y[n]=0, n<0$

$y[0]=1$ $y[4]=3$ $y[8]=2$

$y[1]=2$ $y[5]=4$ $y[9]=2$

$y[2]=2$ $y[6]=3$ $y[10]=1$

$y[3]=2$ $y[7]=2$ $y[n]=0, \; n\geq 11$

(e)
```
x=[1 1 1 1 1 1 0 0];h=[1 1 0 0 1 1]; y=conv(x,h)
```

(f) $$y[n]=\sum_{k=0}^{1}(u[n-k]-u[n-k-2])=u[n]+u[n-1]-u[n-2]-u[n-3]$$

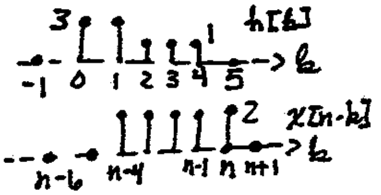
$\therefore y[n]=0, n<0$ $y[2]=1$

$y[0]=1$ $y[n]=0, n\geq 3$

$y[1]=2$

(g)
```
x=[1 1 0 0];h=[1 1 0 0]; y=conv(x,h)
```

10.8 (a)



$y[n]=0, n\leq 0$ $y[5]=18$

$y[1]=6$ $y[6]=12$

$y[2]=12$ $y[7]=6$

$y[3]=14$ $y[8]=4$

$y[4]=16$ $y[9]=2$

$y[n]=0, n\geq 10$

(b) $y[n]=0, n<0$; $y[n]=0, n\geq 8$

n	0	1	2	3	4	5	6	7
y[n]	2	0	2	0	0	-2	0	-2

(c) $y[n]=0, n<0$; $y[n]=0, n\geq 8$

n	0	1	2	3	4	5	6	7
y[n]	6	9	11	12	6	3	1	0

(d) $y[n]=0, n<0$; $y[n]=0, n\geq 8$

n	0	1	2	3	4	5	6	7
y[n]	3	0	1	0	-2	-1	0	-1

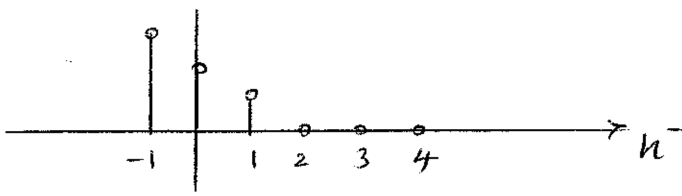
(e) $y[n]=0, n<0$; $y[n]=0, n\geq 8$

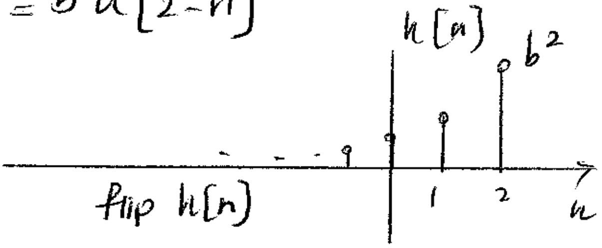
n	0	1	2	3	4	5	6	7
y[n]	-3	-6	-1	4	2	1	2	1

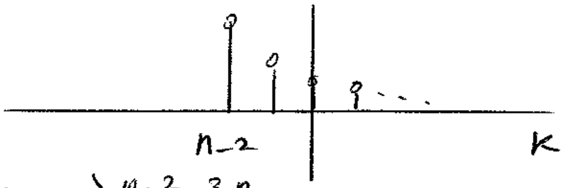
(f)
```
x=[2 2 2 2 2 0 0];h=[3 3 1 1 1 0 0]; y=conv(x,h),pause
x=[2 2 2 2 2 0 0];h=[1 -1 1 -1 0 0]; y=conv(x,h),pause
x=[2 2 2 2 2 0 0];h=[3 1.5 1 0.5 0 0]; y=conv(x,h),pause
x=[3 3 1 1 1 0 0];h=[1 -1 1 -1 0 0]; y=conv(x,h),pause
x=[3 3 1 1 1 0 0];h=[-1 -1 1 1 ]; y=conv(x,h)
```

10.9

a) $x[n] = a^{-3n} u[1-n], \quad h[n] = b^{n} u[2-n]$







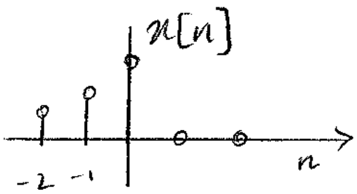
$n-2 > 1, \quad n > 3, \quad y[n] = 0$

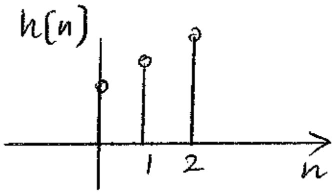
$n-2 \leqslant 1, \quad n \leqslant 3$

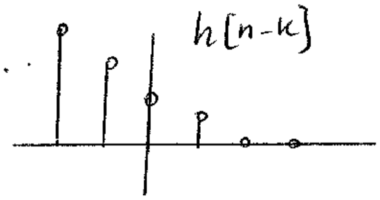
$$\sum_{k=n-2}^{1} b^{n-k} a^{-3k} = b^{n} \sum_{k=n-2}^{1} \left(\frac{1}{a^3 b}\right)^{k} = b^{n} \left(\frac{1}{a^3 b}\right)^{n-2} \sum_{0}^{3-n} \left(\frac{1}{a^3 b}\right)^{k}$$

$$= b^{n} \left(\frac{1}{a^3 b}\right)^{n-2} \left[\frac{1 - \left(\frac{1}{a^3 b}\right)^{4-n}}{1 - \frac{1}{a^3 b}}\right] = b^{2} \left(\frac{1}{a^3}\right)^{n-2} \times \left(\frac{a^3 b - (a^3 b)^{n-3}}{a^3 b - 1}\right) u[3-n]$$

b) $x[n] = a^{n} u[-n], \quad h[n] = b^{n} u[n]$







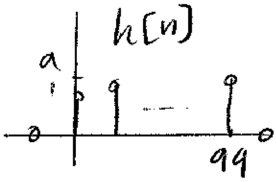
$$n < 0, \quad \sum_{-\infty}^{\infty} a^{k} b^{n-k} = b^{n} \sum_{k=-\infty}^{\infty} \left(\frac{a}{b}\right)^{k} = b^{n} \sum_{-n}^{\infty} (b/a)^{k} = \frac{b^{n} (b/a)^{-n}}{1 - b/a}$$

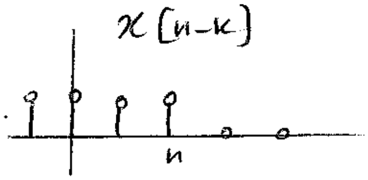
$$= \frac{a^{n}}{1 - b/a}$$

$$\therefore y[n] = \frac{a^{n}}{1 - b/a} u[-n-1] + \frac{b^{n}}{1 - b/a} u[n]$$

$$= \frac{a^{n+1}}{a-b} u[-n-1] + \frac{a b^{n}}{a-b} u[n]$$

c)





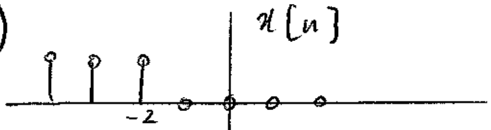
$n<0, \quad y(n)=0$

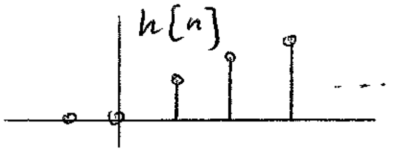
$0 \leqslant n \leqslant 99, \quad y[n] = \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}$

$n \geqslant 100, \quad y[n] = \sum_{0}^{99} a^k = \frac{1-a^{100}}{1-a}$

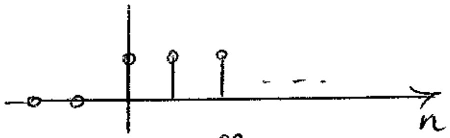
$\therefore \quad y[n] = \left(\frac{1-a^{n+1}}{1-a}\right)\left(u[n]-u[n-100]\right)+\left(\frac{1-a^{100}}{1-a}\right)u(n-100]$

d)





flip x[n]



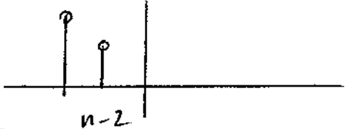
$n+2<1, \quad n<-1, \quad y[n] = \sum_{k=1}^{\infty} b^{-2k} = \sum_{k=1}^{\infty}\left(\frac{1}{b^2}\right)^k = \frac{(1/b^2)}{1-\frac{1}{b^2}}$

$n+2 \geqslant 1, \quad n \geqslant -1, \quad y[n] = \sum_{k=n+2}^{\infty}\left(\frac{1}{b^2}\right)^k = \frac{(1/b^2)^{n+2}}{1-1/b^2}$

$\therefore \quad y[n] = \frac{1}{b^2-1}u[-n-2] + \frac{(1/b^2)^{n+2}}{1-1/b^2}u[n+1]$

$= \frac{1}{b^2-1}u[-n-2] + (1/b^2)^{n+1}\frac{1}{b^2-1}u[n+1]$

e) flip h[n]



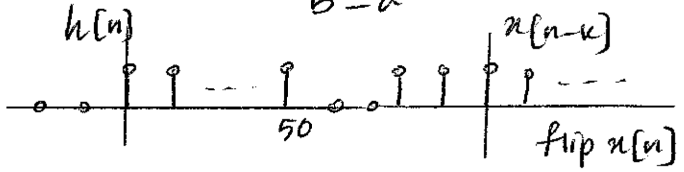
$n-2<0, \quad y[n]=0$

$n-2 \geqslant 0, \quad n \geqslant 2, \quad y[n] = \sum_{k=0}^{n-2} a^{2k} b^{n-k} = b^n \sum_{k=0}^{n-2}\left(\frac{a^2}{b}\right)^k$

$$= b^n \left[\frac{1-\left(\frac{a^2}{b}\right)^{n-1}}{1-a^2/b} \right] = b^n \left[\frac{b - b\left(a^2/b\right)^{n-1}}{b-a^2} \right]$$

$$= \left(\frac{b^{n+1} - b^2 (a^2)^{n-1}}{b-a^2} \right) \therefore y[n] = \left(\frac{b^{n+1} - b^2(a^2)^{n-1}}{b-a^2}\right) u[n-2]$$

f)



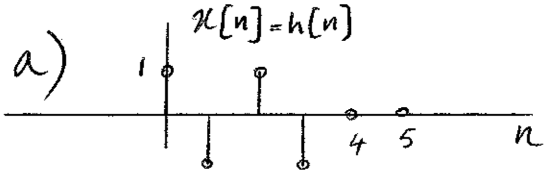
$n-1 > 50, \quad y[n] = 0$

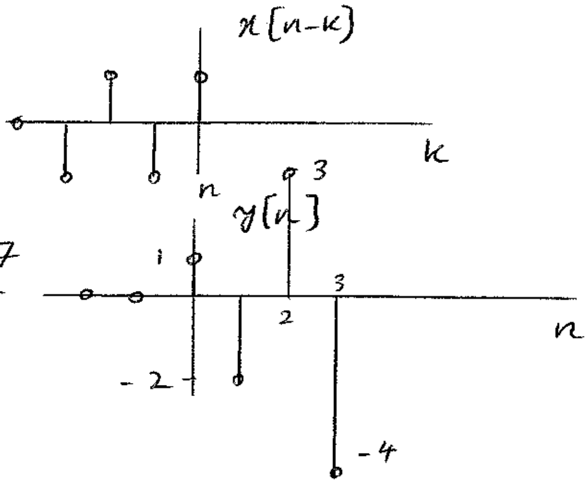
$0 \leq n-1 \leq 50, \quad 1 \leq n \leq 51, \quad y[n] = \sum_{k=n-1}^{50} 1 = 50-(n-1)+1$
$= 52-n$

$n-1 < 0, \quad y[n] = 51$

$n < 1 \quad \therefore \quad y[n] = (52-n)\big(u[n-1]-u[n-52]\big)+51\,u[-n]$

10.10 a)





n	≤-1	0	1	2	3	4	5	6	≥7
y[n]	0	1	-2	3	-4	3	-2	1	0

b) $y[0] = x[0]\,h[0] = (1)(1) = 1$

$y[1] = x[0]\,h[1] + h[0]\,x[1] = (1)(-1)+(1)(-1) = -2$

$y[2] = x[2]h[0] + x[1]h[1] + x[0]h[2] = 1+1+1 = 3$

$y[3] = x[3]h[0] + x[2]h[1] + x[1]h[2] + x[0]h[3] = -1+(-1)$
$+(-1)+(-1) = -4$

$y[4] = x[3]h[1] + x[2]h[2] + x[1]h[3]$
$= (-1)(-1) + (1)(1) + (-1)(-1) = 3$

$y[5] = x[3]h[2] + x[2]h[3] = (-1)(1) + (1)(-1) = -2$

$y[6] = x[3]h[3] = (-1)(-1) = 1$

c) x = [1 -1 1 -1];
conv(x,x)

10.11 a) $h[n] = h_1[n] * h_2[n] = \sum_{k=-\infty}^{\infty} (.8)^k u[k] (.8)^{n-k} u[n-k]$

$$= \sum_{k=0}^{n} (.8)^k (.8)^{n-k} = \left[(.8)^n \sum_{k=0}^{n} (1)\right] u[n] = (n+1)(.8)^n u[n]$$

b) $h[n] = \delta[n-3] * \delta[n-3] = \delta[n-6]$

c) $h[n] = \cdots + \delta[-5]\delta[n-1] + \delta[-4]\delta[n-2] + \cdots + \delta[-1]\delta[n-5]$

$$+ \delta[0]\delta[n-6] + \delta[1]\delta[n-7] + \cdots = \delta[0]\delta[n-6] = \delta[n-6]$$

d) $h[n] = u[n] * u[n] - u[n] * u[n-2) - u[n-2] * u[n]$

$$+ u[n-2] * u[n-2]$$

$u(n-n_1] * u(n-n_2] = (n+1-(n_2-n_1))u[n-(n_1+n_2)]$

$\therefore h[n] = (n+1)u(n] - (n+1-2)u[n-2] + (n+1-2)u(n-2]$

$$+ (n+1-4)u[n-4]$$

$$= (n+1)u[n] - 2(n-1)u[n-2] + (n-3)u(n-4]$$

$h[0] = 1 - 2(0) + (0) = 1$

$h[1] = 2 - 0 + 0 = 2$

$h[2] = 3 - 2 + 0 = 1$

$h[3] = 4 - 4 + 0 = 0$

$\therefore h(n) = \delta[n] + 2\delta[n-1] + \delta[n-2]$

$n \geqslant 4 \quad h[n] = n+1 - 2n + 2 + n - 3 = 0$

10.12 $h[n] = 2^n u[n]$

a) Causal since $h[n]=0, n<0$

b) not stable $\sum_{0}^{\infty} 2^n = \infty$

c) $x[n]=u[n]$ $n<0$ $y=0$

$$y[n] = \sum_{k=0}^{n} 2^k = \frac{1-2^{n+1}}{1-2} = (2^{n+1}-1)u[n]$$

d) x=[1 1 1 1 1];
h=[1 2 4 8];
Conv(x,h)

$y[0]=1$, $y[1]=3$, $y[2]=7$, $y[3]=15$

e) $h[n] = 2^n u[-n]$

a) $h[n] \neq 0$, $n<0$ noncausal

b) stable $\sum_{-\infty}^{\infty} h[n] = \sum_{-\infty}^{\infty} 2^n = \sum_{0}^{\infty} (1/2)^n = \frac{1}{1-1/2} = 2 < \infty$

c) $y[n]=u[n] * 2^n u[-n]$

$n<0$, $y[n] = \sum_{-\infty}^{\infty} 2^n = \sum_{-n}^{\infty} (1/2)^n = \frac{(1/2)^{-n}}{1-1/2} = 2 \cdot 2^n = 2^{n+1}$

$n \geq 0$, $y[n] = \sum_{-\infty}^{0} 2^n = \sum_{0}^{\infty} (1/2)^n = 2$

$\therefore$ $y[n] = 2u[n] + 2^{n+1}u[-n-1]$

f) $h[n] = (.3)^n u[-n]$

a) noncausal since $h[n] \neq 0$, $n<0$

b) $\sum_{-\infty}^{\infty} |h[n]| = \sum_{-\infty}^{0} (.3)^n = \sum_{0}^{\infty} \left(\frac{1}{.3}\right)^n = \infty$

∴ unstable

g) $h[n] = u[-n]$

a) noncausal since $h[n] \neq 0$, $n<0$

b) $\sum_{-\infty}^{\infty} |h[n]| = \sum_{-\infty}^{0} 1 = \infty$ ∴ unstable

10.13 $f[n] * g[n] = \sum_{m=-\infty}^{\infty} f[m]\, g[n-m] = e[n]$

$$f[n] * g[n] * h[n] = \sum_{k=-\infty}^{\infty} e[k]\, h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} f[m]\, g[k-m] \right] h[n-k]$$

$$= \sum_{m=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} g[k-m]\, h[n-k] \right] f[m]$$ let $k-m=p$ or $k=m+p$

$$\therefore \Rightarrow \sum_{m=-\infty}^{\infty} \left[\sum_{p=\infty}^{\infty} g[p]\, h[n-m-p] \right] f[m]$$ let $q=n-p$ or $p=n-q$

$$\Rightarrow \sum_{m=-\infty}^{\infty} \left[\sum_{q=\infty}^{-\infty} g[n-q]\, h[q-m] \right] f[m]$$

$$= \sum_{q=-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} f[m]\, h[q-m] \right] g[n-q]$$

$$= f[n] * h[n] * g[n]$$

10.14 $y[n] = .8\left(x[n+1] + x[n]\right)$

a) let $x[n] = \delta[n]$, then $y(n) = h[n]$

$\therefore\ y[n] = .8\,\delta[n+1] + .8\,\delta[n]$

b) noncausal due to term of $.8\delta[n+1]$

c) $x[n] = u[n+1]$

$y[n] = .8u[n+2] + .8u[n+1]$

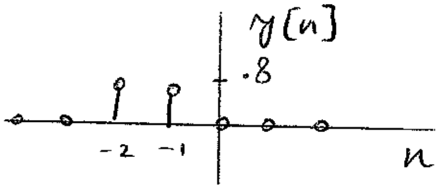
d) $\delta[n-1] * -h[n] = -h[n-1]$

$h_t[n] = h[n] - h[n-1] = .8\delta[n+1] + .8\delta[n] - .8\delta[n]$

$-.8\delta[n-1] = .8\delta[n+1] - .8\delta[n-1]$

e) $x[n] = u[n+1]$

$y[n] = .8u[n+2] - .8u[n]$



10.15 $y[n] = \sin\left(\frac{\pi n}{4}\right) x[n]$

a) linear $\sin\left(\frac{\pi n}{4}\right)\left[x_1[n] + x_2[n]\right] = y_1[n] + y_2[n]$

b) Time-Varying $y[n-n_0] = \sin\left(\frac{\pi(n-n_0)}{4}\right) x[n-n_0]$

$S\left[x[n-n_0]\right] = \sin(\pi n/4)\, x[n-n_0] \quad \neq$

c) $h[n]$ is response to $\delta[n] = \sin\left(\frac{\pi n}{4}\right)\delta[n] = 0$

d) response to $\delta[n-1] = \sin\left(\frac{\pi n}{4}\right)\delta[n-1] = \sin(\pi/4)\,\delta[n-1]$

e) NO, If $x[n] = \delta[n]$, $y[n] = 0$

If $x[n] = \delta[n-1]$, $y[n] = \sin(\pi/4)\,\delta[n-1] \neq h[n]\Big|_{n \leftarrow n+1}$

10.16 a) $h[n] = e^{-n} u[n]$

Causal, $h[n] = 0, \; n < 0$

Stable, $\sum_{-\infty}^{\infty} |h[k]| = \sum_{k=0}^{\infty} e^{-k} = \sum_{k=0}^{\infty} \left(\frac{1}{e}\right)^{k} = \frac{1}{1 - 1/e} < \infty$

b) $h[n] = e^{-n} u[-n]$

Noncausal, $h[n] \neq 0, \; n < 0$

Unstable, $\sum_{-\infty}^{\infty} |h[k]| = \sum_{-\infty}^{0} e^{-k} = \sum_{k=0}^{\infty} e^{k} = \infty$

c) $h[n] = e^{n} u[n]$

Causal, $h[n] = 0, \; n < 0$

Unstable, $\sum_{-\infty}^{\infty} |h[k]| = \sum_{k=0}^{\infty} e^{k} = \infty$

d) $h[n] = \cos(n) u[n]$

Causal, $h[n] = 0, \; n < 0$

Unstable, $\sum_{-\infty}^{\infty} |h[k]| = \sum_{k=0}^{\infty} |\cos(k)| = \infty$

