Basic IIR Digital Filter Structures

- The causal IIR digital filters we are concerned with in this course are characterized by a real rational transfer function of z^{-1} or, equivalently by a constant coefficient difference equation
- From the difference equation representation, it can be seen that the realization of the causal IIR digital filters requires some form of feedback

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Basic IIR Digital Filter Structures

- An N-th order IIR digital transfer function is characterized by $2N+1$ unique coefficients, and in general, requires $2N+1$ multipliers and $2N$ two-input adders for implementation
- **Direct form IIR filters**: Filter structures in which the multiplier coefficients are precisely the coefficients of the transfer function

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Direct Form IIR Digital Filter Structures

- Consider for simplicity a 3rd-order IIR filter with a transfer function

$$H(z) = \frac{P(z)}{D(z)} = \frac{p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3}}{1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}}$$

- We can implement $H(z)$ as a cascade of two filter sections as shown on the next slide

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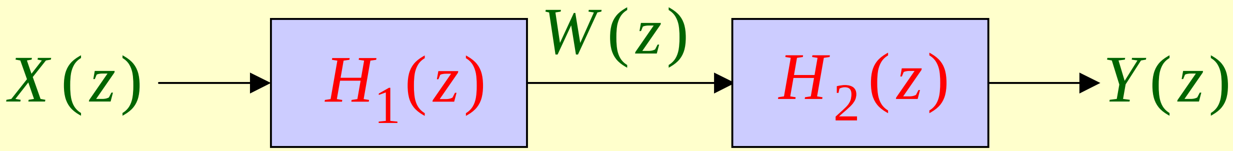
Direct Form IIR Digital Filter Structures



where

$$H_1(z) = \frac{W(z)}{X(z)} = P(z) = p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3}$$

$$H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{D(z)} = \frac{1}{1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}}$$

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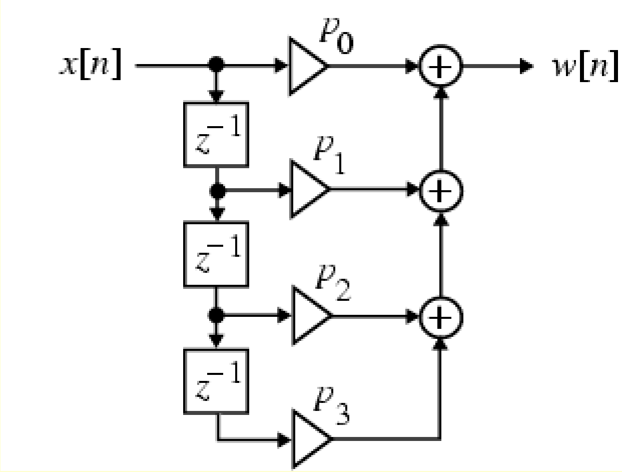
Direct Form IIR Digital Filter Structures

- The filter section $H_1(z)$ can be seen to be an FIR filter and can be realized as shown below

$$w[n] = p_0 x[n] + p_1 x[n-1] + p_2 x[n-2] + p_3 x[n-3]$$



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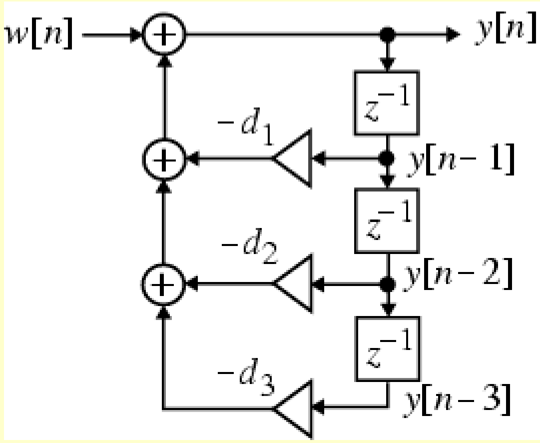
Direct Form IIR Digital Filter Structures

- The time-domain representation of $H_2(z)$ is given by

$$y[n] = w[n] - d_1 y[n-1] - d_2 y[n-2] - d_3 y[n-3]$$

Realization of $H_2(z)$ follows from the above equation and is shown on the right



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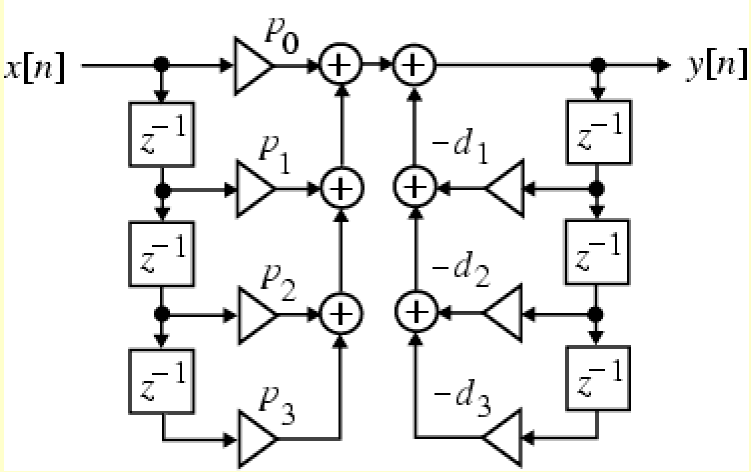
Direct Form IIR Digital Filter Structures

- A cascade of the two structures realizing $H_1(z)$ and $H_2(z)$ leads to the realization of $H(z)$ shown below and is known as the **direct form I** structure



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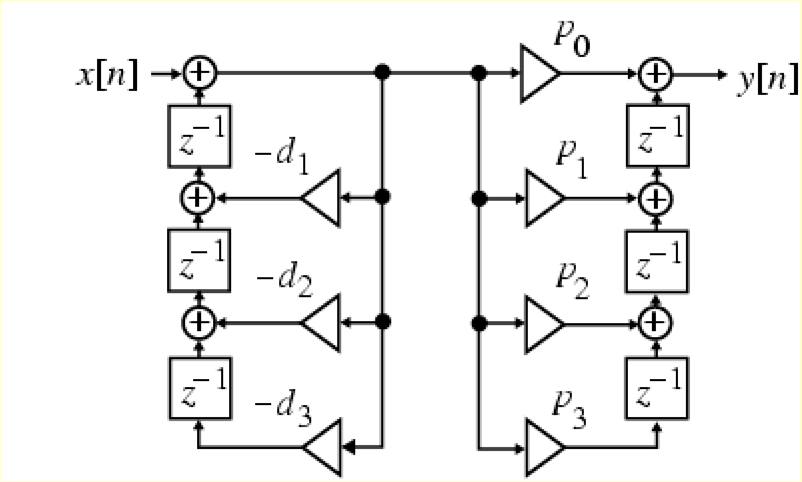
Direct Form IIR Digital Filter Structures

- Note: The direct form I structure is noncanonic as it employs 6 delays to realize a 3rd-order transfer function
- A transpose of the direct form I structure is shown on the right and is called the **direct form I_t structure**



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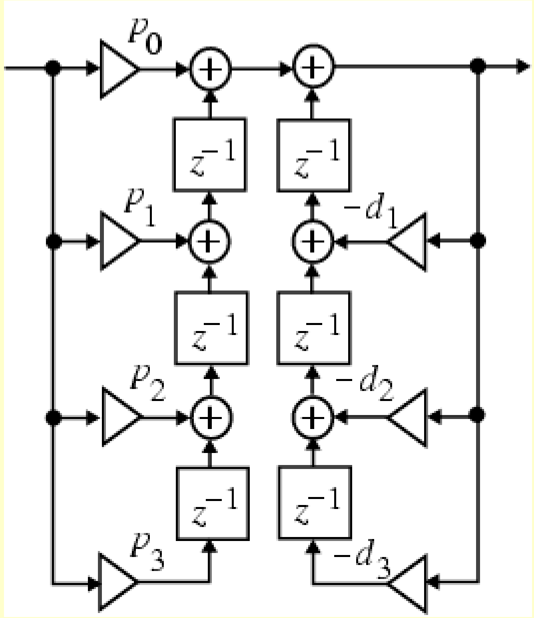
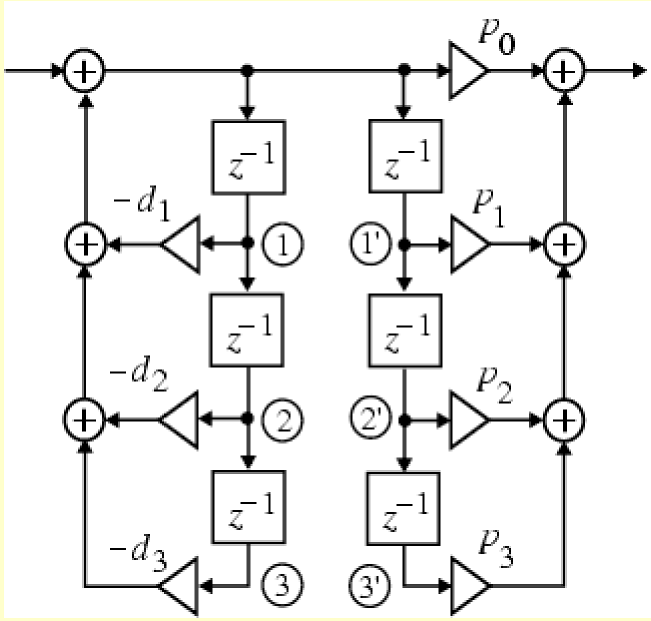
Direct Form IIR Digital Filter Structures

