Parallel Inductor-Resistor-Capacitor (RLC) Circuits

Session 4b for Basic Electricity
A Fairfield University E-Course
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Module: Basic Electronics
(AC Circuits and Impedance: two parts)

- Text: “Electricity One-Seven,” Harry Mileaf, Prentice-Hall, 1996, ISBN 0-13-889585-6 (Covers much more material than this section)
- References:
  - “Digital Mini Test: Principles of Electricity Lessons One and Two,” SNET Home Study Coordinator, (203) 771-5400
  - Electronics Tutorial (Thanks to Alex Pounds)
  - Electronics Tutorial (Thanks to Mark Sokos)
  - Basic Math Tutorial (Thanks to George Mason University)
  - Vector Math Tutorial (Thanks to California Polytec at atom.physics.calpoly.edu)
- Alternating Current and Impedance
  - 5 on-line sessions plus one lab
- Resonance and Filters
  - 5 on-line sessions plus one lab

Section 4:
AC, Inductors and Capacitors

- 0BJECTIVES: This section discusses AC voltage / current and their effects on parallel circuit components (resistors, inductors, transformers and capacitors). The concept of resonance and its use to produce filters is also described.

Section 4 Schedule:

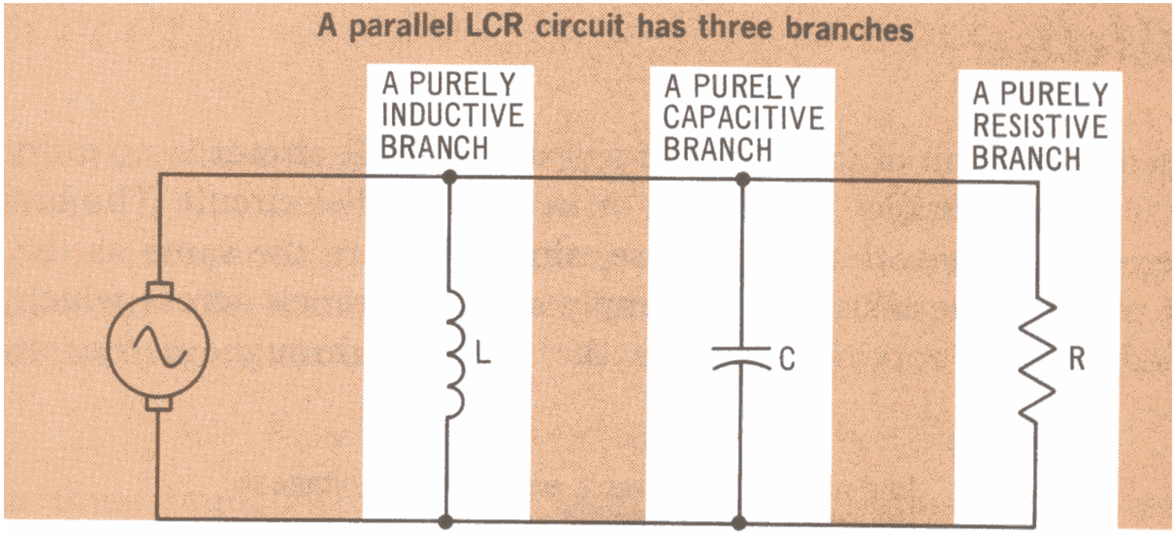
Session 4a	– 07/08	Parallel L-C Circuits	Text 4.114 – 4.122
Session 4b	**– 07/10**	**Parallel R-L-C Circuits**	**Text 4.123 – 4.132**
(break for a week)		**(no class on 07/15 or 07/17)**	
Session 4c	– 07/22	Parallel Resonance	Text 4.133 – 4.146
Session 4d (lab - 07/27, Sat.)	– 07/24	Tuning and Filters	Text 4.147 – 4.151
Session 4e (Quiz 4 due 08/12)	– 07/29	Transformers and Impedance Matching	Text 4.152 – 4.160
Session 4f	– 08/12	Review (Discuss Quiz 4)	
	08/14	MT2 Review	
	08/17	MT2 – AC Circuits	