Since $|\cos(k)|$ does not approach zero as $k \to \infty$

e) $h[n] = n e^{-n} u[n]$

Causal, $h[n] = 0, \; n < 0$

Stable, $\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=0}^{\infty} k e^{-k} = \sum_{0}^{\infty} k (1/e)^{k} = \frac{1/e}{(1 - 1/e)^{2}}$

that is $< \infty$

f) $h[n] = e^{-n} \cos(n)\, u[n]$

Causal, $h[n]=0, n<0$

Stable, $\sum_{-\infty}^{\infty} |h[k]| = \sum_{0}^{\infty} |e^{-k} \cos(k)| \leq \sum_{0}^{\infty} e^{-k} = \frac{1}{1-e} < \infty$

10.17 $y[n] = \sum_{0}^{\infty} e^{-2k} x[n-k]$

a) let $x[n] = \delta[n]$

Then $h[n] = \sum_{k=0}^{\infty} e^{-2k} \delta[n-k] = \sum_{0}^{\infty} e^{-2n} \delta[n-k]$

b) Causal since $h[n]=0, n<0$

c) Stable since $\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=0}^{\infty} e^{-2n} = \frac{1}{1-e^{-2}} < \infty$

d) $y[n] = \sum_{k=-\infty}^{n} e^{-2(n-k)} x[k-1]$

a) $h[n] = \sum_{k=-\infty}^{n} e^{-2(n-k)} \delta[k-1] = \begin{cases} 0, & n<1 \\ e^{-2(n-1)}, & n \geq 1 \end{cases}$

$\therefore h[n] = e^{-2(n-1)} u[n-1]$

b) Causal, since $h[n]=0, n<0$

c) $\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=1}^{\infty} e^{-2(n-1)} = e^{2} \sum_{k=1}^{\infty} e^{-2n} = \frac{e^{2} e^{-2}}{1-e^{-2}}$

$= \frac{1}{1-e^{-2}} < \infty$ ∴ Stable

10.18

a) $y[n]=x[n-3]$ let $x[n]=\delta[n]$ then get

$$h[n]=\delta[n-3]$$

b) $y[n]=\sum_{k=\infty}^{n+3} x[k]$

let $x[n]=\delta[n]$ The $h[n]=0$, $n+3<0$ or $n<-3$

$h[n]=1$, $n+3\geq 0$ or $n\geq -3$

$\therefore h[n]=u[n+3]$

10.19

a) (i) $y[n]-\frac{5}{6}y[n-1]=2^n u[n]$, $y[-1]=0$

$z-\frac{5}{6}=0$ $\therefore y_c[n]=c\left(\frac{5}{6}\right)^n$

$y_p[n]=P(2)^n$

$P2^n-\frac{5}{6}P2^{n-1}=2^n$

$2P-\frac{5}{6}P=2 \longrightarrow \frac{7}{6}P=2 \Longrightarrow P=\frac{12}{7}$

$y[n]=c\left(\frac{5}{6}\right)^n+\frac{12}{7}(2^n)$

$y[-1]=0=c\left(\frac{5}{6}\right)^{-1}+\frac{12}{7}(2^{-1})$ $\quad \frac{6}{5}c+\frac{6}{7}=0$

$\therefore c=-\frac{5}{7}$

$\therefore y[n]=-\frac{5}{7}\left(\frac{5}{6}\right)^n+\left(\frac{12}{7}\right)2^n$

b) $y[-1]=-\frac{5}{7}\left(\frac{6}{5}\right)+\left(\frac{12}{7}\right)\left(\frac{1}{2}\right)=-\frac{6}{7}+\frac{6}{7}=0$ ✓

$n\geq 0$ $\quad y[n]-\frac{5}{6}y[n-1]=-\frac{5}{7}\left(\frac{5}{6}\right)^n+\frac{12}{7}2^n+\frac{5}{6}\left(\frac{5}{7}\right)\left(\frac{5}{6}\right)^{n-1}$

$-\frac{5}{6}\left(\frac{12}{7}\right)2^{n-1}=-\frac{5}{7}\left(\frac{5}{6}\right)^n+\frac{12}{7}2^n+\frac{5}{7}\left(\frac{5}{6}\right)^n-\frac{5}{7}2^n$

$=2^n$

(ii) $y_c[n] = C(.7)^n$

a) $y_p[n] = pe^{-n}$ ∴ $pe^{-n} - .7pe^{-(n-1)} = pe^{-n}[1 - .7e]$

$$= pe^{-n}[-.903] = e^{-n}$$

$$\Longrightarrow p = -1.108$$

∴ $y[n] = c(.7)^n - 1.108e^{-n}$

$y[-1] = 0 = \frac{c}{.7} - 1.108e \Longrightarrow \frac{c}{.7} = 3.012 \Longrightarrow c = 2.108$

∴ $y[n] = -1.108e^{-n} + 2.108(.7)^n$, $n \geq -1$

b) $y[-1] = -1.108e^{-1} + 2.108(.7)^{-1} = -3.012 + 3.01 = 0$ ✓

$y[n] - .7y[n-1] = -1.108e^{-n} + 2.108(.7)^n$

$-.7[-1.108e^{-(n-1)} + 2.108(.7)^{n-1}] = -1.108e^{-n}$

$+2.108(.7)^n + 2.108e^{-n} - 2.108(.7)^n = e^{-n}$ ✓

(iii) $y[n] + 3y[n-1] + 2y[n-2] = 3u[n]$

$y[-1] = 0$, $y[-2] = 0$

$z^2 + 3z + z = (z+2)(z+1)$

∴ $y_c[n] = c_1(-2)^n + c_2(-1)^n$ $y_p[n] = P$

∴ $p + 3p + 2p = 3 \Longrightarrow 6p = 3 \longrightarrow p = 1/2$

∴ $y[n] = 1/2 + c_1(-2)^n + c_2(-1)^n$

use initial conditions to solve for c_1 & c_2

$y[-1] = 0 = 1/2 + c_1(-1/2) + c_2(-1)$

$y[-2] = 0 = 1/2 + c_1(1/4) + c_2$ $\Longrightarrow$ $c_1 = 4$, $c_2 = -3/2$

$\therefore\ y[n] = 1/2 + 4(-2)^n - 3/2(-1)^n$

b) $y[-1] = 1/2 + 4(-2)^{-1} - 3/2(-1)^{-1} = 1/2 + (-4/2) + 3/2 = 0$ ✓

$y[-2] = 1/2 + 4(-2)^{-2} - 3/2(-1)^{-2} = 1/2 + 4/4 - 3/2 = 0$ ✓

$$y[n] + 3y[n-1] + 2y[n-2] = 1/2 + 4(-2)^n - 3/2(-1)^n$$
$$+ 3/2 + 12(-2)^{n-1} - 9/2(-1)^{n-1} + 1 + 8(-2)^{n-2}$$
$$- 3(-1)^{n-2} = 3$$ ✓

10.20

(i) $z - .5 = 0$

a) mode is $(.5)^n$ b) $y_c[n] = C(.5)^n$

(ii)

a) $z^2 - 1.1z + .3 = 0 = (z-.6)(z-.5) \Rightarrow (.5)^n, (.6)^n$

b) $y_c[n] = C_1(.5)^n + C_2(.6)^n$

(iii)

a) $z^2 + 1 = 0 = (z-j)(z+j) \Rightarrow (j)^n, (-j)^n$

b) $y_c[n] = C_1(j)^n + C_2(-j)^n$

(iv)

a) $z^3 + 2z^2 + 1.5z - .5 = 0 = (z-1)(z - 1/2 - 1/2 j)(z - 1/2 + 1/2 j)$

$\Rightarrow (1)^n,\ (1/2 + 1/2 j)^n,\ (1/2 - 1/2 j)^n$

$y_c[n] = C_1 + C_2(1/2 + 1/2 j)^n + C_3(1/2 - 1/2 j)^n$

(v) a) $(z-.5)^3=0 \rightarrow (.5)^n, n(.5)^n, n^2(.5)^n$

b) $y_c[n]=c_1(.5)^n+c_2 n(.5)^n+c_3 n^2(.5)^n$

(vi) a) $(z-.5)(z-1.5)(z+.7)=0$

$\rightarrow (.5)^n, (1.5)^n, (-.7)^n$

b) $y_c[n]=c_1(.5)^n+c_2(1.5)^n+c_3(-.7)^n$

10.21

Stable if all roots of char eqn are inside the unit circle

(i) $z-.9=0$, $z=.9$ stable

(ii) $z=.8, .9$ stable

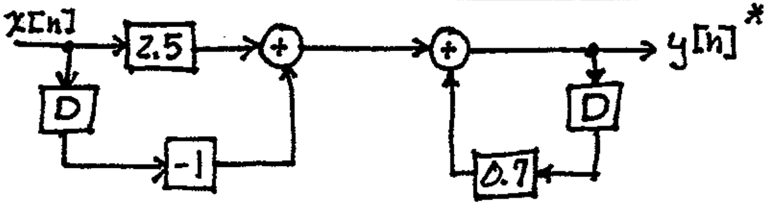
(iii) $z=j\sqrt{2}, -j\sqrt{2}$ not stable

(iv) $z=1, 1\angle\mp 120^\circ$ not stable

(v) $z=.9, .9, .9$ Stable

(vi) $z=.9, 1.2, -.85$ not Stable

*10.22. (a)



10.22. See last page of this chapter

(b) $y[n] = 0.7y[n-1] + 2.5x[n] - x[n-1]$

$y[0] = 0 + 2.5(1) - 0 = \underline{2.5}$

$y[1] = 0.7(2.5) + 0 - 1 = \underline{0.75}$

$y[2] = 0.7(0.75) + 0 - 0 = \underline{0.5250}$

$y[3] = 0.7(0.5250) = \underline{0.3675}$

$y[4] = 0.7(0.3675) = \underline{0.2573}$

(c)

$w[0] = 2.5 \qquad y[0] = \underline{2.5}$

$w[1] = 1 \qquad y[1] = -1 + 0.7(2.5) = \underline{0.75}$

$w[2] = 0 \qquad y[2] = 0.7(0.75) = \underline{0.5250}$

$w[3] = 0 \qquad y[3] = 0.7(0.5250) = \underline{0.3675}$

$w[4] = 0 \qquad y[4] = 0.7(0.3675) = \underline{0.2573}$

(d) $y[n] = h[n+2] - 3h[n] + 2h[n-1]$

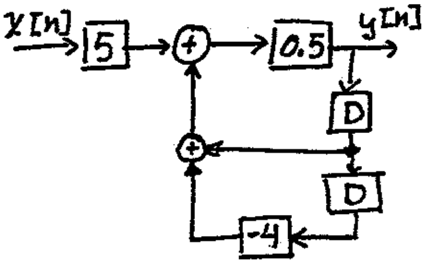
(e) $y[-3] = h[-1] - 3h[-3] + 2h[-4] = \underline{0}$

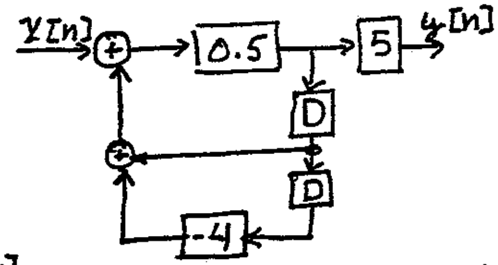
$y[-1] = h[1] - 0 + 0 = \underline{0.75}$

$y[1] = h[3] - 3h[1] + 2h[0]$

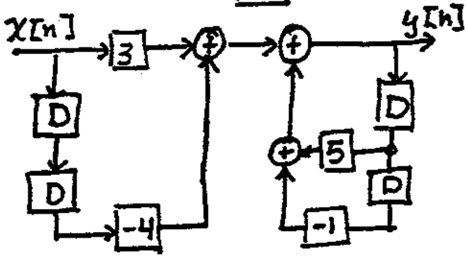
$= 0.3675 - 3(0.75) + 2(2.5) = \underline{3.118}$

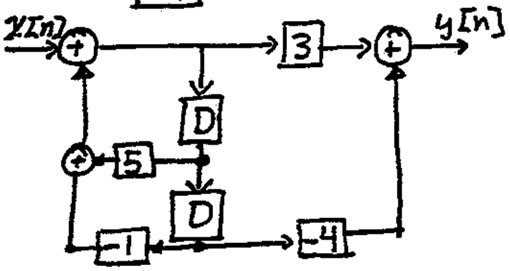
10.23 (a)



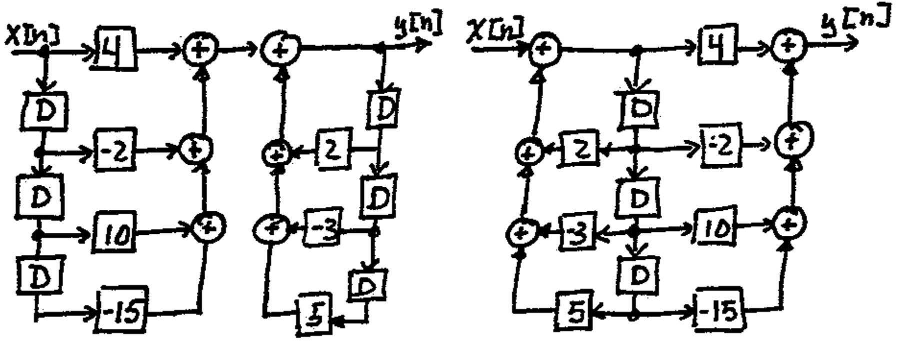


(b)

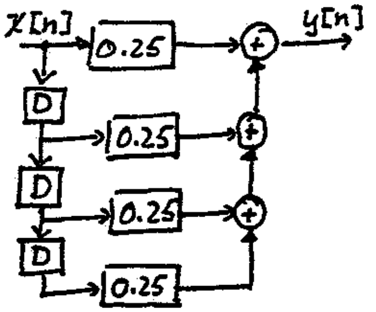




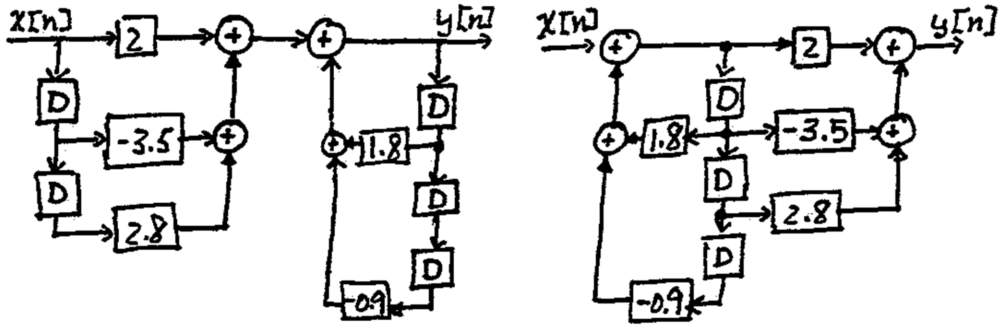
10.23.(C) (cont)



(d)



(e)



10.24(a) $2z^2-z+4=2(z^2-0.5z+2)=2(z-z_1)(z-z_2)$

$\therefore (z_1 z_2)=2$, and at least one root is greater than unity - not stable

(b) $(z^2-5z+1)=(z-4.79)(z-0.21)$ not stable

(c) $z^3-2z^2+3z-5=(z-z_1)(z-z_2)(z-z_3)$

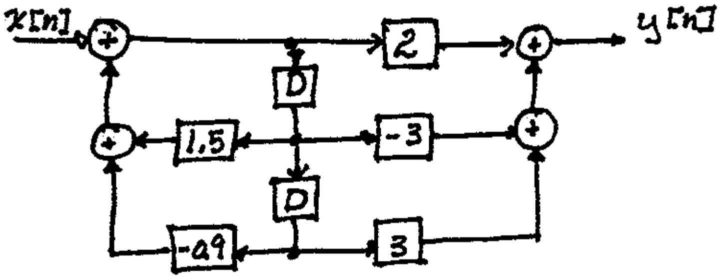
$\therefore (z_1 z_2 z_3)=5$ - not stable (see (a))

(d) stable by inspection (no feedback)

(e) $z^2-1.8z+0.9=(z-0.949\angle 18.4^\circ)(z-0.949\angle -18.4^\circ)$ stable

```
n=[2 -1 4];
roots(n)
pause
n=[1 -5 1];
roots(n)
pause
n=[1 -2 3 -5];
roots(n)
pause
n=[1 -1.8 .9];
roots(n)
```

10.25.(a)

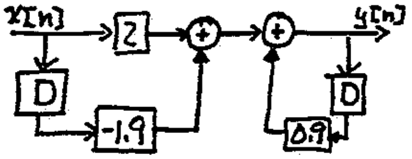


$$y[n]-1.5y[n-1]+0.9y[n-2]=2x[n]-3x[n-1]+4x[n-2]$$

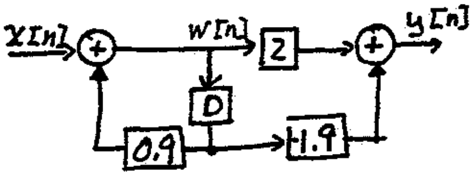
(b) <u>form II</u>

10.26. $y[n]-0.9y[n-1]=2x[n]-1.9x[n-1]$

(a)



(b)



(c) $y[0]=0.9(0)+2-0=\underline{2}$

$z-0.9=0 \Rightarrow y_c[n]=C(0.9)^n$

$y_p[n]=P(0.8)^n \Rightarrow P(0.8)^n-\frac{0.9}{0.8}P(0.8)^n$

$=(P-1.125P)(0.8)^n=(2-2.375)(0.8)^n \Rightarrow P=\underline{3}$

$\therefore y[n]=3(0.8)^n+C(0.9)^n$

$y[0]=2=3+C \Rightarrow C=-1$ and $y[n]=\underline{3(0.8)^n-(0.9)^n}$

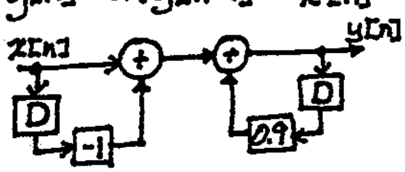
For example: $y[5]=3(0.8)^5-(0.9)^5=0.3926$ checks MATLAB

(d)

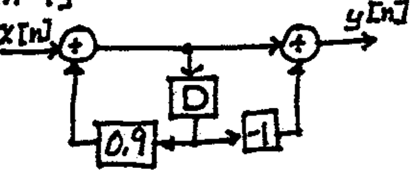
```
y(1)=2;
for n=1:5
  y(n+1)=.9*y(n)+2*((.8)^n)-1.9*((.8)^(n-1));
end
y
```

10.27.(a) $y[n]-0.9y[n-1]=x[n]-x[n-1]$

(b)



(c)



(d) $x[n]=(0.7)^n u[n]$

(e) $z-0.9=0 \Rightarrow y_c[n]=C(0.9)^n$; $y_p[n]=P(0.7)^n$

$P(0.7)^n-\frac{0.9}{0.7}P(0.7)^n=(0.7)^n-\frac{1}{0.7}(0.7)^n \Rightarrow \underline{P=1.5}$

10.27 (e) $\therefore y[n] = C(.9)^n + (1.5)(.7)^n$
cont

$$y[0] = 0 = C + 1.5 \Rightarrow C = -1.5$$

$$\therefore y[n] = 1.5\left[(.7)^n - (.9)^n\right]$$

$y[0] = 0$ $\quad$ $y[2] = -.48$

$y[1] = -.3$ $\quad$ $y[3] = -.579$

10.28

a) $y[n] - .9\, y[n-1] = x[n] - x[n-1]$

b) $y_p[n] = P(1)^n = P$

$P - .9P = 1 - 1 = 0 \Rightarrow P = 0$

c) $H(z) = \dfrac{z-1}{z-.9}$

d) $Y(z) = H(z)X(z) = \dfrac{z-1}{z-.9} \cdot \dfrac{z}{z-1} = \dfrac{z}{z-.9}$

$y[n] = (.9)^n \Rightarrow \lim\limits_{n \to 0} y[n] = 0$

e) in the Second statement : $x[n] = 1$

f)
```
y(1)=0; x(1)=0;
for n=2:6; x(n)=1; end
for n=2:6
    y(n)=.9*y(n-1)+x(n)-x(n-1);
end
```

10.29 a) $y[n] - .7\, y[n-1] = x[n]$

$$Y(z) - .7\, z^{-1} Y(z) = X(z)$$

$$Y(z)\left[1 - .7\, z^{-1}\right] = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - .7 z^{-1}} = \frac{z}{z - .7}$$

b) $x[n] = \cos(n)\, u[n] = \cos(\Omega n)\, u[n] \quad \therefore \Omega = 1$

$$\cos \Omega n \rightarrow (1)\, |H(e^{j\Omega})| \cos(\Omega n + \theta 4)$$

$$e^{j\Omega}\Big|_{\Omega=1} = e^{j} = \cos 1 + j \sin 1 = .54 + .841 j$$

$$\therefore H(e^{j}) = \frac{.54 + .841 j}{.54 + j .841 - .7} = \frac{1 \angle 57.3^\circ}{.856 \angle 100.8^\circ} = 1.168 \angle -43.5^\circ$$

$$\therefore y_{ss}[n] = 1.168 \cos(n - 43.5^\circ)$$

d) $y_{ss}[n] - .7\, y_{ss}[n-1] = 1.168 \cos(n - 43.5^\circ)$

$$- .7(1.168) \cos(n - 43.5^\circ - 57.3^\circ)$$

$$= .847 \cos n + .804 \sin n + .153 \cos n -$$

$$.803 \sin \approx \cos n$$

10.30

for the problem you did not need to do the sums of the convolutions, you could have saved time by ignoring all the time shifts since they do not affect whether or not the sums converge.