- Various other noncanonic direct form structures can be derived by simple block diagram manipulations as shown below





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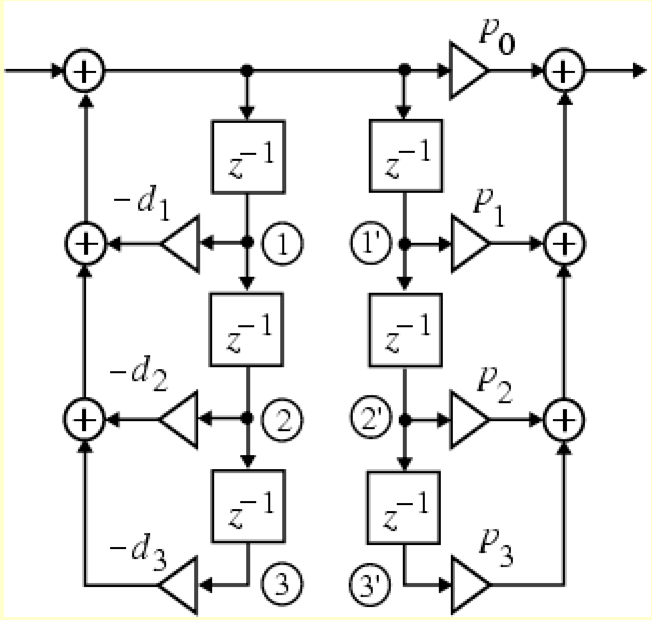
Direct Form IIR Digital Filter Structures

- Observe in the direct form structure shown below, the signal variable at nodes ① and ①' are the same, and hence the two top delays can be shared



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Direct Form IIR Digital Filter Structures

- Likewise, the signal variables at nodes ② and ②' are the same, permitting the sharing of the middle two delays
- Following the same argument, the bottom two delays can be shared
- Sharing of all delays reduces the total number of delays to 3 resulting in a canonic realization shown on the next slide along with its transpose structure

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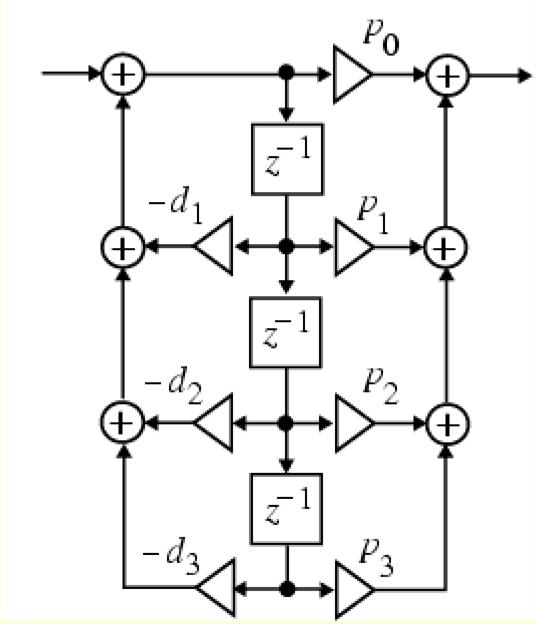
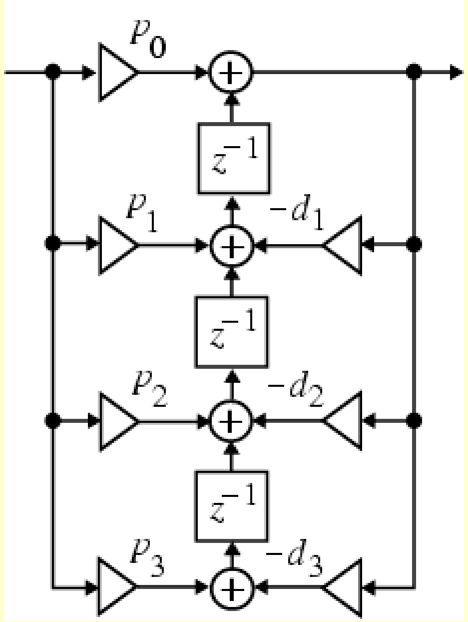
Direct Form IIR Digital Filter Structures





- Direct form realizations of an *N*-th order IIR transfer function should be evident

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Cascade Form IIR Digital Filter Structures

- By expressing the numerator and the denominator polynomials of the transfer function as a product of polynomials of lower degree, a digital filter can be realized as a cascade of low-order filter sections
- Consider, for example, $H(z) = P(z)/D(z)$ expressed as

$$H(z) = \frac{P(z)}{D(z)} = \frac{P_1(z)P_2(z)P_3(z)}{D_1(z)D_2(z)D_3(z)}$$

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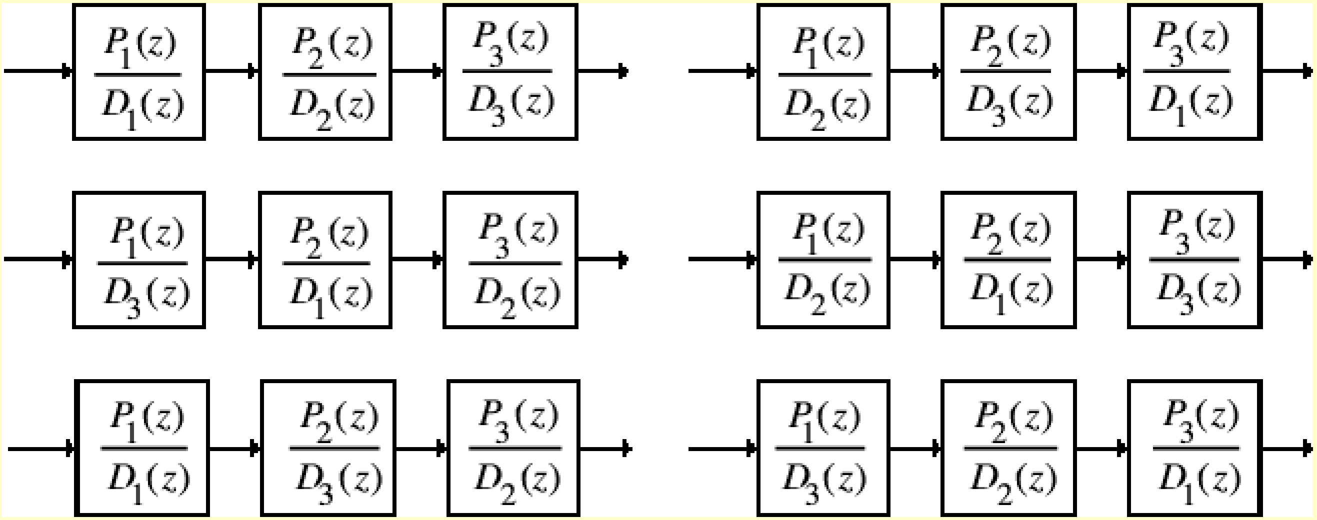
Cascade Form IIR Digital Filter Structures

- Examples of cascade realizations obtained by different pole-zero pairings are shown below



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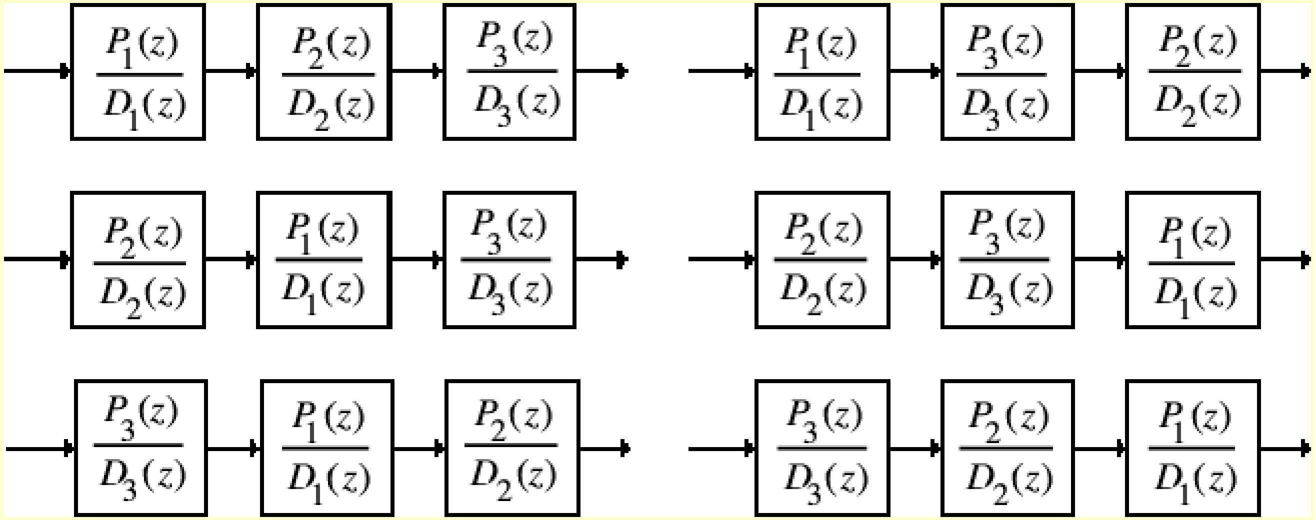
Cascade Form IIR Digital Filter Structures