Session 3 (Parallel L-C) Review

- Capacitive reactance $X_C = 1/2\pi fC$ at -90º
- Inductive reactance $X_L = 2\pi fL$ at 90º
- Impedances in parallel add as inverses
  - Adding Vectors
    - Separately add their horizontal and vertical components
    - Graphically: head-to-tail or parallelogram
    - Here the vectors are in opposite directions; they just subtract.
      - Inductive reactance points up (90º)
      - Capacitive reactance points down (-90º)
  - Multiplying Vectors
    - Multiply their magnitudes (lengths)
    - Add their phases
  - Dividing Vectors
    - Divide their magnitudes (lengths)
    - Subtract their phases
- Ohm's and Kirchoff's laws still work with AC

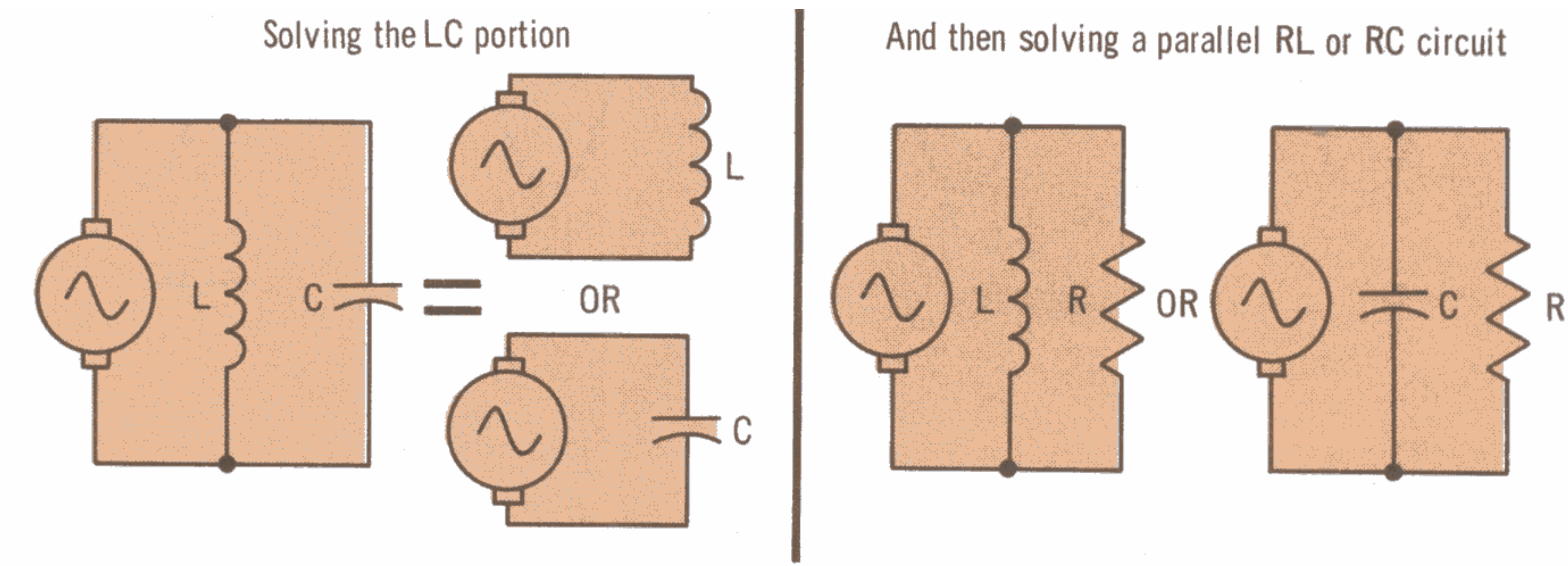
Parallel R-L-C Circuits

- Voltage (ref. phase) is the same across all parallel components
- Branch currents add (vectors) to produce I_{Line}
- Impedances in parallel add (vectors) as inverses