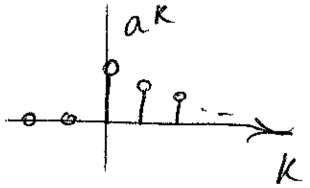
a) $\sum_{i=-\infty}^{1} b^{i}$

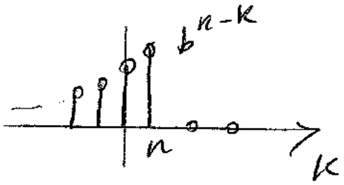
examine: $\sum_{i=-\infty}^{C} b^{i} = \sum_{i=0}^{\infty} (1/b)^{i}$

Sum exsits if $|1/b|<1$ or $|b|>1$

b) $a^{n}u[n] * b^{n}u[n+6]$

examine $= a^{n}u[n] * b^{n}u[n]$



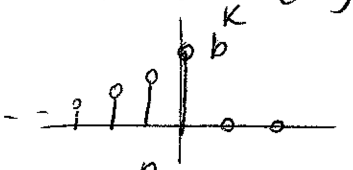


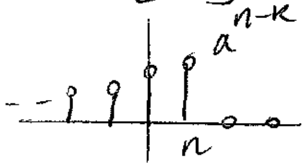
$n<0$, 0

$n \geq 0$ $\sum_{k=0}^{n} a^{k} b^{n-k} \Rightarrow$ This is a finite Sum no ristriction required

c) $a^{n}u[n-2] * b^{n}u[-n-4]$

examine $a^{n}u[n] * b^{n}u[-n]$





$n<0$, $\sum_{k=-\infty}^{n} a^{n-k} b^{k} = a^{n} \sum_{k=-\infty}^{n} (b/a)^{k} = a^{n} \sum_{k=-n}^{\infty} (a/b)^{k}$

this is an infinite sum, it will exsist if $|a/b|<1$ or $|a|<|b|$

d) $a^n u[-n+3] * b^n u[-n-4]$

Since both functions are left-sided, like part ⓑ you only get a finite sum and no restriction is required.

Chapter 11

11.1 a) $Z(.6)^n = \dfrac{z}{z-.6}$

b) $Z(.7^n + 4(1.5)^n) = \dfrac{z}{z-.7} + \dfrac{4z}{z-1.5} = \dfrac{5z^2-4.3z}{z^2-2.2z+1.05}$

c) $Z(5e^{-.5n}) = 5\sum_0^{\infty} e^{-.5n} z^{-n} = 5\sum_0^{\infty}(e^{-.5}z^{-1})^n = \dfrac{5z}{z-e^{-.5}}$

$= \dfrac{5z}{z-.6065}$

d) $Z[5e^{-j.1n}] = \dfrac{5z}{z-e^{-.1j}} = \dfrac{5z}{z-.995+.0998j}$

e) $Z[5\sin 3n] = \dfrac{5z\sin 3}{z^2-2z\cos 3+1}$

f) $Z[20\cos(2n-\pi/4)]$

Using Trig identity $\cos(A-B) = \cos A\cos B + \sin A\sin B$

we get $Z[20\cos 2n\cos\pi/4 + 20\sin 2n\sin\pi/4]$

$= Z[14.14\cos 2n + 14.14\sin 2n] =$

$$\frac{14.14\,z(z-\cos 2)}{z^2-2z\cos 2+1} + \frac{14.14\,z\sin 2}{z^2-2z\cos 2+1} = \frac{14.14z^2+18.75z}{z^2+.832z+1}$$

g) $Z[e^{-.5n}\cos .3n]$ $\quad e^{-.5} = .6065$

from table 11.2, entry 12, we get

$$Z[e^{-.5n}\cos .3n] = \frac{z(z-.6065\cos(.3))}{z^2-2(.6065)z\cos(.3)+(.6065)^2}$$

$$= \frac{z(z-.5794)}{z^2-1.16z+.368}$$

h) $$Z\left[e^{-.5n}\cos(.3n-\pi/4)\right] = Z\left[(.6065)^n .707\left(\sin(.3n)+\cos(.3n)\right)\right]$$

$$= .707\left[\frac{.6065z\sin(.3)}{z^2-2(.6065)z\cos(.3)+(.6065)^2}\right] +$$

$$.707\left[\frac{z(z-(.6065)\cos(.3))}{z^2-2(.6065)z\cos(.3)+(.6065)^2}\right]$$

$$= \frac{.1267z}{z^2-1.16z+.368} + \frac{.707z^2-.4096z}{z^2-1.16z+.368}$$

11.2 $t = nT = .05n$

a) $$2e^{-2t} \text{ or } Z\left[2e^{-2(.05)n}\right] = Z\left[2e^{-.1n}\right] = \frac{2z}{z-e^{-.1}}$$

$$= \frac{2z}{z-.905}$$

b) $$Z\left[2e^{-.1n}+2e^{.05n}\right] = \frac{2z}{z-e^{-.1}} + \frac{2z}{z-e^{.05}} =$$

$$\frac{2z}{z-.905} + \frac{2z}{z-1.05}$$

c) $$Z\left[2e^{-.2(.05)n}\right] = Z\left[2e^{-.01n}\right] = \frac{2z}{z-e^{-.01}} = \frac{2z}{z-.99}$$

d) $$Z\left[5e^{-.5j(.05)n}\right] = Z\left[5e^{-.025jn}\right] = \frac{5z}{z-e^{-.025j}} = \frac{5z}{z-.9997+.025j}$$

e) $Z[5\cos(.05n)] = \frac{5z(z-\cos .05)}{z^2-2z\cos .05+1} = \frac{5z^2-4.99z}{z^2-1.998z+1}$

f) $z[5e^{-.05n}\cos(.05n)] = \frac{5z[z-(e^{-.05})\cos .05]}{z^2-2(e^{-.05})\cos .05z+(e^{-.05})^2}$

$= \frac{5z^2-4.75}{z^2-1.9z+.905}$

11.3 a) $x_a[nT] = e^{-5(.2)n} = e^{-n} = (e^{-1})^n = (.3679)^n$

b) $x_b[nT] = e^{-n} = (.3679)^n$

c) The value of the two signals are equal at each sample instant.

d) $e^{-anT} = (e^{-aT})^n = (e^{-1})^n \therefore aT = 1$

(i) $a = 1/2,\ T = 2$ (ii) $a = 2,\ T = 1/2$

11.4 a) (i) $z[.5^n] = \sum_{n=0}^{\infty} .5^n z^{-n} = \frac{z}{z-.5}$

$\therefore \sum_{0}^{\infty} .5^n = z[.5^n]\Big|_{z=1} = \frac{1}{1-.5} = \underline{2} = x$

(ii) $x = \sum_{n=2}^{\infty} .5^n = \sum_{n=0}^{\infty} .5^n - 1 - .5 = 2 - 1.5 = \underline{.5} = x$

b) $z[(.5)^n\cos .1n] = \frac{z[z-.5(.995)]}{z^2-.5(1.99z)+(.5)^2} = \frac{z(z-.498)}{z^2-.995z+.25}$

$\therefore \sum_{n=0}^{\infty} (.5)^n \cos(.1n) = z[(.5)^n\cos(.1n)]\Big|_{t=1} = \frac{1-.498}{1-.995+.25}$

$= \underline{1.969} = x$

11.5

a) $Z[A\cos\Omega n] = \frac{Az(z-\cos\Omega)}{z^2-2\cos\Omega z+1} = \frac{3z(z-.6967)}{z^2-1.393z+1}$

$\therefore A = \underline{3}$; $\cos\Omega = .6967 \Longrightarrow \Omega = 45.84^\circ = .8 \text{ rad} = \Omega$

b) $A = \underline{3}$; $\cos\Omega n = \cos(\omega T)n$, $\therefore \omega(.0001) = .8$

$\therefore \omega = 8000$

11.6 a) $\lim_{z\to 1}(z-1)F(z) = \lim_{z\to 1}\frac{z^2}{z+1} = 1/2$

b) $\frac{F(z)}{z} = \frac{z}{(z-1)(z+1)} = \frac{1/2}{z-1} + \frac{1/2}{z+1} \Longrightarrow F(z) = \frac{1/2\, z}{z-1} + \frac{1/2\, z}{z+1}$

$\therefore f(n) = 1/2 + 1/2(-1)^n$ ∴ f[n] continues to alternate between 1 and 0, and has no final value.

c) The final value property does not apply

11.7 $Z[4^n] = \frac{z}{z-4}$

a) $f[n] = n(n-1)4^n u[n]$

$Z[n4^n] = -z\frac{df(z)}{dz} = -z\left(\frac{1}{z-4} - \frac{z}{(z-4)^2}\right)$

$= -z\left(\frac{z-4-z}{(z-4)^2}\right) = \frac{4z}{(z-4)^2}$

$Z\left(n^2 2^n\right) = -z\frac{d}{dz}\left(\frac{4z}{(z-4)^2}\right) = -z\left(\frac{4}{(z-4)^2} - \frac{8z}{(z-4)^3}\right)$

$= -z\left(\frac{-4z-16}{(z-4)^3}\right) = \frac{4z^2+16z}{(z-4)^3} = \frac{4z(z+4)}{(z-4)^3}$

$$\therefore F(z) = \frac{4z(z+4)}{(z-4)^3} - \frac{4z}{(z-4)^2} = \frac{4z(z+4)-4z(z-4)}{(z-4)^3} = \frac{32z}{(z-4)^3}$$

b) $$F(z) = \frac{4z(z+4)}{(z-4)^3} - \frac{4z}{(z-4)^2} = \frac{4z(z+4)-4z(z-4)}{(z-4)^3} = \frac{32z}{(z-4)^3} \checkmark$$

11.8 a) $$z\left[y[n-3]u[n-3]\right] = z^{-3}Y(z) = \frac{1}{z^3-3z^2+5z-9} = Y_1(z)$$

b) $$z\left[y[n+3]u[n]\right] = z^3\left[Y(z) - y[0] - y[1]z^{-1} - y[2]z^{-2}\right]$$

$$\begin{array}{r} 1+3z^{-1}+4z^{-2}+6z^{-3}+\cdots \\ z^3-3z^2+5z-9\,\overline{\smash{)}z^3} \end{array}$$

$\therefore y[0]=1,\ y[1]=3,\ y[2]=4$

$$\therefore z\left[y[n+3]u[n]\right] = z^3\left[\frac{z^3}{z^3-3z^2+5z-9} - 1 - \frac{3}{z} - \frac{4}{z^2}\right]$$

$$= \frac{6z^3+7z^2+36z}{z^3-3z^2+5z-9} = Y_2(z)$$

c) $y[0]=1,\ y[3]=6$ from (b)

$y_1[3]=1$, by inspection in a

$y_2[0]=6$, by inspection in b

d) $y_1[3] = y[n-3]u[n-3]\big|_{n=3} = y[0]$ ✓

$y_2[0] = y[n+3]u[n]\big|_{n=0} = y[3]$ ✓

11.9 $f[n] = a^n u[n]$ $\quad z[f[n]] = \frac{z}{z-a} = F(z)$

a) $$z\left[f[n/3]\right] = f(z^3) = \frac{z^3}{z^3-a}$$

b) $$z\left[f[n-3]u[n-3]\right] = z^{-3}F(z) = \frac{z-2}{z-9} = \frac{1}{z^2(z-9)}$$

$$\begin{array}{r l} & z^{-3} + az^{-4} + a^2 z^{-5} + \cdots \\ z^3 - az^2 \big) & 1 \\ & \underline{1 - az^{-1}} \\ & az^{-1} \\ & \underline{az^{-1} - a^2 z^{-2}} \\ & a^2 z^{-2} \end{array}$$

c) $$\mathcal{Z}\left[f[n+3]u[n]\right] = z^3\left[F(z) - f[0] - f[1]z^{-1} - f(2)z^{-2}\right]$$

$$= z^3\left[\frac{z}{z-a} - 1 - az^{-1} - a^2 z^{-2}\right] = \frac{z^3 a^3 z^{-2}}{z-a} = \frac{a^3 z}{z-a}$$

$$\begin{array}{r l} & a^3 + a^4 z^{-1} + a^5 z^{-2} + \cdots \\ z - a \big) & a^3 z \\ & \underline{a^3 z - a^4} \\ & a^4 \\ & \underline{a^4 - a^5 z^{-1}} \\ & a^5 z^{-1} \end{array}$$

d) $b^{2n} f[n]$, $\quad F\left(\frac{z}{b^2}\right) = \frac{z/b^2}{z/b^2 - a} = \frac{z}{z - ab^2}$

or $\quad \mathcal{Z}\left[b^{2n} a^n u[n]\right] = \mathcal{Z}\left[(ab^2)^n u[n]\right] = \frac{z}{z - ab^2}$

11.10 (i) (a) $\frac{X(z)}{z} = \frac{0.4z}{(z-1)(z-0.6)} = \frac{1}{z-1} + \frac{-0.6}{z-0.6} \Rightarrow x[n] = \underline{1-(0.6)^{n+1}}, n \geq 0$

(b)
```
n=[0 .4 0]; d=[1 -1.6 .6]; [r,p,k]=residue(n,d), pause
n=[0 0 .4]; d=[1 -1.6 .6]; [r,p,k]=residue(n,d), pause
n=[0 0 0 .4]; d=[1 -1.6 .6 0]; [r,p,k]=residue(n,d)
```

(c) $x[0]=0.4,\ x[1]=0.64,\ x[2]=0.784$

(d)
$$\begin{array}{r} 0.4 + 0.64z^{-1} + 0.784z^{-2} + \cdots \\ z^2-1.6z+0.6 \overline{\smash{)}\,0.4z^2} \\ \underline{0.4z^2-0.64z+0.24} \\ 0.64z-0.24 \\ \underline{0.64z-1.024+\cdots} \\ 0.784+\cdots \end{array}$$

(e) $x[\infty] = \lim_{z\to 1}(z-1)X(z) = \lim_{z\to 1}\frac{0.4z^2}{z-0.6} = \underline{1}$

(f) $\lim_{n\to\infty}[1-(0.6)^{n+1}] = \underline{1}$

(g) $x[0] = \lim_{z\to\infty} X(z) = \underline{0.4}$ (h) $x[0] = 1-(0.6)^1 = \underline{0.4}$

(ii) (a) $\frac{X(z)}{z} = \frac{0.4}{(z-1)(z-0.6)} = \frac{1}{z-1} + \frac{-1}{z-0.6} \Rightarrow x[n] = \underline{1-(0.6)^n}, n \geq 0$

(c) $x[0]=0,\ x[1]=0.4,\ x[2]=0.64,\ x[3]=0.784$

(d)
$$\begin{array}{r} 0.4z^{-1} + 0.64z^{-2} + 0.784z^{-3} + \cdots \\ z^2-1.6z+0.6 \overline{\smash{)}\,0.4z} \\ \underline{0.4z-0.64+0.24z^{-1}} \\ 0.64-0.24z^{-1} \\ \underline{0.64-1.024z^{-1}} \\ 0.784z^{-1}+\cdots \end{array}$$

(e) $x[\infty] = \lim_{z\to 1}(z-1)X(z)$

$$= \lim_{z\to 1}\frac{0.4z}{z-0.6} = \underline{1}$$

(f) $\lim_{n\to\infty}[1-0.6^n] = \underline{1}$

(g) $x[0] = \lim_{z\to\infty} X(z) = 0$ (h) $x[0] = 1-(0.6)^0 = 0$

11.10. (iii) (a) $\frac{X(z)}{z}=\frac{0.4}{z(z-1)(z-0.6)}=\frac{2/3}{z}+\frac{1}{z-1}+\frac{-5/3}{z-0.6}$

$\therefore x[n]=\underline{\frac{2}{3}\delta[n]+1-\frac{5}{3}(0.6)^n}$

(c) $x[0]=\frac{2}{3}+1-\frac{5}{3}=\underline{0}$; $x[1]=1-\frac{5}{3}(0.6)=\underline{0}$; $x[2]=1-\frac{1}{0.6}(0.6)^2=\underline{0.4}$

$x[3]=1-\frac{1}{0.6}(0.6)^3=\underline{0.64}$, $x[4]=\underline{0.784}$

(d)
$$\begin{array}{r|l} & 0.4z^{-2}+0.64z^{-3}+0.784z^{-4}+\cdots \\ \hline z^2-1.6z+0.6 & 0.4 \\ & \underline{0.4-0.64z^{-1}+0.24z^{-2}} \\ & 0.64z^{-1}-0.24z^{-2} \\ & \underline{0.64z^{-1}-1.024z^{-2}+\cdots} \\ & 0.784z^{-2}+\cdots \end{array}$$

(e) $x[\infty]=\lim_{z\to1}(z-1)X(z)=\lim_{z\to1}\frac{0.4}{z-0.6}=\underline{1}$

(f) $\lim_{n\to\infty}\left[1-\frac{5}{3}(0.6)^n\right]=\underline{1}$

(g) $x[0]=\lim_{z\to\infty}X(z)=\underline{0}$ (h) $x[0]=\frac{2}{3}+1-\frac{5}{3}=\underline{0}$

(iv) (a) $X(z)=\frac{z}{z^2-z+1}$, $\mathcal{Z}[\sin bn]=\frac{(\sin b)z}{z^2-2\cos bz+1}$

$\therefore 2\cos b=1,\ \cos b=\frac{1}{2},\ b=\pi/3$

$\therefore \sin b=0.866,\ x[n]=\frac{1}{0.866}\sin\frac{\pi}{3}n=\underline{1.155\sin\frac{\pi}{3}n}$

(b) $x[0]=0,\ x[1]=1,\ x[2]=1,\ x[3]=0,\ x[4]=-1$

(c)
$$\begin{array}{r|l} & z^{-1}+z^{-2}+z^{-4}+\cdots \\ \hline z^2-z+1 & z \\ & \underline{z-1+z^{-1}} \end{array}$$

(d),(e) $x[n]$ does not have a final value.