- Examples of cascade realizations obtained by different ordering of sections are shown below



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Cascade Form IIR Digital Filter Structures

- There are altogether a total of 36 different cascade realizations of
$$H(z) = \frac{P_1(z)P_2(z)P_2(z)}{D_1(z)D_2(z)D_3(z)}$$
based on pole-zero-pairings and ordering
- Due to finite wordlength effects, each such cascade realization behaves differently from others

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Cascade Form IIR Digital Filter Structures

- Usually, the polynomials are factored into a product of 1st-order and 2nd-order polynomials:

$$H(z) = p_0 \prod_k \left(\frac{1+\beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1+\alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

- In the above, for a first-order factor

$$\alpha_{2k} = \beta_{2k} = 0$$

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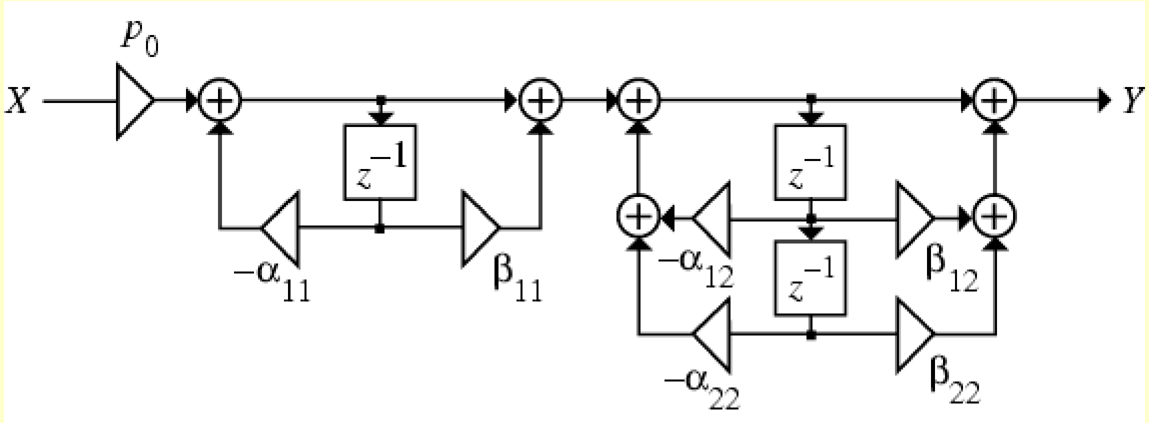
Cascade Form IIR Digital Filter Structures

- Consider the 3rd-order transfer function

$$H(z) = p_0 \left(\frac{1+\beta_{11} z^{-1}}{1+\alpha_{11} z^{-1}} \right) \left(\frac{1+\beta_{12} z^{-1} + \beta_{22} z^{-2}}{1+\alpha_{12} z^{-1} + \alpha_{22} z^{-2}} \right)$$

- One possible realization is shown below



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Cascade Form IIR Digital Filter Structures

- Example - Direct form II and cascade form realizations of

$$H(z)=\frac{0.44z^{-1}+0.362z^{-2}+0.02z^{-3}}{1+0.4z^{-1}+0.18z^{-2}-0.2z^{-3}}$$
$$=\left(\frac{0.44+0.362z^{-1}+0.02z^{-2}}{1+0.8z^{-1}+0.5z^{-2}}\right)\left(\frac{z^{-1}}{1-0.4z^{-1}}\right)$$

are shown on the next slide

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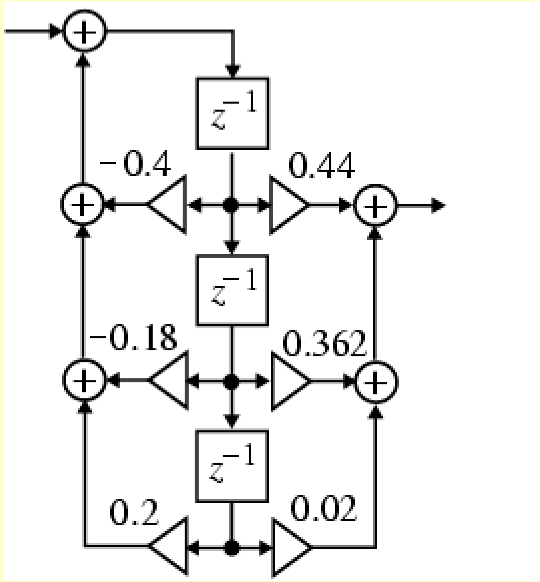
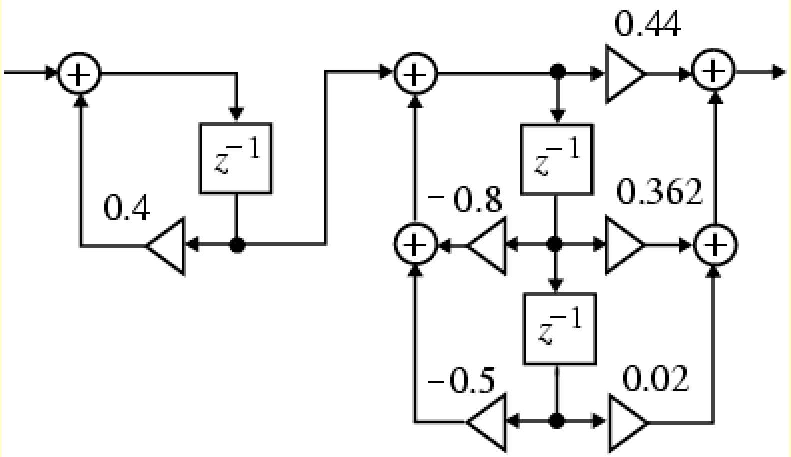
Cascade Form IIR Digital Filter Structures



Direct form II



Cascade form

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Parallel Form IIR Digital Filter Structures

- A partial-fraction expansion of the transfer function in z^{-1} leads to the **parallel form I** structure
- Assuming simple poles, the transfer function $H(z)$ can be expressed as

$$H(z) = \gamma_0 + \sum_k \left(\frac{\gamma_{0k} + \gamma_{1k} z^{-1}}{1 + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

- In the above for a real pole $\alpha_{2k} = \gamma_{1k} = 0$

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Parallel Form IIR Digital Filter Structures

- A direct partial-fraction expansion of the transfer function in z leads to the **parallel form II** structure
- Assuming simple poles, the transfer function $H(z)$ can be expressed as

$$H(z) = \delta_0 + \sum_k \left(\frac{\delta_{0k} z^{-1} + \delta_{2k} z^{-2}}{1 + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