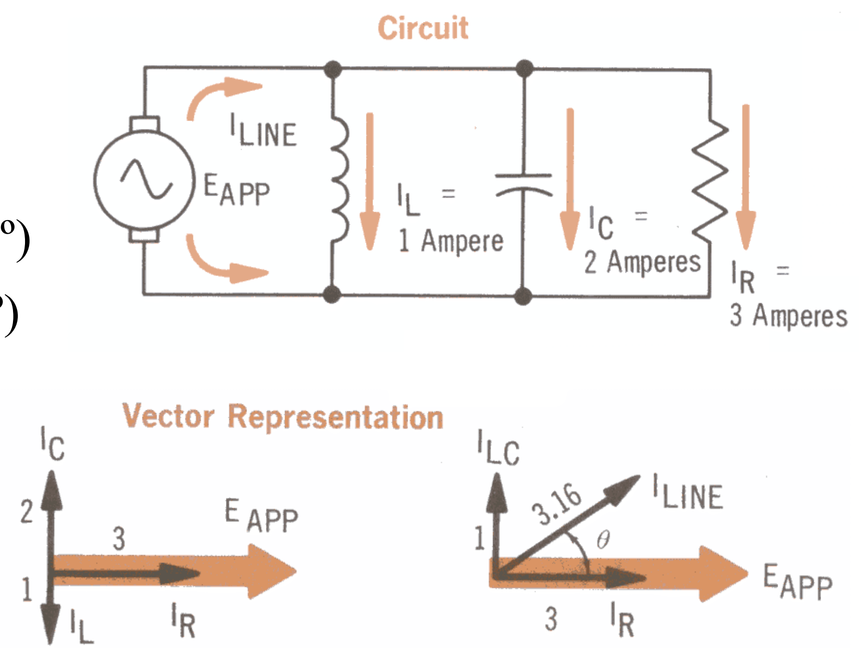
A Two-Step Solution

- Solve an RLC in two steps
  - Combine the L and C branches (both vertical)
  - Add the resistor branch to the result (as a vector)