(f) $x[0]=\lim_{z\to\infty}X(z)=0$

(g) $x[n]=1.155\sin\frac{\pi}{3}n$, $\therefore x[0]=\underline{0}$

11.11. (a) $\mathcal{Z}(x[n-1]u[n-1])=\frac{X(z)}{z}$, $X_1(z)=\frac{0.4z^2}{(z-1)(z-0.6)}$

$\therefore x_2[n]=x_1[n-1]u[n-1]$

$x_3[n]=x_1[n-2]u[n-1]=x_2[n-1]u[n-1]$

(b) See Problem 11.12 (i), (ii), (iii)

11.12 (a) $Y(z)=[1-1.5z^{-1}+0.5z^{-2}]=X(z)\Rightarrow H(z)=\frac{z^2}{z^2-1.5z+0.5}$; $X(z)=z^{-1}$

$\therefore \frac{Y(z)}{z}=\frac{1}{(z-1)(z-0.5)}=\frac{2}{z-1}+\frac{-2}{z-0.5}\Rightarrow y[n]=2-2(0.5)^n$

11.12(b) `n=[0 0 1]; d=[1 -1.5 .5]; [r,p,k]=residue(n,d)`

(cont)

(c) $y[n] = 2 - 2(0.5)^n \Rightarrow y[0] = \underline{0},\ y[1] = \underline{1},\ y[2] = \underline{1.5},$
$y[3] = \underline{1.75},\ y[4] = \underline{1.875}$

$y[n] = 1.5y[n-1] - 0.5y[n-2] + x[n]$

$y[0] = 0 - 0 + 0 = \underline{0}$

$y[1] = 0 - 0 + 1 = \underline{1}$

$y[2] = 1.5(1) - 0 + 0 = \underline{1.5}$

$y[3] = 1.5(1.5) - 0.5(1) = \underline{1.75}$

$y[4] = 1.5(1.75) - 0.5(1.5) = \underline{1.875}$

(d)

```
x=[0 0 0 1 0 0 0 0]; y(1)=0; y(2)=0;
for n=3:7
   y(n)=1.5*y(n-1)-0.5*y(n-2)+x(n)
end
```

(e) $y[0] = \lim_{z\to\infty} Y(z) = \lim_{z\to\infty} \frac{z}{z^2} = \underline{0}$

(f) yes - $\lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1}\frac{z}{z-0.5} = \underline{2}$

11.13(a) $Y(z) - 0.75z^{-1}Y(z) + 0.125z^{-2}Y(z) = X(z) = 1$

$\therefore \frac{Y(z)}{z} = \frac{z}{z^2-0.75z+0.125} = \frac{z}{(z-0.5)(z-0.25)} = \frac{2}{z-0.5} + \frac{-1}{z-0.25}$

$\therefore y[n] = \underline{(2(0.5)^n - (0.25)^n)u[n]}$

(c) $y[0] = \underline{1},\ y[1] = \underline{0.75},\ y[2] = \frac{2}{4} - \frac{1}{16} = \underline{0.4375}$

$y[3] = 2(0.125) - \frac{1}{64} = \underline{0.2344},\ y[4] = \underline{0.1211}$

also $y[n] = 0.75y[n-1] - 0.125y[n-2] + x[n]$

$y[0] = 0 - 0 + 1 = \underline{1}$

$y[1] = 0.75(0) - 0 + 0 = \underline{0.75}$

$y[2] = 0.75(0.75) - 0.125(1) + 0 = \underline{0.4375}$

$y[3] = 0.75(0.4375) - 0.125(0.75) = \underline{0.2344}$

$y[4] = 0.75(0.2344) - 0.125(0.4375) = \underline{0.1211}$

(e) $y[0] = \lim_{z\to\infty} Y(z) = 1$

(f) Yes, $y[\infty] = \underline{0}$ from (a)

$y[\infty] = \lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1}\frac{z^2(z-1)}{(z-0.5)(z-0.25)} = \underline{0}$

(b), (d)

```
n=[0 1 0];  d=[1 -.75 .125];
[r,p,k]=residue(n,d)
pause
x=[0 0 1 0 0 0 0 0];  y(1)=0;  y(2)=0;
for n=3:7
  y(n) = .75*y(n-1)-.125*y(n-2)+x(n);
end
y
```

11.14. (a) $Y(z) - z^{-1}Y(z) + 0.5z^{-2}Y(z) = X(z) = z^{-1}$

$\therefore Y(z) = \frac{z}{z^2 - z + 0.5} = \frac{z}{z^2 - 2a\cos b z + a^2} = \frac{1}{a\sin b} \mathcal{Z}[a^n \sin bn]$

$a = \sqrt{0.5} = 0.707$, $\cos b = \frac{1}{2(0.707)} = 0.707$, $b = 45^0 = \frac{\pi}{4}$

$\therefore y[n] = \frac{(0.707)^n}{0.707(0.707)} \sin\frac{\pi}{4}n = \underline{2(0.707)^n \sin(\frac{\pi}{4}n)\, u[n]}$

(c) $y[0] = \underline{0}$, $y[1] = \underline{1}$, $y[2] = \underline{1}$, $y[3] = \underline{0.5}$, $y[4] = \underline{0}$

also $y[n] = y[n-1] - 0.5y[n-2] + x[n]$

$y[0] = 0 - 0 + 0 = \underline{0}$

$y[1] = 0 - 0 + 1 = \underline{1}$

$y[2] = 1 - 0 + 0 = \underline{1}$

$y[3] = 1 - 0.5 + 0 = \underline{0.5}$

$y[4] = 0.5 - 0.5 + 0 = \underline{0}$

(e) $y[0] = \lim_{z\to\infty} Y(z) = \underline{0}$

(f) Yes, $y[\infty] = \lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1}\frac{z(z-1)}{z^2 - z + 0.5} = \underline{0}$

(b), (d)

```
x=[0 0 0 1 0 0 0 0];  y(1)=0;  y(2)=0;
for n=3:7
  y(n) = y(n-1)-.5*y(n-2)+x(n);
end
```

11.15 a) $Y(z) = az^{-1}X(z) + az^{-1}Y(z)$

$$[1 - az^{-1}]Y(z) = az^{-1}X(z)$$

$$\therefore \quad y[n] = ay[n-1] = ax[n-1]$$

b) from a) $\dfrac{Y(z)}{X(z)} = \dfrac{az^{-1}}{1-az^{-1}} = \dfrac{a}{z-a}$

c) pole at $z=1$ must be inside the unit circle

$\therefore \ |a|<1$ or $-1<a<1$

d) $X(z)=1$, $H(z) = \dfrac{a}{z-a} \Rightarrow \dfrac{H(z)}{z} = \dfrac{a}{z(z-a)} = \dfrac{-1}{z} + \dfrac{1}{z-a}$

$$\therefore \ h[n] = \begin{cases} -1+1=0, & n=0 \\ a^n, & n\geq 1 \end{cases} = a^n u[n-1]$$

Yes, this output is bounded only for $|a|<1$

e) $Y(z)=H(z)X(z) = \dfrac{.5}{z-.5}\left(\dfrac{z}{z-1}\right)$

$$\therefore \ \frac{Y(z)}{z} = \frac{1}{z-1} + \frac{-1}{z-.5} \Rightarrow y[n] = 1 - .5^n, \ n\geq 0$$

f) $\dfrac{Y(z)}{z} = \dfrac{-2}{z-1} + \dfrac{2}{z-2} \Rightarrow y[n] = 2[1-2^n], \ n\geq 0$

↳ since $Y(z)=X(z)H(z)$

g)
```
n=[0 0 .5]; d=[1 -1.5 .5];
[r, p, k] = residue(n,d)
pause
n=[0 0 2]; d=[1 -3 2];
[r,p,k] = residue(n,d)
```

11.16 a) let $x[n]=\delta[n]$, then

$$y[n]=h[n]=a\delta[n-1]+(1-a)\delta[n]$$

b) $H(z) = az^{-1} + 1 - a = \frac{a+(1-a)z}{z}$

Inverse system $H_I(z) = \frac{z}{a+(1-a)z} = \frac{\frac{z}{1-a}}{z+\frac{a}{1-a}}$

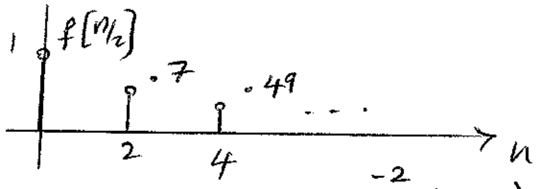
$\therefore \; h_I[n] = \frac{1}{1-a}\left(\frac{-a}{1-a}\right)^n u[n] = \frac{1}{1-a}\left(\frac{a}{a-1}\right)^n u[n]$

11.17 $\mathcal{Z}\left[f[n/k]\right] = F(z^k)$

(i) a) $F(z^2) = \frac{z^2}{z^2-.7}$, $\therefore F(z) = \frac{z}{z-.7}$, $f[n] = (.7)^n$

$f[n/2] = (.7)^{n/2}$, $n = 0, 2, 4, \ldots$ $= 0$, otherwise

b)



c)

$$\begin{array}{r|l} & 1 + .7z^{-2} + (.49)z^{-4} + \cdots \\ \hline z^2 - .7 & z^2 \\ & z^2 - .7 \\ \hline & .7 \\ & .7 - (.49)z^{-2} \\ \hline & .49z^{-2} \end{array}$$

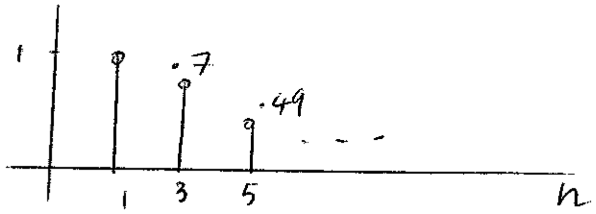
(ii) a) $\frac{z}{z^2-.7} = F_1(z^2) = z^{-1}F(z^2) = z^{-1}\left[\frac{z^2}{z^2-.7}\right]$

$\therefore \; F(z) = \frac{z}{z-.7}$, $f[n] = (.7)^n$

$\therefore \; f_1[n/2] = f\left[\frac{n-1}{2}\right]u[n-1] = (.7)^{\frac{n-1}{2}} u[n-1]$, $n = 1, 3, 5, \ldots$

$= 0$, otherwise

b)



c) $$z^2 - .7 \overline{\smash{)}\, z \qquad}\quad z^{-1} + .7z^{-3} + .49z^{-5} + \cdots$$

$$z - .7z^{-1}$$

$$.7z^{-1} - (.49)z^{-3}$$

$$.7z^{-1}$$

$$(.49)z^{-3}$$

11.18 a) $$F(z) = \frac{z^{-9}}{z-a} = z^{-10}\frac{z}{z-a}$$

$$f[n] = a^{n-10}u[n-10]$$

b) $$F(z) = \frac{z^{-2}}{z-3} = z^{-3}\frac{z}{z-3}$$

$$f[n] = 3^{n-3}u[n-3]$$

11.19 a) $H(z) = \dfrac{z^3}{(z-1.1)^3}$ b) $H(z) = \dfrac{z^4}{(z-.9)^3}$

c) $H(z) = \dfrac{z^4}{(z-1.1)^3}$ d) $H(z) = \dfrac{z^3}{(z-.9)^3}$

11.20

(i) a) poles : $z = \underline{1}, .9$ $\therefore$ not stable

b) unit step function

c) $$\frac{Y(z)}{z} = \frac{3(z-1.2)}{(z-1)(z-.9)} \cdot \frac{z}{z-1}$$

$$\frac{Y(z)}{z} = \frac{3(z-1.2)}{(z-.9)(z-1)^2} = \frac{-6}{(z-1)^2} + \frac{k_2}{z-1} + \frac{k_3}{z-.9}$$

$$\therefore\ y[n] = \underline{\underline{-6n}} + k_2 + k_3(.9)^n,\ n \geq 0$$

(ii) a) poles: $z = 0, .9, \underline{1.2}$; not stable

b) unit impulse function

(c) $$Y(z)=z\left[\frac{3(z+0.9)}{z^2(z-0.9)(z-1.2)}\right]=z\left[\frac{k_1}{z^2}+\frac{k_2}{z}+\frac{k_3}{z-0.9}+\frac{14.6}{z-1.2}\right]$$

$$\therefore y[n]=k_1\delta[n-1]+k_2\delta[n]+k_3(0.9)^n+\underline{\underline{14.6(1.2)^n}}$$

(iii) (a) poles: $z=0,-0.9,\underline{-1.2}$ not stable

(b) unit impulse function

(c) $$Y(z)=z\left[\frac{3(z-0.9)}{z^2(z+0.9)(z+1.2)}\right]=\frac{k_1}{z^2}+\frac{k_2}{z}+\frac{k_3}{z+0.9}+\frac{14.6}{z+1.2}$$

$$\therefore y[n]=k_1\delta[n-1]+k_2\delta[n]+k_3(0.9)^n+\underline{\underline{14.6(-1.2)^n}}$$

(iv) (a) poles: $z=0,0.9,0.9$ $\therefore$ <u>stable</u>

(v) (a) poles: $z=0.9,1.1$

(b) unit impulse function

(c) $$Y(z)=z\left[\frac{2z-1.5}{z^2(z-0.9)(z-1.1)}\right]=z\left[\frac{k_1}{z^2}+\frac{k_2}{z}+\frac{k_3}{z-0.9}+\frac{2.89}{z-1.1}\right]$$

$$\therefore y[n]=k_1\delta[n-1]+k_2\delta[n]+k_3(0.9)^n+2.89(1.1)^n$$

```
d=[1 -1.8 .81 0]
roots(d)
pause
d=[1 -2 .99 0]
roots(d)
```

11.21 Causal: $h[n]=0,\ n<0$

$\underline{a_0\neq 0}$

$$\begin{array}{r l} & \frac{b_0}{a_0}+\left(b_1-\frac{a_1b_0}{a_0}\right)\frac{z^{-1}}{a_1}+\cdots \\ a_0+a_1z^{-1}+\cdots & \overline{\big) \, b_0+b_1z^{-1}+\cdots} \\ & \underline{b_0+\frac{a_1b_0}{a_0}z^{-1}+\cdots} \\ & \left(b_1-\frac{a_1b_0}{a_0}\right)z^{-1} \end{array}$$

$$\therefore h[n]=\frac{b_0}{a_0}+\left[b_1-\frac{a_1b_0}{a_0}\right]\frac{z^{-1}}{a_1}+\cdots \quad \therefore \underline{\text{causal}}$$

$\underline{a_0=0}$

$$\begin{array}{r l} & (b_0/a_0)z+\left(b_1-\frac{b_0a_2}{a_1}\right)\frac{1}{a_2}+\cdots \\ a_1z^{-1}+a_2z^{-2}+\cdots & \overline{\big) \, b_0+b_1z^{-1}+\cdots} \\ & \underline{b_0+\frac{b_0a_2}{a_1}z^{-1}+\cdots} \end{array}$$

$$\therefore h[n]=\underset{\nwarrow\ \underline{\text{not causal}}}{\frac{b_0}{a_1}z}+(\quad)+(\quad)z^{-1}+\cdots$$

~~11.25. Table 11.5 is used~~

~~(a) $\mathcal{Z}_b[0.9^n u[n]]=\frac{z}{z-0.9},\ |z|>0.9$~~

~~(b) $\mathcal{Z}_b[0.9^n u[n-2]]=(0.9)^2\mathcal{Z}_b[0.9^{n-2}u[n-2]]=\frac{0.81z^{-2}z}{z(z-0.9)}$~~

~~$=\frac{0.81}{z(z-0.9)},\quad |z|>0.9$~~

11.22 $f[n] = a^n u[n] - b^{2n} u[-n-1]$

a) $$F(z) = \underset{\substack{\uparrow \\ |z| > |a|}}{\frac{z}{z-a}} + \underset{\substack{\uparrow \\ |z| < |b^2|}}{\frac{z}{z-b^2}}$$

$\therefore \quad |a| < |b^2|$

b) $\frac{z}{z-a} + \frac{z}{z-b^2}$, $|a| < |z| < |b^2|$ or $|a| < |z| < b^2$

11.23

a) $z[.5^n u[n]] = \frac{z}{z-.5}$, $|z| > .5$

b) $$z[.5^n u[n-5]] = \sum_{5}^{\infty} .5^n z^{-n} = \frac{(.5z^{-1})^5}{z-.5}, \; |z| > .5$$

$$= \frac{(.5)^5}{z^4(z-.5)}, \; |z| > .5$$

c) $$z[.5^n u[n+5]] = z\left[\frac{1}{(.5)^5}(.5)^{n+5} u[n+5]\right] =$$

$$\left(\frac{1}{.5}\right)^5 z^5 \frac{z}{z-.5} = \frac{z^6}{(.5)^5(z-.5)} = \frac{32z^6}{z-.5}$$

or $$32z^5 + 16z^4 + 8z^3 + 4z^2 + 2z + \frac{z}{z-.5}, \; |z| > .5$$

d) $z[-(.5)^n u[-n-1]] = \frac{z}{z-.5}$, $|z| < .5$

e) $$z[(.5)^{-n} u[n+5]] = \sum_{n=-5}^{\infty} (.5)^{-n} z^{-n} =$$

$$\sum_{n=-5}^{\infty} (2z^{-1})^n = \frac{(2z^{-1})^{-5}}{1-2z^{-1}} = \frac{z^5 \cdot z}{32(z-2)} = \frac{z^6}{32(z-2)}$$

$|z| > 2$

or $\frac{1}{32}z^5 + \frac{z^4}{16} + \frac{z^3}{8} + \frac{z^2}{4} + \frac{z}{2} + \frac{z}{z-2}$, $|z| > 2$

f) $z[\,.5^n u[-n]] = \sum_{-\infty}^{0} (.5z^{-1})^n = \sum_{n=0}^{\infty} (2z)^n =$

$\frac{1}{1-2z}$, $|2z| < 1$ $= \frac{-1/2}{z-1/2}$, $|z| < 1/2$

11.24 $F_b(z) = \frac{.6z}{(z-1)(z-.6)} = \frac{3/2}{z-1} + \frac{-9/10}{z-.6}$

a) $|z| < .6$, $f[n] = -3/2\,(1)^n u[-n-1] + 9/10\,(.6)^n u[-n-1]$

$= -3/2\, u[-n-1] + 9/10\,(.6)^n u[-n-1]$

b) $|z| > 1$, $f[n] = 3/2\, u[n] - 9/10\,(.6)^n u[n]$

c) $\underset{\substack{\downarrow \\ \text{Right}}}{.6} < |z| < \underset{\substack{\downarrow \\ \text{left}}}{1}$, $f[n] = \frac{-9}{10}(.6)^n u[n] - 3/2\, u[-n-1]$

d) (a) $f[\infty] = 0$, (b) $f[\infty] = 6/10$, (c) $f[\infty] = 0$

11.25 a) $F_b(z) = (1/2)^{-10} z^{10} + (1/2)^{-9} z^9 + \cdots + 1 + (1/2)z + \cdots + (1/2)^{20} z^{20}$

$= (1/2\, z^{-1})^{-10} + (1/2\, z^{-1})^{-9} + \cdots + (1/2\, z^{-1})^{20}$

since: $\sum_{k=n_1}^{n_2} a^k = \frac{a^{n_1} - a^{n_2+1}}{1-a}$

$\therefore F_b(z) = \frac{(1/2\, z^{-1})^{-10} - (1/2\, z^{-1})^{21}}{1 - 1/2\, z^{-1}}$

b) $(1/2)^{-10} z^{+10} + \cdots + (1/2)^{20} \frac{1}{z^{20}}$, $\therefore$ ROC: $|z| \neq 0$

c) $f_1[n] = (1/2)^n$, $-10 \leq n \leq 10$

from a), $F_{b_1}(z) = \frac{(1/2 z^{-1})^{-10} - (1/2 z^{-1})^{11}}{1 - 1/2 z^{-1}}$, $|z| \neq 0$

$f_2[n] = (1/4)^n u[n-21] = (1/4)^{21} (1/4)^{n-21} u[n-21]$

$F_{b_2}(z) = (1/4)^{21} z^{-21} \frac{z}{z - 1/4} = \frac{(1/4)^{21}}{z^{20}(z - 1/4)}$, $|z| > 1/4$

$\therefore$ $F_b(z) = F_{b_1}(z) + F_{b_2}(z)$, $|z| > 1/4$

d) $f_1[n] = (1/2)^n$, $-10 \leq n \leq 0$

$F_{b_1}(z) = 1 + (1/2)^{-1} z + (1/2)^{-10} z^{10} = 1 + (2z) + (2z)^2 + \cdots + (2z)^{10}$

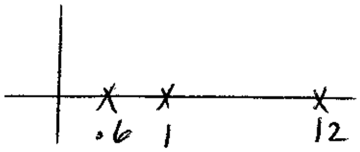
$$= \frac{1 - (2z)^{11}}{1 - 2z}$$

$f_2[n] = (1/4)^n$, $1 \leq n \leq 10$

$F_{b_2}(z) = \left(\frac{1}{4z}\right) + \left(\frac{1}{4z}\right)^2 + \cdots + \left(\frac{1}{4z}\right)^{10} = \frac{\frac{1}{4z} - \left(\frac{1}{4z}\right)^{11}}{1 - \frac{1}{4z}}$, $z \neq 0$