- In the above for a real pole $\alpha_{2k} = \delta_{2k} = 0$

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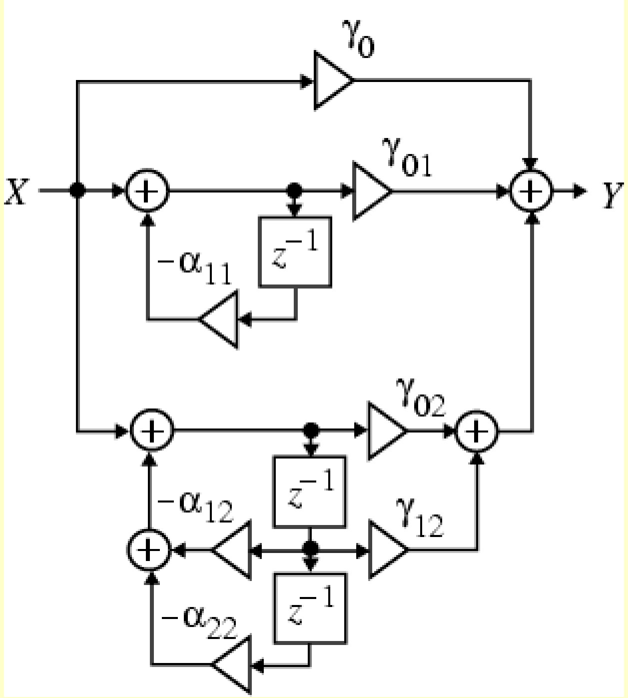
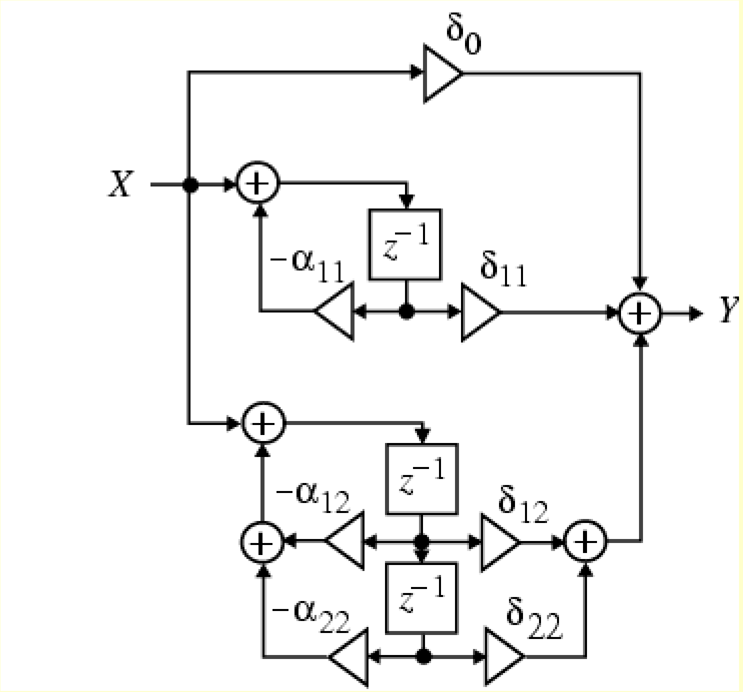
Parallel Form IIR Digital Filter Structures

- The two basic parallel realizations of a 3rd-order IIR transfer function are shown below



Parallel form I



Parallel form II

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Parallel Form IIR Digital Filter Structures

- Example - A partial-fraction expansion of

$$H(z) = \frac{0.44z^{-1} + 0.362z^{-2} + 0.02z^{-3}}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$

in z^{-1} yields

$$H(z) = -0.1 + \frac{0.6}{1 - 0.4z^{-1}} + \frac{-0.5 - 0.2z^{-1}}{1 + 0.8z^{-1} + 0.5z^{-2}}$$

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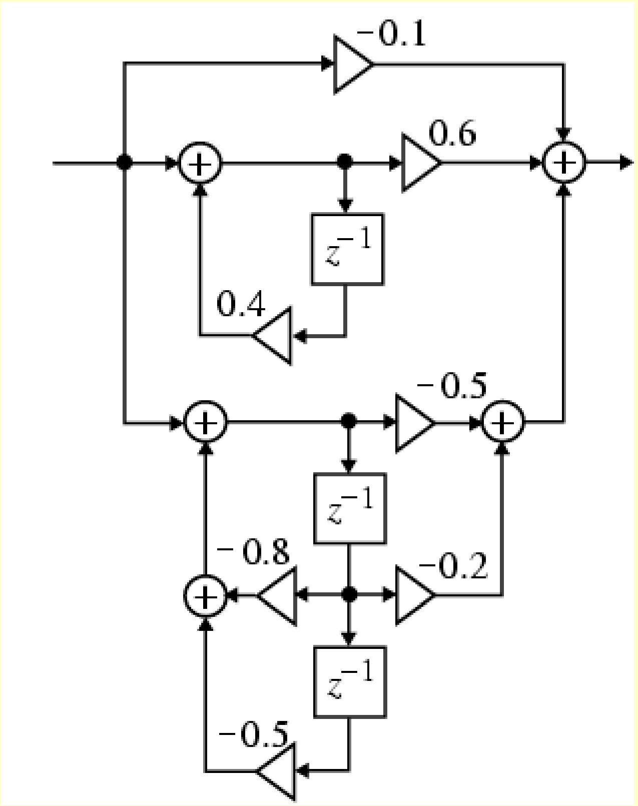
Parallel Form IIR Digital Filter Structures

- The corresponding parallel form I realization is shown below



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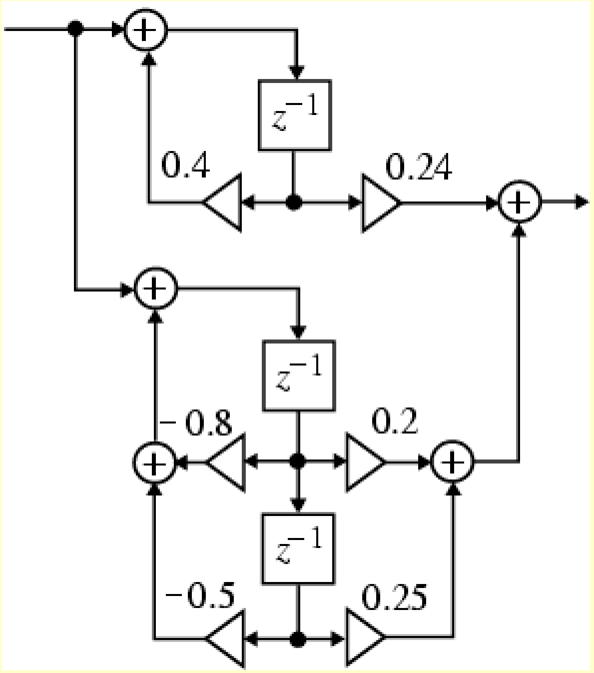
Parallel Form IIR Digital Filter Structures

- Likewise, a partial-fraction expansion of $H(z)$ in z yields

$$H(z) = \frac{0.24z^{-1}}{1-0.4z^{-1}} + \frac{0.2z^{-1}+0.25z^{-1}}{1+0.8z^{-1}+0.5z^{-2}}$$



- The corresponding parallel form II realization is shown on the right

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Realization Using MATLAB

- The cascade form requires the factorization of the transfer function which can be developed using the M-file `zp2sos`
- The statement `sos = zp2sos(z,p,k)` generates a matrix `sos` containing the coefficients of each 2nd-order section of the equivalent transfer function $H(z)$ determined from its pole-zero form

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Realization Using MATLAB

- `sos` is an $L \times 6$ matrix of the form

$$\texttt{sos} = \begin{bmatrix} \texttt{p}_{01} & \texttt{p}_{11} & \texttt{p}_{21} & \texttt{d}_{01} & \texttt{d}_{11} & \texttt{d}_{21} \\ \texttt{p}_{02} & \texttt{p}_{12} & \texttt{p}_{22} & \texttt{d}_{02} & \texttt{d}_{12} & \texttt{d}_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \texttt{p}_{0L} & \texttt{p}_{1L} & \texttt{p}_{2L} & \texttt{d}_{0L} & \texttt{d}_{1L} & \texttt{d}_{2L} \end{bmatrix}$$

whose i-th row contains the coefficients $\{p_{i\ell}\}$ and $\{d_{i\ell}\}$, of the the numerator and denominator polynomials of the i-th 2nd-order section

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Realization Using MATLAB

- L denotes the number of sections
- The form of the overall transfer function is given by

$$H(z) = \prod_{i=1}^{L} H_i(z) = \prod_{i=1}^{L} \frac{p_{0i} + p_{1i} z^{-1} + p_{2i} z^{-2}}{d_{0i} + d_{1i} z^{-1} + d_{2i} z^{-2}}$$

- Program 6_1 can be used to factorize an FIR and an IIR transfer function

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Realization Using MATLAB

- Note: An FIR transfer function can be treated as an IIR transfer function with a constant numerator of unity and a denominator which is the polynomial describing the FIR transfer function

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Realization Using MATLAB

- Parallel forms I and II can be developed using the functions `residuez` and `residue`, respectively
- Program 6_2 uses these two functions

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Realization of Allpass Filters

- An M-th order real-coefficient allpass transfer function $A_M(z)$ is characterized by M unique coefficients as here the numerator is the mirror-image polynomial of the denominator
- A direct form realization of $A_M(z)$ requires $2M$ multipliers
- Objective - Develop realizations of $A_M(z)$ requiring only M multipliers

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Realization Using Multiplier Extraction Approach

- Now, an arbitrary allpass transfer function can be expressed as a product of 2nd-order and/or 1st-order allpass transfer functions
- We consider first the minimum multiplier realization of a 1st-order and a 2nd-order allpass transfer functions