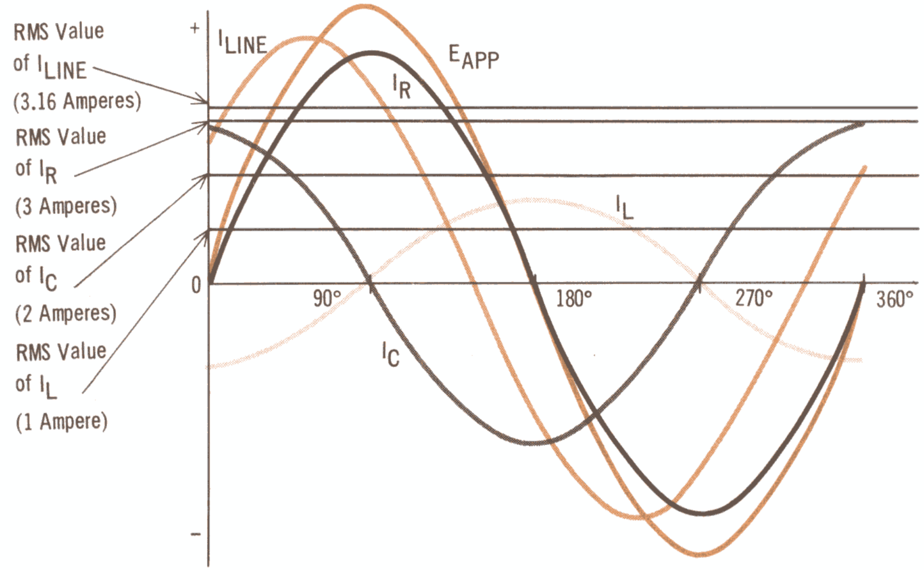
Parallel RLC- Current

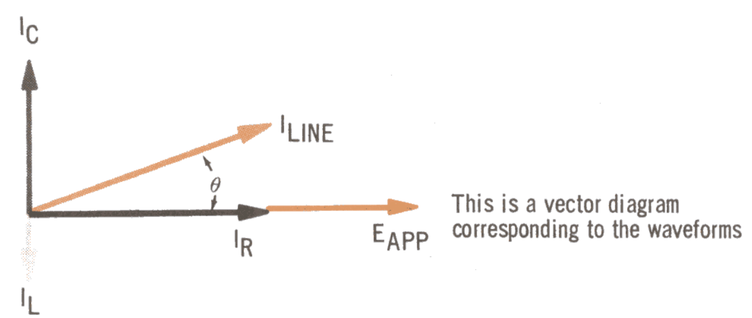
- AC currents always add as vectors
- Voltage (ref. Phase) is the same across all parallel components
- Inductor - I_L points down (Lags voltage by 90º)
- Capacitor - I_C points up (Leads voltage by 90º)
- Add I_L and I_C (they subtract) to get I_{Line}
  - $I_L = 1\angle{-90^{\circ}} = -1\angle 90^{\circ}$
  - $I_C = 2\angle 90^{\circ}$
  - $I_{LC} = (2-1)\angle 90^{\circ} = 1\angle 90^{\circ}$ (capacitive circuit)
- Now add I_R
  - $|I_{Line}| = (1^2 + 3^2)^{½} = 10^{½} = 3.162$
  - $\angle I_{Line} = \arctan(1/3) = 18.435° = 0.322$ Radians



Parallel LC- Current
Waveforms

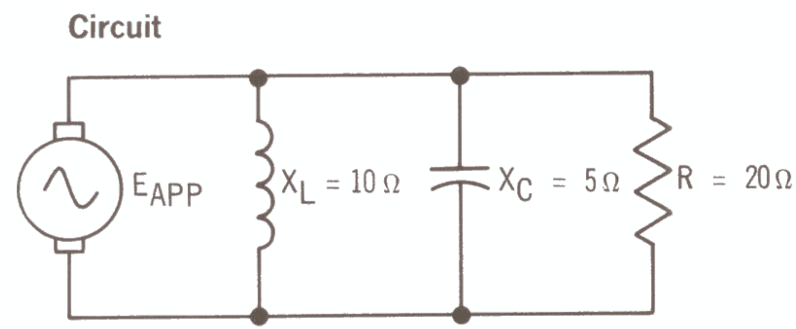
- AC currents always add as vectors
- Voltage (ref. Phase) is the same across all parallel components
- Resistor current is in phase with the voltage
- Inductor - I_L points down (Lags voltage by 90º)
- Capacitor - I_C points up (Leads voltage by 90º)
- Line current - I_{Line} has a phase between I_{LC} and I_R