$\therefore$ $F_b(z) = F_{b_1}(z) + F_{b_2}(z)$, $z \neq 0$

11.26 $F(z) = \frac{3z}{z-1} + \frac{z}{z-12} - \frac{z}{z-.6}$



a) $|z| < .6$, $.6 < |z| < 1$, $1 < |z| < 12$, $|z| > 12$

b) $|z| < .6$, $f[n] = -3u[-n-1] - (12)^n u[-n-1] + (.6)^n u[-n-1]$

$.6 < |z| < 1,\ f[n] = -(.6)^n u[n] - 3u[-n-1] - (12)^n u[-n-1]$

$1 < |z| < 12,\ f[n] = -(.6)^n u[n] + 3u(n] - (12)^n u[-n-1]$

$|z| > 12,\ f[n] = -(.6)^n u(n] + 3u[n] + (12)^n u[n]$

11.27

a) $Y_m(z) = Y(z^m)$

b) $X_m(z) = X(z^m)$

$H_m(z) = H(z^m)$

$\therefore\ Z[x_m[n] * h_m[n]] = X(z^m) H(z^m)$

Chapter 12

12.1 (a)

(i) $f(nT_s) = 8\cos[2\pi(.1n)] + 4\sin[4\pi(.1n)]$

$$f[n] = 8\cos[.2\pi n] + 4\sin[.4\pi n]$$

$$F(\Omega) = 8\pi \sum_{k=-\infty}^{\infty} [\delta(\Omega - .2\pi - 2\pi k) + \delta(\Omega + .2\pi - 2\pi k)]$$

$$- j4\pi \sum_{k=-\infty}^{\infty} [\delta(\Omega - .4\pi - 2\pi k) - \delta(\Omega + .4\pi - 2\pi k)]$$

(ii) $g[n] = 4\cos[.5\pi n]u[n]$

$$4\cos[.5\pi n] \longleftrightarrow 4\pi \sum_{k=-\infty}^{\infty} [\delta(\Omega - .5\pi - 2\pi k) + \delta(\Omega + .5\pi - 2\pi k)]$$

$$u[n] \longleftrightarrow \frac{1}{1 - e^{-j\Omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\Omega - 2\pi k)$$

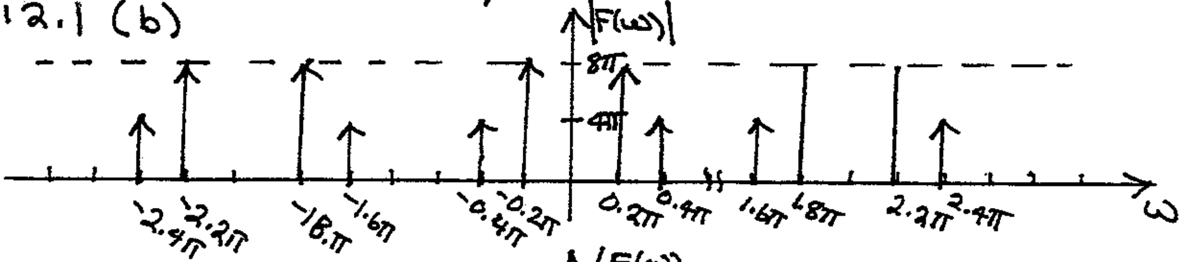
$$x[n]y[n] \longleftrightarrow \frac{1}{2\pi} X(\Omega) * Y(\Omega)$$

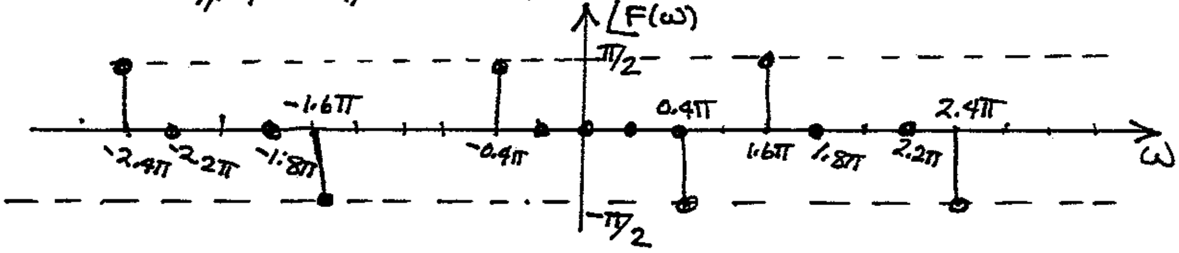
$$G(\Omega) = \frac{4e^{j2\Omega}}{1 + e^{j2\Omega}} + 2\pi \sum_{k=-\infty}^{\infty} [\delta(\Omega - .5\pi - 2\pi k) + \delta(\Omega + .5\pi - 2\pi k)]$$

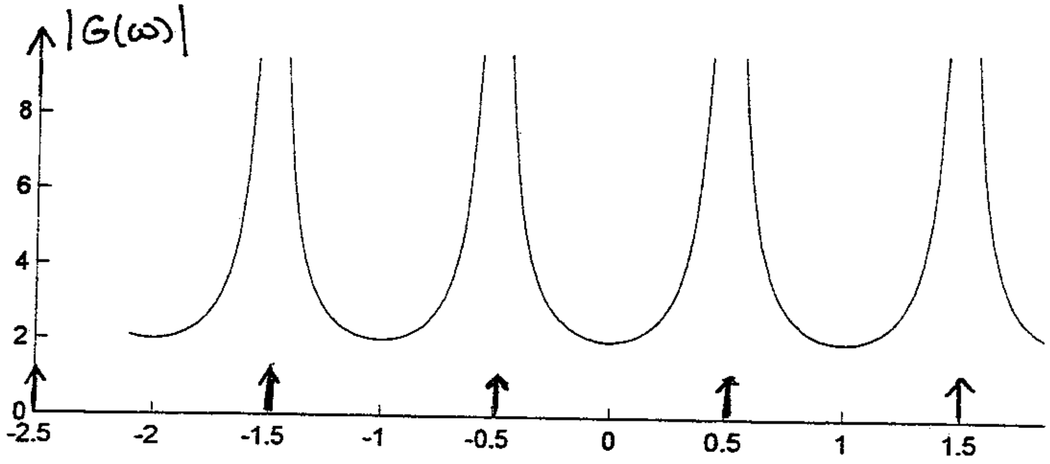
part b) next page

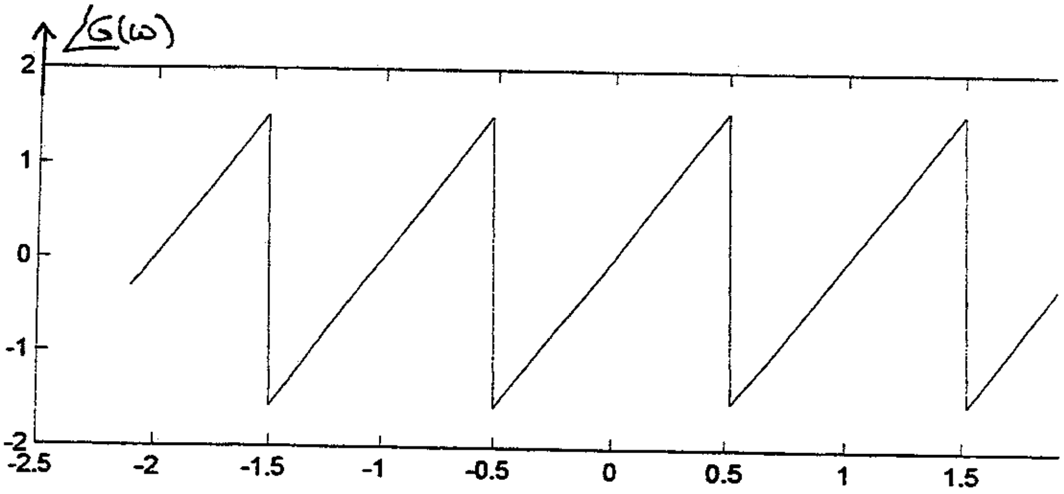
chapter 12

12.1 (b)









12.2 (a)

$$x[n] = \begin{cases} (.5)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}; \quad X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\Omega}$$

$$X(\Omega) = \sum_{n=0}^{\infty} (.5)^n e^{-jn\Omega} = 1 + \underbrace{.5e^{-j\Omega} + (.5e^{-j\Omega})^2 + (.5e^{-j\Omega})^3 + \cdots}_{\text{geometric series}}$$

$$X(\Omega) = \frac{1}{1 - .5e^{-j\Omega}}$$

(b) $y[n] = n(.5)^n u[n] \xleftrightarrow{DTFT} Y(\Omega) = \sum_{n=0}^{\infty} n(.5)^n e^{-jn\Omega}$

From TABLE 12.1 $\quad Y(\Omega) = \dfrac{.5e^{j\Omega}}{(e^{j\Omega} - .5)^2}$

(c) $v[n] = 2\,[u[n] - u[n-5]]$

$$V(\Omega) = \sum_{n=0}^{4} 2e^{-jn\Omega} = 2\left[1 + e^{-j\Omega} + e^{-j2\Omega} + e^{-j3\Omega} + e^{-j4\Omega}\right]$$

$$= 2e^{-j2\Omega}\left[e^{j2\Omega} + e^{j\Omega} + 1 + e^{-j\Omega} + e^{-j2\Omega}\right]$$

$$= 2e^{-j2\Omega}\left\{1 + 2\cos\Omega + 2\cos 2\Omega\right]$$

or from TABLE 12.1 : $V(\omega) = 2\,\dfrac{\sin\left(\frac{5\Omega}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)}\,e^{-j2\Omega}$
(WITH TIME-SHIFT PROPERTY)

(d) $w[n] = \text{rect}(n/4) + \text{rect}(n/10)$

$$W(\Omega) = \sum_{n=-5}^{5} 1e^{-jn\Omega} + \sum_{n=-2}^{2} 1e^{-jn\Omega} =$$

$$W(\Omega) = e^{j5\Omega} + e^{j4\Omega} + e^{j3\Omega} + 2e^{j2\Omega} + 2e^{j\Omega} + 2 + 2e^{-j\Omega} + 2e^{-j2\Omega} + e^{-j3\Omega} + e^{-j4\Omega} + e^{-j5\Omega}$$

or FROM TABLE 12.1

$$W(\Omega) = \frac{\sin\left(\frac{5\Omega}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)} + \frac{\sin\left(\frac{11\Omega}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)} \quad \text{or}$$

$$W(\Omega) = 2\cos 5\Omega + 2\cos 4\Omega + 2\cos 3\Omega + 4\cos 2\Omega + 4\cos\Omega + 2$$

~~12.10 $X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\Omega}, \quad \frac{d}{d\Omega}X(\Omega) = \frac{d}{d\Omega}\sum_{n=-\infty}^{\infty} x[n]e^{-jn\Omega}$~~

~~$\frac{d}{d\Omega}X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]\frac{d}{d\Omega}e^{-jn\Omega} = \sum_{n=-\infty}^{\infty} -jn\,x[n]e^{-jn\Omega}$~~

~~$j\frac{d}{d\Omega}X(\Omega) = \sum_{n=-\infty}^{\infty} (j)(-j)nx[n]e^{-jn\Omega} = \sum_{n=-\infty}^{\infty} n\,x[n]e^{-jn\Omega}$~~

~~$= DTF\{n\,x[n]\}$~~

12.4 $X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]\, e^{-jn\Omega}$, $\frac{d}{d\Omega}X(\Omega)=\frac{d}{d\Omega}\sum_{n=-\infty}^{\infty} x[n]e^{-jn\Omega}$

$$\frac{d}{d\Omega}X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]\,\frac{d}{d\Omega}e^{-jn\Omega}=\sum_{n=-\infty}^{\infty} -jnx[n]e^{-jn\Omega}$$

$$j\frac{d}{d\Omega}X(\Omega)=\sum_{n=-\infty}^{\infty} (j)(-j)nx[n]e^{-jn\Omega}=\sum_{n=-\infty}^{\infty} nx[n]e^{-jn\Omega}$$

$$=DT\mathcal{F}\{nx[n]\}$$

12.5

a) $H(\Omega)=1+2e^{-j\Omega}+e^{-2j\Omega}$

b) $H(\Omega)=e^{-j\Omega}(e^{j\Omega}+2+e^{-2j\Omega})=e^{-j\Omega}(2+2\cos\Omega)$

$\therefore\ \angle H(\Omega)=-\Omega$

12.6 $X(\Omega)=\frac{1}{1-ae^{-j\Omega}}$

$$Y(\Omega)=\frac{a}{a-b}\,\frac{1}{1-ae^{-j\Omega}}+\frac{b}{b-a}\,\frac{1}{1-be^{-j\Omega}}$$

$$H(\Omega)=\frac{Y(\Omega)}{X(\Omega)}=\frac{a}{a-b}+\frac{b}{b-a}=\frac{1-ae^{-j\Omega}}{1-be^{-j\Omega}}$$

$$h[n]=\frac{a}{a-b}\delta[n]+\frac{b}{b-a}b^{n}u[n]-\frac{ab}{b-a}b^{n-1}u[n-1]$$

12.7 $X_0(\Omega)=1+e^{-2j\Omega}+e^{-4j\Omega}$

$$X(\Omega)=\frac{2\pi}{5}\sum_{k=-\infty}^{\infty} X_0\left(\frac{2\pi k}{5}\right)\delta\left(\Omega-\frac{2\pi k}{5}\right)$$

$$X_0(\Omega)=e^{-2j\Omega}(e^{j2\Omega}+1+e^{-2j\Omega})$$

$$=e^{-2j\Omega}(1+2\cos 2\Omega)\ \therefore\ \angle X_0(\Omega)=-2\Omega$$

12.8 $y[n] = x[n/3]$

$Y(\Omega) = \sum_{n=-\infty}^{\infty} x[n/3] e^{-j\Omega n}$ let $l = n/3$

$Y(\Omega) = \sum_{l=-\infty}^{\infty} x[l] e^{-j\Omega 3l} = X(3\Omega)$

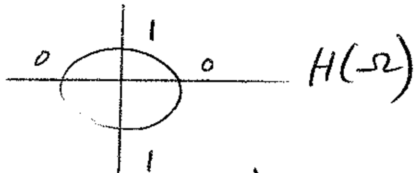
12.9 $x_0[n] = \text{idft}\ [4\ 0\ 4\ 0]$

Since $X[k] = X_0\left(\frac{2\pi k}{4}\right)$ for $N=4$

$x_0[n] = 1/4\left(4 + 4e^{j\pi n}\right)$

$x_0[n] = [2\ 0\ 2\ 0]$

12.10



let $H[k] = H\left(\frac{2\pi k}{4}\right) = [0\ 1\ 0\ 1]$

$h[n]$ is simply IDFT of $H[k]$

$h[n] = 1/4 \sum_{k=0}^{3} H[k] W_4^{-nk} = \frac{1}{4}\left[e^{\frac{j2\pi n}{4}} + e^{\frac{j6\pi n}{4}}\right]$

$h[n] = [1/2, 0, -1/2, 0]$

12.11 next page

12.11 FROM FIGURE P12.6(a): $x[n] = 0.5^n, \; n \geq 0$

$$X[k] = \mathcal{D}\mathcal{F}[x[n]] = \sum_{n=0}^{7} x[n]e^{-j2\pi kn/8} = \sum_{n=0}^{7} (0.5)^n e^{-jkn\pi/4}$$

$$X[k] = 1 + .5e^{-jk\pi/4} + .25e^{-jk\pi/2} + .125e^{-jk3\pi/4} + .0625e^{-jk\pi} + .03125e^{-jk5\pi/4}$$
$$+ .015625e^{-jk3\pi/2} + .0078125e^{-jk7\pi/4}, \; k = 0, 1, \cdots, 7$$

$X[0] = 1.9922$, $X[1] = 1.1861 - j0.6487$, $X[2] = 0.7969 - j0.3984$

$X[3] = 0.6889 - j0.1799$, $X[4] = 0.6641$, $X[5] = 0.6889 + j0.1799$

$X[6] = 0.7969 + j0.3984$, $X[7] = 1.1861 + j0.6487$

12.12 FROM FIGURE 12.6(b): $y[n] = n(.5)^n, \; n \geq 0$

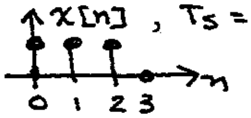
$$Y[k] = \sum_{n=0}^{7} n(.5)^n e^{-jkn\pi/4} = .5e^{-jk\pi/4} + .5e^{-jk\pi/2} + .375e^{-jk3\pi/4}$$
$$+ .25e^{-jk\pi} + .15625e^{-jk5\pi/4} + .09375e^{-jk3\pi/2} + .0546875e^{-jk7\pi/4}$$

$Y[0] = 1.9297$, $Y[1] = -.2334 - j8758$, $Y[2] = -.3438 - j.2266$

$Y[3] = -.2666 - j.0633$, $Y[4] = -.2422$, $Y[5] = -.2666 + j.0633$

$Y[6] = -.3438 + j.2266$, $Y[7] = -.2334 + j.8758$

12.13 (a) $x[n]$, $T_s = 2\text{ms}$



$$X[k] = \sum_{n=0}^{3} x[n]e^{-j\frac{2\pi}{4}nk}, \; k = 0, 1, 2, 3$$

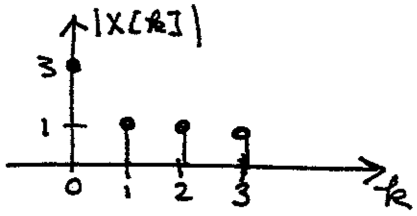
$X[0] = 1 + 1 + 1 + 0 = 3$

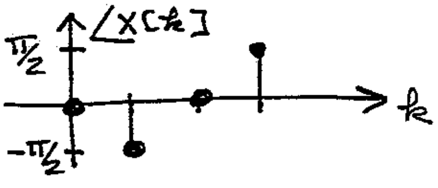
$X[1] = 1 + 1e^{-j2\pi/4} + e^{-j\pi} + 0 = e^{-j\pi/2} = -j1$

$X[2] = 1 + e^{-j\pi} + e^{-j2\pi} + 0 = 1$

$X[3] = 1 + e^{-j3\pi/2} + e^{-j3\pi} + 0 = e^{-j3\pi/2} = j1$

$\therefore X[k] = [3, -j1, 1, j1]$





(b) MATLAB

```
>> x = [1 1 1 1 1 0 0 0];
>> X = fft(x);
>> stem(abs(X))
>> stem (angle (X))
```

part (c) next page

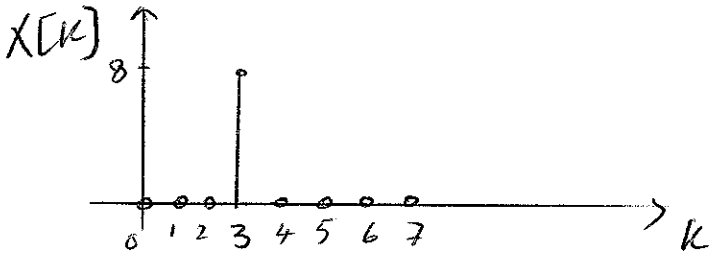
12.13 c) Matlab problem

```
>> x=[1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0];
>> X = fft (x,16);
>> plot (abs(X))
>> plot (angle(X))
```

12.14 $x[n]=e^{\frac{j6\pi n}{8}}$, $N=8$

$$X[k]=\sum_{n=0}^{7} e^{\frac{j6\pi n}{8}} e^{\frac{-j2\pi nk}{8}} = \sum_{n=0}^{7} e^{\frac{j2\pi n(3-k)}{8}}$$

$= 8\,\delta[k-3]$ by orthogonality of exponentials



12.15 a) $x[n]=[5.0, -4.05, 1.55, 1.55, -4.05, 5.0, -4.05, 1.55]$

$$X[k]=\sum_{k=0}^{7} x[n]e^{-j\frac{\pi}{4}nk}, \quad k=0,1,\cdots,7$$

$X[k]=[2.5, 2.65+j.81, 3.45+j2.14, 15.44+j11.98, -5.60, 15.44-j11.98, 3.45-j2.14, 2.65-j.81]$

12.15(b) MATLAB

```
>> for n=1:8
     x(n)= 5*cos((n-1)*8*pi/10);
     end
>> x
>> X = ff(t)(x, 8);
>> X
>> for n=1:8
     w(n) =(n-1)*2*pi*10/8;
      end
>> stem(w, abs(X));
>> stem(w, angle(X));
```

(c) $X(\omega) = \mathfrak{F}\{5\cos(8\pi t)\} = 5\pi[\delta(\omega-8\pi)+\delta(\omega+8\pi)]$

$X(\omega) = 5\pi[\delta(\omega-25.137)+\delta(\omega+25.137)]$

It is seen that the DFT can be used to approximate the Fourier transform. The DFT results in this problem exhibit spectrum spreading.

