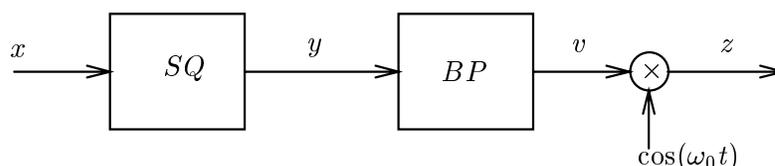


1.



In the above figure:

- The input is  $x(t) = 1 + \cos(\omega_0 t)$ .
- The system  $SQ$  takes the square of the input,  $y(t) = [x(t)]^2$ .
- $BP$  is an ideal band-pass filter, with frequency response function

$$H(i\omega) = u\left(\omega + \frac{3}{2}\omega_0\right) - u\left(\omega + \frac{1}{2}\omega_0\right) + u\left(\omega - \frac{1}{2}\omega_0\right) - u\left(\omega - \frac{3}{2}\omega_0\right)$$

- The last stage is defined by  $z(t) = v(t) \cos(\omega_0 t)$ .

Find the time domain functions  $y(t)$ ,  $v(t)$  and  $z(t)$ , and sketch the Fourier transforms  $Y(i\omega)$ ,  $V(i\omega)$ , and  $Z(i\omega)$ .

2. (a) Find the Fourier transform of  $f(t) = \frac{1}{1-it}$ . *Hint:* consider duality.  
 (b) Find the Fourier transform of

$$\frac{\sin(t)}{\pi t (1-it)}$$

*Hint:* use the previous answer and the convolution properties of the transform.

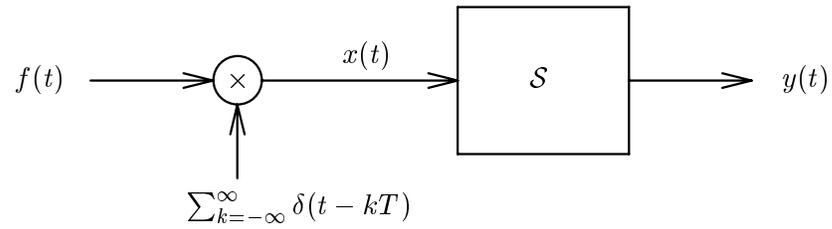
3. We are given a linear time invariant system  $\mathcal{S}$  with impulse response  $h(t) = u(t) - u(t - T)$ , where  $T$  is a fixed constant.

- a) Find the frequency response function  $H(i\omega)$ , and show it can be expressed in the form

$$H(i\omega) = e^{-i\omega \frac{T}{2}} H_R(i\omega)$$

where  $H_R(i\omega)$  is a **real** function of  $\omega$  that you should determine. Sketch  $H_R(i\omega)$ .

- b) If we apply to  $\mathcal{S}$  a periodic input with period exactly equal to  $T$ : what is the output?  
 c) Now suppose we connect  $\mathcal{S}$  in the following configuration:



Taking  $f(t)$  to be the input sketched below, and  $T = 1$ , sketch  $y(t)$  in the same plot.  
 Also write a formula for  $Y(i\omega)$  in terms of  $F(i\omega)$ .