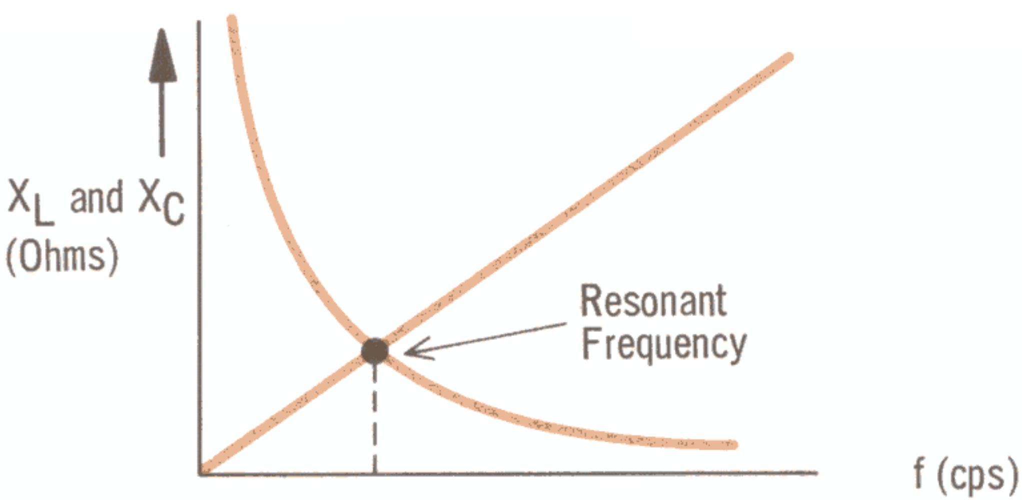
Parallel RLC - Impedance

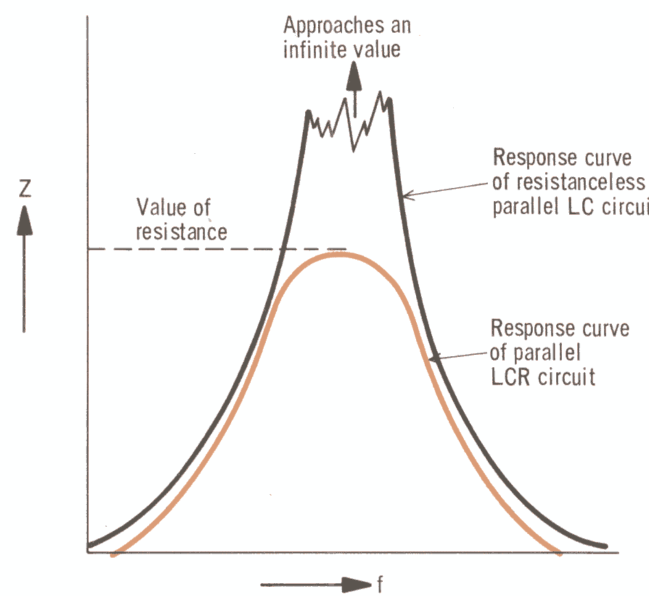
- Impedances in parallel add as inverse vectors
- X_L (up) and X_C (down) are in opposite directions
- First combine the L and C branches
  - $1/X_L = 1/(10\angle 90^o) = 0.1\angle -90^o$
  - $1/X_C = 1/(5\angle -90^o) = 0.2\angle 90^o$
  - $1/X_{LC} = 0.1\angle 90^o$
- $1/R = 0.05\angle 0^o$
- $|1/Z| = (0.05^2 + 0.1^2)^{½} = (0.0025 + 0.01)^{½} = (0.0125)^{½} = 0.1118$
- $\angle 1/Z = \arctan(0.1/0.05) = 63.4^o$
- $Z = 9\angle -63.4^o$ (capacitive)



The Effect of Frequency

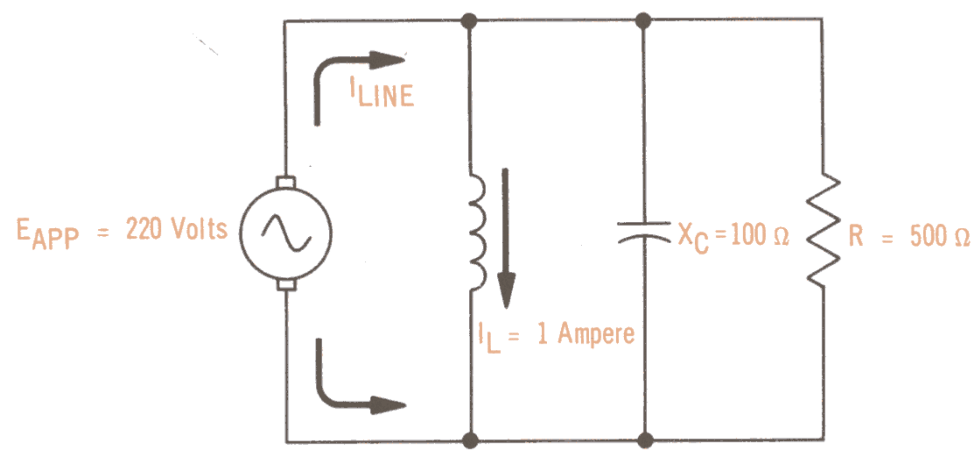
- $Z_L = 2\pi fL$ (rises linearly with frequency)
- $Z_C = 1/2\pi fL$ (decreases with frequency)
- Resonance is when they are equal and cancel; the impedance is then just the resistance