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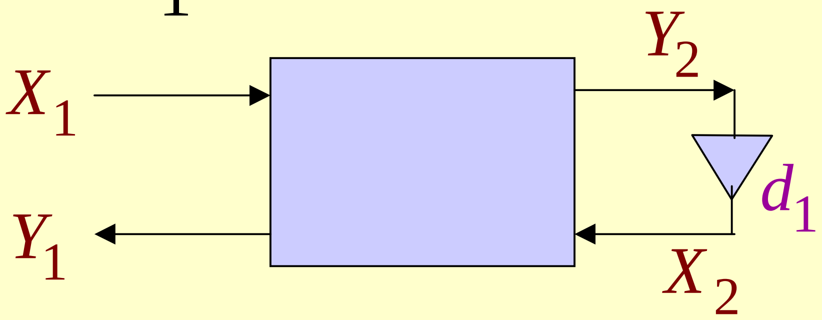
First-Order Allpass Structures

- Consider first the 1st-order allpass transfer function given by

$$A_1(z) = \frac{d_1 + z^{-1}}{1 + d_1 z^{-1}}$$

- We shall realize the above transfer function in the form a structure containing a single multiplier d_1 as shown below



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First-Order Allpass Structures

- We express the transfer function $A_1(z) = Y_1 / X_1$ in terms of the transfer parameters of the two-pair as

$$A_1(z) = t_{11} + \frac{t_{12}t_{21}d_1}{1 - d_1 t_{22}} = \frac{t_{11} - d_1(t_{11}t_{22} - t_{12}t_{21})}{1 - d_1 t_{22}}$$

- A comparison of the above with

$$A_1(z) = \frac{d_1 + z^{-1}}{1 + d_1 z^{-1}}$$

yields

$$t_{11} = z^{-1},\ t_{22} = -z^{-1},\ t_{11}t_{22} - t_{12}t_{21} = -1$$

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First-Order Allpass Structures

- Substituting $t_{11} = z^{-1}$ and $t_{22} = -z^{-1}$ in $t_{11}t_{22} - t_{12}t_{21} = -1$ we get

$$t_{12}t_{21} = 1 - z^{-2}$$

- There are 4 possible solutions to the above equation:

Type 1A: $t_{11} = z^{-1},\ t_{22} = -z^{-1},\ t_{12} = 1 - z^{-2},\ t_{21} = 1$

Type 1B:

$$t_{11} = z^{-1},\ t_{22} = -z^{-1},\ t_{12} = 1 + z^{-1},\ t_{21} = 1 - z^{-1}$$

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First-Order Allpass Structures

- Type 1A_t : $t_{11} = z^{-1},\ t_{22} = -z^{-1},\ t_{12} = 1,\ t_{21} = 1 - z^{-2}$
- Type 1B_t :

$$t_{11} = z^{-1},\ t_{22} = -z^{-1},\ t_{12} = 1 - z^{-1},\ t_{21} = 1 + z^{-1}$$

- We now develop the two-pair structure for the Type 1A allpass transfer function

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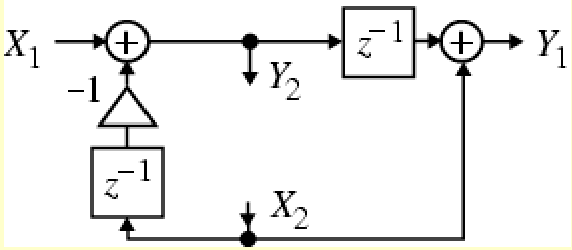
First-Order Allpass Structures

- From the transfer parameters of this allpass we arrive at the input-output relations:

$$Y_2 = X_1 - z^{-1} X_2$$

$$Y_1 = z^{-1} X_1 + (1 - z^{-2}) X_2 = z^{-1} Y_2 + X_2$$

- A realization of the above two-pair is sketched below



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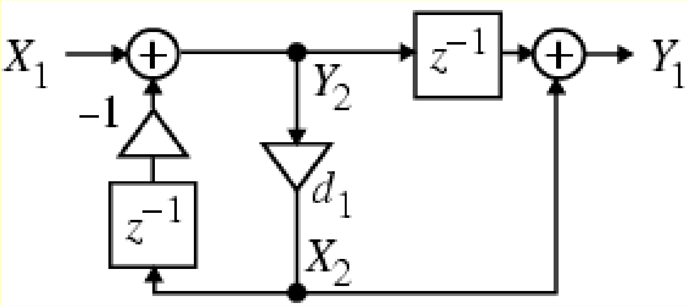
First-Order Allpass Structures

- By constraining the X_2, Y_2 terminal-pair with the multiplier d_1, we arrive at the Type 1A allpass filter structure shown below



Type 1A

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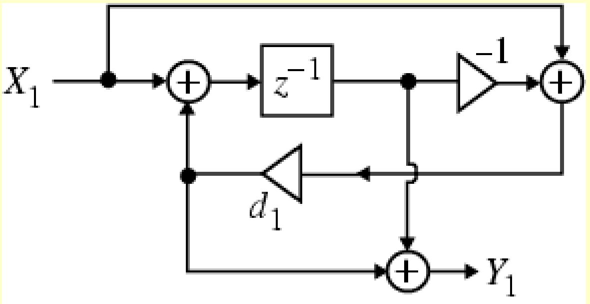
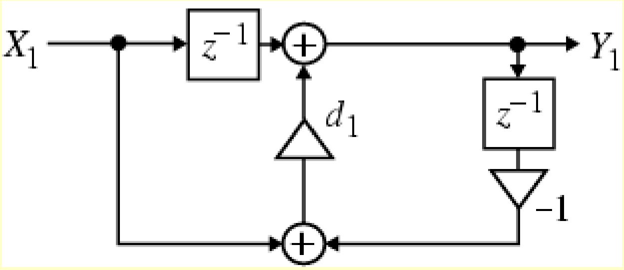
First-Order Allpass Structures

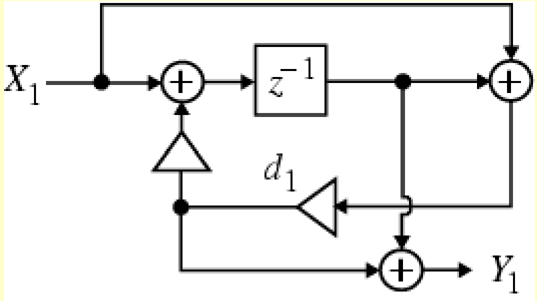
- In a similar fashion, the other three single-multiplier first-order allpass filter structures can be developed as shown below



Type 1B



Type 1A$_t$



Type 1B$_t$

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Second-Order Allpass Structures

- A 2nd-order allpass transfer function is characterized by 2 unique coefficients
- Hence, it can be realized using only 2 multipliers
- Type 2 allpass transfer function:

$$A_2(z) = \frac{d_1 d_2 + d_1 z^{-1} + z^{-2}}{1 + d_1 z^{-1} + d_1 d_2 z^{-2}}$$

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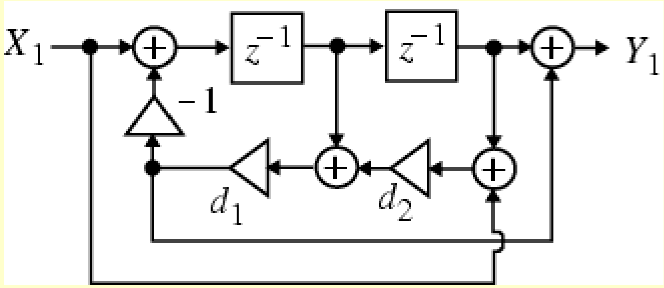
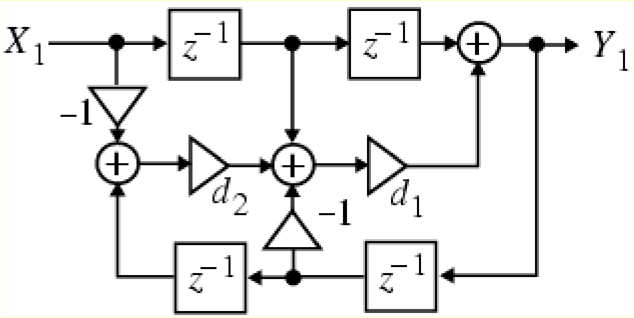
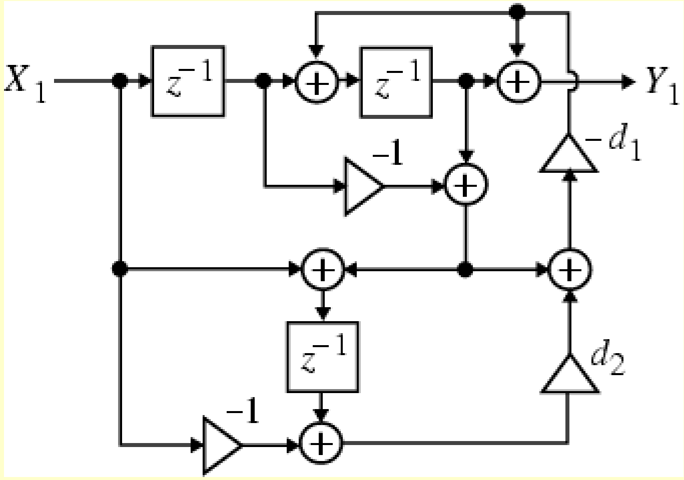
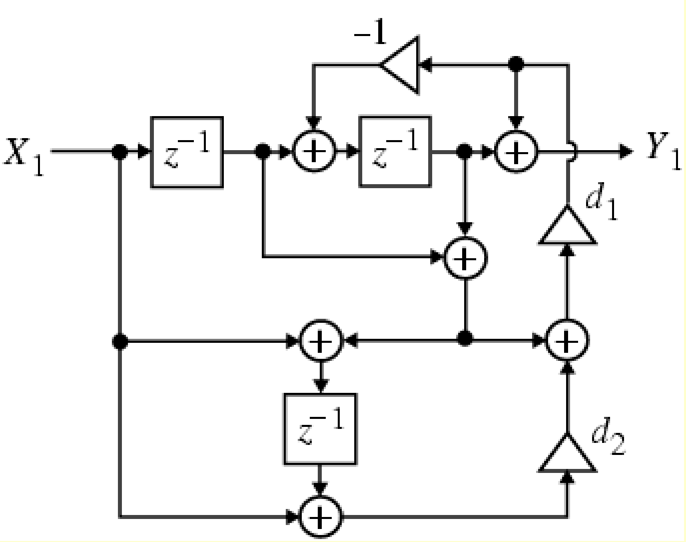
Type 2 Allpass Structures









