EE 102 | # Homework #4 | **Fall 2001**

Professor Paganini | **Due Wednesday 10/24/01**

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1. Use the definition of the Laplace transform
$$F(s) = \int_0^\infty e^{-st} f(t)dt$$
to find the transform of the following functions. Do not invoke properties here; rather, perform the integration. In each case, specify the domain of convergence.

   (a) $u(t-2)e^{2t}$.

   (b) $u(t) - u(t-1) + u(t-2) - u(t-3)$.

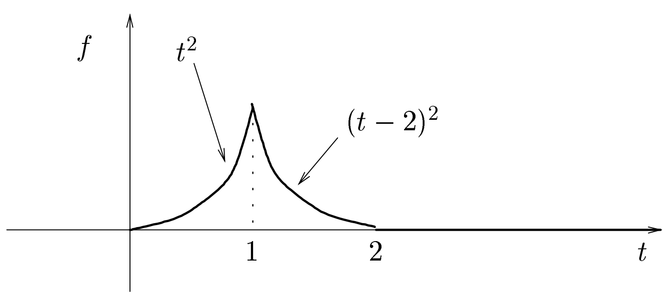
2. Find the Laplace transforms of the following functions using the properties of Laplace transform. Specify the properties being used, and the DOC.

   (a) $e^t u(t) + e^{-2t}u(t)$.

   (b) $u(t-\pi)e^{(t-\pi)}\cos(t)$.

   (c) $\int_0^t \sigma^2 e^{-\sigma} d\sigma$.

3. Consider the function $f(t)$ in the figure.



   (a) Find and sketch the derivatives $\frac{df}{dt}$, $\frac{d^2 f}{dt^2}$.

   (b) Find the Laplace transform of $\frac{d^2 f}{dt^2}$, and deduce the Laplace transforms of $\frac{df}{dt}$ and $f(t)$. Specify the domain of convergence.

4. Find $f(t)$ given $F(s)$.

$$\text{a) } F(s) = \frac{s+11}{s^2-3s-4}; \quad \text{b) } F(s) = \frac{4s+10}{s^3+6s^2+10s}; \quad \text{c) } F(s) = \frac{2s^2-s-5}{(s-1)^2(s+3)}.$$

5. Consider the differential equation for $t \geq 0$:

$$\frac{d^2 f}{dt^2} + \alpha \frac{df}{dt} + f(t) = 1, \qquad f(0-) = \frac{df}{dt}(0-) = 0.$$

Here $\alpha \in \mathbb{R}$ is a parameter.

(a) Find the initial value $\lim_{t \to 0+} f(t)$; does your answer depend on α? Hint: you don't need to solve the differential equation.

(b) Repeat the above for the final value $\lim_{t \to +\infty} f(t)$.

(c) Now let $\alpha = 1$; solve the differential equation.