EE102 - Practice Final Exam

Rules:

- **You have 3 hours.**
- **Only this booklet and <u>Two Sheets</u> of notes may be on your desk. NOT allowed: lecture notes, homeworks, calculators,...**
- **Answer each question in the space provided. EXPLAIN your reasoning. Simply writing down the answer is not adequate.**

Problem 1 [15 pts]

In class we saw that the cascade of two linear time invariant (LTI) systems is also LTI.
Now we ask:

a) Is the cascade of two linear time varying (LTV) systems always LTV?

b) Is the cascade of two time invariant (TI, not necessarily linear) systems always TI?

c) Is the cascade of two nonlinear systems always nonlinear ?

For each case you must give either:

- a proof that the answer is affirmative.
- a counterexample showing it is not the case.

Problem 2 [15 pts]

A linear, time invariant system has impulse response function given by

$$h(t) = a\ \delta(t) + b\ e^{-t}u(t) + c\,t\,e^{-t}u(t)$$

where a, b, c are constants. We are given the following information:

- When the input is $x(t) \equiv 1$ for $t \in (-\infty, \infty)$, the output is the same as the input.
- When the input is $x(t) = \cos(t)$ for $t \in (-\infty, \infty)$, the output is zero.

a) Find a, b, c.

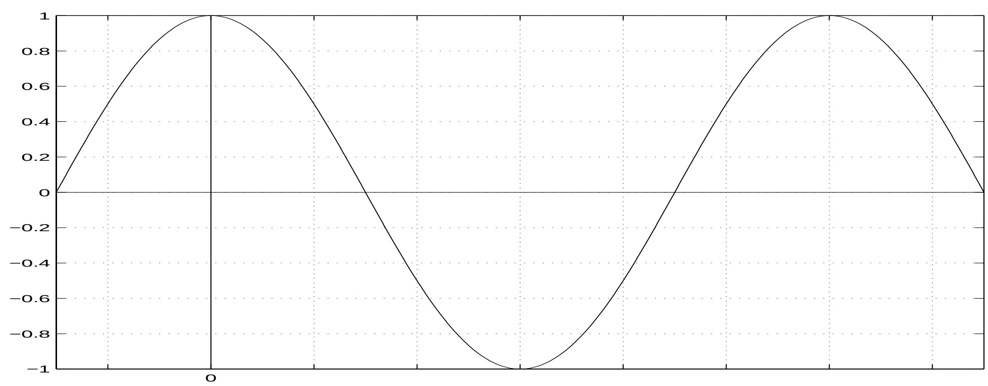
b) Now let the input be $x(t) = \cos(t)u(t)$. Find the output.

Problem 3 [20 pts]

Consider the three signals

$$x_1(t) = \cos(t); \qquad x_2(t) = \cos\left(t - \frac{2\pi}{3}\right); \qquad x_3(t) = \cos\left(t + \frac{2\pi}{3}\right).$$

a) Sketch the signals in the plot below; $x_1(t)$ has been provided for your convenience; you should specify the coordinates in the t-axis.



b) Now let $y(t) = \max\{x_1(t), x_2(t), x_3(t)\}$; in other words $y(t)$ takes the maximum of the three signals at each instant in time.
Sketch $y(t)$ in a separate plot. What is the period of $y(t)$?

c) Find the mean square error $\overline{\epsilon_0^2}$ resulting from approximating $y(t)$ by a constant function.

Problem 4 [15 pts]

We are given an LTI system with impulse response function

$$h(t) = \begin{cases} 1 - \frac{|t|}{\pi} & \text{if } |t| < \pi \\ 0 & \text{otherwise} \end{cases}$$

a) Find the frequency response function $H(i\omega)$.

b) We now apply to this system a periodic input, with period T. Discuss whether the following is true or false:
There exists a value of T such that for every periodic input of this period, the output is a constant function of time.
You should either show it's true and find an appropriate T, or show no such T exists.