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Type 3 Allpass Structures

- Type 3 allpass transfer function:

$$A_3(z) = \frac{d_2 + d_1 z^{-1} + z^{-2}}{1 + d_1 z^{-1} + d_2 z^{-2}}$$

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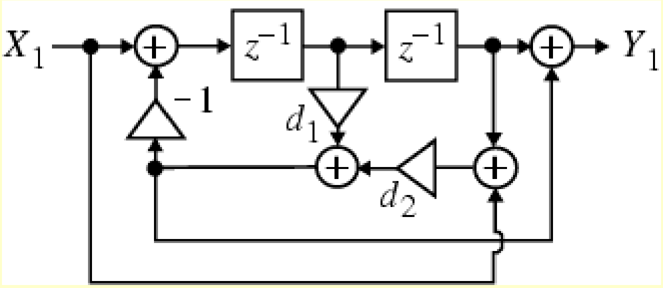
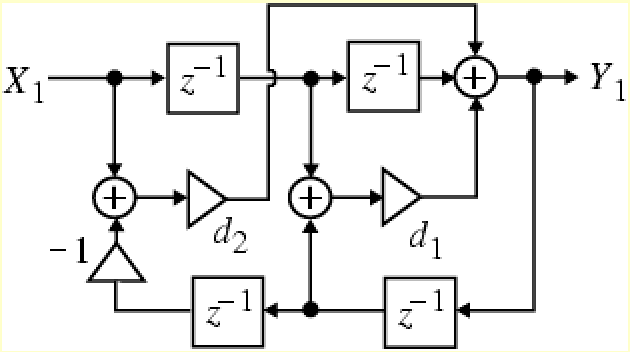
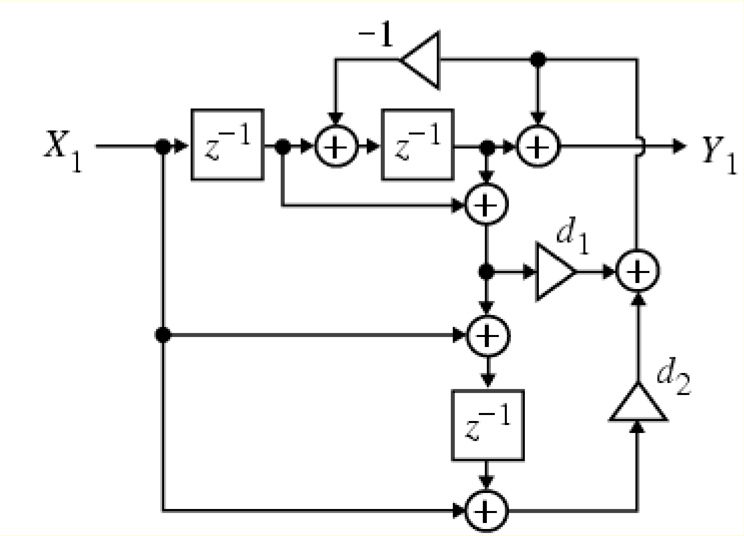
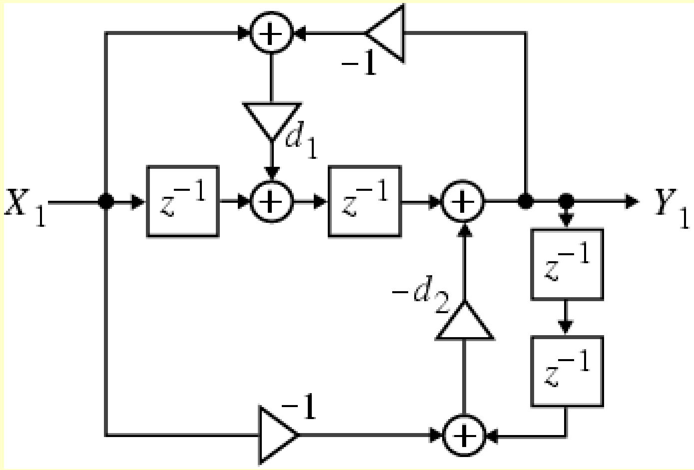
Type 3 Allpass Structures









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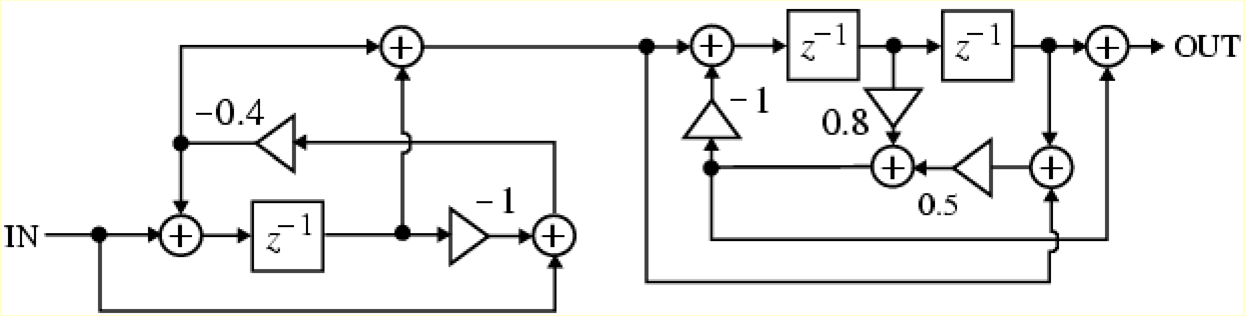
Realization Using Multiplier Extraction Approach

- Example - Realize

$$A_3(z) = \frac{-0.2+0.18z^{-1}+0.4z^{-2}+z^{-3}}{1+0.4z^{-1}+0.18z^{-2}-0.2z^{-3}}$$

$$= \frac{(-0.4+z^{-1})(0.5+0.8z^{-1}+z^{-2})}{(1-0.4z^{-1})(1+0.8z^{-1}+0.5z^{-2})}$$

- A 3-multiplier cascade realization of the above allpass transfer function is shown below



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Realization Using Two-Pair Extraction Approach

- The stability test algorithm described earlier in the course also leads to an elegant realization of an Mth-order allpass transfer function
- The algorithm is based on the development of a series of $(m-1)$th-order allpass transfer functions $A_{m-1}(z)$ from an mth-order allpass transfer function $A_m(z)$ for $m = M, M-1, \ldots, 1$

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Realization Using Two-Pair Extraction Approach

- Let

$$A_m(z) = \frac{d_m + d_{m-1}z^{-1} + d_{m-2}z^{-2} + \cdots + d_1 z^{-(m-1)} + z^{-m}}{1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_{m-1}z^{-(m-1)} + d_m z^{-m}}$$

- We use the recursion

$$A_{m-1}(z) = z\left[\frac{A_m(z) - k_m}{1 - k_m A_m(z)}\right], \quad m = M, M-1, \ldots, 1$$

where $k_m = A_m(\infty) = d_m$

- It has been shown earlier that $A_M(z)$ is stable if and only if

$$k_m^2 < 1 \quad \text{for } m = M, M-1, \ldots, 1$$

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Realization Using Two-Pair Extraction Approach