12.16 The hanning window is given by eq. (12.58)

$han[n] = [0, 0.1883, 0.6113, 0.9505, 0.9505, 0.6113, 0.1883, 0]$

$x_2[n] = han[n] * x[n] = [0, -0.7615, 0.9445, 1.4686, -3.8446, 3.0563, -0.7615, 0]$

$X_2[k] = [0.1015, 0.1068-j0.0448, -4.0278-j0.0826, 7.5829+j3.3671$
$-7.4252, 7.5829-j3.3671, -4.0278+j0.0826, 0.1068+j0.0448]$

THE FREQUENCY COMPONENTS OF $X_2[k]$ ARE AT

$$\omega[k] = \frac{2\pi k}{NT} = 2.5\pi k \quad , \quad k = 0, 1, \cdots 7$$

NOTICE THAT THERE IS A LARGE COMPONENT AT $k = 4$ OR $\omega[k] = 10\pi\ (rad/s) = \omega_s/2$ BECAUSE OF SPECTRUM SPREADING – HOWEVER IT IS LESS THAN FOUND IN P 12.15.

The Hanning window generated by the "hanning(8)" command in MATLAB differs from that given by eq. (12.58). However, the result of using the MATLAB function is similar to the calculated results.

12.16 (continued)

```
>> (generator "x" as in problem 12.15 (b))
>> xh = hanning(8)'.*x;
>>  Xh = fft(xh,8);
>> generate "w" as in problem 12.15 (b)
>> stem(w,abs(Xh)), axis([0,60,0,20])
```

12.17 $A[k] = \sum_{n=0}^{N-1}\left[\frac{1}{2}\left(e^{\frac{j2\pi kn}{N}} + e^{\frac{-j2\pi kn}{N}}\right)\right]\left[\frac{1}{2}\left(e^{\frac{j2\pi pn}{N}} + e^{\frac{-j2\pi pn}{N}}\right)\right]$

by orthogonality of exponentials,

$$= \frac{1}{4}N\delta[k+p] + \frac{1}{4}N\delta[k-p] + \frac{1}{4}N\delta[k-p] + \frac{1}{4}N\delta[k+p]$$

$$= \frac{N}{2}\left[\delta[k+p] + \delta[k-p]\right)$$

12.18 $x[n] = y[n-1] + y[n-3] + y[n-5] + y[n-7]$

$\therefore \quad X[k] = \left(W_8^{k} + W_8^{3k} + W_8^{5k} + W_8^{7k}\right) Y[k]$

12.19 $y[n] = x[n+1] = x[n-3]$

$\therefore \quad Y[k] = X[k]\,e^{\frac{j2\pi k}{4}} = X[k]\,e^{\frac{-3j2\pi k}{4}} = W^{3k}X[k]$

$= W^{-k}X[k]$

12.20 $F(\omega) = 3.5\pi[\delta(\omega-140) + \delta(\omega+140) + \delta(\omega-60) + \delta(\omega+60)]$

the highest frequency component is 140(rad/s)

$\therefore \omega_s > 2 \times 140 \text{ (rad/s)} \Rightarrow \omega_s > 280 \text{ rad/s}$

$T_s < \frac{2\pi}{\omega_s} \quad \therefore T_s < 22.4 \text{ (ms)}$

12.21 a) $\Delta\Omega = \frac{2\pi}{N} = \frac{2\pi}{1024}$

$\Delta\omega = \frac{\Delta\Omega}{T_S} = \frac{\frac{2\pi}{1024}}{\frac{1}{1024}} = 2\pi \text{ rad/sec}$

b) Highest frequency allowed if aliasing Can not occur is

ω_{max}

$\omega_S = \frac{2\pi}{T_S} = \frac{2\pi}{\frac{1}{1024}} = 2048\pi$

$\omega_S > 2 \times \omega_{max} \Rightarrow \omega_{max} < 1024\pi$

12.22 (a) $x[n] = [-2, -1, 0, 2]$, $y[n] = [-1, 2, -1, -3]$

$y[-n] = [-3, -1, 2, -1]$

$$x[n] * y[n] = [(-1)(-2), (-1)(-1) + (2)(2), (2)(-1) + (1)(0) + (-1)(-2),$$
$$(-1)(2) + (2)(0) + (-1)(-1) + (-3)(-2),$$
$$(2)(2) + (-1)(0) + (-3)(-1), (-1)(2) + (3)(0),$$
$$(-3)(2)]$$

$$x[n] * y[n] = [2, -3, 0, 5, 7, -2, -6]$$

(b) $x[n] \circledast y[n] = [9, -5, -6, 5]$

see Example 12.16 for details of the circular convolution process.

(c) Extend both sequences to $N_1 + N_2 - 1$ elements by zero padding.

$x'[n] = [-2, -1, 0, 2, 0, 0, 0]$

$y'[n] = [-1, 2, -1, -3, 0, 0, 0]$

Perform circular convolution as described in Example 12.22, or use the DFT method shown in Example 12.23.

$\mathcal{R}_{xy} = [-6, -3, 5, 6, -2, 4, -1]$

(d) $\mathcal{R}_{yx} = [-6, -1, 4, -2, 6, 5, -3]$

(e) $\mathcal{R}_{xx} = [9, 2, -2, -4, -4, -2, 2]$

12.23 The extended sequences to be convolved must have $N_1 + N_2 - 1$ elements. In this case $N_1 = N_2 = 4$, so 7 elements are required.

$x'[n] = [2, 1, 0, 2, 0, 0, 0]$

$y'[n] = [-1, 2, -1, -3, 0, 0, 0]$

12.24

$X[k]=[12 \quad -2-2j \quad 0 \quad -2+2j]$

$H[k]=[2.3 \quad .51-.81j \quad .68 \quad .51+.81j]$

$y[n] = x[n] \circledast h[n]$

$Y[k] = X[k]H[k] = [27.6 \quad -2.64+.6i \quad 0 \quad -2.64-.6i]$

$y[n] = ifft(Y[k]) = (5.58 \quad 6.6 \quad 8.22 \quad 7.2]$

$y[2] = 8.22$

12.25 (a) $v[n] = x[n] * y[n]$, $V[k] \neq X[k]Y[k]$

$$x[n] = \frac{1}{4}\sum_{k=0}^{3} X[k]e^{j2\pi kn/4}, \quad n=0,1,2,3$$

$$y[n] = \frac{1}{4}\sum_{k=0}^{3} Y[k]e^{j2\pi kn/4}, \quad n=0,1,2,3$$

$$x[n] = [2,\ 6,\ 6,\ 8], \quad y[n] = [1,\ 3,\ 3,\ 1] = y[-n]$$

$$\begin{array}{l} 0\ 0\ 2\ 6\ 6\ 8\ 0 \\ \underline{0\ 1\ 3\ 3\ 1\ 0\ 0} \end{array} \Big\} \text{LINEAR CONVOLUTION}, p=2$$

$$v[2] = 0+0+6+18+6+0+0 = \underline{30}$$

(b) $W[k] = X[k]Y[k] = [176,\ 12+j4,\ 0,\ 12-j4]$

$$w[n] = \mathcal{DF}^{-1}\{W[k]\} = \frac{1}{4}\sum_{k=0}^{3} W[k]e^{j2\pi kn/4}, \quad n=0,1,2,3$$

$$w[2] = \frac{1}{4}\sum_{k=0}^{3} W[k]e^{j\pi n} = \underline{38}$$

(c) $R_{xy} = x[n] * y[-n]$, $R_{xy}[2] = \begin{array}{l} 0\,0\,2\,6\,6\,8\,0 \\ \underline{1\,3\,3\,1\,0\,0\,0} \\ 0+0+6+6+0+0+0 = \underline{12} \end{array}$

(d) $R_{yx} = x[-n] * y[n]$, $R_{yx}[2] = \begin{array}{l} 0\,0\,1\,3\,3\,1\,0 \\ \underline{2\,6\,6\,8\,0\,0\,0} \\ 0+0+6+24+0+0+0 = \underline{30} \end{array}$

(e) $R_{xx} = x[n] * x[-n]$

$$R_{xx}[2] = \begin{array}{l} 0\ 0\ 2\ 6\ 6\ 8 \\ \underline{2\ 6\ 6\ 8\ 0\ 0} \\ 0+6+12+48+0+8 \end{array} = \underline{60}$$

(f) $S_x[k] = \frac{1}{N} X[k]X^*[k]$

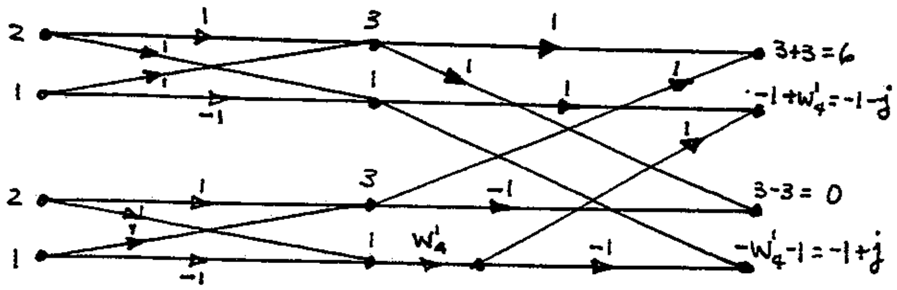
$$= \frac{1}{4}[22 \quad -4+j2 \quad -6 \quad -4-j2][-22 \quad -4-j2 \quad -6 \quad -4+j2]$$

$$= \frac{1}{4}[(22)(22) \quad (-4+j2)(-4-j2) \quad (-6)(-6) \quad (-4-j2)(-4+j2)]$$

$$= \frac{1}{4}[484 \quad 20 \quad 36 \quad 20]$$

$$S_x[k] = \underline{[121 \quad 5 \quad 9 \quad 5]}$$

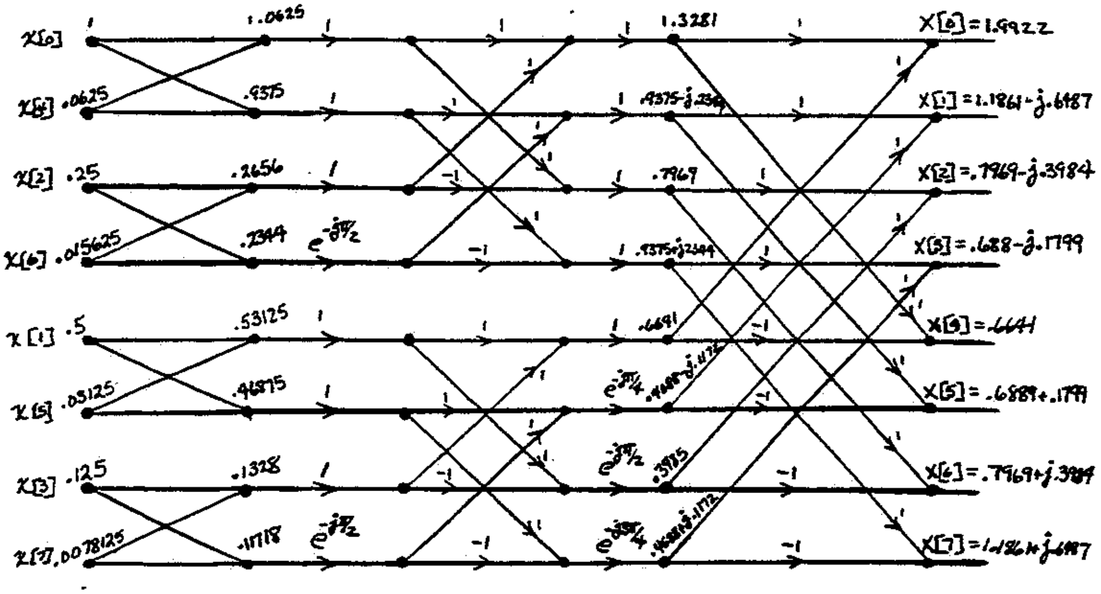
12.26 (a)



(b) MATLAB

```
EDU» f=[1 2 2 1];
EDU» F=fft(f,4)
```

12.27 (a)

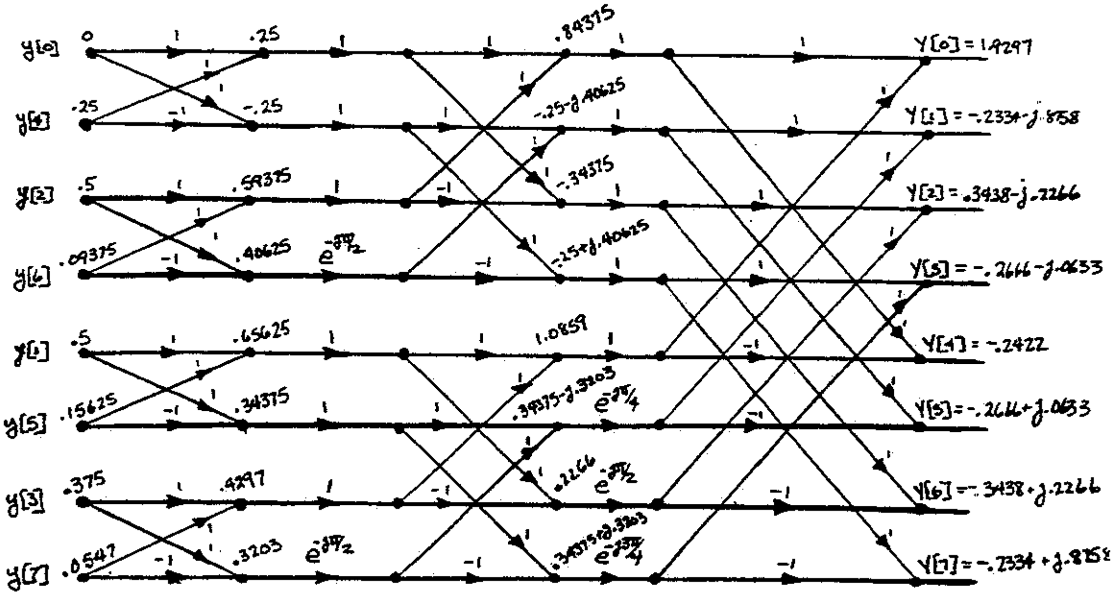


(b)

```
EDU» x=[1 0.5 0.25 0.125 0.0625 0.03125 0.03125/2 0.03125/4]

EDU» X=fft(x,8)
```

12.28 (a)


y[0]
y[4]
y[2]
y[6]
y[1]
y[5]
y[3]
y[7]
0
.25
.5
.09375
.5
.15625
.375
.0547
.25
-.25
.59375
.40625
.65625
.34375
.4297
.3203
$e^{-j\pi/2}$
$e^{-j\pi/2}$
.84375
-.25-j.40625
-.34375
-.25+j.40625
1.0859
.34375-j.3203
.2266
.34375+j.3203
$e^{-j\pi/4}$
$e^{-j\pi/2}$
$e^{-j3\pi/4}$
1
-1
Y[0] = 1.9297
Y[1] = -.2334-j.858
Y[2] = .3438-j.2266
Y[5] = -.2666-j.0633
Y[4] = -.2422
Y[5] = -.2666+j.0633
Y[6] = -.3438+j.2266
Y[7] = -.2334+j.858

CHAPTER 13

13.1. (a) $x[n] = y[n]$

$x[n+1] = 0.8y[n] + 1.9u[n]$

$y[n] = x[n]$

(b) Replace n with $n+2$

$y[n+2] + 0.8y[n] = u[n]$

$x_1[n] = y[n]$

$x_1[n+1] = y[n+1] = x_2[n]$;

$x_2[n+1] = y[n+2] = -0.8\,y[n] + u[n] = -0.8x_1[n] + u[n]$

$$\therefore\ \underline{x}[n+1] = \begin{bmatrix} 0 & 1 \\ -0.8 & 0 \end{bmatrix} \underline{x}[n] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[n]$$

$$y[n] = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}[n]$$

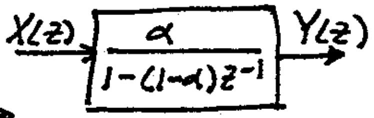
(c)

$x[n] = y[n]$ $\quad \therefore x[n+1] = u[n]$

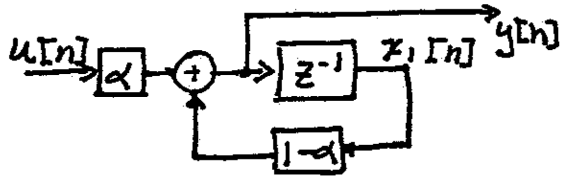
$y[n] = x[n]$

13.2. (a) $zY(z) = (1-\alpha)Y(z) + \alpha z X(z)$

$\therefore Y(z) = \dfrac{\alpha z}{z-(1-\alpha)} X(z) \quad \Longrightarrow$



∴



$\therefore x_1[n+1] = (1-\alpha)x_1[n] + \alpha u[n]$

$y[n] = x_1[n+1] = (1-\alpha)x_1[n] + \alpha u[n]$

(b) (i) see above

(ii) $zX_1(z) = (1-\alpha)X_1(z) + \alpha U(z)$

$\therefore X_1(z) = \dfrac{\alpha}{z-(1-\alpha)} U(z)$

$\therefore Y(z) = (1-\alpha)X_1(z) + \alpha U(z) = \left[\dfrac{\alpha(1-\alpha)}{z-(1-\alpha)} + \alpha\right] U(z)$

$= \dfrac{\alpha z}{z-(1-\alpha)} U(z)$

13.3. (a) $x_1[n+1] = (1-\alpha)x_1[n] + (1-\alpha)Tx_2[n] + \alpha u[n]$

$$x_2[n+1] = x_2[n] + \frac{\beta}{T}\left[-x_1[n] - Tx_2[n]\right] + \frac{\beta}{T}u[n]$$

$$y_1[n] = y[n] = x_1[n+1] = (1-\alpha)x_1[n] + (1-\alpha)Tx_2[n] + \alpha u[n]$$

$$y_2[n] = v[n] = x_2[n+1] = -\frac{\beta}{T}x_1[n] + (1-\beta)x_2[n] + \frac{\beta}{T}u[n]$$

$$\therefore \underline{x}[n+1] = \begin{bmatrix} 1-\alpha & 1-\alpha \\ -\beta/T & 1-\beta \end{bmatrix}\underline{x}[n] + \begin{bmatrix} \alpha \\ \beta/T \end{bmatrix}u[n]$$

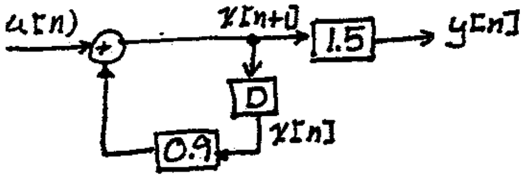
$$\underline{y}[n] = \begin{bmatrix} 1-\alpha & 1-\alpha \\ -\beta/T & 1-\beta \end{bmatrix}\underline{x}[n] + \begin{bmatrix} \alpha \\ \beta/T \end{bmatrix}u[n]$$

(b) With $\beta = 0$, input to $x_2[n+1]$ is zero, $\therefore x_2[n] = 0$

$$\therefore x_1[n+1] = (1-\alpha)x_1[n] + \alpha u[n]$$

$$y[n] = (1-\alpha)x_1[n] + \alpha u[n]$$

13.4. (a) $y[n+1] - 0.9y[n] = 1.5u[n+1]$



(b) $x[n+1] = 0.9x[n] + u[n]$

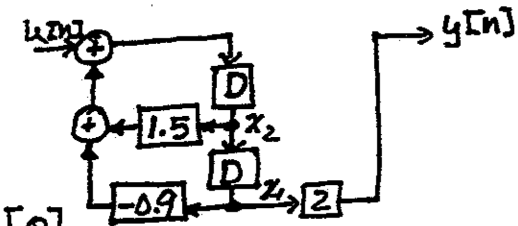
$$y[n] = 1.5x[n+1] = 1.35x[n] + 1.5u[n]$$

(c) $H(z) = \dfrac{1.5z}{z-0.9}$

(d)

```
A=[0.9];B=[1];C=[1.35];D=1.5;
[n,d]=ss2tf(A,B,C,D)
```

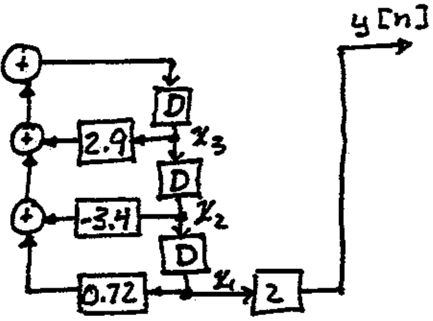
(e) (a) Form 2:



(b) $\underline{x}[n+1] = \begin{bmatrix} 0 & 1 \\ -0.9 & 1.5 \end{bmatrix}\underline{x}[n] + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u[n]$; $y[n] = [2 \quad 0]\,\underline{x}[n]$

(c) $H(z) = \dfrac{2}{z^2 - 1.5z + 0.9}$

(f) (a) Form 2:

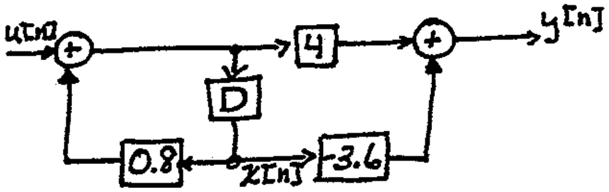


(b) $\underline{x}[n+1] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.72 & -3.9 & 2.9 \end{bmatrix}\underline{x}[n] + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u[n]$

$$y[n] = [2 \quad 0 \quad 0]\,\underline{x}[n]$$

13.4. (c) $H(z) = \dfrac{z}{z^3-2.9z^2+3.4z-0.72}$

13.5. (a)



(b) $x[n+1] = 0.8x[n] + u[n]$

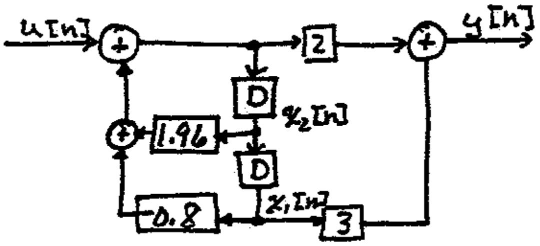
$y[n] = -0.4x[n] + 4u[n]$

(c) $y[n] = 0.8y[n-1] + 4u[n] - 3.6u[n-1]$

(d)

```
n=[4 -3.6];
d=[1 -0.8];
[A,B,C,D]=tf2ss(n,d)
```

(e) (a) Form 2:



(b) $$\underline{x}[n+1] = \begin{bmatrix} 0 & 1 \\ -0.8 & 1.96 \end{bmatrix} \underline{x}[n] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[n]$$

$y[n] = 2x_2[n+1] + 3x_1[n] = [1.4 \quad 3.92]\underline{x}[n] + 2u[n]$

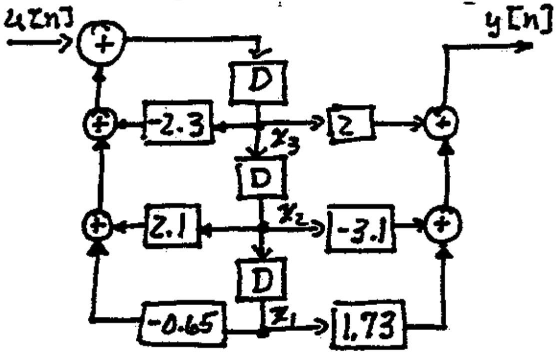
(c) $y[n+2] - 1.96y[n+1] + 0.8y[n] = 2u[n+2] + 3u[n]$

(d) $\underline{x}[n+1] = \begin{bmatrix} 1.96 & -0.8 \\ 1 & 0 \end{bmatrix} \underline{x}[n] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u[n]$; $y[n] = [3.92 \quad 1.4]\underline{x}[n] + 2u[n]$

Simulation diagram same as in (a), with x_1 & x_2 reversed.

(f) Form 2:

(a)



(b) $$\underline{x}[n+1] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.65 & 2.1 & -2.3 \end{bmatrix} \underline{x}[n] + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u[n]$$

$y[n] = [1.73 \quad -3.1 \quad 2]\underline{x}[n]$

13.5.(f) (cont)

(c) $$y[n+3]+2.3y[n+2]-2.1\,y[n+1]+0.65y[n]$$
$$=2u[n+2]-3.1u[n+1]+1.73u[n]$$

(d) MATLAB: $$\underline{x}[n+1]=\begin{bmatrix}-2.3 & 2.1 & -0.65\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\underline{x}[n]+\begin{bmatrix}1\\0\\0\end{bmatrix}u[n]$$
$$y[n]=[2 \quad -3.1 \quad 1.73]$$

Simulation diagram same as (a), with x_1 and x_3 reversed.

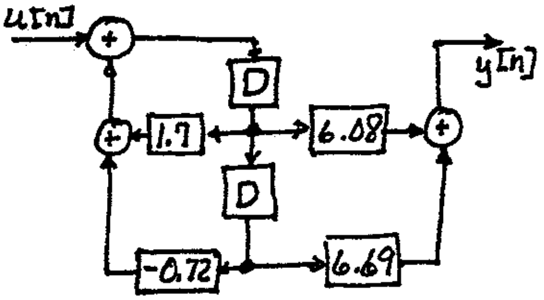
```
n=[2  -1.96];
d=[1  -.99];
[a,b,c,d]=tf2ss(n,d)
pause
n=[2 0 3];
d=[1 -1.96 .8];
[a,b,c,d]=tf2ss(n,d)
pause
n=[0 2 -3.1 1.73];
d=[1 2.3 -2.1 .65];
[a,b,c,d]=tf2ss(n,d)
```

13.6.(a) $$x_1[n+1]=0.8x_1[n]+u[n]$$
$$x_2[n+1]=1.6[2x_1[n+1]+2.2x_1[n]+0.9x_2[n]]$$
$$=1.6[1.6x_1[n]+2u[n]+2.2x_1[n]+0.9x_2[n]]$$
$$=6.08x_1[n]+0.9x_2[n]+3.2u[n]$$
$$y[n]=1.9x_2[n]$$
$$\therefore\ \underline{x}[n+1]=\begin{bmatrix}0.8 & 0\\ 6.08 & 0.9\end{bmatrix}\underline{x}[n]+\begin{bmatrix}1\\3.2\end{bmatrix}u[n]$$
$$y[n]=[0 \quad 1.9]\underline{x}[n]$$

(b) $$(zI-A)=\begin{bmatrix}z-0.8 & 0\\ -6.08 & z-0.9\end{bmatrix};\ |zI-A|=(z-0.8)(z-0.9)=\Delta(z)$$
$$H(z)=C[zI-A]^{-1}B=\frac{1}{\Delta(z)}[0 \quad 1.9]\begin{bmatrix}z-0.9 & 0\\ 6.08 & z-0.8\end{bmatrix}\begin{bmatrix}1\\3.2\end{bmatrix}$$
$$=\frac{1}{\Delta(z)}[11.55 \quad 1.9z-1.52]\begin{bmatrix}1\\3.2\end{bmatrix}=\frac{6.08z+6.69}{(z-0.8)(z-0.9)}$$

(c)
```
A=[0.8 0;6.08 0.9];B=[1;3.2];C=[0 1.9];D=0;
[n,d]=ss2tf(A,B,C,D), pause
A=[0 1;-0.72 1.7];B=[0;1];C=[6.69 6.08];D=0;
[n,d]=ss2tf(A,B,C,D)
```

13.6 (d)
(cont)



(e) $\underline{x}[n+1]=\begin{bmatrix}0 & 1\\ -0.72 & 1.7\end{bmatrix}\underline{x}[n]+\begin{bmatrix}0\\1\end{bmatrix}u[n]$

$y[n]=[6.69 \quad 6.08]\,\underline{x}[n]$

(f) $zI-A=\begin{bmatrix}z & -1\\ 0.72 & z-1.7\end{bmatrix}$; $|zI-A|=z^2-1.7z+0.72=\Delta(z)$

$$H(z)=C[zI-A]^{-1}B=[6.69 \quad 6.08]\frac{1}{\Delta(z)}\begin{bmatrix}z-1.7 & 1\\ -0.72 & z\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$$

$$=\frac{1}{\Delta(z)}[6.69 \quad 6.08]\begin{bmatrix}1\\z\end{bmatrix}=\frac{6.08z+6.69}{z^2-1.7z+0.72}$$

(g) See (c)

13.7 (a) $\underline{x}[n+1]=\begin{bmatrix}0.8 & 1.5\\ 2.3 & 0.7\end{bmatrix}\underline{x}[n]+\begin{bmatrix}1\\2\end{bmatrix}u[n]$

$y[n]=[1.7 \quad 1.6]\,\underline{x}[n]$

(b) $zI-A=\begin{bmatrix}z-0.8 & -1.5\\ -2.3 & z-0.7\end{bmatrix}$; $|zI-A|=\Delta(z)=z^2-1.5z+0.56-3.45$

$=z^2-1.5z-2.89$

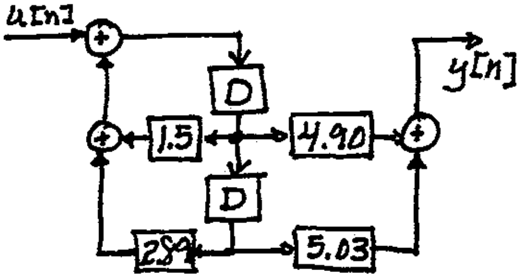
$$H(z)=C(zI-A)^{-1}B=[1.7 \quad 1.6]\frac{1}{\Delta(z)}\begin{bmatrix}z-0.7 & 1.5\\ 2.3 & z-0.8\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$$

$$=\frac{1}{\Delta(z)}[1.7 \quad 1.6]\begin{bmatrix}z+2.3\\2z+0.7\end{bmatrix}=\frac{4.09z+5.03}{z^2-1.5z-2.89}$$

(c)

```
A=[0.8 1.5;2.3 0.7];B=[1;2];C=[1.7 1.6];D=0;
[n,d]=ss2tf(A,B,C,D), pause
A=[0 1;2.89 1.5];B=[0;1];C=[5.03 4.90];D=0;
[n,d]=ss2tf(A,B,C,D)
```

(d)



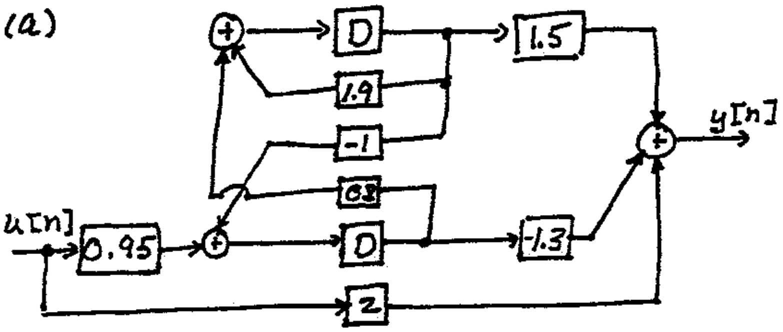
13.7.(e) (cont)

$$\underline{x}[n+1] = \begin{bmatrix} 0 & 1 \\ 2.89 & 1.5 \end{bmatrix} \underline{x}[n] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[n]$$

$$y[n] = [5.03 \quad 4.90]\underline{x}[n]$$

(f) See (c)

13.8.(a)



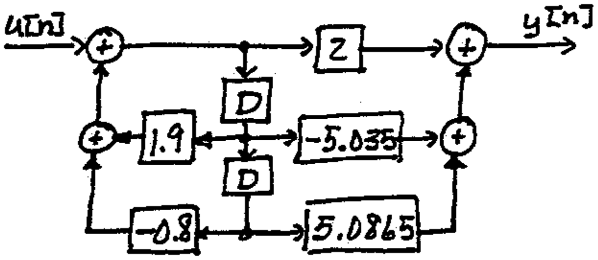
(b) $zI-A = \begin{bmatrix} z-1.9 & -0.8 \\ 1 & z \end{bmatrix}, \quad |zI-A| = \Delta = z^2-1.9z+0.8$

$$H(z) = C(zI-A)^{-1}B+D = [1.5 \quad -1.3]\frac{1}{\Delta}\begin{bmatrix} z & 0.8 \\ -1 & z-1.9 \end{bmatrix}\begin{bmatrix} 0 \\ .95 \end{bmatrix} + 2$$

$$= [1.5 \quad -1.3]\frac{1}{\Delta}\begin{bmatrix} 0.76 \\ 0.95z-1.805 \end{bmatrix} + 2 = \frac{-1.235z+3.4865}{z^2-1.9z+0.8} + 2$$

$$= \frac{2z^2-5.035z+5.0865}{z^2-1.9z+0.8}$$

(d)



(e) $\underline{x}[n+1] = \begin{bmatrix} 0 & 1 \\ -0.8 & 1.9 \end{bmatrix} \underline{x}[n] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[n]$

$$y[n] = [5.0865-1.6]x_1[n] + [-5.035+3.6]x_2[n] + 2u[n]$$

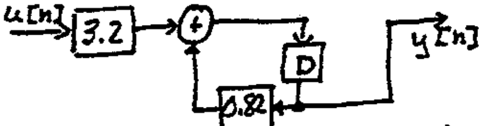
$$= [3.4865 \quad -1.435]\,\underline{x}[n] + 2u[n]$$

(f) $(zI-A) = \begin{bmatrix} z & -1 \\ 0.8 & z-1.9 \end{bmatrix}, \quad |zI-A| = \Delta = z^2-1.9z+0.8$

$$H(z) = C(zI-A)^{-1}B = [3.4865 \, -1.435]\frac{1}{\Delta}\begin{bmatrix} z-1.9 & 1 \\ -0.8 & z \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2$$

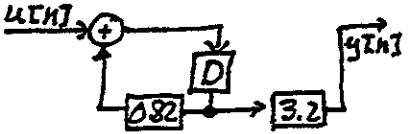
$$= [3.4865 \, -1.435]\frac{1}{\Delta}\begin{bmatrix} 1 \\ z \end{bmatrix} + 2 = \frac{-1.435z+3.4865}{z^2-1.9z+0.8} + 2 = \frac{2z^2-5.035z+5.0865}{z^2-1.9z+0.8}$$

(h)(a)



(b) $H(z) = C(zI-A)^{-1}B = (1)\left(\frac{1}{z-0.82}\right)(3.2) = \frac{3.2}{z-0.82}$

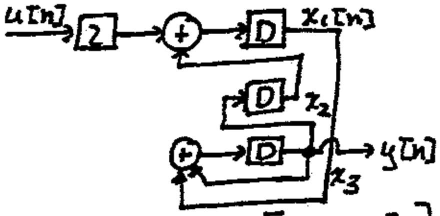
13.8. (cont) (d)



(e) $x[n+1]=0.82x[n]+u[n]$
$y[n]=3.2x[n]$

(f) $H(z)=C(zI-A)^{-1}B=(3.2)(z-0.82)^{-1}(1)=\dfrac{3.2}{z-0.82}$

(i) (a)



(b) $zI-A=\begin{bmatrix} z & -1 & 0 \\ 0 & z & -1 \\ -1 & 0 & z-1 \end{bmatrix}$

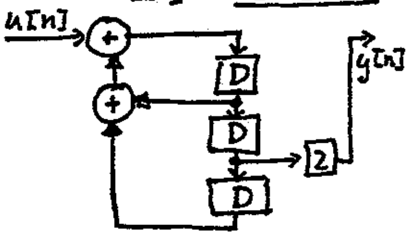
$|zI-A|=\Delta=z^3-z^2-1$

(b) cont. $\text{cof}(zI-A)=\begin{bmatrix} \cdot & \cdot & z \\ \cdot & \cdot & 1 \\ \cdot & \cdot & z^2 \end{bmatrix}$

$\therefore H(z)=C(zI-A)^{-1}B=[0\ 0\ 1]\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ z & 1 & z^2 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}\frac{1}{\Delta}$

$=[z\ 1\ z^2]\frac{1}{\Delta}\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}=\dfrac{2z}{z^3-z^2-1}$

(d) <u>Form II</u>:



(e) $x[n+1]=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}x[n]+\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u[n]$

$y[n]=[0\ 2\ 0]\underline{x}[n]$

(f) From (i)(b),

$H(z)=[0\ \ 2\ \ 0]\frac{1}{\Delta}\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ z & 1 & z^2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}=[0\ \ 2\ \ 0]\begin{bmatrix} 1 \\ z \\ z^2 \end{bmatrix}\frac{1}{\Delta}=\dfrac{2z}{z^3-z^2-1}$

MATLAB:
```
a=[1.9 0.8;-1 0]; b=[0; .95]; c=[1.5 -1.3]; d=2;
[n,d]=ss2tf(a,b,c,d)

a=[.82]; b=[3.2]; c=[1];
[n,d]=ss2tf(a,b,c,0)
```

13.9. (a) $\underline{x}[n+1]=\begin{bmatrix} 0 & 1 \\ -0.8 & 1.7 \end{bmatrix}\underline{x}[n]+\begin{bmatrix} 0 \\ 1 \end{bmatrix}m[n]$; $y[n]=[-1.3\ \ 1.5]\underline{x}[n]$

(b) $H_p(z)=\dfrac{1.5z-1.3}{z^2-1.7z+0.8}$

(c) $y[n+2]-1.7y[n+1]+0.8y[n]=1.5m[n+1]-1.3m[n]$

(d) $x_3[n+1]=0.98x_3[n]+e[n]$; $m[n]=2x_3[n]$

(e) $H_c(z)=\dfrac{2}{z-0.98}$

(f) $m[n+1]-0.98m[n]=2e[n]$

(g) $\underline{x}[n+1]=\begin{bmatrix} 0 & 1 & 0 \\ -0.8 & 1.7 & 2 \\ 1.3 & -1.5 & 0.98 \end{bmatrix}\underline{x}[n]+\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u[n]$

$y[n]=[-1.3\ \ 1.5\ \ \ 0]\underline{x}[n]$

13.9.(h) $(zI-A) = \begin{bmatrix} z & -1 & 0 \\ 0.8 & z-1.7 & -2 \\ -1.3 & 1.5 & z-0.98 \end{bmatrix}$

$$|zI-A| = \Delta = z^3 - 2.68z^2 + 1.666z - 2.6 - [-3z - 0.8z + 0.748]$$
$$= z^3 - 2.68z^2 + 5.466z - 3.384$$

$$\mathrm{cof}(zI-A) = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 2 & 2z & z^2-1.7z+0.8 \end{bmatrix}$$

$$H(z) = C(zI-A)^{-1}B = [-1.3 \quad 1.5 \quad 0]\frac{1}{\Delta}\begin{bmatrix} \cdot & \cdot & z \\ \cdot & \cdot & 2z \\ \cdot & \cdot & z^2-1.7z+0.8 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\Delta}[-1.3 \quad 1.5 \quad 0]\begin{bmatrix} z \\ 2z \\ \cdot \end{bmatrix} = \underline{\frac{3z-2.6}{z^3-2.68z^2+5.466z-3.384}}$$

(i)

```
a=[0 1 0;-0.8 1.7 2;1.3 -1.5 .98];
b=[0; 0; 1]; c=[-1.3 1.5 0];
[n,d]=ss2tf(a,b,c,0)
```

(j) $$H = \frac{H_c H_p}{1 + H_c H_p} = \frac{\left(\frac{z}{z-0.98}\right)\left(\frac{1.5z-1.3}{z^2-1.7z+0.8}\right)}{1 + (\cdot)(\cdot)} = \underline{\frac{3z-2.6}{z^3-2.68z^2+5.466z-3.384}}$$

(k) $y[n+3] - 2.68y[n+2] + 5.466y[n+1] - 3.384y[n]$
$= 3u[n+1] - 2.6u[n]$

13.10.(a) From Problem 13.2: $x_1[n+1] = (1-\alpha)x_1[n] + \alpha u[n]$
$y[n] = (1-\alpha)x_1[n] + \alpha u[n]$

(b) $$H(z) = C(zI-A)^{-1}B + D = (1-\alpha)\frac{1}{z-(1-\alpha)}\alpha + \alpha$$
$$= \frac{\alpha(1-\alpha) + \alpha z - \alpha(1-\alpha)}{z-(1-\alpha)} = \underline{\frac{\alpha z}{z-(1-\alpha)}}$$

13.11.(a) From Prob. 13.3: $\underline{x}[n+1] = \begin{bmatrix} 1-\alpha & 1-\alpha \\ -\beta/T & 1-\beta \end{bmatrix}\underline{x}[n] + \begin{bmatrix} \alpha \\ \beta/T \end{bmatrix}u[n]$
$y[n] = [1-\alpha \quad 1-\alpha]\underline{x}[n] + \alpha u[n]$