Example

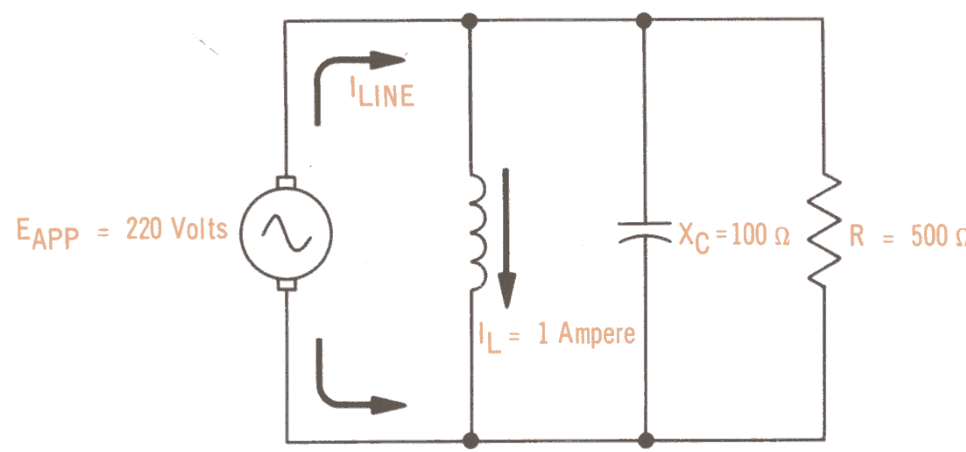
- First find the Branch currents
  - $I_C = 220/100$
    $= 2.2\angle 90^o$
  - $I_R = 220/500$
    $= 0.44\angle 0^o$



- $I_{LC} = (2.2\text{-}1)\angle 90^o = 1.2\ \angle 90^o$
- $|\ I_{Line}| = (0.44^2 + 1.2^2)^{½} = (0.1936 + 1.44)^{½}$
  $= (1.6336)^{½} = 1.278$ amps
- $\angle\ I_{Line}$ = arctan(1.2/0.44) = arctan(2.72) = 69.9º

Example (continued)

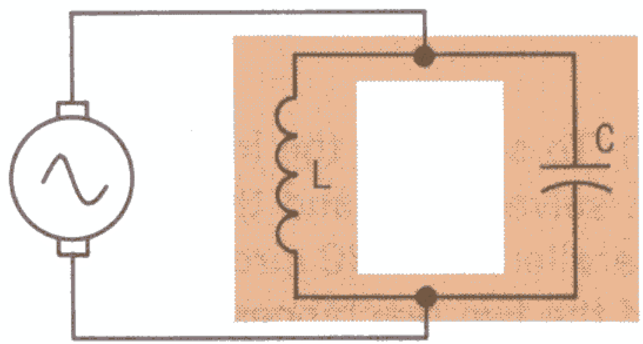
- First find the Inductor reactance
  - X_L = 220 volts/1 amp
    = $220\angle 90^\circ$ Ohms
- Now add the impedances as inverses



- $1/X_{LC} = 1/220\angle 90^\circ + 1/100\angle -90^\circ$
  $= 0.0045\angle 90^\circ + 0.01\angle -90^\circ = 0.0055\angle -90^\circ$
- $|1/X_t| = (0.0055^2 + 0.002^2)^{1/2} = (0.00003 + 0.000004)^{1/2}$
  $= (0.000034)^{1/2} = 0.0058$
  ($|X_t|$ = 172 ohms, I_{line} = 220/172 = 1.28 amps)
- $\angle 1/X_t$ = arctan(-0.0055/0.002) = arctan(-2.75) = -70º

Resonance

- X_L and X_C cancel
- Parallel Resonance
  - High Impedance
  - Low line current
    (high current in the LC loop!)
- Series Resonance
  - Low impedance
  - High line current
- Resonant frequency
  $2\pi fL = 1/2\pi fC$
  $f = 1/2\pi(LC)^{½}$



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