- If the allpass transfer function $A_{m-1}(z)$ is expressed in the form

$$A_{m-1}(z) = \frac{d'_{m-1} + d'_{m-2}z^{-1} + \cdots + d'_1 z^{-(m-2)} + z^{-(m-1)}}{1 + d'_1 z^{-1} + \cdots + d'_{m-2} z^{-(m-2)} + d'_{m-1} z^{-(m-1)}}$$

then the coefficients of $A_{m-1}(z)$ are simply related to the coefficients of $A_m(z)$ through

$$d'_i = \frac{d_i - d_m d_{m-i}}{1 - d_m^2}, \;\; 1 \le i \le m-1$$

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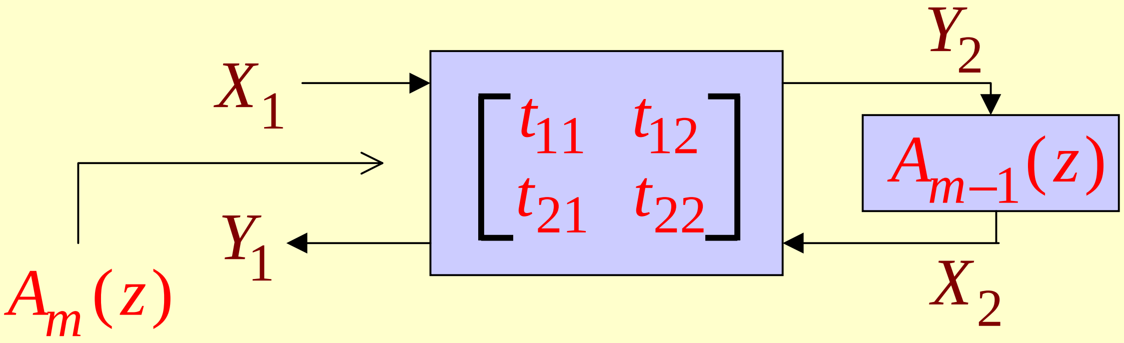
Realization Using Two-Pair Extraction Approach

- To develop the realization method we express $A_m(z)$ in terms of $A_{m-1}(z)$:

$$A_m(z) = \frac{k_m + z^{-1}A_{m-1}(z)}{1 + k_m z^{-1}A_{m-1}(z)}$$

- We realize $A_m(z)$ in the form shown below



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Realization Using Two-Pair Extraction Approach

- The transfer function $A_m(z) = Y_1/X_1$ of the constrained two-pair can be expressed as

$$A_m(z) = \frac{t_{11} - (t_{11}t_{22} - t_{12}t_{21})A_{m-1}(z)}{1 - t_{22}A_{m-1}(z)}$$

- Comparing the above with

$$A_m(z) = \frac{k_m + z^{-1}A_{m-1}(z)}{1 + k_m z^{-1}A_{m-1}(z)}$$

we arrive at the two-pair transfer parameters

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Realization Using Two-Pair Extraction Approach

$$t_{11} = k_m, \quad t_{22} = -k_m z^{-1}$$

$$t_{11}t_{22} - t_{12}t_{21} = -z^{-1}$$

- Substituting $t_{11} = k_m$ and $t_{22} = -k_m z^{-1}$ in the equation above we get

$$t_{12}t_{21} = (1 - k_m^2) z^{-1}$$

- There are a number of solutions for t_{12} and t_{21}

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Realization Using Two-Pair Extraction Approach

- Some possible solutions are given below:

$$t_{11}=k_m,\ t_{22}=-k_m z^{-1},\ t_{12}=z^{-1},\ t_{21}=1-k_m^2$$

$$t_{11}=k_m,\ t_{22}=-k_m z^{-1},\ t_{12}=(1-k_m)z^{-1},\ t_{21}=1+k_m$$

$$t_{11}=k_m,\ t_{22}=-k_m z^{-1},\ t_{12}=\sqrt{1-k_m^2}\,z^{-1},\ t_{21}=\sqrt{1-k_m^2}$$

$$t_{11}=k_m,\ t_{22}=-k_m z^{-1},\ t_{12}=(1-k_m^2)z^{-1},\ t_{21}=1$$

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Realization Using Two-Pair Extraction Approach

- Consider the solution

$$t_{11} = k_m, \; t_{22} = -k_m z^{-1}, \; t_{12} = (1 - k_m^2) z^{-1}, \; t_{21} = 1$$

- Corresponding input-output relations are

$$Y_1 = k_m X_1 - (1 - k_m^2) z^{-1} X_2$$
$$Y_2 = X_1 - k_m z^{-1} X_2$$

- A direct realization of the above equations leads to the 3-multiplier two-pair shown on the next slide

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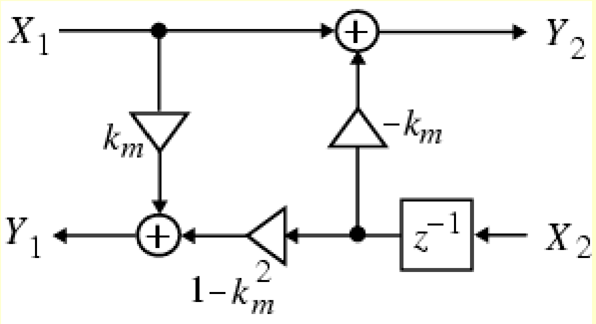
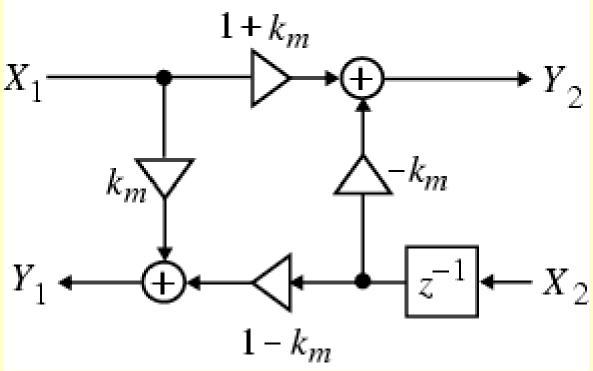
Realization Using Two-Pair Extraction Approach



- The transfer parameters

$$t_{11} = k_m,\ t_{22} = -k_m z^{-1},\ t_{12} = (1-k_m)z^{-1},\ t_{21} = 1+k_m$$

lead to the 4-multiplier two-pair structure shown below



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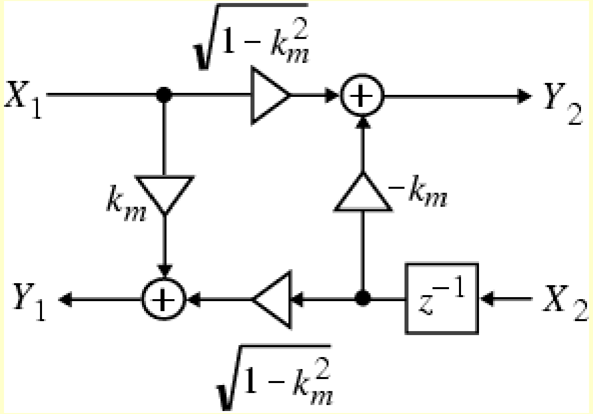
Realization Using Two-Pair Extraction Approach

- Likewise, the transfer parameters

$$t_{11} = k_m,\ t_{22} = -k_m z^{-1},\ t_{12} = \sqrt{1-k_m^2}\, z^{-1},\ t_{21} = \sqrt{1-k_m^2}$$

lead to the 4-multiplier two-pair structure shown below



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Realization Using Two-Pair Extraction Approach

- A 2-multiplier realization can be derived by manipulating the input-output relations:

$$Y_1 = k_m X_1 - (1 - k_m^2) z^{-1} X_2$$

$$Y_2 = X_1 - k_m z^{-1} X_2$$

- Making use of the second equation, we can rewrite the first equation as

$$Y_1 = k_m Y_2 + z^{-1} X_2$$

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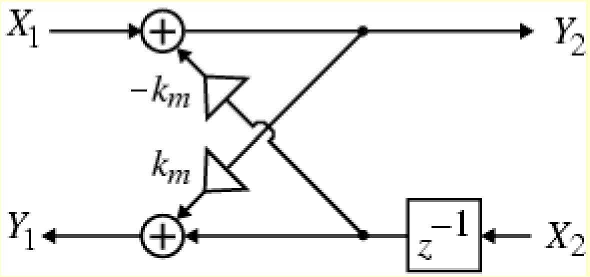
Realization Using Two-Pair Extraction Approach

- A direct realization of

$$Y_1 = k_m Y_2 + z^{-1} X_2$$
$$Y_2 = X_1 - k_m z^{-1} X_2$$

lead to the 2-multiplier two-pair structure, known as the **lattice structure**, shown below



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Realization Using Two-Pair Extraction Approach

- Consider the two-pair described by

$$t_{11} = k_m,\ t_{22} = -k_m z^{-1},\ t_{12} = (1-k_m)z^{-1},\ t_{21} = 1+k_m$$