(b) $zI-A = \begin{bmatrix} z-(1-\alpha) & -(1-\alpha) \\ \beta/T & z-(1-\beta) \end{bmatrix}$

$$|zI-A| = \Delta = z^2 - (2-\alpha-\beta)z + (1-\alpha-\beta+\alpha\beta+\beta/T-\alpha\beta/T)$$

$$H(z) = C(zI-A)^{-1}B + D = [1-\alpha \quad 1-\alpha]\frac{1}{\Delta}\begin{bmatrix} z-(1-\beta) & 1-\alpha \\ -\beta/T & z-(1-\alpha) \end{bmatrix}\begin{bmatrix} \alpha \\ \beta/T \end{bmatrix} + \alpha$$

$$= \frac{(1-\alpha)}{\Delta}[z-(1-\beta)-\beta/T \quad z]\begin{bmatrix} \alpha \\ \beta/T \end{bmatrix} + \alpha$$

$$= \frac{1-\alpha}{\Delta}\left[\alpha z - \alpha(1-\beta) - \frac{\alpha\beta}{T} + \frac{\beta}{T}z\right] + \alpha = \frac{(1-\alpha)[(\alpha+\beta/T)z - \alpha(1-\beta-\beta/T)}{z^2-(2-\alpha-\beta)z+(1-\alpha-\beta+\alpha\beta+\frac{\beta}{T}-\frac{\alpha\beta}{T})}$$

(c) $\beta = 0$, $$H(z) = \frac{(1-\alpha)[z\alpha-\alpha]}{z^2-(2-\alpha)z+(1-\alpha)} + \alpha = \frac{\alpha(1-\alpha)[z-1] + \alpha z^2 - \alpha z - \alpha(1-\alpha)z + \alpha(1-\alpha)}{}$$
$$= \frac{\alpha z(z-1)}{(z-1)(z-(1-\alpha))} = \underline{\frac{\alpha z}{z-(1-\alpha)}}$$

13.12. (a) From Prob. 13.6(a): $\underline{x}[n+1]=\begin{bmatrix}0.8 & 0\\ 6.08 & 0.9\end{bmatrix}\underline{x}[n]+\begin{bmatrix}1\\ 3.2\end{bmatrix}u[n]$

$y[n]=[0 \quad 1.9]\underline{x}[n]$

(b) From Prob 13.6(b):

$$z(zI-A)^{-1}=z\begin{bmatrix}\frac{z-0.9}{(z-0.8)(z-0.9)} & 0\\ \frac{6.08}{(z-0.8)(z-0.9)} & \frac{z-0.8}{(z-0.8)(z-0.9)}\end{bmatrix}=z\begin{bmatrix}\frac{1}{z-0.8} & 0\\ \frac{-60.8}{z-0.8}+\frac{6.08}{z-0.9} & \frac{1}{z-0.9}\end{bmatrix}$$

$$\therefore \Phi[n]=\begin{bmatrix}0.8^n & 0\\ 60.8[0.9^n-0.8^n] & 0.9^n\end{bmatrix}$$

(c) $\underline{x}[n]=\Phi[n]\underline{x}[0]=\Phi[n]\begin{bmatrix}1\\ 2\end{bmatrix}=\begin{bmatrix}0.8^n\\ 62.8(0.9)^n-60.8(0.8)^n\end{bmatrix}$

$y[n]=1.9x_2[n]=\underline{119.3(0.9)^n-115.5(0.8)^n}, \quad n\geq 0$

(d) $\underline{X}(z)=(zI-A)^{-1}BU(z)=\begin{bmatrix}\frac{1}{z-0.8} & 0\\ \frac{6.08}{(z-0.8)(z-0.9)} & \frac{1}{z-0.9}\end{bmatrix}\begin{bmatrix}1\\ 3.2\end{bmatrix}\frac{z}{z-1}$

$$=z\begin{bmatrix}\frac{1}{(z-1)(z-0.8)}\\ \frac{6.08}{(z-0.8)(z-0.9)(z-1)}+\frac{3.2}{(z-1)(z-0.9)}\end{bmatrix}$$

$$=z\begin{bmatrix}\frac{5}{z-1}+\frac{-5}{z-0.8}\\ \frac{304}{z-1}+\frac{304}{z-0.8}+\frac{-608}{z-0.9}+\frac{32}{z-1}+\frac{-32}{z-1}\end{bmatrix}$$

$$=\begin{bmatrix}1-5(0.8)^n\\ 336+304(0.8)^n-640(0.9)^n\end{bmatrix}$$

$\therefore y[n]=\underline{1.9x_2[n]=638.4+577.6(0.8)^n-1216(0.9)^n;\ n\geq 0}$

(e) From Prob 13.6, $H(z)=\frac{6.08z+6.69}{(z-0.8)(z-0.9)}$

$$\frac{Y(z)}{z}=\frac{H(z)U(z)}{z}=\frac{6.08z+6.69}{(z-1)(z-0.8)(z-0.9)}=\frac{638.6}{z-1}+\frac{577.7}{z-0.8}+\frac{-1216.2}{z-0.9}$$

$\therefore y[n]=\underline{638.5+577.7(0.8)^n-1216.2(0.9)^n}$

(f) $y[n]=638.5+462.2(0.8)^n-1{,}096.9(0.9)^n$

13.13. (a) $\underline{x}[n+1]=\begin{bmatrix}0.8 & 0\\ 0 & 0.7\end{bmatrix}\underline{x}[n]+\begin{bmatrix}1\\ 2\end{bmatrix}u[n]$, $\quad y[n]=[1.7 \quad 1.6]\underline{x}[n]$

(b) $zI-A=\begin{bmatrix}z-0.8 & 0\\ 0 & z-0.7\end{bmatrix}$; $|zI-A|=(z-0.8)(z-0.7)=\Delta$

$$\Phi(z)=z(zI-A)^{-1}=\frac{z}{\Delta}\begin{bmatrix}z-0.7 & 0\\ 0 & z-0.8\end{bmatrix}=\begin{bmatrix}\frac{z}{z-0.8} & 0\\ 0 & \frac{z}{z-0.7}\end{bmatrix}$$

13.13.(b) (cont) $\Phi[n]=\mathcal{Z}^{-1}[\Phi(z)]=\begin{bmatrix}0.8^n & 0\\ 0 & 0.7^n\end{bmatrix}$

(c) $\underline{x}[n]=\Phi[n]\underline{x}[0]=\begin{bmatrix}\downarrow\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}0.8^n\\2(0.7)^n\end{bmatrix}$, $\therefore y[n]=[1.7\ \ 1.6]\underline{x}[n]$
$=\underline{1.7(0.8)^n+3.2(0.7)^n}$

(d) $\underline{X}(z)=(zI-A)^{-1}BU(z)=\begin{bmatrix}\frac{z}{z-0.8} & 0\\ 0 & \frac{z}{z-0.7}\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}\frac{z}{z-1}$

$$=\begin{bmatrix}\frac{z}{(z-1)(z-0.8)}\\ \frac{z}{(z-1)(z-0.7)}\end{bmatrix}=\begin{bmatrix}\frac{5z}{z-1}+\frac{-5z}{z-0.8}\\ \frac{6.67z}{z-1}+\frac{-6.67z}{z-0.7}\end{bmatrix}\Rightarrow \underline{x}[n]=\begin{bmatrix}5(1-0.8^n)\\ 6.67(1-0.7^n)\end{bmatrix}$$

$\therefore y[n]=C\underline{x}[n]=[1.7\ \ 1.6][\cdot]=8.5(1-0.8^n)+10.67(1-0.7)^n$

(e) $H(z)=C(zI-A)^{-1}B=[1.7\ \ 1.6]\begin{bmatrix}\frac{1}{z-0.8} & 0\\ 0 & \frac{1}{z-0.7}\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=[1.7\ \ 1.6]\begin{bmatrix}\frac{1}{z-0.8}\\ \frac{2}{z-0.7}\end{bmatrix}$

$\therefore H(z)=\frac{1.7}{z-0.8}+\frac{3.2}{z-0.7}$

$Y(z)=H(z)U(z)=\frac{1.7z}{(z-1)(z-0.8)}+\frac{3.2z}{(z-1)(z-0.8)}=\frac{8.5z}{z-1}+\frac{-8.5z}{z-0.8}+\frac{10.67z}{z-1}-\frac{10.67z}{z-0.7}$

$\therefore y[n]=8.5(1-0.8^n)+10.67(1-0.7)^n$

(f) $y[n]=1.7(0.8)^n+3.2(0.7)^n+8.5-8.5(0.8)^n+10.67-10.67(0.7)^n$
$=19.17-6.8(0.8)^n-7.47(0.7)^n$

(g) $y[0]=\underline{4.9}^{\checkmark}$, $y[2]=\underline{11.158}^{\checkmark}$

```
x1(1)=1; x2(1)=2;
for n=1:4
  y(n)=1.7*x1(n) + 1.6*x2(n);
  x1(n+1)=.8*x1(n)+0*x2(n)+1;
  x2(n+1)=0*x1(n)+.7*x2(n)+2;
end
y
```

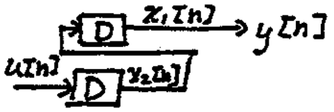
13.14.(a)(1) $zI-A=\begin{bmatrix}z & -1\\ 0 & z\end{bmatrix}$, $|zI-A|\ z^2$, $(zI-A)^{-1}=\begin{bmatrix}\frac{1}{z} & \frac{1}{z^2}\\ 0 & \frac{1}{z}\end{bmatrix}$

$\Phi[n]=\mathcal{Z}^{-1}(z(zI-A)^{-1})=\mathcal{Z}^{-1}\begin{bmatrix}1 & \frac{1}{z}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\delta[n] & \delta[n-1]\\ 0 & \delta[n]\end{bmatrix}$

(2) $\Phi[n]=A^n$; $\Phi(0)=I\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $\Phi[1]=A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$

$\Phi[2]=A^2=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$\therefore n\geq 2$, $\Phi[n]=\Phi[2]\Phi[n-2]=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ $\therefore \Phi[n]=\begin{bmatrix}\delta[n] & \delta[n-1]\\ 0 & \delta[n]\end{bmatrix}$

(b) 

$\therefore$ Realize by two cascaded delays.

13.15. (a) $zI-A=\begin{bmatrix}z & 0\\ -1 & z\end{bmatrix}$, $|zI-A|=z^2$, $(zI-A)^{-1}=\begin{bmatrix}\frac{1}{z} & 0\\ 1/z^2 & 1/z\end{bmatrix}$

13.15.(a) (CONT) $\therefore \Phi[n]\ \mathcal{Z}^{-1}\left[z(zI-A)^{-1}\right] = \mathcal{Z}^{-1}\begin{bmatrix} 1 & 0 \\ 1/z & 1 \end{bmatrix} = \underline{\begin{bmatrix} \delta[n] & 0 \\ \delta[n-1] & \delta[n] \end{bmatrix}}$

(b) $\Phi[n] = A^n;\ \Phi[0] = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Phi[1] = A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},\ \Phi[2] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \therefore n \geq 2, \Phi[n] = \underline{0}$

$\therefore \Phi[n] = \underline{\begin{bmatrix} \delta[n] & 0 \\ \delta[n-1] & \delta[n] \end{bmatrix}}$

(d) $\underline{x}[1] = A\underline{x}[0] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\underline{x}[2] = A\underline{x}[1] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \therefore \underline{x}[n] = \underline{0},\ n \geq 2$

(c) $x[n] = \Phi[n]\,\underline{x}[0] = \begin{bmatrix} \delta[n] & 0 \\ \delta[n-1] & \delta[n] \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \delta[n] \\ 2\delta[n] + \delta[n-1] \end{bmatrix}$

$\therefore y[n] = [0 \quad 1]\,\underline{x}[n] = \underline{2\delta[n] + \delta[n-1]}$

(e) $y[0] = C\,x[0] = \underline{0}$

$\underline{x}[1] = A\underline{x}[0] + Bu[0] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}(1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix};\ y[1] = x_2[1] = \underline{1}$

$\underline{x}[2] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad ; y[2] = x_2[2] = \underline{2}$

$\underline{x}[3] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad ; y[3] = \underline{2}$

$\underline{x}[4] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad ; y[4] = \underline{2}$

$\therefore y[n] = \begin{cases} 0, & n = 0 \\ 1, & n = 1 \\ 2, & n \geq 2 \end{cases}$

(f) $\underline{X}(z) = (zI - A)^{-1} B\, U(z) = \begin{bmatrix} \frac{1}{z} & 0 \\ 1 & \frac{1}{z} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\frac{z}{z-1} = \begin{bmatrix} \frac{1}{z} \\ \frac{1}{z^2} + \frac{1}{z} \end{bmatrix}\frac{z}{z-1} = \begin{bmatrix} \frac{1}{z-1} \\ \frac{1}{z(z-1)} + \frac{1}{z-1} \end{bmatrix}$

$Y(z) = C\underline{X}(z) = [0 \quad 1][\cdot] = \frac{1}{z(z-1)} + \frac{1}{z-1}$

$\therefore y[n] = \mathcal{Z}^{-1}\, Y(z) = \underline{u[n-2] + u[n-1]}$

(g) $H(z) = C(zI - A)^{-1}B = [0 \quad 1]\begin{bmatrix} \frac{1}{z} & 0 \\ \frac{1}{z^2} & \frac{1}{z} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{z^2} & \frac{1}{z} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \underline{\frac{1}{z^2} + \frac{1}{z}}$

$\therefore Y(z) = H(z)U(z) = \left[\frac{1}{z^2} + \frac{1}{z}\right]\left[\frac{z}{z-1}\right] \Rightarrow y[n] = \underline{u[n-2] + u[n-1]}$

(h)

```
x1(1)=0; x2(1)=0;
for n=1:4
  y(n)=0*x1(n)  + 1*x2(n);
  x1(n+1)=0*x1(n)+0*x2(n)+1;
  x2(n+1)=1*x1(n)+0*x2(n)+1;
end
y
```

13.16. (a) $\Phi(z) = z(zI - A)^{-1} = z\frac{1}{z - 0.95} = \frac{z}{z - 0.95} \Rightarrow \Phi[n] = \underline{0.95^n}$

(b) $x[n] = \Phi[n]\, x[0] = 0.95^n;\ y[n] = C\,x[n] = \underline{3(0.95)^n}$

13.16 (c) $x[1] = 0.95\,x[0] = 0.95 \qquad x[3] = 0.95\,x[2] = (0.95)^3$
(cont) $x[2] = 0.95\,x[1] = (0.95)^2 \quad \therefore\ x[n] = \underline{(0.95)^n}$

(d) $X(z) = (zI-A)^{-1}BU(z) = \frac{1}{z-0.95}(1)\left(\frac{z}{z-1}\right)$

$$\frac{X(z)}{z} = \frac{1}{(z-1)(z-0.95)} = \frac{20}{z-1} + \frac{-20}{z-0.95} \Rightarrow x[n] = \underline{20(1-0.95^n)}, n \geq 0$$

$$y[n] = 3x[n] = \underline{60(1-0.95^n)},\ n \geq 0$$

(e) $H(z) = C(zI-A)^{-1}B = \frac{3}{z-0.95}$

$$\therefore \frac{Y(z)}{z} = \frac{3}{(z-1)(z-0.95)} = \frac{60}{z-1} + \frac{-60}{z-0.95} \Rightarrow y[n] = \underline{60(1-0.95^n)}, n \geq 0$$

(f)

```
u=1;x(1)=0;
for n=1:5
   y=3*x(n)
   x(n+1)=0.95*x(n)+u;
end
```

13.17.(a) From Prob 13.16, $H(z) = \frac{1}{z} + \frac{1}{z^2} = \underline{\frac{z+1}{z^2}}$

(b) let $P = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, $P^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

$$A_v = P^{-1}AP = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$B_v = P^{-1}B = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C_v = CP = [0 \quad 1]\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = [1 \quad 2]$$

$$\therefore \underline{v}[n+1] = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}\underline{v}[n] + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u[n]\ ;\ y[n] = [1 \quad 2]\,v[n]$$

(d) $zI - A_v = \begin{bmatrix} z+1 & 1 \\ -1 & z-1 \end{bmatrix}$; $|zI-A| = z^2 = \Delta$

$$H(z) = C_v(zI-A_v)^{-1}B_v = [1 \quad 2]\begin{bmatrix} \frac{z-1}{z^2} & -1/z^2 \\ 1/z^2 & \frac{z+1}{z^2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = [1 \ 2]\begin{bmatrix} \frac{z-1}{z^2} \\ 1/z^2 \end{bmatrix} = \underline{\frac{z+1}{z^2}}$$

(f) $\lambda_1 = \lambda_2 = 0$

(13.67) $|zI-A| = z^2 = |zI-A_v| = (z-0)(z-0)$

(13.68) $\det A = 0 = \det A_v = (0)(0)$

(13.69) $\operatorname{tr} A = 0 = \operatorname{tr} A_v = 0+0$

(c)(e)

```
a=[0 0;1 0]; b=[1;1]; c=[0 1]; d=0; q=[2 -1;-1 1];
p=inv(q);
av=q*a*p
bv=q*b
cv=c*p
pause
[n,d1]=ss2tf(av,bv,cv,d)
```

13.18. (a) From Prob. 13.8, $H(z) = \dfrac{2z^2 - 5.035z + 5.0865}{z^2 - 1.9z + 0.8}$

(b) Let $P = \begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$, $P^{-1} = \begin{bmatrix}2 & -1\\ -1 & 1\end{bmatrix}$

$$A_v = P^{-1}AP = \begin{bmatrix}2 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}1.9 & 0.8\\ -1 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}4.8 & 1.6\\ -2.9 & -0.8\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix} = \begin{bmatrix}6.4 & 8\\ -3.7 & -4.5\end{bmatrix}$$

$$B_v = P^{-1}B = \begin{bmatrix}2 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}0\\ 0.95\end{bmatrix} = \begin{bmatrix}-0.95\\ 0.95\end{bmatrix}$$

$$C_v = CP = [1.5 \;\; -1.3]\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix} = [0.2 \;\; -1.1] \; ; \; D_v = D = 2$$

$$\therefore \underline{v}[n+1] = \begin{bmatrix}6.4 & 8\\ -3.7 & -4.5\end{bmatrix}\underline{v}[n] + \begin{bmatrix}-0.95\\ 0.95\end{bmatrix}u[n]$$

$$y[n] = [0.2 \;\; -1.1]\underline{v}[n] + 2$$

(d) $zI - A_v = \begin{bmatrix}z-6.4 & -8\\ 3.7 & z+4.5\end{bmatrix}$, $|zI - A| = z^2 - 1.9z + 0.8$

$$H(z) = C_v(zI - A_v)^{-1}B_v + D_v = [0.2 \;\; -1.1]\frac{1}{\Delta}\begin{bmatrix}z+4.5 & 8\\ -3.7 & z-6.4\end{bmatrix}\begin{bmatrix}-0.95\\ 0.95\end{bmatrix} + 2$$

$$= \frac{1}{\Delta}[0.2z+4.97 \;\; -1.1z+8.64]\begin{bmatrix}-0.95\\ 0.95\end{bmatrix} + 2 = \underline{\frac{2z^2 - 5.035z + 5.0865}{z^2 - 1.9z + 0.8}}$$

(f) (13.67) $|zI - A| = z^2 - 1.9z + 0.8 = |zI - A_v| = \underline{(z-1.27)(z-0.63)}$

(13.68) $\det A = 0.8 = \det A_v = (1.27)(0.63)$

(13.69) $\operatorname{tr} A = 1.9 = \operatorname{tr} A_v = 1.27 + 0.63$

(c)(e)

```
a=[1.9 .8;-1 0]; b=[0;.95]; c=[1.5 -1.3]; d=2; q=[2 -1;-1 1];
p=inv(q);
av=q*a*p
bv=q*b
cv=c*p
pause
[n,d1]=ss2tf(av,bv,cv,d)
```

13.19. (a) From Prob. 13.18, C.E.: $z^2 - 1.9z + 0.8 = (z-1.27)(z-0.63) = 0$

<u>not stable</u>

(b) modes: $(1.27)^n$, $(0.63)^n$

(c)
```
a=[1.9 0.8;-1 0];
eig(a)
```

13.20. (a)
```
a=[0 1 0;0 0 1;1 0 1];
eig(a)
```

From MATLAB, $z = 1.4656,\ 0.826\underline{/\pm 106.4^\circ}$

$\therefore$ unstable

(b) modes: $(1.4656)^n$, $(-0.2328 + j0.7926)^n$, $(-0.2328 - j0.7926)^n$