Problem 5 [20 pts]

a) Given a periodic function $f(t)$ with Fourier series expansion

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{in\omega_0 t}$$

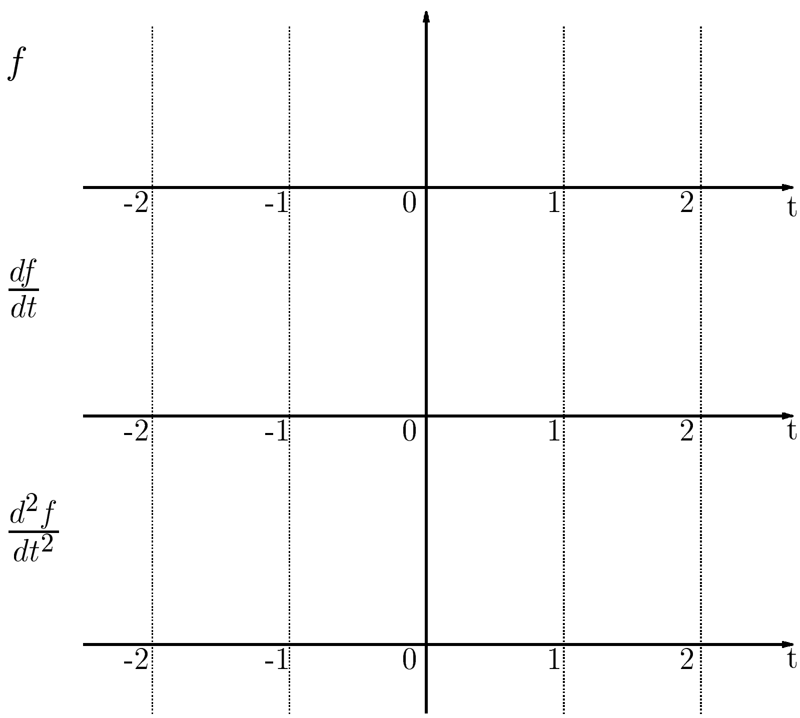
derive a formula for the Fourier series expansion of $\frac{df}{dt}(t)$.

b) Now consider $f(t)$ of period $T = 2$ and such that

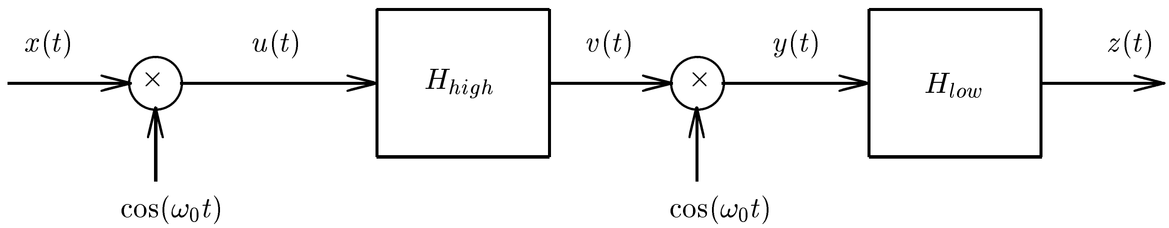
$$f(t) = \begin{cases} (t+1)^2 \text{ for } t \in [-1, 0] \\ (t-1)^2 \text{ for } t \in [0, 1] \end{cases}$$

Sketch $f(t)$, $\frac{df}{dt}(t)$ and $\frac{d^2 f}{dt^2}(t)$ in the space provided below.

c) Find the Fourier series expansions of $f(t)$, $\frac{df}{dt}(t)$ and $\frac{d^2 f}{dt^2}(t)$.



Problem 6 [15 pts]



In the above system

- H_{high} is an ideal high-pass filter with cutoff frequency ω_0.
- H_{low} is an ideal low-pass filter, also with cutoff frequency ω_0.
- $u(t) = x(t)\cos(\omega_0 t)$ and $y(t) = v(t)\cos(\omega_0 t)$.
- $x(t)$ is band-limited to $[-B, B]$, as depicted in the figure below. $X(0) = A$.
- $\omega_0 > 2B$.

Sketch the Fourier transforms $U(i\omega)$, $V(i\omega)$, $Y(i\omega)$ and $Z(i\omega)$, and relate $z(t)$ to $x(t)$. Justify your answer.