- Its input-output relations are given by

$$Y_1 = k_m X_1 + (1-k_m)z^{-1}X_2$$

$$Y_2 = (1+k_m)X_1 - k_m z^{-1}X_2$$

- Define

$$V_1 = k_m(X_1 - z^{-1})X_2$$

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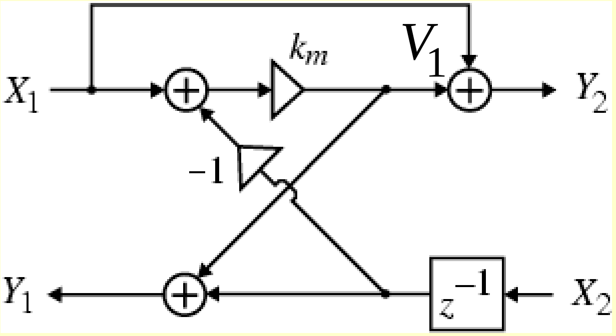
Realization Using Two-Pair Extraction Approach

- We can then rewrite the input-output relations as $Y_1 = V_1 + z^{-1}X_2$ and $Y_2 = X_1 + V_1$
- The corresponding 1-multiplier realization is shown below



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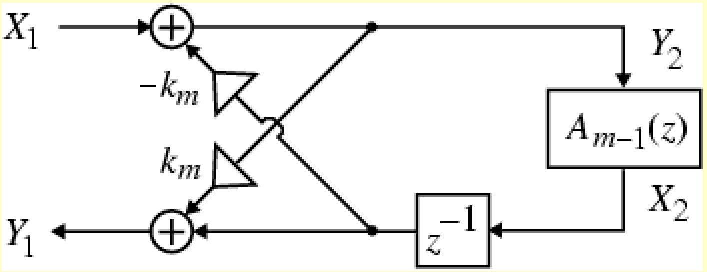
Realization Using Two-Pair Extraction Approach

- An mth-order allpass transfer function $A_m(z)$ is then realized by constraining any one of the two-pairs developed earlier by the $(m-1)$th-order allpass transfer function $A_{m-1}(z)$



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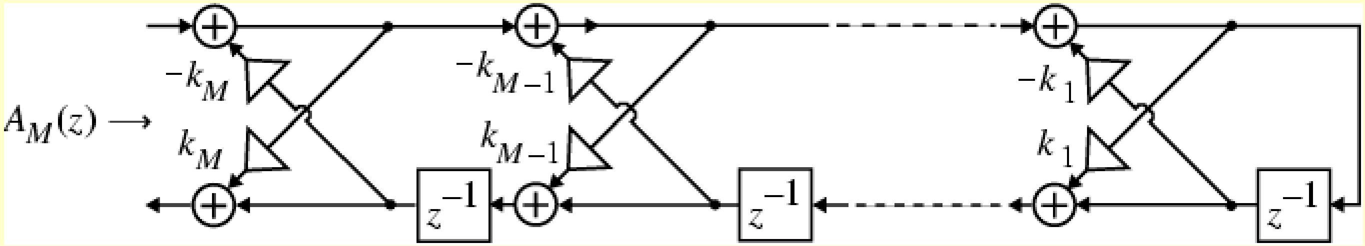
Realization Using Two-Pair Extraction Approach

- The process is repeated until the constraining transfer function is $A_0(z) = 1$
- The complete realization of $A_M(z)$ based on the extraction of the two-pair lattice is shown below



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Realization Using Two-Pair Extraction Approach

- It follows from our earlier discussion that $A_M(z)$ is stable if the magnitudes of all multiplier coefficients in the realization are less than 1, i.e., $|k_m| < 1$ for $m = M, M-1, \ldots, 1$
- The cascaded lattice allpass filter structure requires $2M$ multipliers
- A realization with M multipliers is obtained if instead the single multiplier two-pair is used

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Realization Using Two-Pair Extraction Approach

- Example - Realize

$$A_3(z) = \frac{-0.2 + 0.18z^{-1} + 0.4z^{-2} + z^{-3}}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$

$$= \frac{d_1 + d_2 z^{-1} + d_1 z^{-2} + z^{-3}}{1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}}$$

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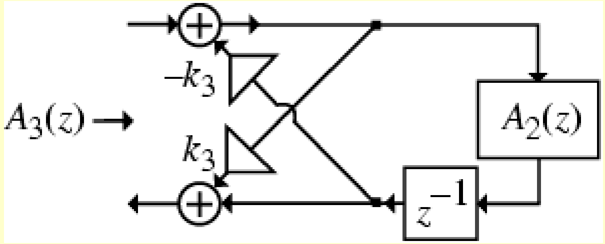
Realization Using Two-Pair Extraction Approach

- We first realize $A_3(z)$ in the form of a lattice two-pair characterized by the multiplier coefficient $k_3 = d_3 = -0.2$ and constrained by a 2nd-order allpass $A_2(z)$ as indicated below



$k_3 = -0.2$

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Realization Using Two-Pair Extraction Approach

- The allpass transfer function $A_2(z)$ is of the form

$$A_2(z) = \frac{d_2' + d_1' z^{-1} + z^{-2}}{1 + d_1' z^{-1} + d_2' z^{-2}}$$

- Its coefficients are given by

$$d_1' = \frac{d_1 - d_3 d_2}{1 - d_3^2} = \frac{0.4 - (-0.2)(0.18)}{1 - (-0.2)^2} = 0.4541667$$

$$d_2' = \frac{d_2 - d_3 d_1}{1 - d_3^2} = \frac{0.18 - (-0.2)(0.4)}{1 - (-0.2)^2} = 0.2708333$$

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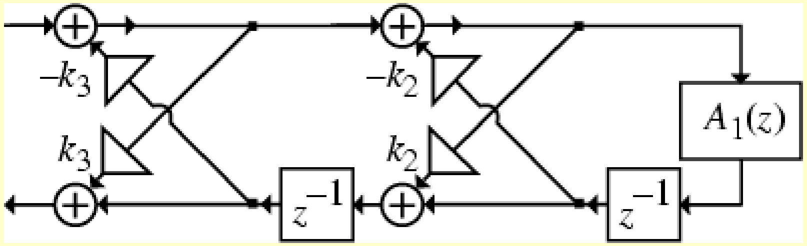
Realization Using Two-Pair Extraction Approach

- Next, the allpass $A_2(z)$ is realized as a lattice two-pair characterized by the multiplier coefficient $k_2 = d_2^{'} = 0.2708333$ and constrained by an allpass $A_1(z)$ as indicated below



$k_3 = -0.2, \quad k_2 = 0.2708333$

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Realization Using Two-Pair Extraction Approach

- The allpass transfer function $A_1(z)$ is of the form

$$A_1(z) = \frac{d_1'' + z^{-1}}{1 + d_1'' z^{-1}}$$

- It coefficient is given by

$$d_1'' = \frac{d_1' - d_2' d_1'}{1 - (d_2')^2} = \frac{d_1'}{1 + d_2'} = \frac{0.4541667}{1.2708333} = 0.3573771$$

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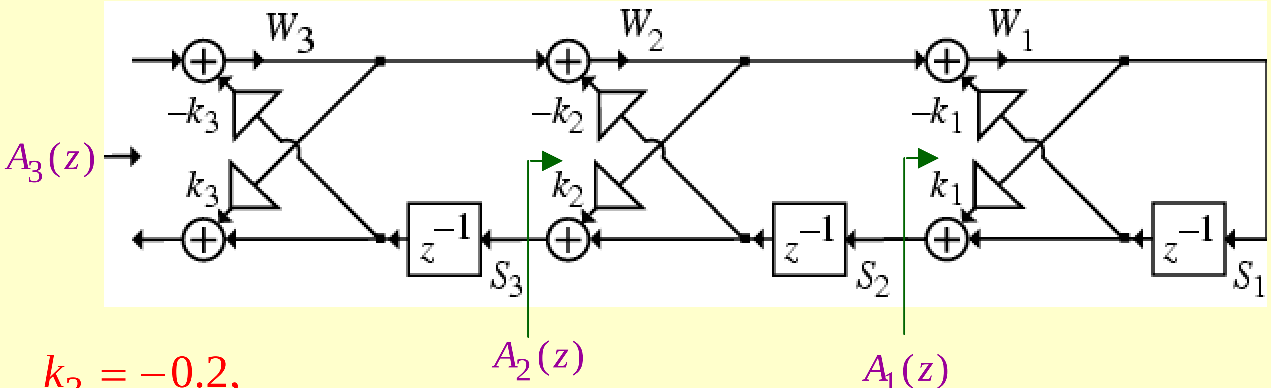
Realization Using Two-Pair Extraction Approach

- Finally, the allpass $A_1(z)$ is realized as a lattice two-pair characterized by the multiplier coefficient $k_1 = d_1'' = 0.3573771$ and constrained by an allpass $A_0(z) = 1$ as indicated below



$k_3 = -0.2,$

$k_2 = 0.2708333, \quad k_1 = 0.3573771$

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Cascaded Lattice Realization Using MATLAB

- The M-file `poly2rc` can be used to realize an allpass transfer function in the cascaded lattice form
- To this end Program 6_3 can be employed

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